9. Some Applications of Trigonometry: Heights and Distances

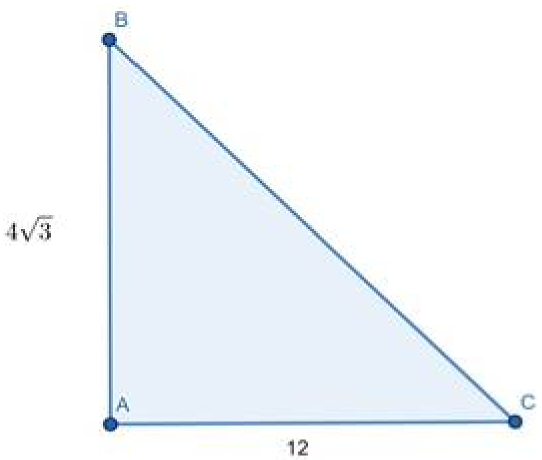
Exercise 9.1

1. Question

In ΔABC, ∠ A = 90°, AB = 12 cm and $AC = 4\sqrt{3}$ cm, then find ∠B.

Answer

Let us draw a diagram according to the question specification so that it gives us a better understanding of the problem.



We know that $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$$\tan\theta = \frac{AC}{AB}$$

According to the question, it is asked that we find the angle of B.

So, from the diagram, we can see that,

$$\tan B = \frac{AC}{AB}$$

$$= \frac{4\sqrt{3}}{12}$$

$$= \frac{\sqrt{3}}{3}$$

$$= \frac{1}{\sqrt{3}}$$

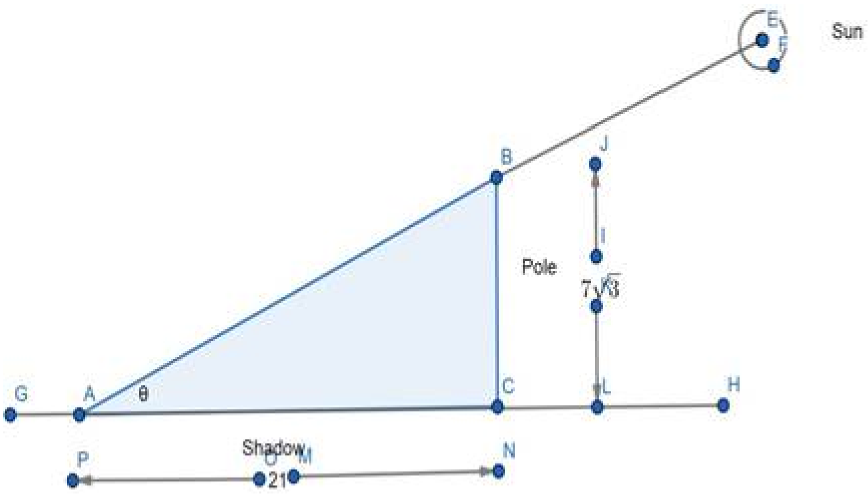
$$B = \arctan(\frac{1}{\sqrt{3}})$$

B = 30°

2. Question

A vertical pole is $7\sqrt{3}$ high and the length of its shadow is 21m. Find the angle of elevation of the source of light.

Answer



We will first draw the schematic diagram to get a better understanding.

In the given diagram the angle of elevation is θ.

Actually, the angle of elevation in the question asked is of the sun with respect to the end of the shadow.

Note that if the elevation was asked of any other point like from the bottom edge of the pole, the answer would have been different.

We know that, $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

applying this trigonometric ratio in the problem,

$$\tan\theta = \frac{CB}{AB}$$

$$= \frac{7\sqrt{3}}{21}$$

$$= \frac{1}{\sqrt{3}}$$

$$\theta = \arctan(\frac{1}{\sqrt{3}})$$

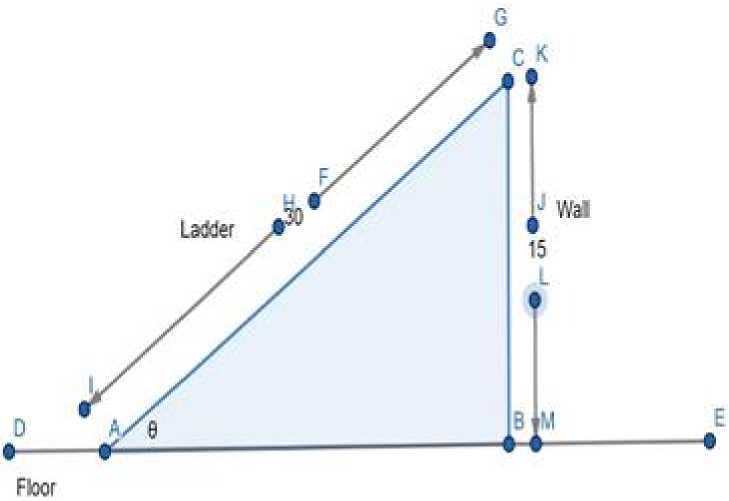
= 30°

3. Question

A ladder of length 30 m is placed against a wall such that it just reaches the top of the 15 m high wall. At what angle is the ladder inclined to the ground?

Answer

Drawing the given problem so that we get a better understanding,



In the given diagram θ is the required angle as asked in the problem

We know that,

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$= \frac{AC}{BC}$$

$$= \frac{15}{30}$$

$$= \frac{1}{2}$$

$$\theta = \arc\sin\left(\frac{1}{2}\right)$$

= 30°

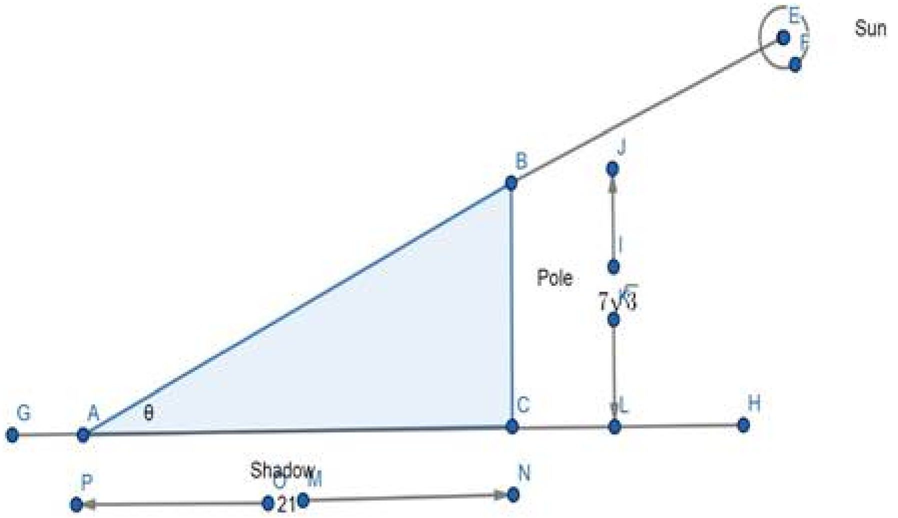
Therefore, the required angle the ladder makes with the wall is 30°.

4. Question

When the ratio of the height of a telephone pole and the length of its shadow is $\sqrt{3}:1$, find the angle of elevation of the sun.

Answer

Drawing the given problem so that we get a better understanding,



In the problem, it is given that,

$$\frac{\text{length of pole}}{\text{length of shadow}} = \frac{\sqrt{3}}{1} = \frac{BC}{AC}$$

We know that,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

using this formula in the question,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{BC}{AC} = \frac{\sqrt{3}}{1}$$

(θ is the angle of elevation as discussed in question 2)

$\Theta = \arctan(\sqrt{3})\theta = 60°$.

Therefore, the angle of elevation is 60°.

5. Question

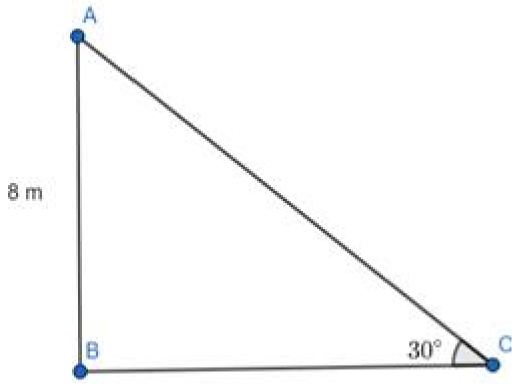
In the figure, ABC is a right triangle in which AB = 8 m, ∠BCA = 30°, then find

(i) the angle of elevation of A at C.

(ii) the angle of depression of C at A.

(iii) BC and AC

Answer



(i) From the diagram, it is clear that the angle of elevation of A with respect to C is 30° where BC becomes the horizontal.

(ii) DC is a vertical line drawn from A so that we get a reference line from which we can measure the depression angle.

As AD and BC are both vertical lines therefore they are parallel.

Now ∠ACB = ∠CAD as they are alternate angles along two parallel lines

therefore, ∠ CAD = 60°

Now, from the diagram drawn before we can easily say that

The angle of depression of C at A is 60°

(iii) $\tan A = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{BC} = \frac{8}{BC}$

$$\tan 30° = \frac{8}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{8}{BC}$$

BC = $8\sqrt{3}$m

Now,

$$\sin C = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{8}{AC}$$

$$\sin 30° = \frac{8}{AC}$$

or,

$$\frac{1}{2} = \frac{8}{AC}$$

AC = 16m.

6. Question

ABC is a right triangle in which BC is horizontal, AB = 8 m, ∠BAC = 60°, then find

(i) the angle of elevation of A at C

(ii) the angle of depression of C at A

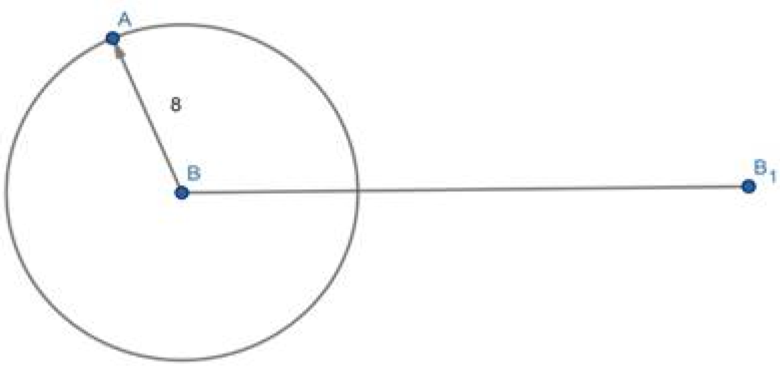
(iii) the distance of B from C

Answer

First constructing the triangle to get the diagram,

Drawing BC a horizontal line,

Drawing a locus of A from B.



But actually nothing is mentioned about BC's length.

But we only know that ∠BAC = 60°

And knowing that it is a right-angled triangle we can say that one angle is 90° and another is 180° - 90° - 60° = 30°

So, two possible cases arise,

if ∠ B = 90° and ∠C = 30°, (Condition 1)

then BC > 8m

if ∠B =30° and ∠C= 90°, (Condition 2)

then BC < 8m

Actual understanding of this requires knowing equation of circles and lines and equating them which results in two variables and because of the two predefined conditions which are independent we get two conditions.

Solving condition 1,

Here, AB = 8m and ∠CBA= 90˚ and ∠BAC= 60˚ and ∠ACB=30˚.

Angle of elevation of A from C will actually be equal to the ∠BCA

where BC acts as the reference line from which the elevated angle is measured.

Therefore, angle of elevation of A from C = ∠ BCA = 60˚

A horizontal line parallel to BC is drawn from A such that it acts like a reference line for calculating the angle of depression of C from A.

Angle of depression of C from A = ∠ CAD

Now, ∠CAD = ∠ACB as they are alternate angles where AD and BC acts as the parallel lines.

Therefore, ∠CAD = ∠ACB = 30˚

Therefore, the angle of depression = 60˚

Now distance of B from C i.e; BC can be calculated from trigonometric ratios.

Using,

$$\tan A = \frac{BC}{AB} = \frac{BC}{8}$$

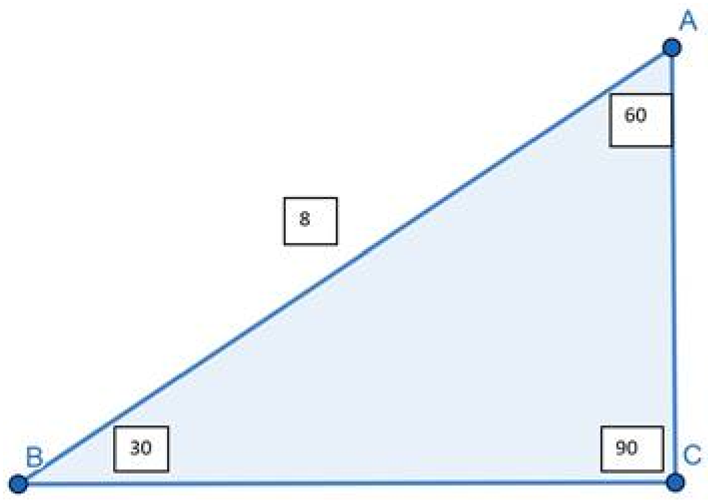
$$\tan 60° = \frac{BC}{8}$$

BC = 8√3

Now,

Solving for Condition 2,

Here, angle BAC = 60˚



$\angle$ ABC = 30°

$\angle$ACB = 90°.

Also, here, BC < AB.

Now, using the above formulas and concepts for these cases we will again get the corresponding answers.

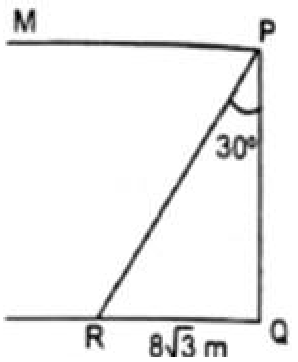
Angle of elevation of A at C = 30°

Angle of depression of C at A= 60°

Length of BC = 4√3

7. Question

In the figure PQR is a right triangle in which $QR = 8\sqrt{3}\,m$ and ∠QPR = 30°. Find QP.



Answer

From the figure we can see that for ∠RPQ, RQ is the perpendicular and PQ is the base. Applying the formula for tangent of an angle we get,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$\Rightarrow \tan P = \frac{\text{perpendicular}}{\text{base}} = \frac{RQ}{QP} = \frac{8\sqrt{3}}{QP}$

$\Rightarrow QP = \frac{8\sqrt{3}}{\sqrt{3}}$

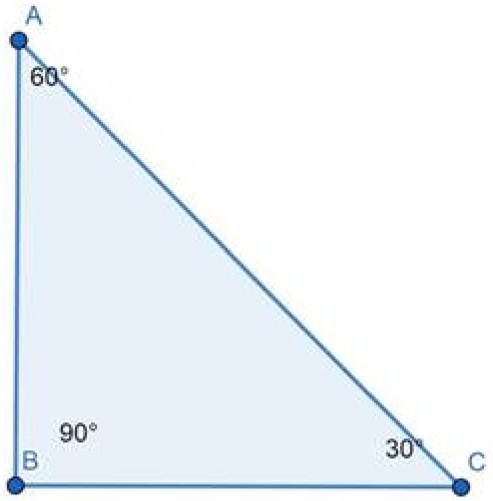
QP = 8 m.

8. Question

In AABC, hypotenuse AC = 12 cm and ∠A = 60°, then find the length of remaining sides.

Answer

Drawing the triangle for better reference of the problem.



Here we are actually sure that B is the 90 degrees angle as the ends of the hypotenuse can never have 90 degrees.

Also, when the ends are produced backwards (where angle A is produced backwards at an angle of 60 degrees) they seem to form a 90 degree angle.

The remaining sides length can be figured out by simple trigonometry,

Using $\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$\sin C = \frac{AB}{AC}$

$\Rightarrow \sin 30° = \frac{AB}{12}$

or, AB = 6cm.

Now, using $\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$

$\Rightarrow \cos C = \frac{BC}{AB} = \frac{BC}{12}$

or, BC= $12 \times \frac{\sqrt{3}}{2}$

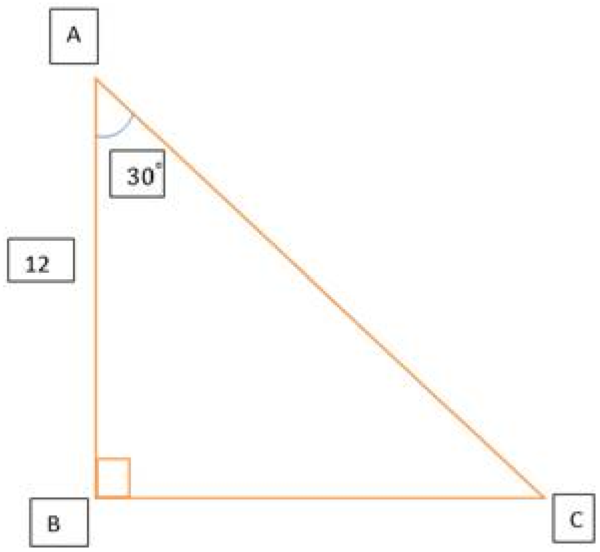
= 6√3 cm.

9. Question

In right angled triangle ABC, AC is the hypotenuse, AB = 12 cm and ∠ BAC = 30°, then find the length of the side BC.

Answer

Drawing the given triangle so that we get a better view of the problem.



As in the question it is given that AC is the hypotenuse, therefore it is evident that angle BAC and angle ACB cannot form 90° as the ends of the hypotenuse never form the right angle in a right-angled triangle.

With regard to the above written point, we can say that angle ABC will form the 90° or B will form the right angle in this right-angled triangle. Also, we can say that on producing the two ends of the hypotenuse (given one predefined angle is given) we will always get the right-angled point at the intersection of these lines (where A is produced at an angle of 30°) which in this case is point B.

Now using the trigonometric ratio,

$$\tan\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{BC}{12}$$

$$\tan 30^{\circ} = \frac{BC}{12}$$

$$\frac{1}{\sqrt{3}} = \frac{BC}{12}$$

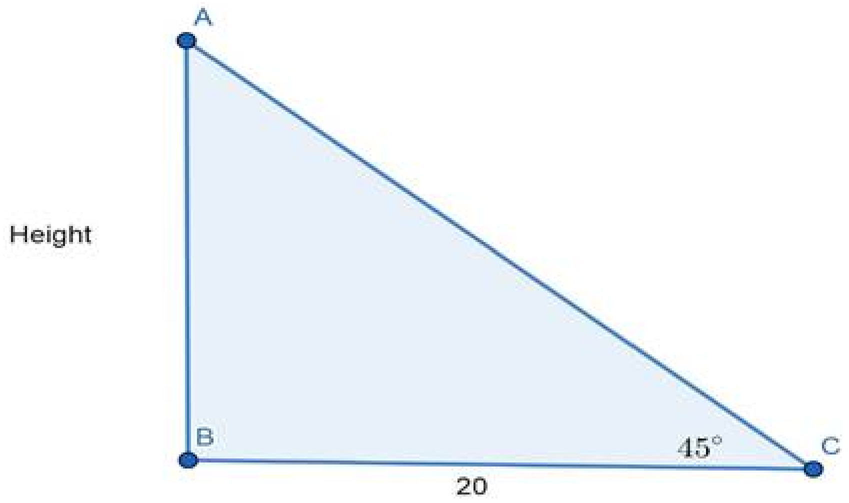
$$BC = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

10. Question

The top of a tower makes an angle of 45° at a point in the horizontal plane at a distance of 20 m. Find the height of the tower.

Answer

Drawing the diagram so that we get a better perspective of the question,



We know that from trigonometric ratios that,

$$\tan\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

Therefore, using this ratio in this problem,

$$\tan\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{BC} = \frac{\text{height}}{20}$$

$$1 = \frac{\text{height}}{20}$$

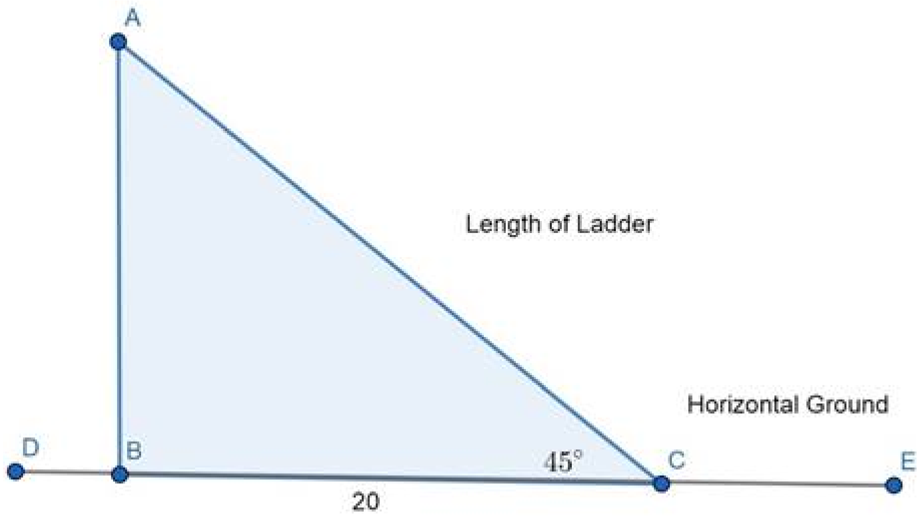
Height = 20

Therefore, the height of the pole is 20m.

11. Question

AB is a vertical wall and B is on the ground. The ladder AC is resting at C on the ground. ∠ACB = 45°, BC = 5m, find the length of the ladder.

Answer



We know that from trigonometric ratios that,

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{BC}{\text{length of ladder}}$$

$$\cos 45^{\circ} = \frac{5}{\text{length of ladder}}$$

$$\frac{1}{\sqrt{2}} = \frac{5}{\text{length of ladder}}$$

length of ladder = $5\sqrt{2}$

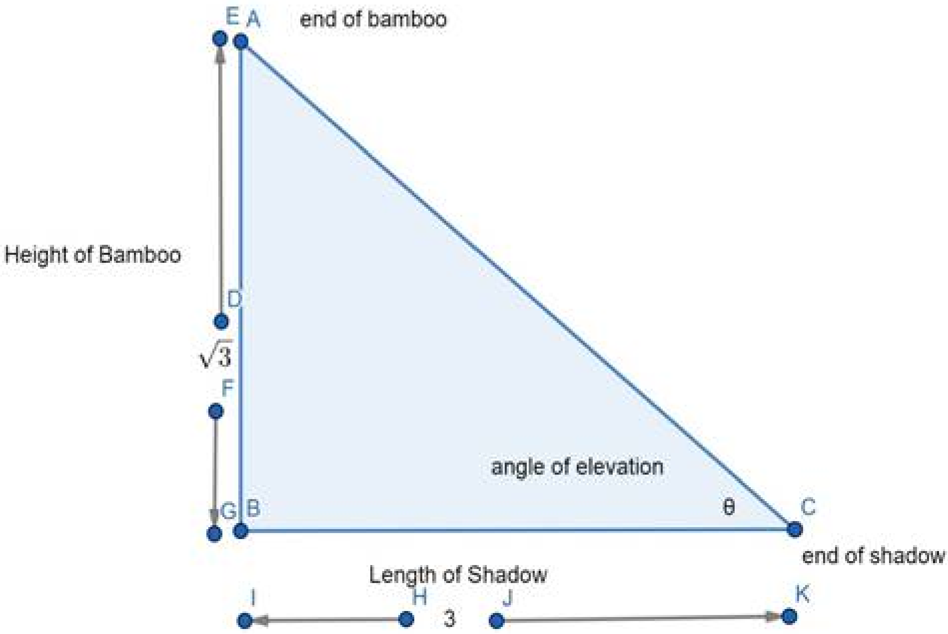
Hence, the length of the ladder is $5\sqrt{2}$.

12. Question

The length of the shadow of a $\sqrt{3}\,\text{m}$ high bamboo tree is 3m, then what will be the angle of elevation of the top of the bamboo tree at the end of the shadow.

Answer

Drawing the given diagram so that we get a better understanding of the problem,



We have to find the angle of elevation of the end of bamboo from the end of the shadow with the horizontal as the reference line from which the angle is measured.

We know that from trigonometric ratios that,

$$\tan\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

Therefore, using this ratio in this problem,

$$\tan\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{BC}$$

$$\tan\theta = \frac{\text{height of bamboo}}{\text{length of shadow}} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$\theta = \arctan(\frac{1}{\sqrt{3}})$$

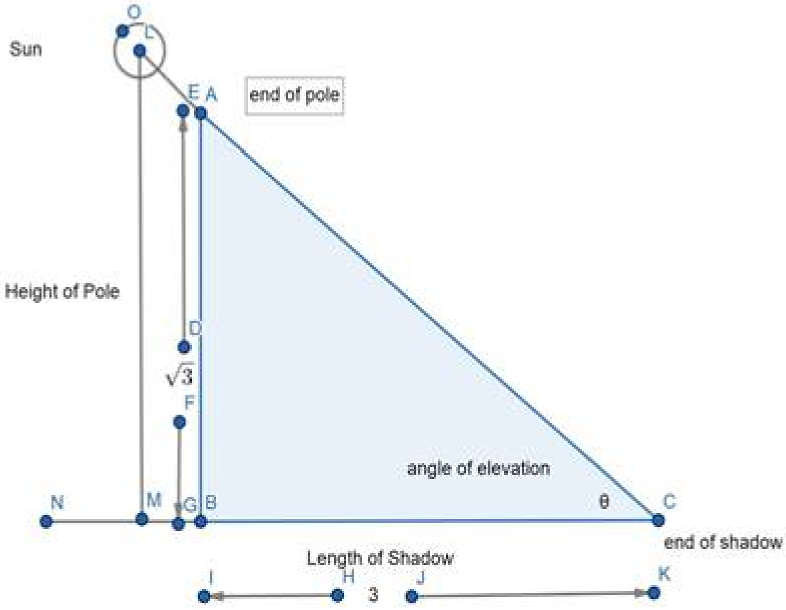
$\Theta = 30°$

Therefore, the angle of elevation is 30°.

13. Question

The height of a telephone pole is $\frac{1}{\sqrt{3}}$ times the length of its shadow, then find the angle of elevation of the source of light.

Answer



We have to measure the angle of elevation of the source of light from the end of the bamboo tree with the horizontal line as the reference.

i,e; We have to find ∠ LCM

But from the diagram it is clear that, ∠LCM = ∠ACB

Now, using trigonometric ratios,

$$\tan\theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{BC}$$

$$\tan\theta = \frac{\text{height of pole}}{\text{length of shadow}} = \frac{\frac{1}{\sqrt{3}} \times \text{length of shadow}}{\text{length of shadow}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \arctan(\frac{1}{\sqrt{3}})$$

$\Theta = 30°$

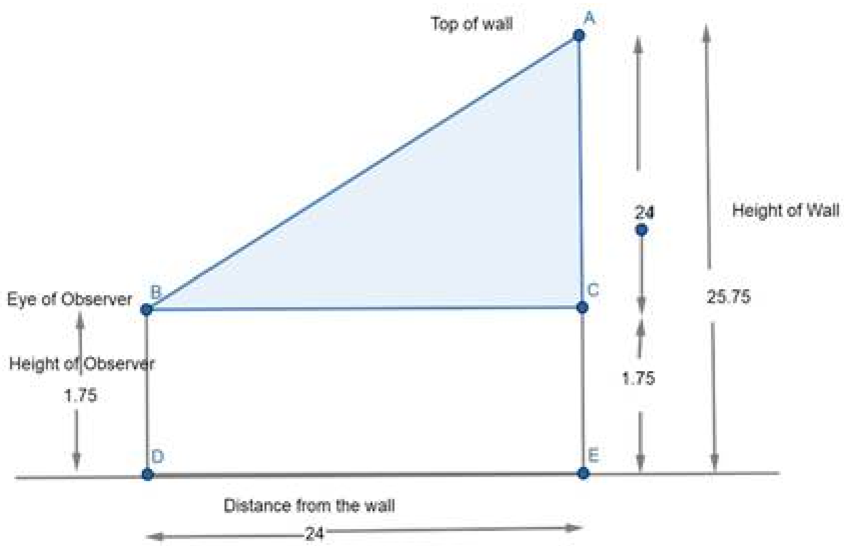
Therefore, the angle of elevation of the source of light is 30°.

14. Question

An observer 1.75 m tall is at a distance of 24 m from a wall 25.75 m high. Find the angle of elevation of the top of the wall at the observer's eye.

Answer

Drawing a diagram for a better perspective of the problem,



We know that from trigonometric ratios that,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{AC}{BC}$$

The angle of elevation of top of the wall will be ∠ABC.

Now Let ∠ABC = θ

Therefore,

$$\tan\theta = \frac{AC}{BC}$$

From the figure we can see that,

AC = BC = 24 m.

Therefore,

$$\tan\theta = \frac{1}{1}$$

θ = arctan(1)

θ =45°

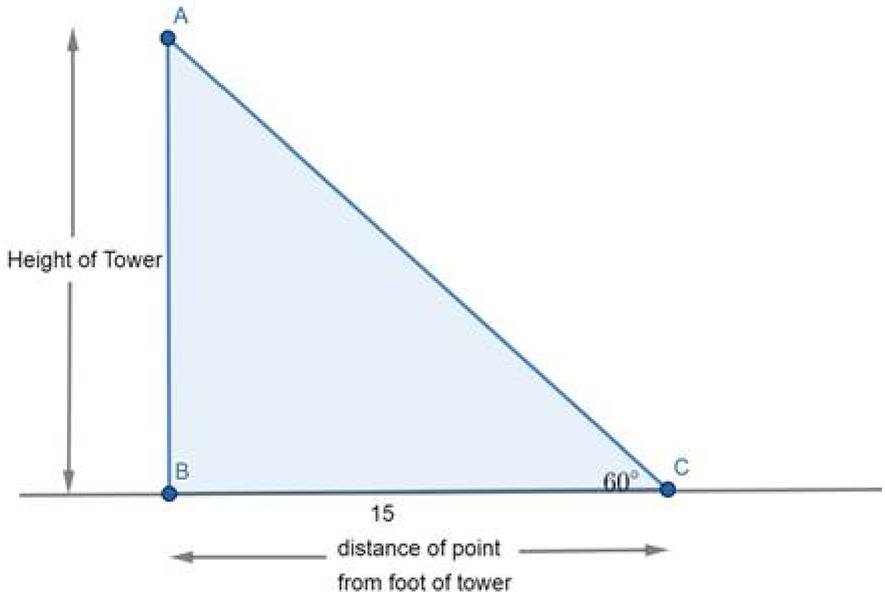
Therefore, the angle of elevation of the top of the wall from the eyes of the observer is 45°.

15. Question

A tower stands vertically on the ground. At a point on the ground, 15 m away from the foot of the tower, the angle of elevation of the top of the tower is 60°. What is the height of the tower?

Answer

A diagram of the situation explained is drawn,



We know that from trigonometric ratios that,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan 60^{\circ} = \sqrt{3} = \frac{\text{perpendicular}}{\text{base}} = \frac{\text{height of tower}}{\text{distance of point from foot of tower}}$$

$$\sqrt{3} = \frac{\text{height of tower}}{15}$$

$$\text{height of tower} = 15\sqrt{3}$$

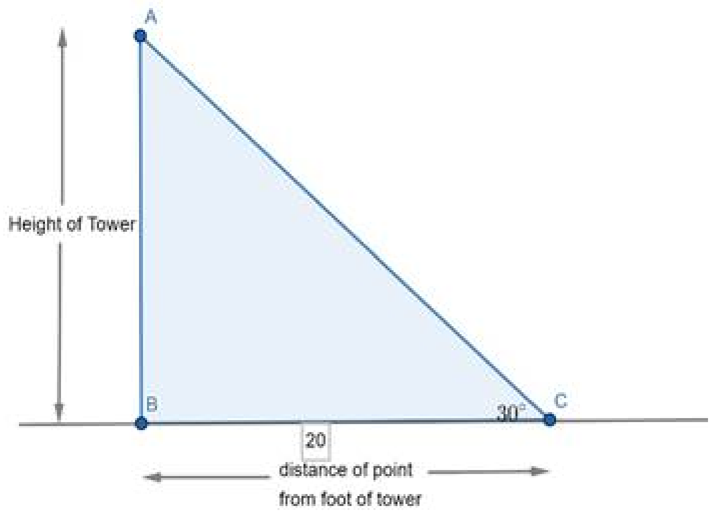
Therefore, the calculated height of tower is 15√3m.

16. Question

At a point 20 m away from the foot of a tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower.

Answer

A diagram of the given situation is drawn,



We know that from trigonometric ratios that,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan 30^{\circ} = \frac{1}{\sqrt{3}} = \frac{\text{perpendicular}}{\text{base}} = \frac{\text{height of tower}}{\text{distance of point from foot of tower}}$$

$$\frac{1}{\sqrt{3}} = \frac{\text{height of tower}}{20}$$

$$\text{height of tower} = \frac{20}{\sqrt{3}}$$

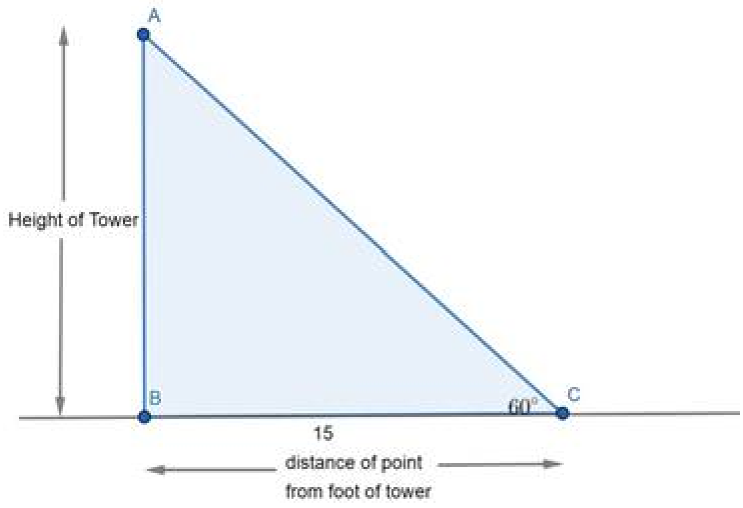
Therefore, the calculated height of tower is $\frac{20\sqrt{3}}{3}$m.

17. Question

The angle of elevation of the top of a tower at a distance of 50 m from its foot is 60°. Find the height of the tower.

Answer

A diagram of the given situation is drawn,



Using a diagram to explain why angle BCA is used as the angle of elevation.

We know that from trigonometric ratios that,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan 60^{\circ} = \sqrt{3} = \frac{\text{perpendicular}}{\text{base}} = \frac{\text{height of tower}}{\text{distance of point from foot of tower}}$$

$$\sqrt{3} = \frac{\text{height of tower}}{50}$$

$$\text{height of tower} = 50\sqrt{3}$$

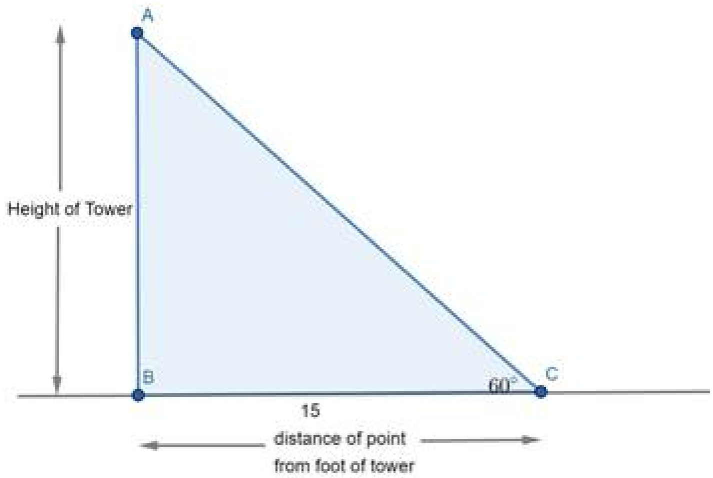
Therefore, the calculated height of tower is $50\sqrt{3}$m.

18. Question

A ladder is placed against a vertical wall such that it just reaches the top of the wall. The foot of the ladder is 1.5 m away from the wall and the ladder is inclined at an angle of 60° with the ground. Find the height of the wall.

Answer

Drawing a diagram for better understanding of the problem,



We know that from trigonometric ratios that,

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$\tan 60^{\circ} = \sqrt{3} = \frac{\text{perpendicular}}{\text{base}} = \frac{\text{height of vertical wall}}{\text{distance of base of wall from foot of ladder}}$$

$$\sqrt{3} = \frac{\text{height of vertical wall}}{1.5}$$

height of vertical wall = 1.5√3m

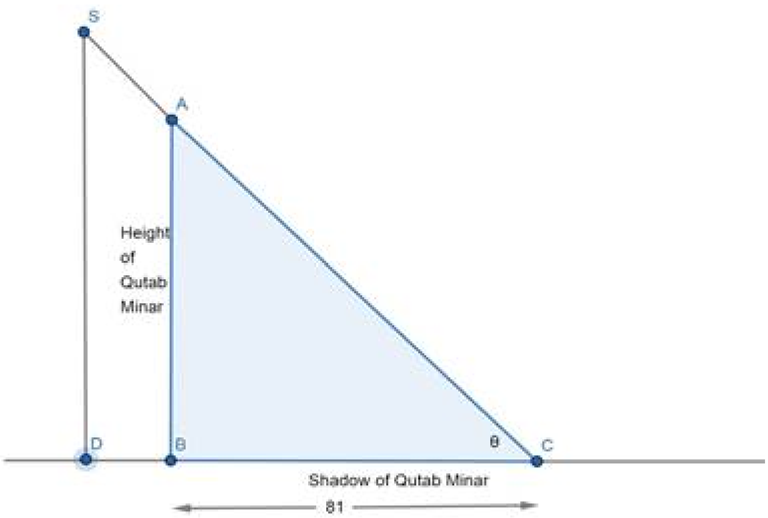
Therefore, the calculated height of vertical wall is 1.5√3m.

19. Question

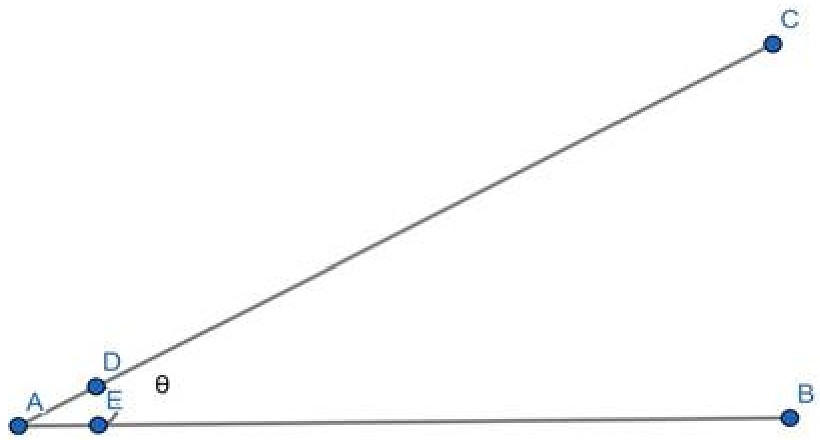
The shadow of Qutab Minar is 81 m long when the angle of elevation of the Sun is θ. Find the height of the Qutab Minar if tanθ= 0.89.

Answer

Drawing a diagram of the given condition,



Using a diagram to explain why angle BCA is used as the angle of elevation.



θ is the angle of elevation.

In the diagram θ is the angle of elevation of point C with respect to point A where AB is the reference line from which the angle is measured.

From the theory explained above it easily understood that angle SCD is the angle of elevation

But from the diagram is also seen that angle SCD= angle ACB as both represent the same angle.

$$\tan\theta = \frac{\text{perpendicular}}{\text{base}}$$

$$0.89 = \frac{\text{height of qutab minar}}{\text{shadow of qutab minar}}$$

height of qutab minar = 0.89 × shadow of qutab minar

height of qutab minar= 0.89 × 81

$= 72.09$

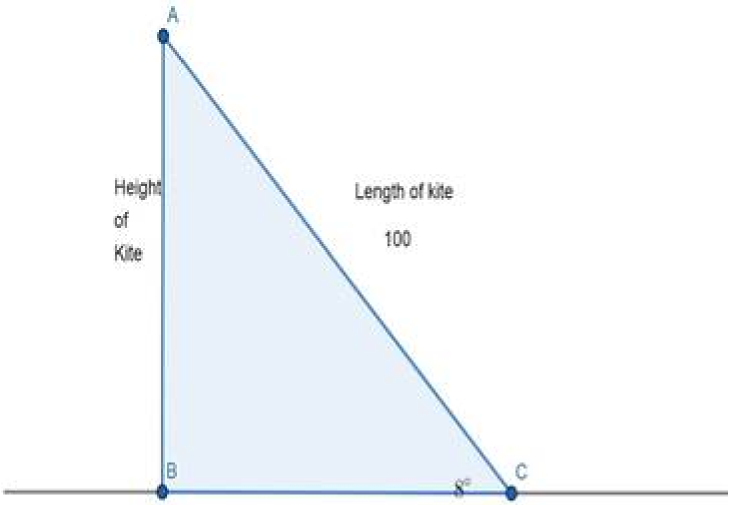
Therefore, the height of the qutab minar is 72.09 m.

20. Question

The string of a kite is 100 m long. If the string is in the form of a straight line (there is no slack in the string) and makes an angle of 8° with the level ground such that $\sin\theta = \frac{8}{15}$ then find the height of the kite.

Answer

Drawing the given situation for a better understanding of the question,



Using the trigonometric ratio,

$$\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\sin 8^{\circ} = \frac{\text{height of kite}}{\text{length of kite}} = \frac{\text{height of kite}}{100}$$

$$\frac{8}{15} = \frac{\text{height of kite}}{100}$$

$$\text{height of kite} = \frac{8 \times 100}{15} = 53.33$$

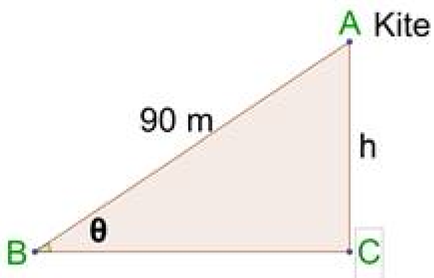
Therefore, the height of the kite is 53.33m.

(no figure was given in question 5 although the question was based from diagram, had to draw the diagram based on the answer given.)

1. Question

The length of a string between a kite and a point on the ground is 90 m. If the string makes an angle θ with the level ground such that $\tan\theta = \frac{15}{8}$. Find the height of the kite.

Answer



Given, $\tan\theta = \frac{15}{8}$

So from the ΔABC

$\sec^2\theta = \tan^2\theta + 1$

$$\sec^2\theta = \frac{225}{64} + 1$$

$$\sec^2\theta = \frac{64}{289}$$

$$1 - \sin^2\theta = \frac{64}{289}$$

$$\sin^2\theta = 1 - \frac{64}{289}$$

$$\sin^2\theta = \frac{225}{289}$$

$$\sin\theta = \frac{15}{17}$$

From the Triangle, $\sin\theta = \frac{h}{90}$

H= 90 × sinθ

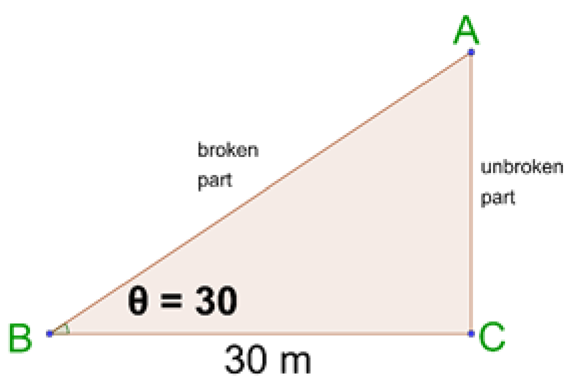
$$h = 90 \times \frac{15}{17} = 79.41$$

Therefore, the height of the kite is 79.41 m.

22. Question

The upper part of a tree is broken over by the strong wind makes an angle of 30° with the ground. The top of the broken tree meets the ground at a distance of 25 m from the foot of the tree. Find the original height of the tree.

Answer



Let in ΔABC,

AB broken part the of Tree

AC unbroken part of the Tree

BC the distance between root and the top of the Tree

$$\tan\theta = \frac{AC}{BC}$$

$$\tan 30 = \frac{AC}{30}$$

$$AC = 30 \times \tan 30 = 30 \times \frac{1}{\sqrt{3}}$$

$$AC = 10\sqrt{3}$$

Now, $\sin\theta = \frac{AC}{AB} = \frac{10\sqrt{3}}{AB}$

$$AB = \frac{10\sqrt{3}}{\sin 30}$$

$$AB = 20\sqrt{3} = 34.64\ \text{m}$$

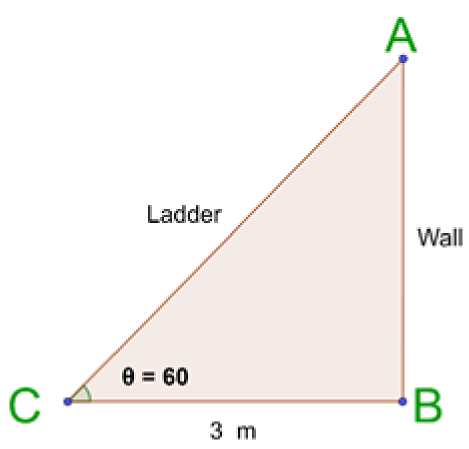
So, Height of the Tree $= AC + AB = 10\sqrt{3} + 20\sqrt{3} = 30\sqrt{3} = 51.96$

Therefore, the height of the tree is 51.96 m.

23. Question

AB is a vertical wall and B is on the ground. A ladder AC is resting at point C on the ground. If ∠ACB = 60°, BC = 3m, then find the length of the ladder.

Answer



From the ΔABC,

$$\cos\theta = \frac{BC}{AC}$$

$$\cos 60 = \frac{3}{AC}$$

$$\frac{1}{2} = \frac{3}{AC}$$

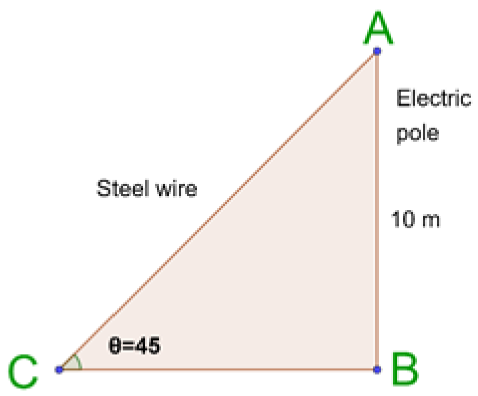
AC = 6 m.

Therefore, the length of the ladder is 6 m.

24. Question

An electric pole is 10 m high. A steel wire tied to the top of the pole is affixed at a point on the ground to keep the pole upright. If the steel wire makes an angle of 45° with the horizontal through the foot of the pole. Find the length of the steel wire.

Answer



From the ΔABC,

$$\sin\theta = \frac{AB}{AC}$$

$$\sin 45 = \frac{10}{AC}$$

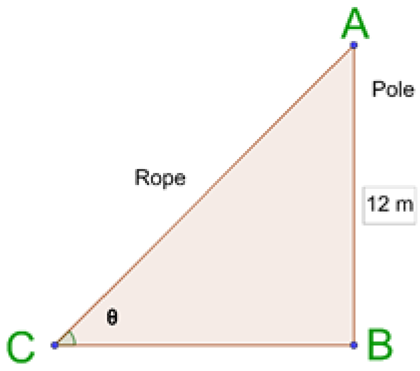
$$AC = \frac{10}{1/\sqrt{2}} = 10\sqrt{2}$$

Therefore, the length of the rope is $10\sqrt{2}$ m.

25. Question

A circus artist climbs on a rope which is tied between the top of a pole and a fixed point on the level ground. The height of the pole is 12 m and the rope makes an angle of θ with the ground. Find the distance covered by the artist to climb to the top of the pole. [sinθ= 0.5783]

Answer



From the ΔABC,

$$\text{Sin}\,\theta = \frac{AB}{AC}$$

$$0.5783 = \frac{12}{AC}$$

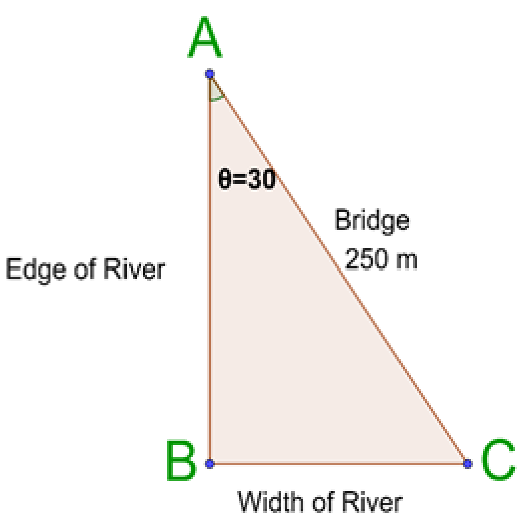
$$AC = \frac{12}{0.5783} = 20.75$$

Therefore, distance covered by the artist to climb to the top of the pole is 20.75 m.

26. Question

In order to cross a river, a person has to cover a distance of 250 m along the straight bridge from one end to the other. I f the bridge makes an angle of 30° with the edge of the river, find the width of the river.

Answer



From the ΔABC,

$$\text{Sin}\,\theta = \frac{BC}{AC}$$

$$\text{Sin}\,30 = \frac{250}{BC}$$

$$\frac{1}{2} = \frac{250}{BC}$$

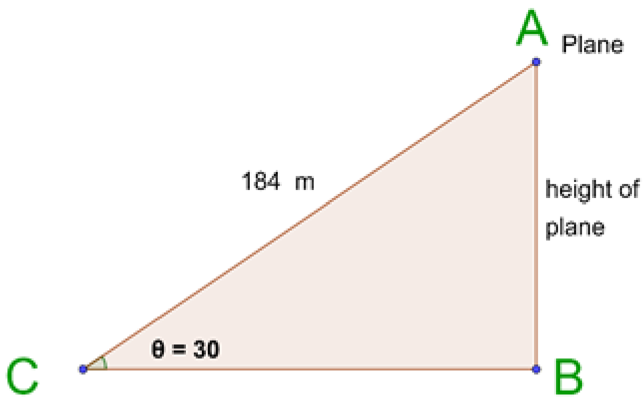
$$BC = \frac{250}{2} = 125$$

Therefore, the width of the river is 125 m.

27. Question

An aeroplane flies from the ground making an angle of 30° with the ground and covers a distance of 184 m. What will be the height of the aeroplane above the ground?

Answer



From the ΔABC,

$$\text{Sin}\,\theta = \frac{AB}{AC}$$

$$\text{Sin}\,30 = \frac{AB}{184}$$

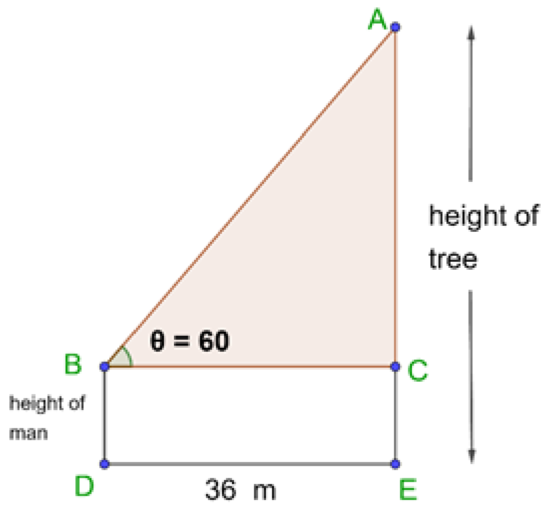
$$AB = 184 \times \frac{1}{2} = 92$$

Therefore, height of the plane is 92 m.

28. Question

A man of height 1 .5 m sees the top of a tree and the angle of elevation of the top at his eye is 60°. Find the height of the tree if the distance of the man from the tree is 36 m.

Answer



Here distance of the man from tree is given

BC = DE = 36 m

And height of man = BD = CE =1.5 m

From the ΔABC,

$$\text{Tan}\,\theta = \frac{AC}{BC}$$

$$\tan 60 = \frac{AC}{36}$$

$$AC = 36 \times \sqrt{3} = 62.35$$

Now, height of the tree = AC + CE

= 62.35 + 1.5

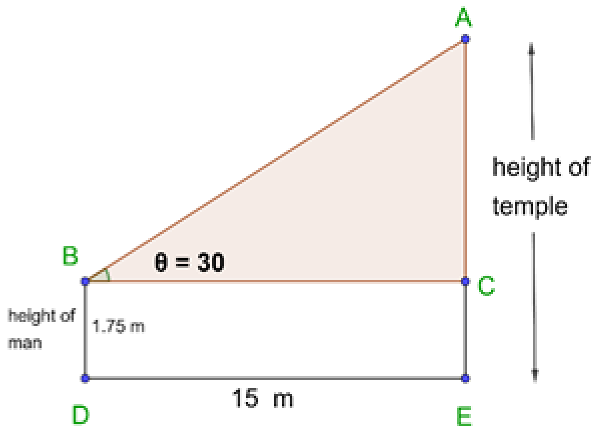
= 63.85

Therefore, height of the tree is 63.85 m.

29. Question

A man who is $1\frac{3}{4}\text{m}$ tall sees that angle of elevation of the top of a temple is 30°. If the distance of the man from the temple is 15 m, find the height of the temple.

Answer



Here the distance of the man from the temple is given,

BC = DE = 15 m.

And height of man =BD = CE =$1\frac{3}{4}$ = 1.75 m

From the ΔABC,

$$\text{Tan}\,\theta = \frac{AC}{BC}$$

$$\text{Tan}\,30 = \frac{AC}{15}$$

$$\frac{1}{\sqrt{3}} = \frac{AC}{15}$$

So, $AC = \frac{15}{\sqrt{3}} = 8.66$

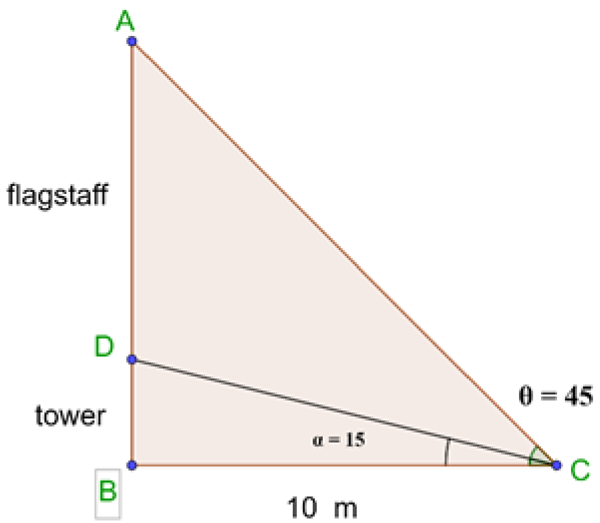
Now, height of the temple = AC + CE = 8.66 + 1.75 = 10.41

Therefore, height of the temple is 10.41 m.

30. Question

A flagstaff stands on a vertical tower. At a point distant 10 m from the base of the tower, the tower and the flagstaff make angles 45° and 15° respectively. Find the length of the flagstaff.

Answer



Here, point distance from base pf tower = BC = 10 m

From the ΔABC,

$$\text{Tan}\,\theta = \frac{AB}{BC}$$

$$\text{Tan}\,45 = \frac{AB}{10} = 1$$

$$AB = 10$$

Now, ΔDBC,

$$\text{Tan}\,\alpha = \frac{DB}{BC}$$

$$\text{Tan}\,15 = \frac{DB}{10}$$

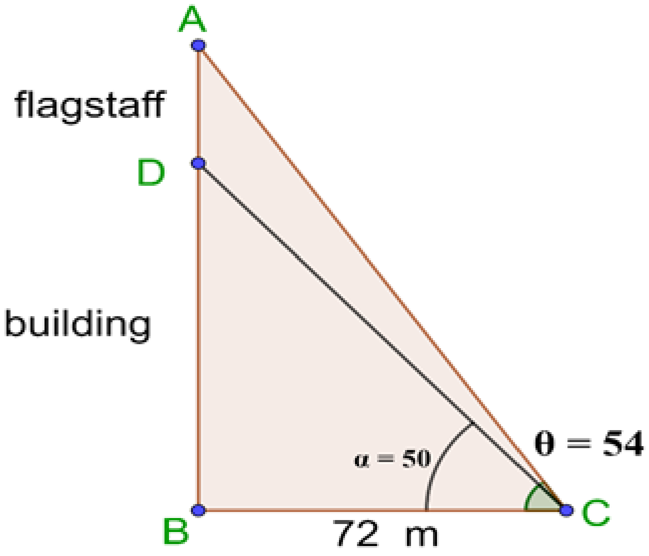
$$DB = 10 \times \tan 15 = 2.67$$

Therefore, the length of flagstaff = AB – DB = 10 – 2.67 = 7.32 m.

31. Question

An observer standing at a distance of 72 m from a building measures the angles of elevation of the top and foot of a flagstaff on the building as 54° and 50°. Find the height of the flagstaff. [tan54° = 1.376, tan50° = 1.192]

Answer



From the ΔABC,

$$\text{Tan}\,\theta = \frac{AB}{BC}$$

$$\text{Tan}\,54 = \frac{AB}{72}$$

$$AB = 1.376 \times 72 = 99.07$$

From the ΔDBC,

$$\text{Tan}\,\alpha = \frac{DB}{BC}$$

$$\text{Tan}\,50 = \frac{DB}{72}$$

$$DB = 1.192 \times 72 = 85.82$$

Now, the length of the flagstaff = AB – DB

=99.07 – 85.82

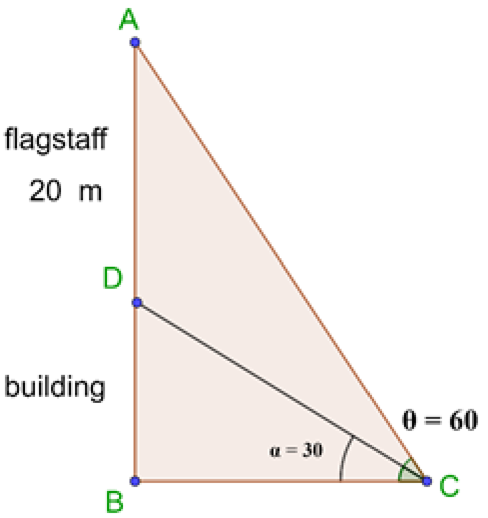
=13.25

Therefore, the length of the flagstaff is 13.25 m.

32. Question

A 20 m long flagstaff stands on a tower. At a point on the level ground the angles of elevations of the foot and top of the flagstaff are 30° and 60° respectively. Find the height of the tower.

Answer



From the ΔABC,

$$\text{Tan}\,\theta = \frac{AB}{BC}$$

$$\tan 60 = \frac{AB}{BC} = \sqrt{3}$$

$BC = \frac{AB}{\sqrt{3}} = \frac{AD+DB}{\sqrt{3}}$……….equation (i)

Now, from the ΔDBC,

$$\text{Tan}\,\alpha = \frac{DB}{BC} = \tan 30$$

$$DB = BC \times \tan 30$$

Put value of BC from equation (i)

$$DB = \frac{AD + DB}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{AD + DB}{3}$$

$$3 \times DB = 20 + DB$$

$$2 \times DB = 20$$

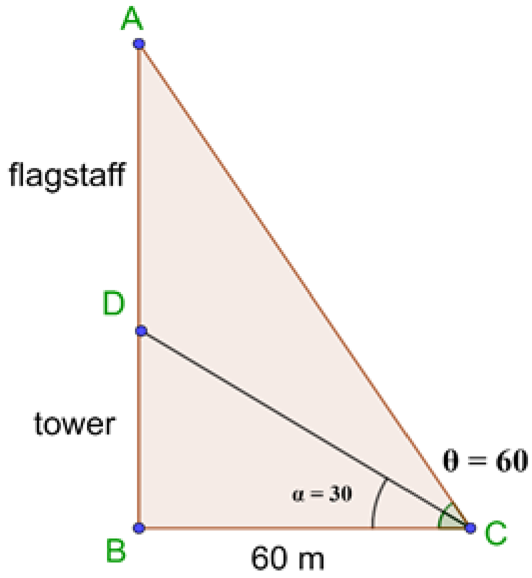
$$DB = 10$$

Therefore, the height of the tower is 10 m.

33. Question

A flagstaff stands on a tower. At a point distant 60 m from the base of the tower, the top of the flagstaff makes an angle of 60° and the tower makes an angle of 30° at that very point. Find the height of the flagstaff

Answer



From the ΔDBC,

$$\text{Tan}\,\alpha = \frac{DB}{BC}$$

$$\text{Tan}\,30 = \frac{DB}{60}$$

$$DB = 60 \times \frac{1}{\sqrt{3}} = 34.64$$

Now from the ΔABC,

$$\text{Tan}\,\theta = \frac{AB}{BC}$$

$$\text{Tan}\,60 = \frac{AB}{60}$$

$$AB = 60 \times \sqrt{3} = 103.92$$

So, the height of the flagstaff

= AB – DB

=103.92 – 34.64

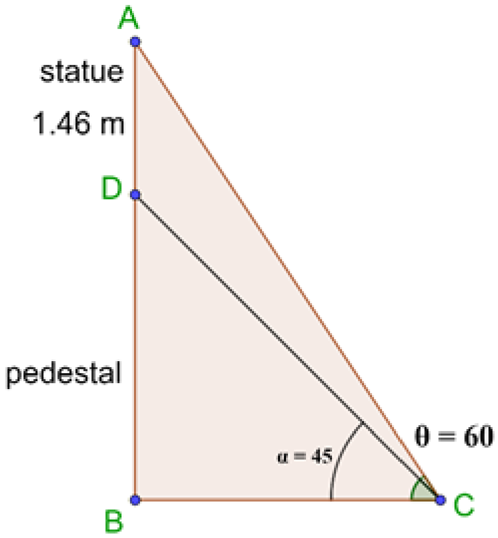
=69.28 m

Therefore, the height of the flagstaff is 69.28 m.

34. Question

A statue 1.46 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal (use $\sqrt{3}$ = 1.73)

Answer



From the ΔDBC,

$$\text{Tan}\,\alpha = \frac{DB}{BC}$$

$$\text{Tan}\,45 = \frac{DB}{BC} = 1$$

$$DB = BC \ldots \ldots \text{equation(i)}$$

Now from the ΔABC,

$$\text{Tan}\,\theta = \frac{AB}{BC}$$

Put BC = DB from equation(i)

$$\text{Tan}\,60 = \frac{AD + DB}{DB}$$

$$\sqrt{3} \times DB = AD + DB$$

$$\sqrt{3} \times DB - DB = 1.46$$

$$DB(\sqrt{3} - 1) = 1.46$$

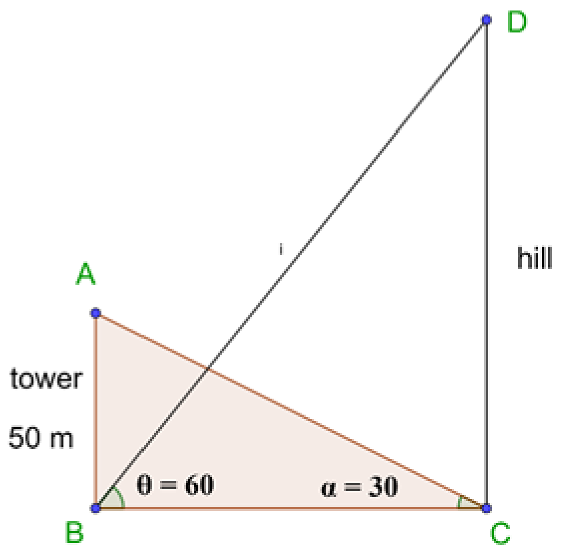
$$DB = \frac{1.46}{0.73} = 2$$

Therefore, the height of pedestal is 2 m.

35. Question

The angle of elevation of the top of a hill at the foot of a tower is 60° and the angle of elevation of the top of the tower at the foot of the hill is 30°. If the tower is 50 m tall, what is the height of the hill?

Answer



From the ΔABC,

$$\text{Tan}\,\alpha = \frac{AB}{BC}$$

$$\text{Tan}\,30 = \frac{50}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{50}{BC}$$

$$BC = 50\sqrt{3} \ldots \ldots \text{equation(i)}$$

Now, from the ΔDBC,

$$\text{Tan}\,\theta = \frac{DC}{BC}$$

$$\text{Tan}\,60 = \frac{DC}{BC} = \sqrt{3}$$

Put value of BC from equation (i)

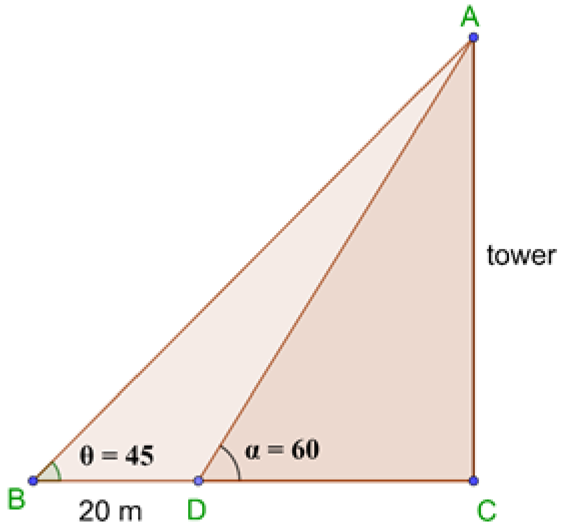
$$DC = 50\sqrt{3} \times \sqrt{3} = 50 \times 3 = 150$$

Therefore, height of the hill is 150 m.

36 A. Question

At a point on the level ground, the angle of elevation of the top of a tower is 45°. On moving 20 m towards the tower, the angle of elevation becomes 60°. Find the height of the tower.

Answer



From the ΔADC,

$$\text{Tan}\,\alpha = \frac{AC}{DC} = \sqrt{3}$$

$$DC = \frac{AC}{\sqrt{3}} \ldots .\text{equation(i)}$$

Now, from the ΔABC,

$$\text{Tan}\,\theta = \frac{AC}{BC} = \frac{AC}{BD + DC}$$

$$\text{Tan}\,45 = \frac{AC}{BD + DC} = 1$$

$$BD + DC = AC$$

Put value of DC from equation(i) and BD = 20 that is given,

So $20 + \frac{AC}{\sqrt{3}} = AC$

$$20 = AC - \frac{AC}{\sqrt{3}} = AC\left(1 - \frac{1}{\sqrt{3}}\right) = AC \times 0.422$$

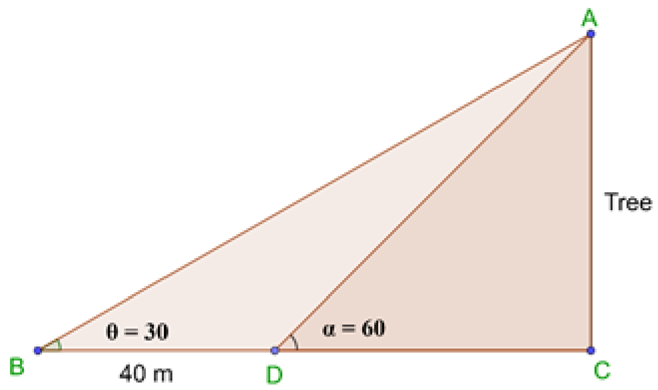
$$AC = \frac{20}{0.422} = 47.32$$

Therefore, height of the tower is 47.32 m.

36 B. Question

A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 m away from the bank, he finds the angle of elevation to be 30°. Find the height of the tree and the width of the river.[Use $\sqrt{3}$ = 1.732]

Answer



From the ΔADC,

$$\text{Tan}\,\alpha = \frac{AC}{DC}$$

$$\text{Tan}\,60 = \frac{AC}{DC} = \sqrt{3}$$

$$DC = \frac{AC}{\sqrt{3}} \ldots..\text{equation(i)}$$

Now, from the ΔABC,

$$\text{Tan}\,\theta = \frac{AC}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{AC}{BD + DC}$$

$$BD + DC = \sqrt{3} \times AC$$

Put vale of DC from equation(i) and BD = 40 that is given.

So, $40 + \frac{AC}{\sqrt{3}} = \sqrt{3} \times AC$

$$40 = \sqrt{3} \times AC - \frac{AC}{\sqrt{3}} = AC\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = AC \times \frac{2}{\sqrt{3}}$$

$AC = 20\sqrt{3} = 34.64$

Now, put value of AC in equation(i)

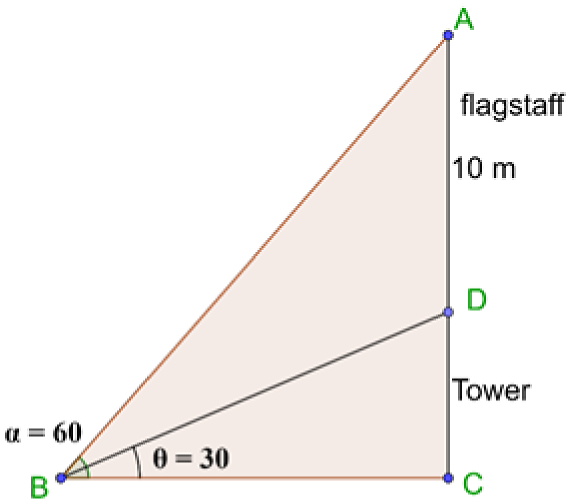
$$DC = \frac{AC}{\sqrt{3}} = \frac{20\sqrt{3}}{\sqrt{3}} = 20$$

Therefore, height of the tree is 34.64 m and width of the river is 20 m.

37 A. Question

A 10 m high flagstaff stands on a tower. From a point on the level ground, the angles of elevation of the foot and top of the flagstaff are 30° and 60° respectively. Find the height of the tower.

Answer



From the ΔDBC,

$$\text{Tan}\,\theta = \frac{DC}{BC} = \frac{1}{\sqrt{3}}$$

$BC = \sqrt{3} \times DC \ldots.\text{equation(i)}$

Now, from the ΔABC,

$$\text{Tan}\,\alpha = \frac{AC}{BC}$$

$$\text{Tan}\,60 = \frac{AD + DC}{BC} = \sqrt{3}$$

$AD + DC = \sqrt{3} \times BC$

Put the value of BC from the equation(i) and AD = 10 that is given.

So, $10 + DC = \sqrt{3} \times \sqrt{3}\,DC = 3 \times DC$

$10 = 3 \times DC - DC = 2 \times DC$

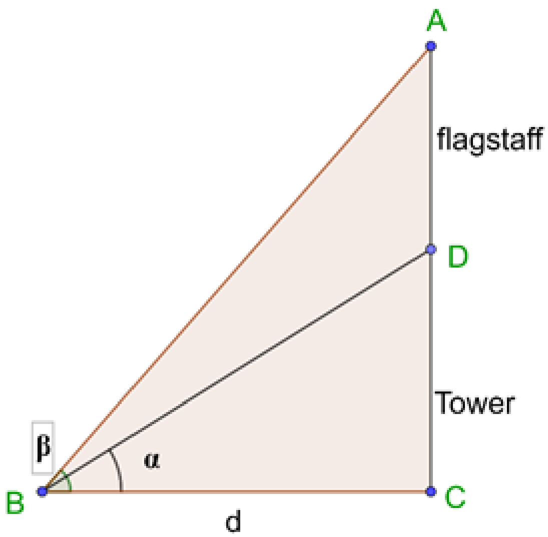
$$DC = \frac{10}{2} = 5$$

Therefore, the height of the tower is 5 m.

37 B. Question

A flagstaff stands on the top of a tower. At a point distant d from the base of the tower, the angles of elevation of the top of the flagstaff and that of the tower are]3 and a respectively. Prove that the height of the flagstaff is = d (tanβ – tan α).

Answer



From the ΔDBC,

$$\text{Tan}\,\alpha = \frac{DC}{BC} = \frac{DC}{d}$$

$DC = d \times \tan\alpha \ldots\ldots \text{equation(i)}$

Now from the ΔABC,

$$\tan\beta = \frac{AC}{BC} = \frac{AD + DC}{d}$$

Put the value of DC from the equation(i)

$d \times \tan\beta = AD + (d \times \tan\alpha)$

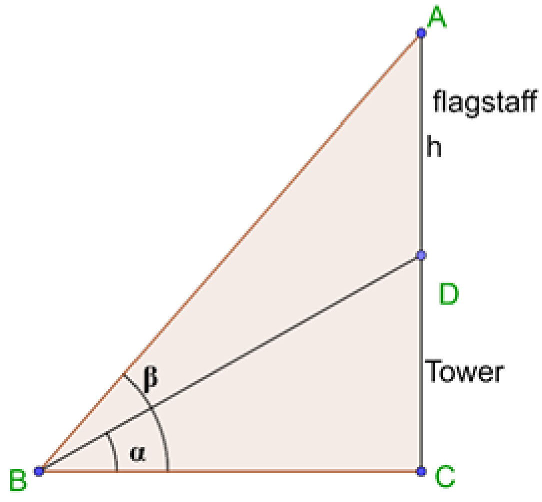
So, $AD = d\tan\beta - d\tan\alpha = d\,(\tan\beta - \tan\alpha)$

Therefore, height of the flagstaff is $= \mathbf{d(\tan\beta - \tan\alpha)}$.

37 C. Question

A vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height h. At a point on the plane, the angle of elevation of the bottom of the flagstaff is α and that of the top of the flagstaff is β. Prove that the height of the h tan a tower is $\frac{h \tan \alpha}{\tan \beta - \tan \alpha}$.

Answer



From the ΔDBC,

$$\text{Tan}\,\alpha = \frac{DC}{BC}$$

$$BC = \frac{DC}{\tan \alpha} \ldots\ldots \text{equation(i)}$$

From the ΔABC,

$$\text{Tan}\,\beta = \frac{AC}{BC} = \frac{AD + DC}{BC}$$

$$BC = \frac{h + DC}{\tan \beta}$$

Put value of BC from equation(i),

$$\frac{DC}{\tan \alpha} = \frac{h + DC}{\tan \beta}$$

$$DC \times \tan \beta = \tan \alpha\,(h + DC) = h \tan \alpha + DC \tan \alpha$$

$$DC\,(\tan \beta - \tan \alpha) = h \tan \alpha$$

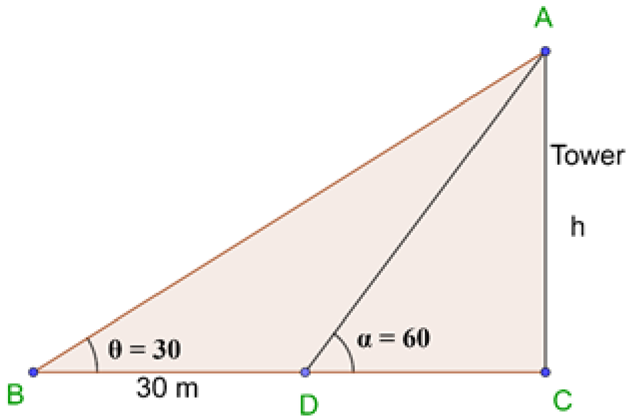
$$DC = \frac{h \tan \alpha}{\tan \beta - \tan \alpha}$$

Therefore, height of the tower is $\frac{h \tan \alpha}{\tan \beta - \tan \alpha}$.

38 A. Question

From a point on the level ground, the angle of elevation of the top of a tower is 30°. On proceeding 30 m towards the tower the angle of elevation becomes 60°. Find the height of the tower.

Answer



From the ΔADC,

$$\text{Tan}\,\alpha = \frac{AC}{DC}$$

$$\text{Tan}\,60 = \frac{h}{DC}$$

$$DC = \frac{h}{\sqrt{3}} \ldots \ldots \text{equation(i)}$$

From the ΔABC,

$$\tan \theta = \frac{AC}{BC}$$

$$\text{Tan}\,30 = \frac{h}{30 + DC}$$

Put the value of DC from the equation(i)

$$\frac{1}{\sqrt{3}} = \frac{h}{30 + \frac{h}{\sqrt{3}}}$$

$$h = \frac{30}{\sqrt{3}} + \frac{h}{3}$$

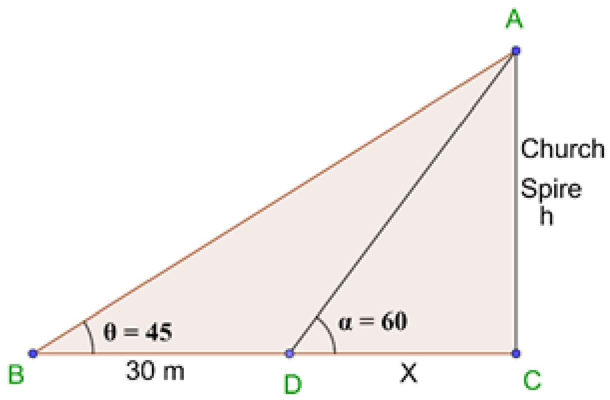
$$h = \frac{10\sqrt{3}}{\left(1 + \frac{1}{3}\right)} = 15\sqrt{3}$$

Therefore, the height of the tower is $15\sqrt{3}$ m.

38 B. Question

The angle of elevation of a church-spire at some point in the plane is 45°. On proceeding 30 m towards the church, the angle of elevation becomes 60°. Find the height of the church-spire.

Answer



From the ΔACD,

$$\text{Tan}\,\alpha = \frac{AC}{DC} = \frac{h}{x} = \tan 60 = \sqrt{3}$$

$$x = \frac{h}{\sqrt{3}} \ldots\ldots \text{equation(i)}$$

From the ΔABC,

$$\text{Tan}\,\theta = \frac{AC}{BC} = \frac{h}{30 + x} = \tan 45 = 1$$

$$h = 30 + x = 30 + \frac{h}{\sqrt{3}}$$

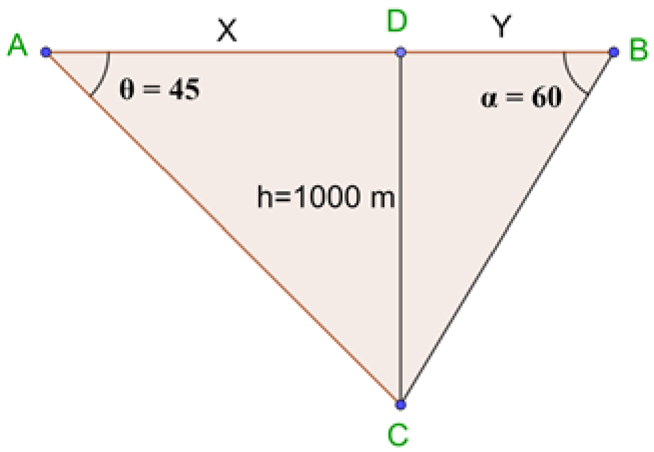
$$h = \frac{30}{\left(1 - \frac{1}{\sqrt{3}}\right)} = 70.80$$

Therefore, the height of the church-spire is 70.80 m.

39 A. Question

The pilot of helicopter at an altitude of 1000 m sees two aeroplanes, one on his left and the other on his right at the same height and finds their angles of depression as 45° and 60°. Find the distance between the two aeroplanes.

Answer



From the ΔADC,

$$\tan\theta = \frac{DC}{AD} = \frac{h}{x} = \frac{1000}{x} = \tan 45 = 1$$

$x = 1000$ m.

From the ΔDBC,

$$\text{Tan}\,\alpha = \frac{DC}{DB} = \frac{h}{y} = \frac{1000}{y}$$

$$\tan 60 = \sqrt{3} = \frac{1000}{y}$$

$$y = 1000\sqrt{3} = 577.4\text{ m}$$

So, Distance between the two aeroplanes = AD +DB

= x + y = 1000+577.4

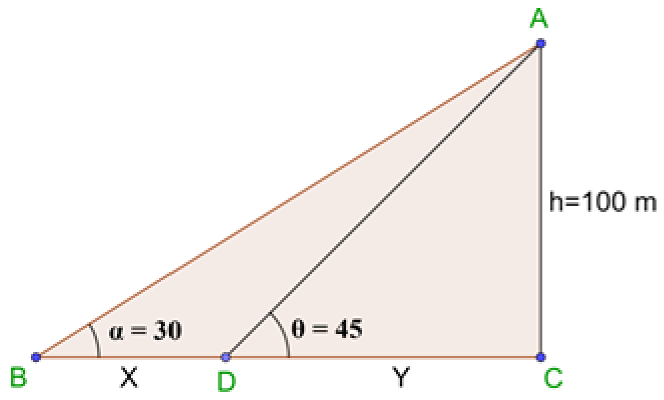
= 1577.4 m

Therefore, the distance between the two aeroplanes is 1577.4 m.

39 B. Question

As observed from the top of a 100 m tall light house, the angles of depression of two ships approaching it are 30° and 45°. If one ship is directly behind the other, find the distance between the two ships.

Answer



From the ΔADC,

$$\text{Tan}\,\theta = \frac{AC}{DC} = \frac{100}{y} = \tan 45 = 1$$

$$y = 100 \ldots \ldots \text{equation(i)}$$

From the ΔABC,

$$\text{Tan}\,\alpha = \frac{AC}{BC} = \frac{h}{x + y} = \frac{100}{x + 100} = \frac{1}{\sqrt{3}}$$

$$100\sqrt{3} = x + 100$$

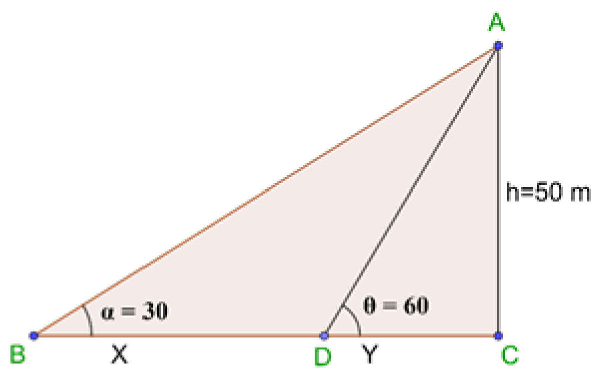
$$x = 73.2$$

Therefore, the distance between the two ships is 73.2 m.

39 C. Question

A straight highway leads to the foot of a 50 m tall tower. From the top of the tower, the angles of depression of two cars on the highway are 30° and 60°. What is the distance between the two cars and how far is each car from the tower?

Answer



From the ΔADC,

$$\text{Tan}\,\theta = \frac{AC}{DC} = \frac{h}{y} = \frac{50}{y} = \tan 60 = \sqrt{3}$$

$$y = \frac{50}{\sqrt{3}} = 28.86 \ldots \ldots \text{equation(i)}$$

From the ΔABC,

$$\text{Tan}\,\alpha = \frac{AC}{BC} = \frac{h}{x+y} = \frac{50}{x+\frac{50}{\sqrt{3}}} = \frac{1}{\sqrt{3}}$$

$$50\sqrt{3} = x + \frac{50}{\sqrt{3}}$$

$$x = \frac{100}{\sqrt{3}} = 57.73$$

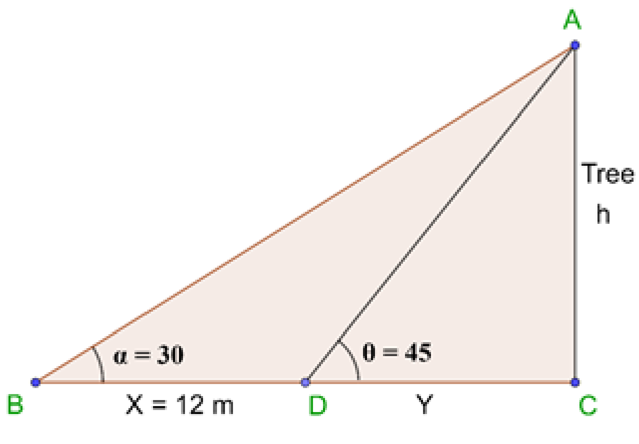
Therefore, the distance between two cars is 57.73 m—

And car D is 28.87 m and car B is x+y = 86.6 m far from the tower.

40 A. Question

When the altitude of the Sun increases from 30° to 45°, the length of the shadow of a palm tree decreases by 12 m. Find the length of the palm tree.

Answer



From the ΔADC,

$$\text{Tan}\,\theta = \frac{AC}{DC} = \frac{h}{y} = \tan 45 = 1$$

$$h = y \ldots \ldots \text{equation(i)}$$

From the ΔABC,

$$\mathrm{Tan}\,\alpha = \frac{AC}{BC} = \frac{h}{x+y} = \frac{h}{12+h}$$

$$\mathrm{Tan}\,30 = \frac{h}{12+h} = \frac{1}{\sqrt{3}}$$

$$h\sqrt{3} = 12 + h$$

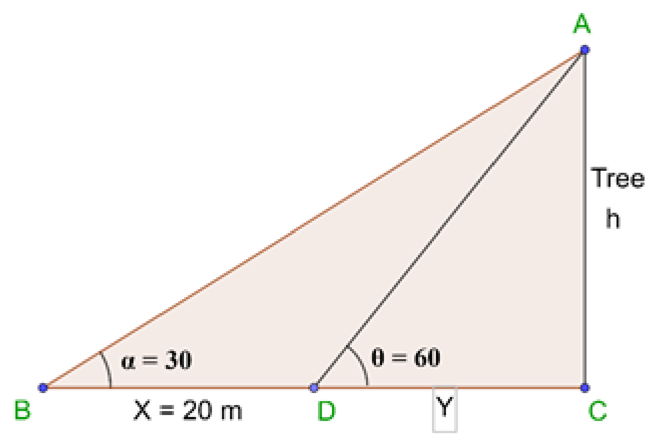
$$h = \frac{12}{\sqrt{3} - 1} = 16.39$$

Therefore, the height of the palm tree is 16.39 m.

40 B. Question

A tall tree stands vertically on a bank of a river. At the point on the other bank directly opposite the tree, the angle of elevation of the top of the tree is 60°. At a point 20 m behind this point on the same bank, the angle of elevation of the top of the tree is 30°. Find the height of the tree and the width of the river.

Answer



From the ΔADC,

$$\mathrm{Tan}\,\theta = \frac{AC}{DC} = \frac{h}{y} = \tan 60 = \sqrt{3}$$

$$y = \frac{h}{\sqrt{3}} \ldots \ldots \text{equation(i)}$$

From the ΔABC,

$$\mathrm{Tan}\,\alpha = \frac{AC}{BC} = \frac{h}{x+y} = \frac{h}{20 + \frac{h}{\sqrt{3}}} = \frac{1}{\sqrt{3}}$$

$$\sqrt{3}\,h = 20 + \frac{h}{\sqrt{3}}$$

$$h = \frac{20}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)} = 10\sqrt{3} = 17.32$$

put value of h in equation(i)

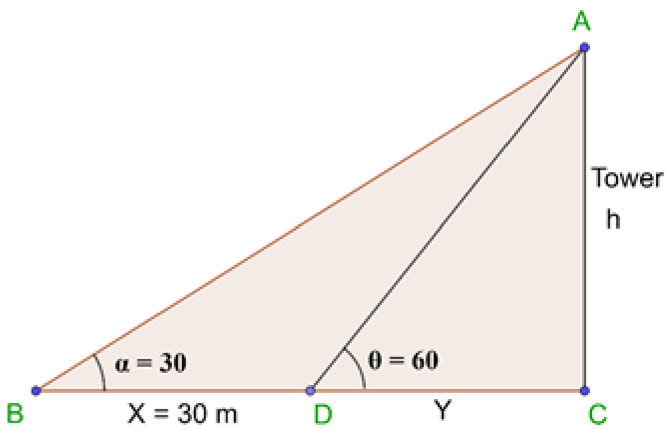
$$\text{So, } y = \frac{h}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}} = 10$$

Therefore, the height of tree is 17.32 m and width of the river is 10 m.

40 C. Question

The angle of elevation of the top of a tower from a point on the ground is 30°. After walking 30 m towards the tower, the angle of elevation becomes 60°. What is the height of the tower?

Answer



From the ΔADC,

$$\text{Tan}\,\theta = \frac{AC}{DC} = \frac{h}{y} = \tan 60 = \sqrt{3}$$

$$y = \frac{h}{\sqrt{3}} \dots \dots \text{equation(i)}$$

From the ΔABC,

$$\text{Tan}\,\alpha = \frac{AC}{BC} = \frac{h}{x + y}$$

$$\text{Tan}\,30 = \frac{h}{30 + \frac{h}{\sqrt{3}}} = \frac{1}{\sqrt{3}}$$

$$h\sqrt{3} = 30 + \frac{h}{\sqrt{3}}$$

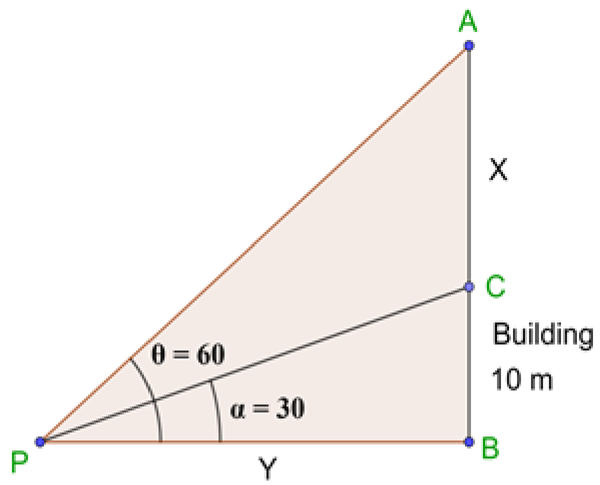
$$h = \frac{30}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)} = 15\sqrt{3}$$

Therefore, the height of the tower is $\mathbf{15\sqrt{3}}$ m.

40 D. Question

At a point P on the ground, the angles of elevation of the top of a 10 m tall building, and of a helicopter covering some distance over the top of the building, are 30° and 60° respectively. Find the height of the helicopter above the ground.

Answer



From the ΔPBC,

$$\mathrm{Tan}\,\alpha = \frac{BC}{PB} = \frac{10}{y} = \tan 30 = \frac{1}{\sqrt{3}}$$

$$y = 10\sqrt{3} \ldots \ldots \text{equation(i)}$$

From the ΔAPB,

$$\mathrm{Tan}\,\theta = \frac{AB}{PB} = \frac{x+10}{y} = \frac{x+10}{10\sqrt{3}}$$

$$\mathrm{Tan}\,60 = \frac{x+10}{10\sqrt{3}} = \sqrt{3}$$

$$x + 10 = 30$$

$$x = 20$$

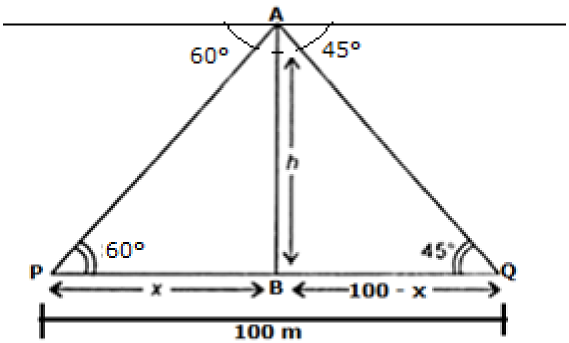
So, the height of the helicopter above the ground = x+10 = 20+10 = 30

Therefore, the height of the helicopter is 30 m.

41 A. Question

From an aeroplane, the angles of depression of two ships in a river on left and right of it are 60° and 45° respectively. If the distance between the two ships is 100m, find the height of the aeroplane.

Answer



In right Δ ABP, we have

$$\tan 60° = \frac{AB}{PB}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \ldots(i)$$

In the right Δ ABQ, we have

$$\tan 45° = \frac{AB}{BQ}$$

$$\Rightarrow 1 = \frac{h}{100 - x}$$

⇒ 100 – x = h

⇒ 100 = h + x

$$\Rightarrow 100 = h + \frac{h}{\sqrt{3}} \text{[from (i)]}$$

$$\Rightarrow 100 = \frac{h\sqrt{3} + h}{\sqrt{3}}$$

$$\Rightarrow h = \frac{100\sqrt{3}}{\sqrt{3}+1}$$

Multiplying and divide by the conjugate of √3 + 1, we get

$$\Rightarrow h = \frac{100\sqrt{3}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$\Rightarrow h = \frac{100\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3})^2-(1)^2}$ [∵ $(a - b)(a + b) = (a^2 - b^2)$]

$$\Rightarrow h = \frac{100(3-\sqrt{3})}{3-1}$$

⇒ h = 50 (3 - √3)

⇒ h = 50 (3 – 1.732) [∵ √3 = 1.732]

⇒ h = 50 (1.268)

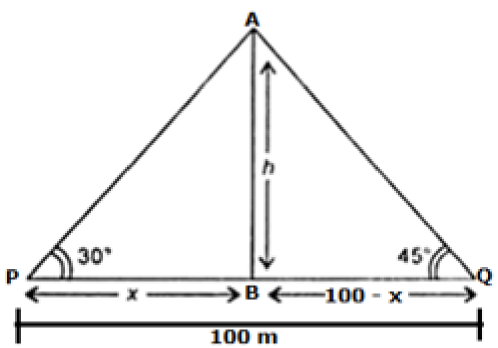
⇒ h = 63.4 m

Hence, the height of the aeroplane is 63.4m

41 B. Question

There is a small island in the middle of 100 m wide river. There is a tall tree on the island. Points P and Q are points directly opposite each other on the two banks and in line with the tree. If the angles of elevation of the top of the tree at P and Q are 30° and 45°, find the height of the tree.

Answer



In right Δ ABP, we have

$$\tan 30° = \frac{AB}{PB}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$\Rightarrow x = \sqrt{3}h$ ...(i)

In the right Δ ABQ, we have

$$\tan 45° = \frac{AB}{BQ}$$

$$\Rightarrow 1 = \frac{h}{100 - x}$$

$\Rightarrow 100 – x = h$

$\Rightarrow 100 = h + x$

$\Rightarrow 100 = h + \sqrt{3}h$ [from (i)]

$\Rightarrow 100 = h(\sqrt{3} + 1)$

$$\Rightarrow h = \frac{100}{\sqrt{3} + 1}$$

Multiplying and divide by the conjugate of $\sqrt{3} + 1$, we get

$$\Rightarrow h = \frac{100}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$\Rightarrow h = \frac{100(\sqrt{3}-1)}{(\sqrt{3})^2-(1)^2}$ [$\because (a – b)(a + b) = (a^2 – b^2)$]

$$\Rightarrow h = \frac{100(\sqrt{3} - 1)}{3 - 1}$$

$\Rightarrow h = 50 (\sqrt{3} – 1)$

$\Rightarrow h = 50 (1.732 – 1)$

$\Rightarrow h = 50 (0.732)$

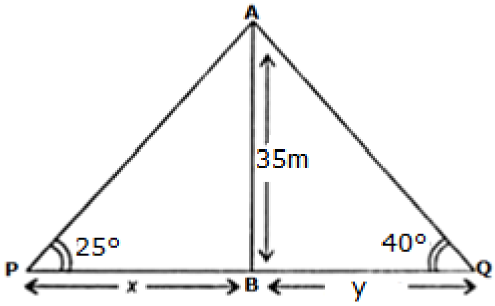
$\Rightarrow h = 36.6m$

Hence, the height of the tree is 36.6m

41 C. Question

Two men are on the opposite sides of a tower. They measure the angles of elevation of the tower as 25° and 40° respectively. If the height of the tower is 35 m, find the distance between two men; having given tan 25° = 0.4663 and tan 40° = 0.8391.

Answer



In right Δ ABP, we have

$$\tan 25^\circ = \frac{AB}{PB}$$

$$\Rightarrow 0.4663 = \frac{h}{x}$$

$$\Rightarrow 0.4663 = \frac{35}{x}$$

$$\Rightarrow x = \frac{35}{0.4663}$$

⇒ x = 75.058 m

⇒ x = 75.06

In the right Δ ABQ, we have

$$\tan 40^\circ = \frac{AB}{BQ}$$

$$\Rightarrow 0.8391 = \frac{h}{y}$$

$$\Rightarrow y = \frac{35}{0.8391}$$

⇒ y = 41.71 m

So, the distance between two men = x + y

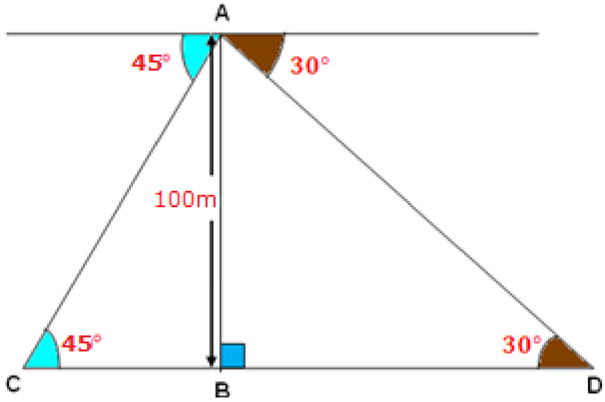
= 75.06 + 41.71

= 116.77 m (approx.)

41 D. Question

From a light-house, the angles of depression of two ships on opposite sides of the light-house are 30° and 45°. If the height of the light-house is 100 m, find the distance between the ships, if the line joining them passes through the foot of the light-house.

Answer



Let the two ships be at C and D with angles of depression 45° and 30° from point A.

The height of the light house, AB = 100m

In the right Δ ABD, we have

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{BD}$$

⇒ BD = $100\sqrt{3}$ ...(i)

In right Δ ABC, we have

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow 1 = \frac{100}{BC}$$

⇒ BC = 100m

Hence, the distance between the two ships = BC + BD

= $100 + 100\sqrt{3}$

= $100(1 + \sqrt{3})$

= 100(1 + 1.732)

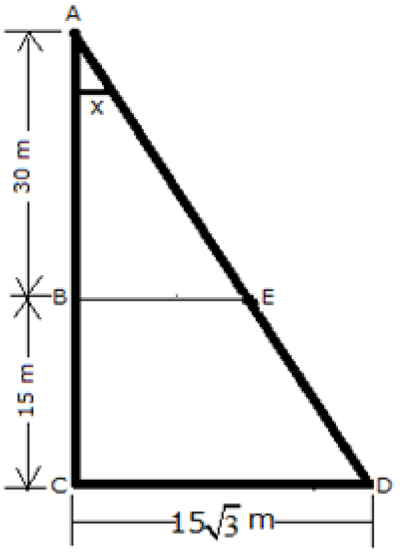
= 100(2.732)

= 273.2 m (approx.)

42. Question

An idol 30m tall stands on a pillar 15 m high. Find the angle in degrees which the idol subtends at a point distant $15\sqrt{3}$ m from the base of the pillar.

Answer



Let AB be the idol and BC be the pillar

So, AB = 30 m

and BC = 15 m

Distance from the base of pillar = $15\sqrt{3}$ m

Now, In right ΔACD, we have

$$\tan x = \frac{CD}{AC}$$

$$\Rightarrow \tan x = \frac{15\sqrt{3}}{30 + 15}$$

$$\Rightarrow \tan x = \frac{15\sqrt{3}}{45}$$

$$\Rightarrow \tan x = \frac{\sqrt{3}}{3}$$

$$\Rightarrow \tan x = \frac{1}{\sqrt{3}}$$

$\Rightarrow \tan x = \tan 30°$

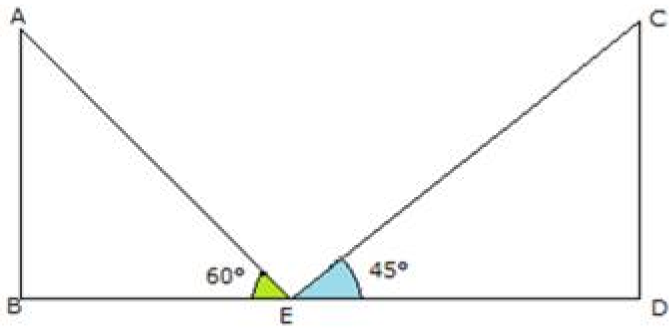
$\Rightarrow x = 30°$

Hence, the angle subtends from the base of the pillar is 30°

43. Question

A ladder is placed against a building, and the angle of elevation of the top of the ladder is 60°. The ladder is turned so that it is placed against another building on the other side of the lane and the angle of elevation, in this case, is 45°. If the ladder is 26 m long, then find the width of the lane.

Answer



Let AB and CD are the two buildings and AE and CE are the ladder

Hence, AE and CE = 26 m (given)

In the right Δ ABE, we have

$$\cos 60° = \frac{BE}{AE}$$

$$\Rightarrow \frac{1}{2} = \frac{BE}{26}$$

⇒ BE = 13 m

Now, In ΔCED, we have

$$\cos 45° = \frac{DE}{CE}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{DE}{26}$$

$$\Rightarrow DE = \frac{26}{\sqrt{2}}$$

$$\Rightarrow DE = \frac{26}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\Rightarrow DE = \frac{26\sqrt{2}}{2}$$

⇒ DE = 13√2 m

So, the width of the lane = BE + DE

= 13 + 13√2

= 13 (1 + √2)

= 13 (1 + 1.414) [∵√2 = 1.414]

= 13 × 2.414

= 31.38

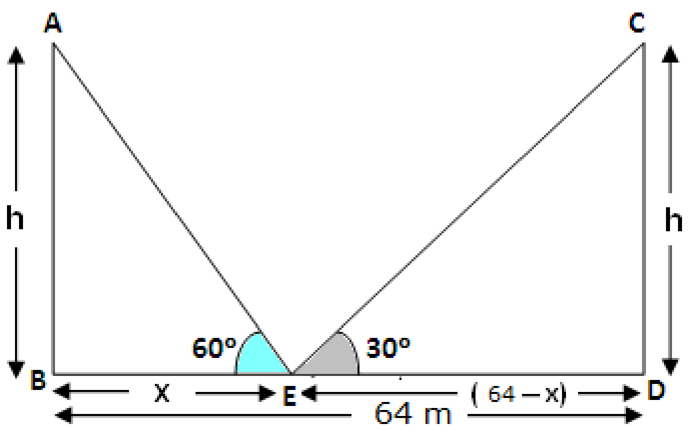
= 31.4 m (approx.)

Hence, the width of the lane is 31.4 m (approx.)

44. Question

Two pillars of equal height are 64 m apart. The angles of elevation of their tops from any point joining their feet are respectively 30° and 60°. Find the height of the pillars.

Answer



Let the height of the equal pillars AB = CD = h

Given the width of the road, BD = 64m

Let BE = x. Hence, DE = 64 – x

Now, In right ΔABE, we have

$$\tan 60° = \frac{AB}{BE}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}}$$

In the right ΔCDE, we have

$$\tan 30° = \frac{CD}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{64 - x}$$

$$\Rightarrow 64 - x = \sqrt{3}h$$

$$\Rightarrow 64 - \frac{h}{\sqrt{3}} = \sqrt{3}h$$

$$\Rightarrow 64 = \frac{h}{\sqrt{3}} + \sqrt{3}h$$

$$\Rightarrow 64 = \frac{h + 3h}{\sqrt{3}}$$

$$\Rightarrow h = \frac{64\sqrt{3}}{4}$$

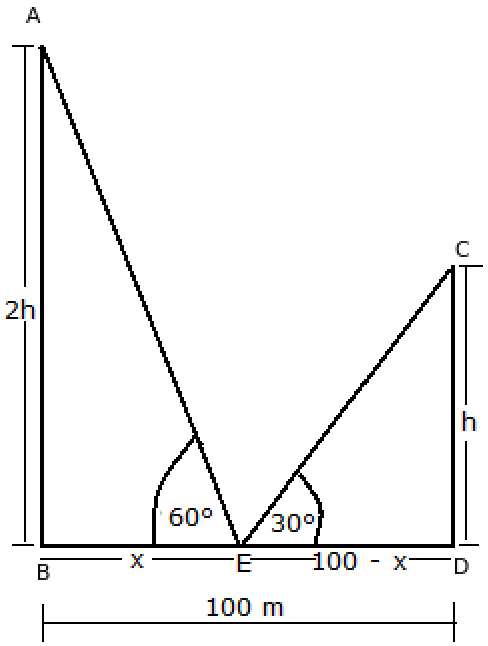
$$\Rightarrow h = 16\sqrt{3} \text{ m}$$

Hence, the height of the equal pillars AB = CD = $16\sqrt{3}$m

45. Question

The distance between two vertical pillars is 100 m, and the height of one is double of the other. The angles of elevation of their tops at a point on the line joining the foot of the two pillars are 60° and 30° respectively. Find their heights.

Answer



Let the height of 1st pillar CD = h and height of the 2nd pillar = 2h

It is given that the distance between two vertical pillars is 100m

Now, In right ΔABX, we have

$$\tan 60° = \frac{AB}{BE}$$

$$\Rightarrow \sqrt{3} = \frac{2h}{x}$$

$$\Rightarrow x = \frac{2h}{\sqrt{3}}$$

In the right ΔCDE, we have

$$\tan 30° = \frac{CD}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100 - x}$$

$\Rightarrow 100 - x = \sqrt{3}h$

$$\Rightarrow 100 - \frac{2h}{\sqrt{3}} = \sqrt{3}h$$

$$\Rightarrow 100 = \frac{2h}{\sqrt{3}} + \sqrt{3}h$$

$$\Rightarrow 100 = \frac{2h + 3h}{\sqrt{3}}$$

$$\Rightarrow h = \frac{100\sqrt{3}}{5}$$

$\Rightarrow$ h = $20\sqrt{3}$ m

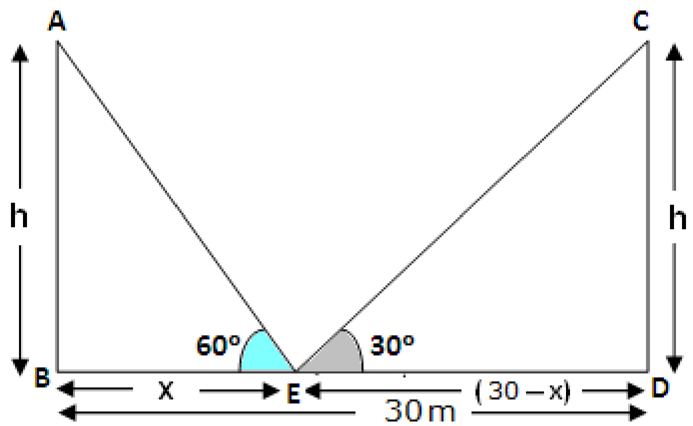
Hence, the height of the 1^{st} vertical pole, CD = $20\sqrt{3}$ m

and the height of the 2^{nd} vertical pole, AB = 2 × $20\sqrt{3}$ = $40\sqrt{3}$ m

46. Question

Two pillars of equal height stand on either side of roadway which is 30 m wide. At a point in the roadway between the pillars, the elevations of the tops of the pillars are 60° and 30°. Find the heights of the pillars and the position of the point.

Answer



Let the height of the equal pillars AB = CD = h

Given the width of the road, BD = 30m

Let BE = x. Hence, DE = 30 – x

Now, In right ΔABE, we have

$$\tan 60° = \frac{AB}{BE}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}}$$

In the right ΔCDE, we have

$$\tan 30° = \frac{CD}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{64 - x}$$

⇒ 30 – x = √3h

$$\Rightarrow 30 - \frac{h}{\sqrt{3}} = \sqrt{3}h$$

$$\Rightarrow 30 = \frac{h}{\sqrt{3}} + \sqrt{3}h$$

$$\Rightarrow \ 30 \ = \ \frac{h + 3h}{\sqrt{3}}$$

$$\Rightarrow h = \frac{30\sqrt{3}}{4}$$

$$\Rightarrow \ h \ = \ \frac{15\sqrt{3}}{2}$$

$$\Rightarrow h = \frac{15 \times 1.732}{2}$$

⇒ h = 12.99m

Hence, the height of the equal pillars AB = CD = 12.99 m

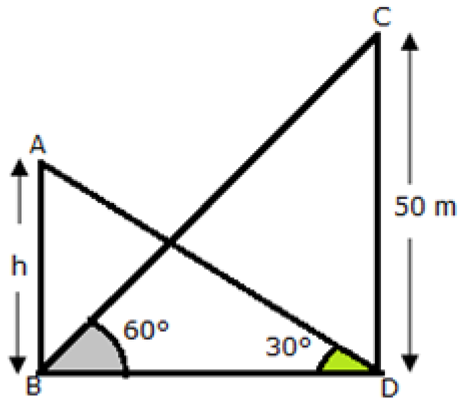
The distance of a point from one pillar is

$$x = \frac{12.99}{\sqrt{3}} = \frac{12.99}{1.732} = 7.5 \text{ m}$$

47. Question

The angle of elevation of the top of a tower from the bottom of a tree is 60°, and the angle of elevation of the top of the tree from the foot of the tower is 30°. If the tower is 50 m tall, what is the height of the tree?

Answer



Let tree be AB and tower be CD

Given: Height of the tower = 50 m

Hence, CD = 50 m

The angle of elevation of the top of the tower from the bottom of a tree = 60°

Hence, $\angle$ CBD = 60°

The angle of elevation of the top of the tree from the foot of tower = 30°

Hence, $\angle$ ADB = 30°

Now, In right ΔCBD, we have

$$\tan 60° = \frac{CD}{BD}$$

$$\Rightarrow \sqrt{3} = \frac{50}{BD}$$

$$\Rightarrow BD = \frac{50}{\sqrt{3}}$$

In the right ΔADB, we have

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{\frac{50}{\sqrt{3}}}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}h}{50}$$

$$\Rightarrow h = \frac{50}{3}$$

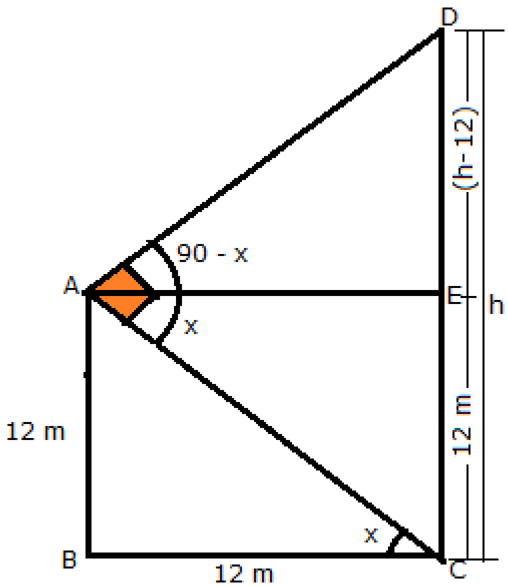
$$\Rightarrow h = 16\frac{2}{3}\ m$$

Hence, the height of the tree is $16\frac{2}{3}$ m

48. Question

A vertical tower of height 12m subtends a right angle at the top of a flagstaff If the distance between them is 12 m, find the height of the tower.

Answer



Let AB be the Flagstaff and CD be the vertical tower

let the height of the tower, CD = h

∵ AB = EC = 12 m

and the distance between Flagstaff and tower = 12 m

Hence, BC = AE = 12 m

Now, In Δ ABC, we have

$$\tan x = \frac{AB}{BC}$$

$$\Rightarrow \tan x = \frac{12}{12} = 1$$

⇒ tan x = tan 45°

⇒ x = 45° ...(i)

Now, In ΔADE, we have

$$\tan(90° - x) = \frac{DE}{AE}$$

$$\Rightarrow \tan(90° - 45°) = \frac{DE}{12} \text{ [from(i)]}$$

$$\Rightarrow \tan 45° = \frac{DE}{12}$$

$$\Rightarrow 1 = \frac{DE}{12}$$

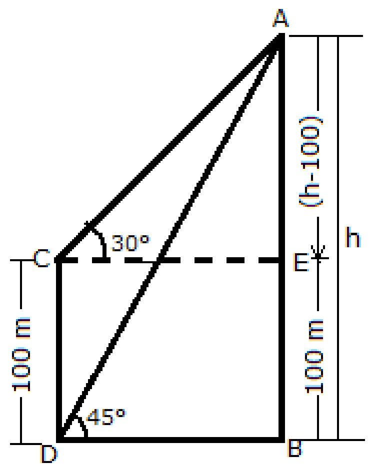
⇒ DE = 12 m

Hence, the height of the tower, CD = DE + CE = 12 + 12 = 24m

49. Question

The angles of elevation of the top of a rock at the top and foot of a 100 m high tower, at respectively 30° and 45°. Find the height of the rock.

Answer



Given: Height of the tower = 100 m

Hence, CD = 100 m = BE

Let the height of the rock = h

Hence, AB = h

In the right ΔABD, we have

$$\therefore \tan 45° = \frac{AB}{BD}$$

$$\Rightarrow 1 = \frac{h}{BD}$$

$\Rightarrow$ BD = h

$\Rightarrow$ CE = h

In the right ΔAEC, we have

$$\therefore \tan 30° = \frac{AE}{CE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h - 100}{h}$$

$\Rightarrow h = \sqrt{3}(h - 100)$

$\Rightarrow h = \sqrt{3}h - \sqrt{3} \times 100$

$\Rightarrow 100 \times \sqrt{3} = \sqrt{3}h - h$

$\Rightarrow 100 \times \sqrt{3} = h(\sqrt{3} - 1)$

$$\Rightarrow h = \frac{100\sqrt{3}}{\sqrt{3} - 1}$$

Multiplying and divide by the conjugate of $\sqrt{3} - 1$

$$\Rightarrow h = \frac{100\sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$\Rightarrow h = \frac{100\sqrt{3}(\sqrt{3} + 1)}{(\sqrt{3})^2 - (1)^2}$$

$$\Rightarrow h = \frac{100(3 + \sqrt{3})}{3 - 1}$$

$\Rightarrow h = 50(3 + 1.732)$

$\Rightarrow h = 50(4.732)$

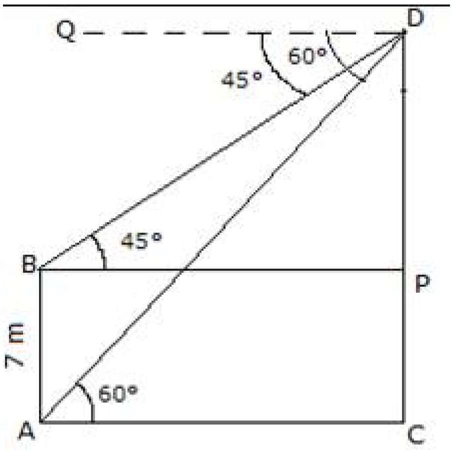
$\Rightarrow h = 236.6$ m

Hence, the height of the rock = 236.6 m (approx.)

50. Question

The angles of depression of the top and the bottom of a 7 m tall building from the top of a tower are 45° and 60° respectively. Find the height of the tower.

Answer



Let building be AB and tower be CD

The height of building, AB = 7 m

Let the height of tower = CD

and, the distance between tower and building = AC

The angle of depression to top of the building, ∠QDB = 45°

Angle of depression to bottom of building, ∠QDA = 60°

In the right Δ BDP, we have

$$\tan 45° = \frac{DP}{BP}$$

$$\Rightarrow 1 = \frac{DP}{BP}$$

⇒ BP = DP ...(i)

In the right Δ ADC, we have

$$\tan 60° = \frac{CD}{AC}$$

$$\Rightarrow \sqrt{3} = \frac{CD}{AC}$$

$\Rightarrow \sqrt{3} = \frac{CD}{BP}$ [∵ AC = BP]

$$\Rightarrow BP = \frac{CD}{\sqrt{3}}$$

$$\Rightarrow DP = \frac{CD}{\sqrt{3}}$$

∵ CD = DP + PC

⇒ CD = DP + AB [∵ AB = PC]

⇒ CD = DP + 7

$$\Rightarrow CD = \frac{CD}{\sqrt{3}} + 7$$

$$\Rightarrow CD - \frac{CD}{\sqrt{3}} = 7$$

$$\Rightarrow \frac{\sqrt{3}CD - CD}{\sqrt{3}} = 7$$

$$\Rightarrow \frac{CD(\sqrt{3} - 1)}{\sqrt{3}} = 7$$

$$\Rightarrow CD = \frac{7\sqrt{3}}{\sqrt{3} - 1}$$

$$\Rightarrow CD = \frac{7\sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$\Rightarrow CD = \frac{7\sqrt{3}(\sqrt{3} + 1)}{3 - 1}$$

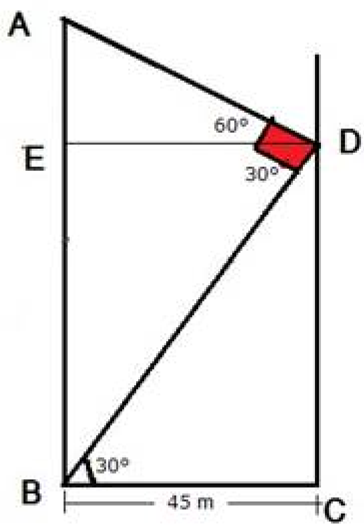
$$\Rightarrow CD = \frac{7(3 + 1.732)}{2}$$

⇒ CD = 16.56 (approx.)

51. Question

A building subtends a right angle at the top of a pole on the other side of the road. The line joining the top of the pole and the top of the building makes an angle of 60° with the vertical. If the width of the road is 45 m, find the height of the building.

Answer



Given: Width of the road = 45m

In right Δ BCD, we have

$$\tan 30° = \frac{CD}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{CD}{45}$$

$\Rightarrow$ CD = 15√3 m

∴ CD = BE = 15√3 m

Now, In ΔAED, we have

$$\tan 60° = \frac{AE}{DE}$$

$$\Rightarrow \sqrt{3} = \frac{AE}{45}$$

$\Rightarrow$ AE = 45√3 m

Now, the height of the building = AE + BE = 45√3 + 15√3

= √3(45 + 15)

= 60√3 m

52 A. Question

From the top and bottom of a building of height h, the angles of elevation of the top of a tower are α and β respectively. Prove that the height of the tower is $\frac{h \tan \beta}{\tan \beta - \tan \alpha}$

Answer

[Hint: Let AB be the tower and CD be the building. We draw CE ⊥ AB. According to the question,

CD = h, ∠BDE = α, ∠BCA = β

Let AB =y

Then, BE = BA — EA =y— h

Let CA = x. Then DE =x

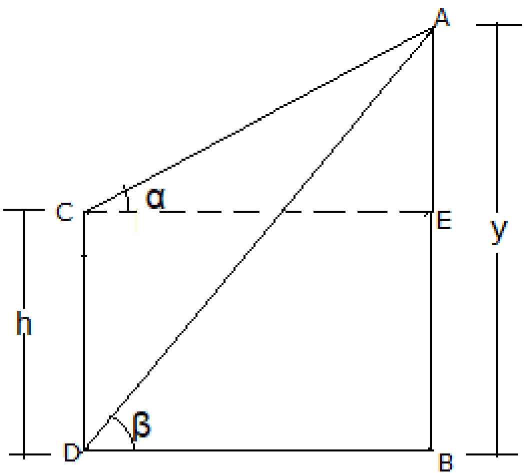
From right Δ BDE, $\tan\alpha = \frac{BE}{DE} = \frac{y-h}{x}$ .....(i)

Also, from right Δ BCA, $\tan\beta = \frac{AB}{AC} = \frac{y}{x}$ .....(ii)

52 B. Question

From the top and bottom of a building of height h, the angles of elevation of the top of a tower are α and β respectively. Prove that the height of the tower is $\frac{h\tan\beta}{\tan\beta - \tan\alpha}$.

Answer



Let AB be the tower and CD be the building.

We draw CE ⊥ AB.

According to the question,

CD = h = BE

Let AB =y

Then, AE = AB — BE =y— h

Let CE =x. Then DB =x

In right $\triangle$ ACE, we have

$$\tan\alpha = \frac{AE}{CE}$$

$$\Rightarrow \tan\alpha = \frac{y-h}{x}$$

$$\Rightarrow x = \frac{y-h}{\tan\alpha} \text{ ...(i)}$$

Also, In right $\triangle$ ABD,

$$\tan\beta = \frac{AB}{DB}$$

$$\Rightarrow \tan\beta = \frac{y}{x}$$

$$\Rightarrow x = \frac{y}{\tan\beta} \text{ ...(ii)}$$

From eq. (i) and (ii), we have

$$\frac{y-h}{\tan\alpha} = \frac{y}{\tan\beta}$$

$$\Rightarrow \frac{y}{\tan\alpha} - \frac{h}{\tan\alpha} = \frac{y}{\tan\beta}$$

$$\Rightarrow \frac{y}{\tan\alpha} - \frac{y}{\tan\beta} = \frac{h}{\tan\alpha}$$

$$\Rightarrow y\left[\frac{1}{\tan\alpha} - \frac{1}{\tan\beta}\right] = \frac{h}{\tan\alpha}$$

$$\Rightarrow y\left[\frac{\tan\beta - \tan\alpha}{\tan\alpha\tan\beta}\right] = \frac{h}{\tan\alpha}$$

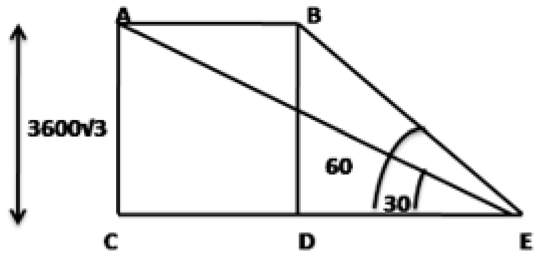
$$\Rightarrow y = \frac{h\tan\beta}{\tan\beta - \tan\alpha}$$

Hence, the height of the tower = $\frac{h\tan\beta}{\tan\beta-\tan\alpha}$

53. Question

The angle of elevation of an airplane from a point A on the ground is 60° after a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of 3600 $\sqrt{3}$ m, find the speed, in km/hour, of the plane.

Answer



Let us suppose that DE = x and CD = y

Now, In right ΔBED, we have

$$\tan 60^\circ = \frac{BD}{DE}$$

$$\Rightarrow \sqrt{3} = \frac{3600\sqrt{3}}{x}$$

⇒ x = 3600

In right ΔACE, we have

$$\tan 30^\circ = \frac{AC}{CE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3600\sqrt{3}}{CE}$$

⇒ CE = 10800

⇒ CD + DE = 10800

⇒ y + x = 10800

⇒ y + 3600 = 10800

⇒ y = 10800 – 3600

⇒ y = 7200

∵ AB = CD = 7200

We know that,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\Rightarrow \text{Speed} = \frac{7200}{30} = 240 \text{ m/s}$$

$$= 240 \times \frac{18}{5} = 864 \text{ km/hr}$$

Hence, the speed of the aeroplane is 864 km/hr

54. Question

An aeroplane left 30 minutes later than its scheduled time; and in order to reach its destination 1500 km away in time, it has to increase its speed by 250 km/hour from its usual speed. Determine its usual speed.

Answer

Let the usual speed of the plane = x km/hr

Total distance = 1500km

$$\therefore \text{Time taken} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Time taken at usual speed } = \frac{1500}{x} \text{ hr.}$$

Actual Speed of the plane = (x + 250) km/hr

$$\text{Time taken at actual speed } = \frac{1500}{(x+250)} \text{ hr}$$

$$\text{Difference between the two times taken } = \frac{1}{2} \text{ hr.}$$

$$\therefore \frac{1500}{x} - \frac{1500}{(x+250)} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{x} - \frac{1}{(x+250)} = \frac{1}{3000}$$

$$\Rightarrow \frac{x+250-x}{x(x+250)} = \frac{1}{3000}$$

$$\Rightarrow \frac{250}{x^2+250x} = \frac{1}{3000}$$

$\Rightarrow x^2 + 250x - 750000 = 0$

$\Rightarrow x^2 + 1000x - 750x - 750000 = 0$

$\Rightarrow x(x + 1000) - 750(x + 1000) = 0$

$\Rightarrow (x - 750)(x + 1000) = 0$

$\Rightarrow x + 1000 = 0$ or $x - 750 = 0$

$\Rightarrow x = -1000$ or $x = 750$

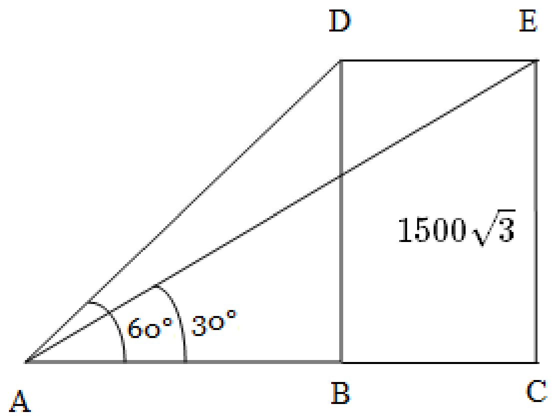
$\Rightarrow x = 750$ [∵ speed can't be negative]

Hence, the usual speed of the aeroplane was 750km/hr

55. Question

The angle of elevation of an aeroplane from a point on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the aeroplane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the aeroplane.

Answer



Let D and E be the initial and final positions of the plane respectively.

It is given that BD = $1500\sqrt{3}$ m

In right ΔABD, we have

$$\tan 60° = \frac{BD}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{1500\sqrt{3}}{AB}$$

$\Rightarrow AB = 1500$

In right ΔACE, we have

$$\tan 30° = \frac{CE}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{AC}$$

⇒ AC = 4500

Now, Distance = BC = AC – AB = 4500 – 1500 = 3000 m

DE = BC = 3000 m

i.e. the plane travels a distance of 3000m in 15 seconds

$$\therefore \text{, the speed of the plane} = \frac{\text{distance}}{\text{time}} = \frac{3000}{15} = 200\text{m/s}$$

$$= 200 \times \frac{18}{5} \text{ km/hr}$$

= 720km/hr

Hence, the speed of the aeroplane is 720km/hr.