12th MATHS MARKING SCHEME (PRACTICE PAPER- 2)

PART – A
SECTION – I

1. {(1, 1), (2, 2), (3, 3), (4, 4)} **OR** 1

2. R = R^{-1}

3. 2 **OR** 60

4. 2 , 4

5. I **OR** Null Matrix

6. 32

7. $e^x \sec x + C$ **OR** $\tan x - \sec x + C$

8. 2 sq units

9. cot x **OR** 3

10. $4\hat{i} - 2\hat{j} + 4\hat{k}$

11. 5/2

12. $\sqrt{2}$

13. $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

14. $\sqrt{14}$

15. 0.75

16. 1/8

SECTION II

17. (i) B (ii) C (iii) D (iv) A (v) B

18. (i) D (ii) C (iii) B (iv) D (v) A

PART – B
SECTION III

19. Given that, $\frac{dy}{dx} = e^{-2y} \Rightarrow \frac{dy}{e^{-2y}} = dx$ **2**

$\Rightarrow \int e^{2y} dy = \int dx \Rightarrow \frac{e^{2y}}{2} = x + C$

When $x = 5$ and $y = 0$, then substituting these values in Eq. (i), we get

$$\frac{e^0}{2} = 5 + C$$

$\Rightarrow \frac{1}{2} = 5 + C \Rightarrow C = \frac{1}{2} - 5 = -\frac{9}{2}$

Eq. (i) becomes $e^{2y} = 2x - 9$

When $y = 3$, then $e^6 = 2x - 9 \Rightarrow 2x = e^6 + 9$

$\therefore x = \frac{(e^6 + 9)}{2}$.

20. The equation of the line passing through $A(-1, 3, 2)$ and $B(-4, 2, -2)$ is **2**

$$\frac{x+1}{-4+1} = \frac{y-3}{2-3} = \frac{z-2}{-2-2}$$

$$\Rightarrow \quad \frac{x+1}{-3} = \frac{y-3}{-1} = \frac{z-2}{-4}$$

$$\Rightarrow \quad \frac{x+1}{3} = \frac{y-3}{1} = \frac{z-2}{4} \quad \dots \text{(i)}$$

If the points $A(-1, 3, 2)$, $B(-4, 2, -2)$ and $C(5, 5, \lambda)$ are collinear, then the coordinates of C must satisfy equation (i). Therefore,

$$\frac{5+1}{3} = \frac{5-3}{1} = \frac{\lambda-2}{4}$$

$$\Rightarrow \quad \frac{\lambda-2}{4} = 2$$

$$\Rightarrow \quad \lambda = 10.$$

21. Firstly, we find a unit vector in the direction of $\vec{a} = \frac{\vec{a}}{|\vec{a}|}$ **2**

$$= \frac{2\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{(2)^2 + (-1)^2 + (2)^2}}$$

$$= \frac{2\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{9}}$$

$$= \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

Now, vector of magnitude 6 units $= 6\left[\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right]$

$$= 4\hat{i} - 2\hat{j} + 4\hat{k}$$

22. We know that the principal value branch of $\tan^{-1}x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\cot^{-1}x$ is $(0, \pi)$. **2**

$\therefore$ Principal value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) + \cot^{-1}\left(\cot\frac{7\pi}{6}\right)$

$$= \tan^{-1}\left[\tan\left(\pi + \frac{\pi}{6}\right)\right] + \cot^{-1}\left[\cot\left(\pi + \frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left(\tan\frac{\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{6}\right)$$

$$= \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}.$$

23. 2

$$I = \int_0^{\frac{\pi}{2}} \log\left(\frac{3+5\cos\left(\frac{\pi}{2}-x\right)}{3+5\sin\left(\frac{\pi}{2}-x\right)}\right) dx \qquad \left[\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \log\left(\frac{3+5\sin x}{3+5\cos x}\right) dx = -I$$

$$2I = 0 \Rightarrow I = 0$$

OR The differentiation of $\log(\sin x)$ is $\tan x$, which exists in denominator. So solve by substitution method. 2

Given integral is $\int \frac{\log(\sin x)}{\tan x} dx$

Putting $\log \sin x = t$

$\Rightarrow \quad \frac{1}{\sin x} \cdot \cos x\, dx = dt$

$\Rightarrow \quad \cot x\, dx = dt$

$\Rightarrow \quad \frac{1}{\tan x} dx = dt \qquad \left[\because \int \sin x\, dx = -\cos x + C\right]$

$\therefore$ We get $\quad \int \frac{\log(\sin x)}{\tan x} dx = \int t\, dt$

$$= \frac{t^2}{2} + C$$

$$= \frac{(\log \sin x)^2}{2} + C$$

24. We have, $x = a\cos^3\theta,\ y = a\sin^3\theta$ 2

$\Rightarrow \quad \frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$

and $\quad \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$

Now, $\quad \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$

$\therefore$ Slope of the normal at any point on the curve $= -\frac{1}{\frac{dy}{dx}} = \frac{-1}{-\tan\theta} = \cot\theta$

Hence, $\left(\text{Slope of the normal at } \theta = \frac{\pi}{4}\right) = \cot\frac{\pi}{4} = 1.$

25. **2**

$$y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$$

$$= \tan^{-1}\left\{\frac{\sin\left(\frac{\pi}{2}+x\right)}{1-\cos\left(\frac{\pi}{2}+x\right)}\right\}$$

$$= \tan^{-1}\left\{\frac{\not{2}\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\not{2}\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right\}$$

$$= \tan^{-1}\left\{\cot\left(\frac{\pi}{4}+\frac{x}{2}\right)\right\}$$

$$= \tan^{-1}\left\{\tan\frac{\pi}{2}-\left(\frac{\pi}{4}+\frac{x}{2}\right)\right\}$$

$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\}$$

$$= \frac{\pi}{4}-\frac{x}{2}$$

$$\therefore \quad \frac{dy}{dx} = -\frac{1}{2}.$$

26. **2**

We have, $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

$$\Rightarrow \quad (X+Y)+(X-Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow \quad 2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow \quad X = \frac{1}{2}\begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

and, $(X + Y) - (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$

$\Rightarrow \qquad 2Y = \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$

$Y = \frac{1}{2}\begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$

OR Let A (5,5), B (k, 1) and C (11, 7) be three vertices of a ΔABC. **2**

Then area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ k & 1 & 1 \\ 11 & 7 & 1 \end{vmatrix}$

$= \frac{1}{2}[5(1-7) - 5(k-11) + 1(7k-11)]$

$= \frac{1}{2}[-30 - 5k + 55 + 7k - 11]$

$= \frac{1}{2}[2k + 14] = (k + 7)$ sq. units

But it is given that A (5, 5), B (k, 1) and C (11, 7) are collinear.

$\therefore$ area of $\Delta ABC = 0$

$\therefore$ $k + 7 = 0 \Rightarrow k = -7$

27. Let A and B be the events of drawings an even number ticket in the first and second drawn respectively. **2**

In the first draw, there are 7 even numbers out of 15 numbers.

$\therefore \qquad P(A) = \frac{7}{15}$

After first draw, there are 14 tickets left.

In the second drawn, one even number ticket is drawn out of 14 tickets.

$\therefore \qquad P(B/A) = \frac{6}{14} = \frac{3}{7}$

$\therefore \qquad P(A \cap B) = P(A) \cdot P(B/A) = \frac{7}{15} \times \frac{3}{7} = \frac{1}{5}$

OR When a die is thrown, there are 3 odd numbers on the die out of 6 numbers. **2**

$\therefore$ Probability of getting odd number $= \frac{3}{6} = \frac{1}{2}$

Probability of getting even number $= 1 - \frac{1}{2} = \frac{1}{2}$

Now probability of getting no odd number when the die is tossed thrice

= Probability of getting even number when the die is tossed thrice

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$\therefore$ Probability of getting an odd number at least once when the die is tossed thrice

$= 1 - \frac{1}{8} = \frac{7}{8}.$

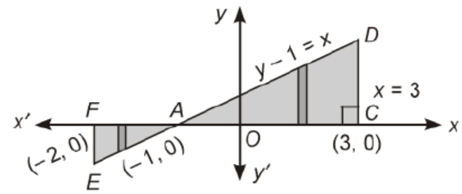
28. $y - 1 = x$ or $y = x + 1$ is the given line DE. ... (i) **2**

$x = -2$ is the line EF.

$x = 3$ is the line CD.

Let A be a point of intersection of (i) and x-axis.

Limits are $x = -2$ and $x = -1$ for the area AEF and the limits for the area ACD are $x = -1$ and $x = 3$.



The required area = Shaded area

$$= |\Delta AFE| + |\Delta ACD|$$

$$= \left|\int_{-2}^{-1}(x+1)\,dx\right| + \left|\int_{-1}^{3}(x+1)\,dx\right|$$

$$= \left|\left[\frac{x^2}{2}+x\right]_{-2}^{-1}\right| + \left|\left[\frac{x^2}{2}+x\right]_{-1}^{3}\right|$$

$$= \left|\left(\frac{1}{2}-1\right)-(2-2)\right| + \left|\left(\frac{9}{2}+3\right)-\left(\frac{1}{2}-1\right)\right|$$

SECTION IV

29. $R = \{(a, b) : a - b$ is divisible by 5, $a, b \in Z\}$ **3**

For reflexive: $(a, a) \in R \Rightarrow a - a$ is divisible by 5, true. Hence R is reflexive.

For symmetric: $(a, b) \in R \Rightarrow a - b$ divisible by 5 $\Rightarrow b - a$ is divisible by 5 $\Rightarrow (b, a) \in R$, Hence R is symmetric.

For transitive: Let for $(a, b), (b, c) \in R$

$(a, b) \in R \Rightarrow a - b$ divisible by 5

$(b, c) \in R \Rightarrow b - c$ divisible by 5

As $a - b$ divisible by 5 and $b - c$ divisible by 5. Hence $a - c$ is also divisible by 5

i.e., $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$. Hence R is transitive.

From above R is reflexive, symmetric, transitive, therefore R is an equivalence relation.

30. Given curve is **3**

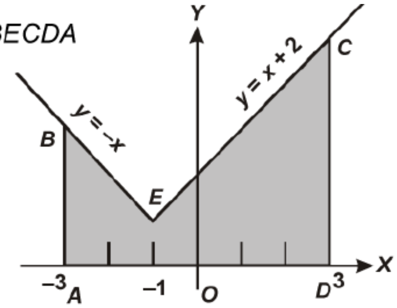
$$y = 1 + |x+1| = \begin{cases} 1 + x + 1, & \text{if } x + 1 \geq 0 \\ 1 - (x+1), & \text{if } x + 1 < 0 \end{cases}$$

or, $$y = \begin{cases} x + 2, & \text{if } \quad x \geq -1 \\ -x, & \text{if } \quad x < -1 \end{cases} \quad \text{... (i)}$$

Given lines are $x = -3, \quad x = 3, \quad y = 0$ ... (ii)

The rough sketch of (i) has been shown in the figure.

$\therefore$ The required area = the area of the shaded region $ABECDA$



$$= \int_{-3}^{-1} y_{BE}\, dx + \int_{-1}^{3} y_{EC}\, dx$$

$$= \int_{-3}^{-1} (-x)\, dx + \int_{-1}^{3} (x+2)\, dx$$

$$= -\left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$$

$$= -\frac{1}{2}(1-9) + \left[\left(\frac{9}{2}+6\right) - \left(\frac{1}{2}-2\right)\right]$$

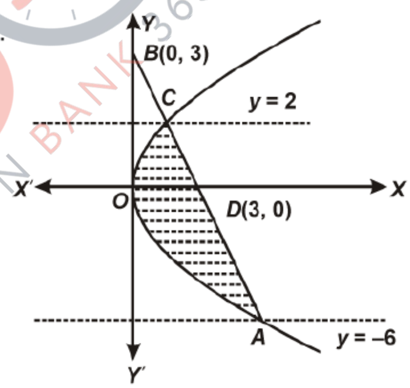
= 4 + 12 = 16 sq. units.

OR Given curves are $y^2 = 4x$ ... (i) **3**

and $x + y = 3$ ... (ii)

Curve (i) is a right handed parabola whose vertex is (0, 0) and axis is $y = 0$.

Line (ii) cuts x-axis at (3, 0) and y-axis at (0, 3). Here required area $OCDAO$ is bounded by curves (i) and (ii) and abscissa at A and C.



Hence we will find the values of y from equations (i) and (ii).

Putting the value of x from equation (ii) in (i), we get

$$y^2 = 4(3-y)$$

or, $y^2 + 4y - 12 = 0$

$\therefore$ $y = -6, 2$

Required area $OCDAO = \int_{-6}^{2} (x_1 - x_2)\, dy$

$$= \int_{-6}^{1} \left(x_{line\,(i)} - x_{curve\,(ii)}\right) dy$$

$$= \int_{-6}^{2} \left[(3-y) - \frac{y^2}{4}\right] dy = \left[3y - \frac{y^2}{2} - \frac{y^3}{12}\right]_{-6}^{2}$$

$$= \left(6 - 2 - \frac{2}{3}\right) - \left(-18 - 18 + \frac{216}{12}\right) = \frac{10}{3} + 18 = \frac{64}{3} \text{ sq. units.}$$

31. Let $I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ ... (i) **3**

$I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)} dx$ $\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x} dx$ ... (ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi} \frac{(x + \pi - x)\sin x}{1+\cos^2 x} dx$$

$$I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx \qquad [\because \text{Let } t = \cos x,\ dt = -\sin x dx]$$

$$\begin{bmatrix}\text{upper limit} \to \cos\pi = -1 \\ \text{Lower limit} \to \cos 0 = 1\end{bmatrix}$$

$$I = \frac{\pi}{2}\int_1^{-1} \frac{-dt}{1+t^2} = \frac{\pi}{2}\int_{-1}^{1}\frac{dt}{1+t^2} \qquad \left[\because \int_a^b f(x)dx = -\int_b^a f(x)dx\right]$$

$$= \frac{\pi}{2}[\tan^{-1} t]_{-1}^{1} = \frac{\pi}{2}[\tan^{-1}(1) - \tan^{-1}(-1)]$$

$$= \frac{\pi}{2}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \frac{\pi}{2}\left(\frac{\pi}{4}+\frac{\pi}{4}\right) = \frac{\pi}{2}.\frac{\pi}{2}$$

$$= \frac{\pi^2}{4}$$

32. We have, **3**

$$f(x) = (x+1)^3 (x-3)^3$$

$$\Rightarrow \quad f'(x) = (x-3)^3 \cdot 3(x+1)^2 \frac{d}{dx}(x+1) + (x+1)^3 \cdot 3(x-3)^2 \frac{d}{dx}(x-3)$$

$$\Rightarrow \quad f'(x) = 3(x+1)^2 (x-3)^3 + 3(x+1)^3 (x-3)^2$$

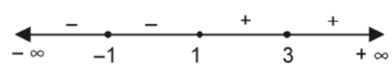
$$\Rightarrow \quad f'(x) = 3(x+1)^2 (x-3)^2 (x+1+x-3)$$

$$\Rightarrow \quad f'(x) = 6(x+1)^2 (x-3)^2 (x-1)$$

For $f(x)$ to be increasing, we must have

$$f'(x) \geqslant 0$$

$$\Rightarrow \quad 6(x+1)^2 (x-3)^2 (x-1) \geqslant 0$$



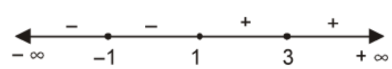
$$\Rightarrow \quad x \in [1, \infty)$$

So, $f(x)$ is increasing on $[1, \infty)$.

For $f(x)$ to be decreasing, we must have

$$f'(x) \leqslant 0$$

$$\Rightarrow \quad 6(x+1)^2 (x-3)^2 (x-1) \leqslant 0$$



$$\Rightarrow \quad x \in (-\infty, 1]$$

So, $f(x)$ is decreasing on $(-\infty, 1]$.

33. Given that **3**

$$x = \tan\left(\frac{1}{a}\log y\right)$$

$$\Rightarrow \quad \tan^{-1} x = \frac{1}{a}\log y$$

$$\Rightarrow \quad a\tan^{-1}x = \log y$$

Now, differentiating both sides w.r.t. x, we get

$$a \times \frac{1}{1+x^2} = \frac{1}{y}\cdot\frac{dy}{dx} \qquad \left[\because \frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}\right]$$

$$\Rightarrow \quad (1+x^2)\frac{dy}{dx} = ay \quad \text{[By cross multiplication]}$$

Differentiating again on both sides w.r.t. x, we get

$$(1+x^2)\cdot\frac{d}{dx}\left(\frac{dy}{dx}\right)+\frac{dy}{dx}\frac{d}{dx}(1+x^2) = \frac{d}{dx}(ay) \quad \left[\because \frac{d}{dx}(u\cdot v)=u\frac{dv}{dx}+v\frac{du}{dx}\right]$$

$$\Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot(2x) = a\cdot\frac{dy}{dx}$$

$$\Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-a\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2}+(2x-a)\frac{dy}{dx} = 0.$$ **Hence proved.**

34. $x = ae^{\theta}(\sin\theta - \cos\theta)$ **3**

Diff. w.r.t. θ

$$\frac{dx}{d\theta} = a[e^{\theta}(\cos\theta + \sin\theta) + (\sin\theta - \cos\theta)e^{\theta}]$$

$$= ae^{\theta}[\cos\theta + \sin\theta + \sin\theta - \cos\theta]$$

$$= 2ae^{\theta}\sin\theta$$

Now, $y = ae^{\theta}(\sin\theta + \cos\theta)$

Diff. w.r.t. θ

$$\frac{dy}{d\theta} = a[e^{\theta}(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta)e^{\theta}]$$

$$= ae^{\theta}[\cos\theta - \sin\theta + \sin\theta + \cos\theta]$$

$$= 2ae^{\theta}\cos\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2ae^{\theta}\cos\theta}{2ae^{\theta}\sin\theta}$$

$$= \cot\theta.$$

OR Given that $y^x = e^{y-x}$ **3**

Taking log on both sides, we get

$$\log y^x = \log e^{(y-x)}$$

$$\Rightarrow \quad x\log y = (y-x)\log e \quad [\because \log e = 1]$$

$$\Rightarrow \quad x\log y = y - x \quad \text{... (i)}$$

$$\Rightarrow \quad x = \frac{y}{1+\log y}$$

Differentiating eq. (i) both sides w.r.t. x, we get

$$\Rightarrow \quad x\cdot\frac{d}{dx}(\log y)+\log y\frac{d}{dx}(x) = \frac{d}{dx}(y)-\frac{d}{dx}(x)$$

$\Rightarrow \quad x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot 1 = \frac{dy}{dx} - 1$

$\Rightarrow \quad (1 + \log y) = \frac{dy}{dx}\left(1 - \frac{x}{y}\right)$

$\Rightarrow \quad \frac{dy}{dx} = \frac{y(1+\log y)}{(y-x)}$ ... (ii)

Put the value of x from Eq. (i) in Eq. (ii), we get

$$\frac{dy}{dx} = \frac{y(1+\log y)}{y - \left(\frac{y}{1+\log y}\right)} \qquad \left[\because x \log y = y - x \text{ or } x = \frac{y}{1+\log y}\right]$$

$\Rightarrow \quad \frac{dy}{dx} = \frac{y(1+\log y)^2}{(y + y\log y - y)}$

35. Given differential equation is **3**

$$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2-1}; \ |x| \neq 1$$

Dividing both sides by $x^2 - 1$, we get

$$\frac{dy}{dx} + \frac{2xy}{x^2-1} = \frac{1}{(x^2-1)^2} \quad \text{... (i)}$$

This is a linear differential equation and is of the form

$$\frac{dy}{dx} + Py = Q \quad \text{... (ii)}$$

Comparing Eqs. (i) and (ii), we get

$$P = \frac{2x}{x^2-1} \text{ and } Q = \frac{1}{(x^2-1)^2}$$

Now, solution of above equation is given by

$$y \times \text{I.F.} = \int (Q \times \text{I.F.})\, dx + c \quad \text{... (iii)}$$

where, I.F. = Integrating factor and I.F. = $e^{\int P dx}$

$\therefore \quad \text{I.F.} = e^{\int \frac{2x}{x^2-1}dx} = e^{\log(x^2-1)} = x^2 - 1 \qquad [\because e^{\log x} = x]$

$$\left[\because \int \frac{2x}{x^2-1}dx \text{ Put } x^2 - 1 = t \Rightarrow 2x\,dx = dt \ \therefore \int \frac{dt}{t} = \log|t| = \log|x^2-1| + c\right]$$

Putting I.F. = $x^2 - 1$ and $Q = \frac{1}{(x^2-1)^2}$ in Eq. (iii),

we get

$$y(x^2 - 1) = \int (x^2-1) \cdot \frac{1}{(x^2-1)^2}dx$$

$\Rightarrow \quad y(x^2 - 1) = \int \frac{1}{x^2-1}dx$

$\Rightarrow \quad y(x^2 - 1) = \int \frac{dx}{x^2 - (1)^2}$

$$y(x^2-1) = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C$$

SECTION – V

36. Let $P(2, 3, 4)$ be the given point and given equation of line be

$$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$$

Any random point T on the given lines is calculated as

$$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2} = \lambda \quad \text{[Say]}$$

or $x = 3\lambda - 3$, $y = 6\lambda + 2$, $z = 2\lambda$

$\therefore$ Coordinates of T are $(3\lambda - 3, 6\lambda + 2, 2\lambda)$

Now, DR's of line PT are

$(3\lambda - 3 - 2, 6\lambda + 2 - 3, 2\lambda - 4) = (3\lambda - 5, 6\lambda - 1, 2\lambda - 4)$

Since, the line PT is parallel to the plane

$$3x + 2y + 2z - 5 = 0$$

$$\therefore \quad a_1a_2 + b_1b_2 + c_1c_2 = 0$$

[$\because$ Line is parallel to the plane, therefore normal to the plane is perpendicular to the line]

where, $a_1 = 3\lambda - 5$, $b_1 = 6\lambda - 1$, $c_1 = 2\lambda - 4$

and $a_2 = 3$, $b_2 = 2$, $c_2 = 2$

[$\because$ a_2, b_2, c_2 are DR's of plane whose equation is $3x + 2y + 2z - 5 = 0$]

$\therefore$ We get,

$$3(3\lambda - 5) + 2(6\lambda - 1) + 2(2\lambda - 4) = 0$$

$$\Rightarrow \quad 9\lambda - 15 + 12\lambda - 2 + 4\lambda - 8 = 0$$

$$\Rightarrow \quad 25\lambda - 25 = 0$$

$$\Rightarrow \quad 25\lambda = 25$$

or $\lambda = 1$

$\therefore$ Coordinates of $T = (3\lambda - 3, 6\lambda + 2, 2\lambda) = (0, 8, 2)$ [Put $\lambda = 1$]

Finally, the required distance between points $P(2, 3, 4)$ and $T(0, 8, 2)$ is given by

$$PT = \sqrt{(0-2)^2 + (8-3)^2 + (2-4)^2}$$

[$\because$ $(x_1, y_1, z_1) = (2, 3, 4)$ and $(x_2, y_2, z_2) = (0, 8, 2)$]

$$= \sqrt{4 + 25 + 4} = \sqrt{33} \text{ units.}$$

OR We have, $\vec{n}_1 = (\hat{i} + 3\hat{j})$, $d_1 = 6$ and $\vec{n}_2 = (3\hat{i} - \hat{j} - 4\hat{k})$, $d_2 = 0$

Using the relation, $\vec{r} \cdot (\vec{n}_1 + \lambda\vec{n}_2) = d_1 + d_2\lambda$

$$\Rightarrow \quad \vec{r} \cdot [(\hat{i} + 3\hat{j}) + \lambda(3\hat{i} - \hat{j} - 4\hat{k})] = 6 + 0 \cdot \lambda$$

$$\Rightarrow \quad \vec{r} \cdot [(1 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + \hat{k}(-4\lambda)] = 6 \quad \text{... (i)}$$

On dividing both sides by $\sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + (-4\lambda)^2}$, we get

$$\frac{\vec{r} \cdot [(1 + 3\lambda)\hat{i} + (3 - \lambda)\hat{j} + \hat{k}(-4\lambda)]}{\sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + (-4\lambda)^2}} = \frac{6}{\sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + (-4\lambda)^2}}$$

Since, the perpendicular distance from origin is unity.

$$\therefore \quad \frac{6}{\sqrt{(1+3\lambda)^2 + (3-\lambda)^2 + (-4\lambda)^2}} = 1$$

$$\Rightarrow \quad (1 + 3\lambda)^2 + (3 - \lambda)^2 + (-4\lambda)^2 = 36$$

$$\Rightarrow \quad 1 + 9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2 = 36$$

$$\Rightarrow \quad 26\lambda^2 + 10 = 36$$

$$\Rightarrow \quad \lambda^2 = 1$$

$$\therefore \quad \lambda = \pm 1$$

Using Eq. (i), the required equation of plane is

$$\vec{r} \cdot [(1 \pm 3)\hat{i} + (3 \mp 1)\hat{j} + (\mp 4)\hat{k}] = 6$$

$$\Rightarrow \quad \vec{r} \cdot [(1 + 3)\hat{i} + (3 - 1)\hat{j} + (-4)\hat{k}] = 6$$

and $$\vec{r} \cdot [(1 - 3)\hat{i} + (3 + 1)\hat{j} + 4\hat{k}] = 6$$

$$\Rightarrow \quad \vec{r} \cdot (4\hat{i} + 2\hat{j} - 4\hat{k}) = 6$$

and $$\vec{r} \cdot (-2\hat{i} + 4\hat{j} + 4\hat{k}) = 6$$

$$\Rightarrow \quad 4x + 2y - 4z - 6 = 0$$

and $$-2x + 4y + 4z - 6 = 0$$

37. Maximise $z = 1000x + 600y$

Subject to $x + y \leq 200,\ x \geq 20,\ y \geq 4x,\ x, y \geq 0.$

Consider the linear constraint defined by the inequality

$$x + y \leq 200$$

First draw the graph of the line $x + y = 200$

x	100	80
y	100	120

Putting (0, 0) in the inequality $x + y \leq 200$, we have

$$0 + 0 \leq 200 \Rightarrow 0 \leq 200, \text{ which is true}$$

So the half plane of $x + y \leq 200$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$x \geq 20$$

First draw the graph of the line $x = 20$

Putting (0, 0) in the inequality $x \geq 20$, we have

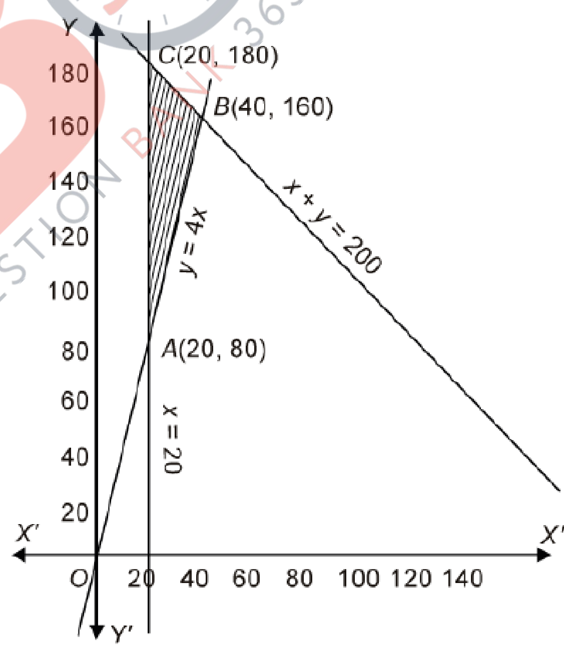
$$0 \geq 20, \text{ which is false}$$

So the half plane of $x \geq 20$ is away from origin.

Now consider the linear constraint defined by the inequality

$$y \geq 4x$$

First draw the graph of the line $y = 4x$



x	10	20
y	40	80

Putting (10, 0) in the inequality $y \geq 4x$, we have

$$0 \geq 4 \times 10 \Rightarrow 0 \geq 40, \text{ which is false}$$

So the half plane of $y \geq 4x$ is away from the point (10, 0).

Since $x, y \geq 0$

So the feasible region lies in the first quadrant.

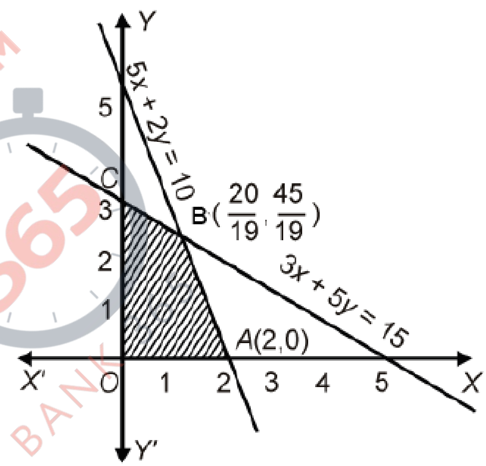
The coordinates of the corner points of the feasible region are $A(20, 80)$, $B(40, 160)$ and $C(20, 180)$. These points have been obtained by solving equations of the corresponding intersecting lines simultaneously.

Now	$z = 1000x + 600y$
At $A(20, 80)$	$z = 1000 \times 20 + 600 \times 80 = 20000 + 48000 = 68000$
At $B(40, 160)$	$z = 1000 \times 40 + 600 \times 160 = 40000 + 96000 = 136000$
At $C(20, 180)$	$z = 1000 \times 20 + 600 \times 180 = 20000 + 108000 = 128000$

Thus z is maximum at (40, 160) and maximum value = 136000

OR.

The given objective function is $z = 6x + 5y$



Consider the linear constraint defined by the inequality

$$3x + 5y \leq 15$$

First draw the graph of the line $3x + 5y = 15$

x	0	5
y	3	0

Putting (0, 0) in the inequality $3x + 5y \leq 15$, we have

$$3 \times 0 + 5 \times 0 \leq 15 \quad \Rightarrow \quad 0 \leq 15, \text{ which is true}$$

So the half plane of $3x + 5y \leq 15$ is towards the origin.

Now consider the linear constraint defined by the inequality

$$5x + 2y \leq 10$$

First draw the graph of the line $5x + 2y = 10$

x	0	2
y	5	0

Putting (0, 0) in the inequality $5x + 2y \leq 10$, we have

$$5 \times 0 + 2 \times 0 \leq 10 \quad \Rightarrow \quad 0 \leq 10, \text{ which is true}$$

So the half plane of $5x + 2y \leq 10$ is towards the origin.

Since $\qquad x, y \geq 0$

So the feasible region lies in first quadrant.

The coordinates of the corner points of the feasible region are $O(0, 0)$

$A(2, 0)$, $B\left(\frac{20}{19}, \frac{45}{19}\right)$ and $C(0, 3)$. These points have been obtained by solving equations of the

corresponding intersecting lines simultaneously.

Now $\qquad z = 6x + 5y$

At $O(0, 0)$ $\qquad z = 6 \times 0 + 5 \times 0 = 0$

At $A(2, 0)$ $\qquad z = 6 \times 2 + 5 \times 0 = 12 + 0 = 12$

At $B\left(\frac{20}{19}, \frac{45}{19}\right)$ $\qquad z = 6 \times \frac{20}{19} + 5 \times \frac{45}{19} = \frac{120}{19} + \frac{225}{19} = \frac{345}{19}$

At $C(0, 3)$ $\qquad z = 6 \times 0 + 5 \times 3 = 0 + 15 = 15$

Thus z is maximum at $\left(\frac{20}{19}, \frac{45}{19}\right)$ and maximum value $= \frac{345}{19}$.

38. Given that

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$$

The given system of equations can be written as

$$AX = B$$

where, $\qquad A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$

Solution of above system of equations is given by

$$X = A^{-1}B \qquad \text{... (i)}$$

So, now we find A^{-1},

where $\qquad A^{-1} = \frac{\text{adj}(A)}{|A|}$

Now, $\qquad |A| = 3(3 - 6) - 2(-12 - 14) + 1(12 + 7)$

$= 3(-3) - 2(-26) + 1(19)$

$= -9 + 52 + 19 = 62$

$|A| \neq 0$, $\qquad$ hence unique solution.

and $$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$\therefore$ $$A_{11} = (-1)^2 \begin{vmatrix} -1 & 2 \\ 3 & -3 \end{vmatrix} = (-1)^2 \times (3-6) = -3$$

$$A_{12} = (-1)^3 \begin{vmatrix} 4 & 2 \\ 7 & -3 \end{vmatrix} = -1(-12-14) = 26$$

$$A_{13} = (-1)^4 \begin{vmatrix} 4 & -1 \\ 7 & 3 \end{vmatrix} = 1(12+7) = 19$$

$$A_{21} = (-1)^3 \begin{vmatrix} 2 & 1 \\ 3 & -3 \end{vmatrix} = -1(-6-3) = 9$$

$$A_{22} = (-1)^4 \begin{vmatrix} 3 & 1 \\ 7 & -3 \end{vmatrix} = 1(-9-7) = -16$$

$$A_{23} = (-1)^5 \begin{vmatrix} 3 & 2 \\ 7 & 3 \end{vmatrix} = -1(9-14) = 5$$

$$A_{31} = (-1)^4 \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = 1(4+1) = 5$$

$$A_{32} = (-1)^5 \begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix} = -1(6-4) = -2$$

$$A_{33} = (-1)^6 \begin{vmatrix} 3 & 2 \\ 4 & -1 \end{vmatrix} = 1(-3-8) = -11$$

$\therefore$ $$\text{adj}(A) = \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}^T$$

$$= \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$$

$\Rightarrow$ $$A^{-1} = \frac{1}{62} \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$ $\left[\because A^{-1} = \frac{\text{adj}(A)}{|A|}\right]$

Now, by using Eq. (i), we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{62}\begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}\begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}$$

$$= \frac{1}{62}\begin{bmatrix} -18+45+35 \\ 156-80-14 \\ 114+25-77 \end{bmatrix} = \frac{1}{62}\begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix}$$

$$\therefore \qquad x = \frac{62}{62}; \quad y = \frac{62}{62}; \quad z = \frac{62}{62}$$

Hence, $x = 1$, $y = 1$ and $z = 1$.

OR

First find the product AB and then premultiply both sides of product AB by A^{-1} and obtain A^{-1}. Then, using the relation $X = A^{-1}C$ and simplify it to get the result.

First we find the product AB

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4+0 & 2-2-0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 6I$$

$$\therefore \qquad AB = 6I \qquad \text{...(i)}$$

Now, given system of equations can be written as

$$AX = C \Rightarrow X = A^{-1}C \qquad \text{... (ii)}$$

where $$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

Now, again from Eq. (i)

$$AB = 6I$$

$$\Rightarrow \quad A^{-1}AB = 6A^{-1}I \qquad \text{[Premultiplying by } A^{-1} \text{ on both sides]}$$

$$\Rightarrow \quad B = 6A^{-1} \qquad [\because A^{-1}A = I \text{ and } IB = B]$$

$$\therefore \quad A^{-1} = \frac{1}{6}B = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Now from Eq. (ii), we get

$$X = A^{-1}C \quad \text{where} \quad C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$\therefore \quad x = 2, \quad y = -1 \quad \text{and} \quad z = 4.$$