6th Standard Maths

Algebra

The branch of mathematics in which we study numbers is called arithmetic. The branch of mathematics in which we study shapes is called geometry. The study to use the letters and symbols in mathematics is called Algebra.

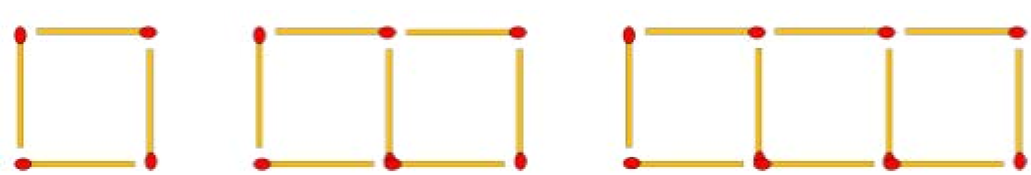
Algebra:

Algebra is a part of mathematics in which the letter and symbols are used to represent numbers in equations. It helps us to study about unknown quantities.

In algebra, we use letters. Use of letters helps us in numerous ways as follows:

- Using letters we can write rules and formulae in a general way.
- Using letters we can talk about any number and not just a particular number.
- The letters may stand for unknown quantities. By learning methods of determining unknowns, we develop powerful tools for solving puzzles and many problems from daily life.
- Since letters stand for numbers, operations can be performed on them as on numbers. This leads to the study of algebraic expressions and their properties.

Matchstick Patterns



No. of matchsticks used to make 1^{st} square = 4

No. of matchsticks used to make 2^{nd} square = 7

No. of matchsticks used to make 3^{rd} square = 10

So, the pattern that we observe here is $3n + 1$

Example 1

How many matchsticks will be used in the 15^{th} figure?

Solution

$3n + 1$

$3 \times 15 + 1 = 46$ matchsticks

Example 2

How many matchsticks will be used in the 20^{th} figure?

Solution

$3n + 1$

$3 \times 20 + 1 = 61$ matchsticks

With this pattern, we can easily find the number of matchsticks required in any number of squares.

We can make letters and other shapes using matchsticks. We can write a general relation between the number of matchsticks required for repeating a given shape. The number of times a given shape is repeated varies; it takes on values 1, 2, 3,... . It is a variable, denoted by some letter n.

The Idea of a Variable

Variable means something that can vary (or change). The value of a variable is not fixed. It can take different values. The length of a square can have any value. It is a variable. But the number of angles of a triangle has a fixed value 3. It is not a variable. We may use any letter n, l, m, p, x, y, z, etc. to show a variable.

One such example of the same is the rule that we used in the matchstick pattern

$3n + 1$

Here the value of **n** is unknown and it can vary from time to time.

More Matchstick Patterns

We can make many letters of the alphabet and other shapes from matchsticks. For example U, V, triangle, Square, etc. In matchstick patterns, we use the variable n to give us the general rule for die number of matchsticks required to make a pattern. This is an important use of variables in Mathematics.

More Examples of Variables

To show a variable, we may use any letter as n, m, l, p, x,y, z, etc. Recall that a variable is a number, which does not have a fixed value. It can take on various values. For example, the number 10, or the number 100 or any other given number is not a variable. They have fixed values. Similarly, the number of comers of a quadrilateral (4) is fixed; it is also not a variable.

- We can use any letter as a variable, but only lowercase English alphabets.
- Numbers cannot be used for the variable as they have a fixed value.
- They can also help in solving some other problems.

Use of Variables in Common Rules

Rules from geometry

- Perimeter of a square (p) = 4s, where s is the length of the side of the square
- Perimeter of a rectangle (p) = 2l + 2b, where l is the length and b is the breadth of the rectangle.

Perimeter of Square:

The perimeter of a square = Sum of all sides

= 4 × side

= 4s

Here s is variable, so the perimeter changes as the value of side change.

Perimeter of Rectangle:

Perimeter of rectangle = 2(length + breadth)

= 2 (l + b) or 2l + 2b

Thus, p + 2 × (l + b) or 2l + 2b

Where, l and b are variable and the value of perimeter changes with the change in l and b.

Rules from Arithmetic
Commutativity of Addition Let a and b be two variables, which can take any numerical value. Then, a + b = b + a **Example** a = 56 and b = 20 According to commutative property 56 + 20 = 20 + 56 76 = 76
Commutativity of Multiplication Let a and b be two variables. Then, a × b = b × a **Example** 18 ×12 = 216, 12 ×18 = 216 Thus, 18 × 12 = 12 × 18
Distributivity of multiplication over addition

Rules from Arithmetic
Let a, b and c be three variables. Then a × (b + c) = a × b + a × c **Example** 6 × (40 + 2) = 240 + 12 = 252 Thus, 6 × 42 = 252 A × (b + c) = a × b + a × c Here, a, b and c are different variables.
Associativity of addition Let a, b and c be three variables. Then, (a + b) + c = a + (b + c). **Example** (4 + 3) + 7 = 4 + (3 + 7) 7 + 7 = 4 + 10 14 = 14

Expressions With Variables

We know that variables can take different values; they have no fixed value. But they are numbers. That is why as in the case of numbers, operations of addition, subtraction, multiplication, and division can be performed on them. Using different operations, we can form expressions with variables like x – 2, x + 1, 3n, 2m, p/4, 2y + 5, 3l – 7, etc.

Note: A number of expressions can be immediately evaluated.

For example: $3 \times 4 + 6 = 12 + 6 = 18$

But an expression containing the variable x cannot be evaluated until x is assigned same value.

For example,

When $x = 1$, $4x + 3 = 4 \times 1 + 3 = 4 + 3 = 7$.

Using Expressions Practically

Many statements described in ordinary language can be changed to statements using expressions with variables.

What is the Equation?

An equation is a condition on a variable. It is expressed by saying that expression with a variable is equal to a fixed number.

For example, $x - 3 = 2$.

An equation has two sides, LHS and RHS and between them, is file sign of equality (=).

The equation states that the value of the left-hand side (LHS) is equal to the value of the right-hand side (RHS).

If the LHS is not equal to the RHS, we do not get an equation.

For example, the statement $2n > 10$ or $2n < 10$ is not an equation.

Note: An equation like 10 – 1 = 9 is called a numerical equation as neither of its two sides contains a variable. Usually, the word equation is used only for equations involving one or more variables.

Solution of an Equation

The value of the variable in an equation for which LHS of the equation becomes equal to RHS of the equation is said to satisfy the equation and itself is called a solution of the equation. For example, $n = 2$ is a solution to the equation $2n = 4$, where $n = 3$ is not a solution of the equation $3n = 13$.

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Getting a Solution to the Equation

For getting the solution of an equation, one method is trial and error method. In this method, we assign some value to the variable and check whether it satisfies the equation. We go on assigning this way different values to the variable until we find the right value which satisfies the equation.

But this is not a direct and practical way of finding a solution. We need a more systematic way of getting a solution of the equation than the trial and error method. In case of very simple equations, the variable is replaced by a place holder I and its value is determined by usual methods. Thus the value of the variable obtained is the solution of the equation.

Using an Equation

Originally, we are given an equation in a variable whose value is unknown to us. To solve the equation means to find the unknown value. Thus a variable in an equation is looked upon as unknown and starting from the unknown, we can see up the equation. Solving the equation is thus a method of finding the unknown. It is, therefore, a powerful method of solving puzzles and problems.

Introduction to Algebra:

Variable: A variable is an unknown quantity that is prone to change with the context of a situation. Example: In the expression 2x+5, x is the variable.

Constant: Constant is a quantity which has a fixed value. In the given example 2x+5, 5 is the constant.

Terms of an Expression: Parts of an expression which are formed separately first and then added or subtracted, are known as terms. In the above-given example, terms 2x and 5 are added to form the expression (2x+5).

Factors of a term: Parts of an expression which are formed separately first and then added or subtracted, are known as terms.

- Factors of a term are quantities which cannot be further factorised.
- In the above-given example, factors of the term 2x are 2 and x.

Coefficient of a term: The numerical factor of a term is called the coefficient of the term. In the above-given example, 2 is the coefficient of the term 2x.

Like and Unlike Terms:

Like terms

Terms having the same variables are called like terms. Example: 8xy and 3xy are like terms.

Unlike terms

Terms having different variables are called, unlike terms. Example: 7xy and -3x are unlike terms.

Monomial, Binomial, Trinomial and Polynomial Terms

Name	Monomial	Binomial	Trinomial	Polynomial
No. of terms	1	2	3	>3
Example	7xy	(4x−3)	(3x+5y−6)	(6x+5yx−3y+4)