11th Standard - Mathematics

Permutations and Combinations

Fundamental Principles of Counting

Multiplication Principle: Suppose an operation A can be performed in m ways and associated with each way of performing of A, another operation B can be performed in n ways, then total number of performance of two operations in the given order is mxn ways. This can be extended to any finite number of operations.

Addition Principle:

If an operation A can be performed in m ways and another operation S, which is independent of A, can be performed in n ways, then A and B can performed in (m + n) ways. This can be extended to any finite number of exclusive events.

Factorial

The continued product of first n natural number is called factorial ‘n’.
It is denoted by n! or $n! = n(n - 1)(n - 2)\dots 3 \times 2 \times 1$ and $0! = 1! = 1$

Permutation

Each of the different arrangement which can be made by taking some or all of a number of objects is called permutation.

Permutation of n different objects

The number of arranging of n objects taking all at a time, denoted by ^nP_n, is given by $^nP_n = n!$

The number of an arrangement of n objects taken r at a time, where $0 < r \leq n$, denoted by nP_r is given by

$^nP_r = n!(n-r)!$

Properties of Permutation

(i) $^nP_n = n(n-1)(n-2)\ldots 3\times 2\times 1 = n!$

(ii) $^nP_0 = \frac{n!}{n!} = 1$

(iii) $^nP_1 = n$

(iv) $^nP_{n-1} = n!$

(v) $^nP_r = n\,.\,^{n-1}P_{r-1} = n(n-1)\,^{n-2}P_{r-2}$

(vi) $^{n-1}P_r + r\,.\,^{n-1}P_{r-1} = {^nP_r}$

(vii) $\frac{^nP_r}{^nP_{r-1}} = n - r + 1$

Important Results on Permutation

The number of permutation of n things taken r at a time, when repetition of object is allowed is nr.

The number of permutation of n objects of which p1 are of one kind, p2 are of second kind,… pk are of kth kind such that $p_1 + p_2 + p_3 + \ldots + p_k = n$

is n!p1!p2!p3!.....pk!

Number of permutation of n different objects taken r at a time,

When a particular object is to be included in each arrangement is r. $^{n-1}P_{r-1}$

When a particular object is always excluded, then number of arrangements $= {}^{n-1}P_r$.

Number of permutations of n different objects taken all at a time when m specified objects always come together is m! (n – m + 1)!.

Number of permutation of n different objects taken all at a time when m specified objects never come together is n! – m! (n – m + 1)!.

Combinations

Each of the different selections made by taking some or all of a number of objects irrespective of their arrangements is called combinations. The number of selection of r objects from; the given n objects is denoted by ${}^nC_r$, and is given by

${}^nC_r = n!r!(n-r)!$

Properties of Combinations

(i) ${}^nC_0 = {}^nC_n = 1$

(ii) ${}^nC_1 = {}^nC_{n-1} = n$

(iii) ${}^nC_r = \frac{{}^nP_r}{r!}$

(iv) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(v) ${}^nC_r = {}^nC_{n-r}$

(vi) $r\,{}^nC_{r-1} = (n-r+1)\,{}^nC_{r-1}$