# 11th Standard - Mathematics

## Straight Lines

### Distance Formula

The distance between two points A($x_1$, $y_1$) and B ($x_2$, $y_2$) is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance of a point A(x, y) from the origin 0 (0, 0) is given by OA = x2+y2

### Section Formula

The coordinates of the point which divides the joint of ($x_1$, $y_1$) and ($x_2$, $y_2$) in the ratio m : n internally, is

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right) \text{ and externally is}$$

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right).$$

Mid-point of the joint of ($x_1$, $y_1$) and ($x_2$, $y_2$) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

X-axis divides the line segment joining ($x_1$, $y_1$) and ($x_2$, $y_2$) in the ratio $-y_1 : y_2$.

Y-axis divides the line segment joining ($x_1$, $y_1$) and ($x_2$, $y_2$) in the ratio $-x_1 : x_2$.

The coordinates of the centroid of the triangle whose vertices are ($x_1$, $y_1$), ($x_2$, $y_2$) and ($x_3$, $y_3$) is

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

## Area of Triangle

The area of the triangle, the coordinates of whose vertices are $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ is the absolute value of

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

or
$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

If the points $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ are collinear, then $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$.

## Shifting of Origin

Let the origin is shifted to a point O'(h, k). If P(x, y) are coordinates of a point referred to old axes and P'(X, Y) are the coordinates of the same points referred to new axes, then $x = X + h$, $y = Y + k$.

## Straight Line

Any curve is said to be a straight line if two points are taken on the curve such that every point on the line segment joining any two points on it lies on the curve. General equation of a line is $ax + by + c = 0$.

## Slope or Gradient of Line

The inclination of angle θ to a line with a positive direction of X-axis in the anti-clockwise direction, the tangent of angle θ is said to be slope or gradient of the line and is denoted by m.
i.e. $m = \tan\theta$
The slope of a line passing through points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

**Note:** Slope of a line parallel to X-axis is zero and slope of a line parallel to Y-axis is not defined.

## Angle between Two Lines

The angle θ between two lines having slope $m_1$ and $m_2$ is

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

- If two lines are parallel, their slopes are equal i.e. $m_1 = m_2$.
- If two lines are perpendicular to each other, then their product of slopes is -1 i.e. $m_1m_2 = -1$.

## Various Forms of the Equation of a Line

If a line is at a distance k and parallel to X-axis, then the equation of the line is $y = \pm k$.

If a line is parallel to Y-axis at a distance c from Y-axis, then its equation is $x = \pm c$.

Slope-intercept form: The equation of line with slope m and making an intercept c on the y-axis, is $y = mx + c$.

One point-slope form: The equation of a line which passes through the point $(x_1, y_1)$ and has the slope of m is given by $y - y_1 = m(x - x_1)$.

Two points form: The equation of a line passing through the points $(x_1, y_1)$ and $(x_2, y_2)$ is given by

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1).$$

The Intercept form: The equation of a line which cuts off intercepts a and b respectively on the x and y-axes is given by xa+yb=1

$$\text{i.e.} \quad \frac{x}{x\text{-intercept}} + \frac{y}{y\text{-intercept}} = 1$$

The normal form: The equation of a straight line upon which the length of the perpendicular from the origin is p and angle made by this perpendicular to the x-axis is α, is given by $x\cos\alpha + y\sin\alpha = p$.

General Equation of a Line
Any equation of the form Ax + By + C = 0, where A and B are simultaneously not zero is called the general equation of a line.

## Different Forms of Ax + By + C = 0

Slope intercept form: If B ≠ 0, then Ax + By + C = 0 can be written as

$$y = \frac{-A}{B}x - \frac{C}{B} \text{ or } y = mx + c$$

$$\text{where } m = \frac{-A}{B} \text{ and } c = -C/B$$

If B = 0, then x = – C / A which is a vertical line, whose slope is not defined and x-intercept is – C/A.

Intercept form If C ≠ 0, then Ax + By + C = 0 can be written as

$$\frac{x}{-C/A} + \frac{y}{-C/B} = 1 \text{ or } \frac{x}{a} + \frac{y}{b} = 1$$

where a = – C / A and b = – C/B
If C = 0, then Ax + By + C = 0 can be written as Ax + By = 0 which is a line passing through origin and therefore has zero intercept on the axes.

Normal form: The normal form of equation Ax + By + C = 0 is x cos α + y sin α = p where

$$\cos\alpha = \pm\frac{A}{\sqrt{A^2+B^2}}, \sin\alpha = \pm\frac{B}{\sqrt{A^2+B^2}}$$

$$\text{and} \quad p = \pm\frac{C}{\sqrt{A^2+B^2}}$$

**Note:** Proper choice of signs to be made so that p should be always positive.

## Position of Points is Relative to a Given Line

Let the equation of the given line be ax + by + c = 0 and let the coordinates of the two given points be $P(x_1, y_1)$ and $Q(x_2, y_2)$.

The two points are on the same side of the straight line $ax + by + c = 0$, If $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have the same sign.

The two points are on the opposite sides of the straight line $ax + by + c = 0$, If $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have opposite sign.

## Condition of concurrency for three given lines

$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ is $a_3(b_1c_2 - b_2c_1) + b_3(a_2c_1 - a_1c_2) + c_3(a_1b_2 - a_2b_1) = 0$

## Point of intersection of two lines

Let equation of lines be $ax_1 + by_1 + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, then their point of intersection is

$$\left(\frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}\right)$$

## Distance of a Point from a Line

The perpendicular distanced of a point $P(x_1, y_1)$from the line $Ax + By + C = 0$ is given by

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

## Distance Between Two Parallel Lines

The distance d between two parallel lines $y = mx + c_1$ and $y = mx + c_2$ is given by

$$d = \frac{|c_1 - c_2|}{\sqrt{1 + m^2}}.$$