TOPIC 11
LINEAR PROGRAMMING

KEY CONCEPTS

Linear Programming Problem : Linear programming problem is one that is concerned with finding the optimal value (maximum or minimum value) of a linear function of several variables called **objective function**

Feasible Region: Feasible region is the region which is common to all the linear constraints (linear inequalities)

Important LPP are

1. Diet Problems
2. Manufacturing Problems
3. Transportation Problems

Steps for solving a LPP

Solving linear programming problem using **Corner Point Method**. The method comprises of the following steps:

1. Convert the word problem into mathematical formulation by using given constraints.

2. Solve the linear inequations formed in step 1 and plot the graph.

3. Find the feasible region of the linear programming problem and determine its corner points either by inspection or by solving the two equations of the lines intersecting at that point.

4. Evaluate the objective function Z = $ax + by$ at each corner point. Let M and m, respectively denote the largest and smallest values of these points.

5. (i) When the feasible region is **bounded**, M and m are the maximum and minimum values of Z.

(ii) In case, the feasible region is **unbounded**, we have:

6. (a) M is the maximum value of Z, if the open half plane determined by ax + by > M has no point in common with the feasible region. Otherwise, Z has no maximum value.

(b) Similarly, *m* is the minimum value of Z, if the open half plane determined by ax + by < m has no point in common with the feasible region. Otherwise, Z has no minimum value.

Problems of LPP

1. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs.5 and Rs.4 per unit respectively. One unit of the food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories, while one unit of the food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the foods A and B should be used to have least cost, but it must satisfy the requirements of the sick person. Form the question as LPP and solve it graphically. Explain the importance of balanced diet.

Hint : Let x units of food A, y units of food B are mixed $Z = 5x + 4y$

$200x+100y \geq 4000$, $40x + 40y \geq 1400$, $x \geq 0$, $y \geq 0$ 2m

Correct graph 2m

Points (50,0), (20,15), (5,30), (0, 40) and minimum cost is at (5, 30) 2m

2. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000.Which computer would you prefer to buy?

Solution:-Let the merchant stock *x* desktop models and *y* portable models. Therefore,

$x \geq 0$ and $y \geq 0$

The cost of a desktop model is Rs 25000 and of a portable model is Rs 4000. However, the merchant can invest a maximum of Rs 70 lakhs.

$\therefore 25000x + 40000y \leq 7000000$

$5x + 8y \leq 1400$

The monthly demand of computers will not exceed 250 units.

$\therefore x + y \le 250$

The profit on a desktop model is Rs 4500 and the profit on a portable model is Rs 5000.

Total profit, Z = 4500*x* + 5000*y*

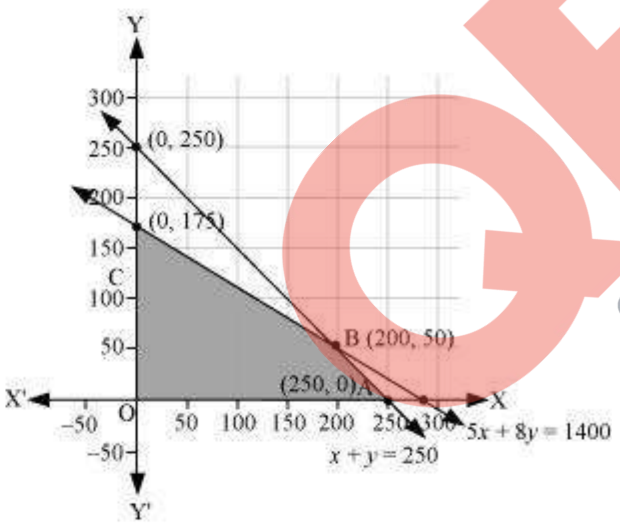
Thus, the mathematical formulation of the given problem is

$$\text{Maximum } Z = 4500x + 5000y \qquad \ldots(1)$$

subject to the constraints,

$$5x + 8y \le 1400 \qquad \ldots(2)$$
$$x + y \le 250 \qquad \ldots(3)$$
$$x,\ y \ge 0 \qquad \ldots(4)$$

The feasible region determined by the system of constraints is as follows.



The corner points are A (250, 0), B (200, 50), and C (0, 175).

The values of Z at these corner points are as follows.

Corner point	Z = 4500*x* + 5000*y*	
A(250, 0)	1125000	

B(200, 50)	1150000	→ Maximum
C(0, 175)	875000	

The maximum value of Z is 1150000 at (200, 50).

Thus, the merchant should stock 200 desktop models and 50 portable models to get the maximum profit of Rs 1150000.

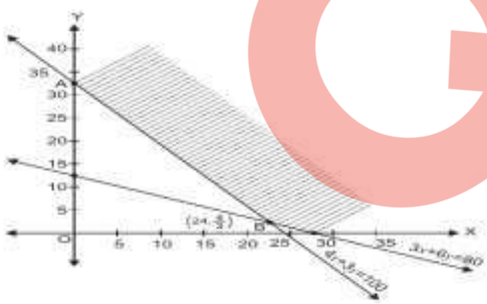
3. A diet is to contain at least 80 units of Vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F_1costs Rs. 4 per unit and F_2costsRs. 6 per unit. One unit of food F_1 contains 3 units of Vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of Vitamin A and 3 units of minerals. Formulate this as a linear programming problem and find graphically the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.

Solution:-

. Minimize Z = 4x + 6y

$3x + 6y \geq 80, \quad 4x + 3y \geq 80, \quad x \geq 0, y \geq 0$

Cost will be minimum when 24 units of F_1& 4/3 units of F_2 will be mixed and minimum cost will be Rs 104



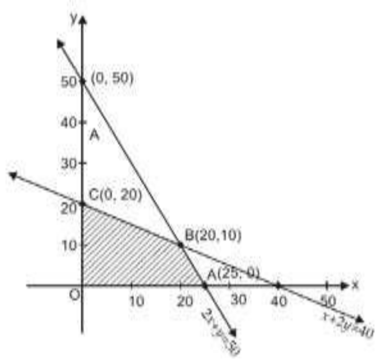
4. One kind of cake requires 200 g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Formulate the above as a linear programming problem and solvegraphically.

Solution:-Let x and y be the no. of cakes of type I & II respectively. Then according to question

Cake	Flour	Fat	
Type I	200 g	25 g	x
Type II	100 g	50 g	y
	≤ 5000 g	≤ 1000 g	

$. Z = x + y, 2x + y \leq 50,$

$x + 2y \leq 40, \quad x \geq 0, y \geq 0$



20 cakes of first kind and 10 cakes of 2nd kind should be made to get max. numbers of cake.

5: A firm has to transport 1200 packages using large vans,which can carry 200 packages each& small vans which can carry 80 packages each.The cost of engaging each large van is Rs 400 7 each small van is Rs 200. Not more than 3000 is to be spent on the job& the no. of large vans cannot exceed the no. of small vans. Formulate the problem as CPP, given that the objective is to minimize the cost.

ANSWER

	package	Cost(in Rs)
Large van(x)	200	400
Small van(y)	80	200
	≥1200	≤3000

Let x large vans and y small vans are engaged
Then LPP is
To minimize Z = 400 x + 200y
Subject to constraints
$x \geq o, y \geq 0$

$400x+200y \leq 3000 \Rightarrow 2x+y \leq 15$
$200x+80y \geq 1200 \Rightarrow 5x+2y \geq 30$
and $x \leq y$

HOTS (6 marks)

Q1 : A library has to accommodate two different types of books on the shelf.The books are 6cm & 4cm thick and weigh 1kg & 1.5 kg respt. The shelf is 90cm long 7 at most can support a weight of 21kg. How the shelf should be filled with two types of books in order to include maximum no. of books. Form LPP & solve it graphically.

(Answer: 10 boks of type I & 7 books of type II can be place).

Q2: Anil wants to invest at most Rs 12000 in two bonds A & B. According to the rules,he has to invest Rs 2000 in bond A &at least Rs 4000 in bond B. If the rate of interest on bond A is 8% & bond B is 10% , per annum.How should he invest his Form a LPP & solve it graphically money for maximum interest? Solve the problem graphically?

(Answer : Rs 2000 should be invested in bond A &Rs 10000 in bond B for maximum interest of Rs1160)

Q3:One kind of cake requires 200 g of flour and 25 g of fat another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat, assuming that there is no shortage of other ingredients used in making the cake.

(Answer : 20 cakes of type I and 10 cakes of type II must be made for maximum number of 30 cakes.)

Q4:An oil company has two depots A and B with capacities 7,000 L and 4,000 L respectively. The company is to supply oil to three petrol pumps P, Q and R whose requirements are 4500L,3000Land 3500L respectively. The distance (in km) between the depots and the petrol pumps is given below:

From↓/ To →	P	Q	R
A	7	6	3
B	3	4	2

Assuming that the transportation cost is Rs 1 per litre, per Km.

How should the delivery be scheduled so as to keep the transportation cost minimum? From a LPP and solve it graphically.

(Answer : 500 L, 3000L and 3500 L must be transported from A to P,Q ,R respectively and 4000 L 0 L and 0 L must be transported from B to P,Q,R for a minimum cost of Rs 440000).

Q5: A small scale factory makes two types of dolls. One doll of type I takes 1.5 hours of electronic machine and 3 hours of hand operated machine whereas one doll of type II takes 3 hours of electronic machine and 1 hour of hand operated machine . In a day a factory has the ability of not more than 42 hours of electronic machine and 24 hours of hand operated machines .If the profit on one doll of type 1 is Rs 20 and one doll of type II is Rs 40 . How may dolls of each type the factory should manufacture to earn the maximum profit ? Form an LPP and solve it graphically .

HINT : Let X dolls of type I and Y dolls of type II are manufactured . Then LPP is

To maximize $Z = 20X + 40Y$.
Z is maximum for B (4,12) or C (0,14) i.e X= 4 ,Y=12 i.e, 4 dolls of type I and 12 dolls of type II are manufactured and C(0,14) i.e, X=0 ,Y=14 i.e., only 14 dolls of type II are manufactured for maximum profit of Rs 560 .

Q6: A packet of plane biscuits costs Rs6 and that of chocolate biscuits costsRs9 . A housewife has Rs72 and wants to buy at least 3 packets of plane biscuits and at least 4 packets of chocolate biscuits . How many of each type ? Form a LPP and solve it graphically . should she buy so that she can have maximum number of packets

Answer : 6 packets of plane biscuits and 4 packets of chocolate biscuits must be bought to get maximum 10 packets

Q7:A person has 30 units and 17 units of labour and capital respectively ,which he can use to produce two types of goods X and Y . To produce one unit of X, 2 units of labour and 3 units of capital are required . Similarly 3 units of labour and 1 unit of capital are required to produce 1 unit of Y . If X and Y are priced at Rs 100 and Rs 120 per unit respectively, how should the producer use his resources to maximise the revenue ? Solve the problem graphically .

Answer : 3 units of goods x and 8units of goods y must be produced for a maximum profit of Rs1,260.