Very Short Answer Questions (PYQ)

Q. 1. Define capacitor reactance. Write its S.I. units? [CBSE Delhi 2015]

Ans. The imaginary/virtual resistance offered by a capacitor to the flow of an alternating current is called capacitor reactance, $X_C = \frac{1}{\omega C}$. Its SI unit is ohm.

Q. 2. Explain why current flows through an ideal capacitor when it is connected to an ac source but not when it is connected to a dc source in a steady state. [CBSE (East) 2016]

Ans. For ac source, circuit is complete due to the presence of displacement current in the capacitor. For steady dc, there is no displacement current, therefore, circuit is not complete.

Mathematically, Capacitive reactance XC = $\frac{1}{2x} = \frac{1}{\omega\ C}$

So, capacitor allows easy path for ac source.

For dc, v = 0, so X_c = infinity,

So capacitor blocks dc.

Q. 3. Define 'quality factor' of resonance in series LCR circuit. What is its SI unit? [CBSE Delhi 2016]

Ans. The quality factor (Q) of series LCR circuit is defined as the ratio of the resonant frequency to frequency band width of the resonant curve.

$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r L}{R}$$

Clearly, smaller the value of R, larger is the quality factor and sharper the resonance. Thus quality factor determines the nature of sharpness of resonance.

It has no units.

Q. 4. In a series LCR circuit, $V_L = V_C \neq V_R$.

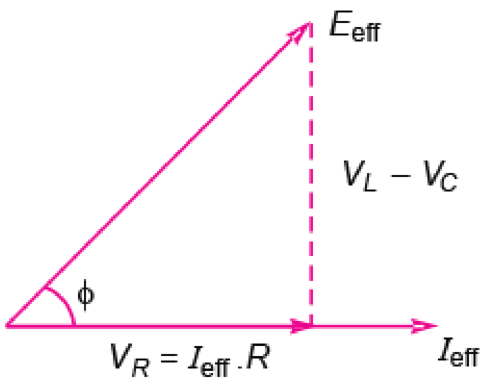
What is the value of power factor for this circuit? [CBSE Panchkula 2015]

Ans. Power factor,

$$\cos \varphi = \frac{V_R}{\sqrt{V_R^2 + (V_L - V_C)^2}}$$

$$\text{Since } V_L = V_C, \quad \cos \varphi = \frac{V_R}{V_R} = 1$$

The value of power factor is 1.



Q. 5. The power factor of an AC circuit is 0.5. What is the phase difference between voltage and current in this circuit? [CBSE (F) 2015, (South) 2016]

Ans. Power factor between voltage and current is given by φ, where φ is phase difference

$$\cos \varphi = 0.5 = \frac{1}{2} \Rightarrow \quad \varphi = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Q. 6. What is wattless current? [CBSE Delhi 2011, Chennai 2015]

Ans. When pure inductor and/or pure capacitor is connected to ac source, the current flows in the circuit, but with no power loss; the phase difference between voltage and current is $\frac{x}{2}$. Such a current is called the wattless current.

Q. 7. Mention the two characteristic properties of the material suitable for making core of a transformer. [CBSE (AI) 2012]

Ans. Two characteristic properties:

(i) Low hysteresis loss
(ii) Low coercivity

Q. 8. A light bulb and a solenoid are connected in series across an ac source of voltage. Explain, how the glow of the light bulb will be affected when an iron rod is inserted in the solenoid. [CBSE (F) 2017]

Ans. When iron rod is inserted in the coil, the inductance of coil increases; so impedance of circuit increases and hence, current in circuit $I=\frac{V}{\sqrt{R^2+(\omega L)^2}}$ decreases. Consequently, the glow of bulb decreases.

Q. 9. Why is the use of AC voltage preferred over DC voltage? Give two reasons. [CBSE (AI) 2014]

Ans. (i) The generation of AC is more economical than DC.

(ii) Alternating voltage can be stepped up or stepped down as per requirement during transmission from power generating station to the consumer.

(iii) Alternating current in a circuit can be controlled by using wattless devices like the choke coil.

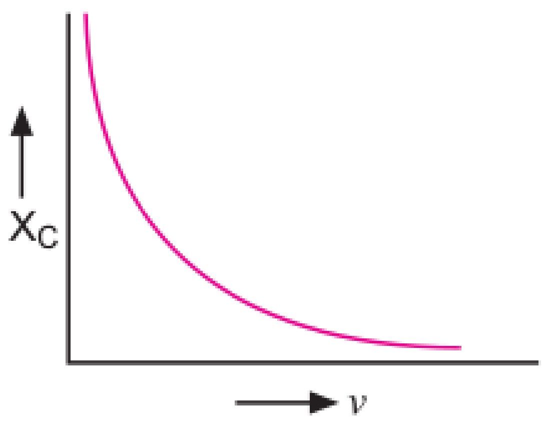
(iv) Alternating voltages can be transmitted from one place to another, with much lower energy loss in the transmission line.

Very Short Answer Questions (OIQ)

Q. 1. Sketch a graph showing variation of reactance of a capacitor with frequency in an AC circuit.

Ans. Capacitive reactance,

The graph of capacitive reactance X_C and frequency v is shown in figure.



Q. 2. What will be the effect on inductive reactance XL and capacitive reactance XC if frequency of ac source is increased?

Ans.

The inductive reactance $X_L = \omega L = 2\pi \nu L$ will increase with the increase of frequency ν, while capacitive reactance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$ will decrease with the increase of frequency ν.

Q. 3. State which of the two, the capacitor or an inductor, tends to become a SHORT when the frequency of the applied alternating voltage has a very high value.

Ans.

The capacitor $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$

When $\nu \to \infty$ (very high)

$X_C = 0$

Q. 4. What is the phase difference between voltage and current in a LCR series circuit at resonance?

Ans. At resonance voltage and current are in the same phase, i.e., phase difference between voltage and current at resonance is **zero.**

Q. 5. What is the average value of ac voltage

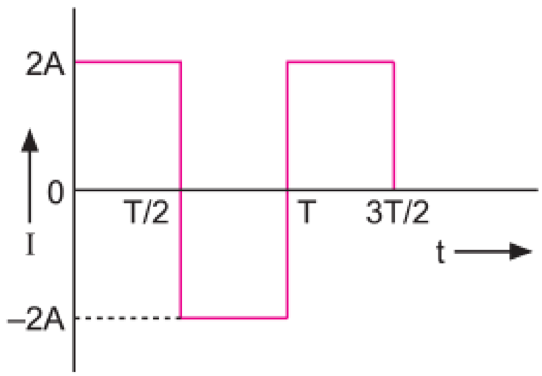
V= V0 sin ωt

Over the time interval t = 0 to t = $\frac{\pi}{\omega}$. **[HOTS]**

Ans.

$$V_{av} = \frac{\int_0^{\pi/\omega} V dt}{\int_0^{\pi/\omega} dt} = \frac{\int_0^{\pi/\omega} V_0 \sin \omega t \; dt}{[t]_0^{\pi/\omega}} = \frac{V_0\left\{-\frac{\cos \omega t}{\omega}\right\}_0^{\pi/\omega}}{\pi/\omega} = -\frac{V_0}{\pi}[\cos \pi - \cos 0]$$

Q. 6. What is the rms value of alternating current shown in figure? [HOTS]



Ans.

$$(I^2)_{\text{mean}} = \frac{\int_0^T I^2 \text{ dt}}{\int \text{ dt}} = \frac{\int_0^{T/2} (2)^2 \text{ dt} + \int_{T/2}^{T} (-2)^2 \text{ dt}}{T} = \frac{\int_0^T 4 \text{ dt}}{T} = 4$$

$$I_{\text{rms}} = \sqrt{4} = 2A$$

Short Answer Questions – I (PYQ)

Q. 1. Define power factor. State the conditions under which it is (i) maximum and (ii) minimum. [CBSE Delhi 2010]

Ans. The power factor (cos φ) is the ratio of resistance and impedance of an ac circuit i.e.,

$$\text{Power factor, } \cos\varphi = \frac{R}{Z}$$

Maximum Power factor is 1 when Z = R i.e., when circuit is purely resistive. Minimum power factor is 0 when R = 0 i.e., when circuit is purely inductive or capacitive.

Q. 2. When an AC source is connected to an ideal inductor show that the average power supplied by the source over a complete cycle is zero. [CBSE (Central) 2016]

Ans. For an ideal inductor phase difference between current and applied voltage = π/2

$$\therefore \quad \text{Power, } P = V_{rms}\, I_{rms} \cos\varphi = V_{rms}\, I_{rms} \cos\frac{\pi}{2} = 0.$$

Thus the power consumed in a pure inductor is **zero**.

Q. 3. When an AC source is connected to an ideal capacitor, show that the average power supplied by the source over a complete cycle is zero. [CBSE (North) 2016]

Ans.

Power dissipated in ac circuit, $P = V_{rms}\, I_{rms} \cos\varphi$ where $\cos\varphi = \frac{R}{Z}$

For an ideal capacitor $R = 0 \quad \therefore \cos\varphi = \frac{R}{Z} = 0$

$$\therefore P = V_{rms}\, I_{rms} \cos\varphi = V_{rms}\, I_{rms} \times 0 = 0 \text{ (zero)}.$$

i.e., power dissipated in an ideal capacitor is zero.

Q. 4. The current flowing through a pure inductor of inductance 2 mH is i=15 cos 300t ampere. What is the (i) rms and (ii) average value of current for a complete cycle?
[CBSE (F) 2011]

Ans.

Peak value of current (i_0) = 15 A

i. $i_{\text{rms}} = \frac{i_0}{\sqrt{2}} = \frac{15}{\sqrt{2}} = \frac{15}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = 7.5\sqrt{2}\,A$

ii. $i_{av} = 0$

Q. 5. In a series LCR circuit with an ac source of effective voltage 50 V, frequency v =50/π Hz, R = 300Ω, C = 20 μF and L = 1.0 H. Find the rms current in the circuit. [CBSE (F) 2014]

Ans.

Given, L = 1.0 H; C = 20μF = 20 × 10^{-6}F

$$R = 300\Omega; V_{\text{rms}} = 50V; \nu = \frac{50}{\pi}\text{Hz}$$

Inductive reactance $X_L = \omega L = 2\pi\nu\, L = 2 \times \pi \times \frac{50}{\pi} \times 1 = 100\ \Omega$

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2\times\pi\times\frac{50}{\pi}\times 20\times 10^{-6}} = 500\Omega$

Impedance of circuit

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$= \sqrt{(300)^2 + (500 - 100)^2} = \sqrt{90000 + 160000} = \sqrt{250000} = 500\Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{z} = \frac{50}{500} = 0.1A$$

Q. 6. Calculate the quality factor of a series LCR circuit with L = 2.0 H, C= 2 μF and R = 10Ω. Mention the significance of quality factor in LCR circuit.
[CBSE (F) 2012]

Ans.

We have, $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$= \frac{1}{10}\sqrt{\frac{2}{2\times 10}} = 100$

It signifies the sharpness of resonance.

Short Answer Questions – I (OIQ)

Q. 1. The instantaneous current in an ac circuit is i = 0.5 sin 314 t, what is (i) rms value and (ii) frequency of the current.

Ans.

Given $I = 0.5 \sin 314\, t$ ... (*i*)

Standard equation of current is $I = I_0 \sin \omega t$ ... (*ii*)

Comparing (*i*) and (*ii*), we get $I_0 = 0.5$ A, $\omega = 314$

∴ (*i*) rms value $I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{0.5}{\sqrt{2}} A = 0.35\, A$

(*ii*) Frequency $\nu = \frac{\omega}{2\pi} = \frac{314}{2\times 3.14} = 50\ \text{Hz}$

Q. 2. The instantaneous voltage from an ac source is given by E = 300 sin 314t; what is the rms voltage of the source.

Ans. Given equation is E = 300 sin 314 t

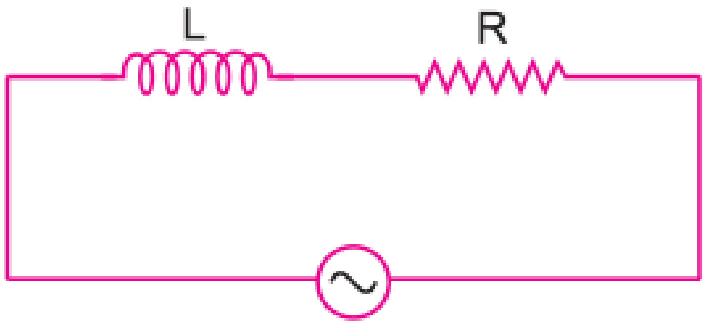
Comparing with standard equation $E = E_0 \sin \omega t$, we have

$E_0 = 300$ volt

So $E_{\text{rms}} = \frac{E_0}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 150\sqrt{2}\,\text{volt} = 150 \times 1.414 = 212\, V$

Q. 3. In the given circuit, the potential difference across the inductor L and resistor R are 200 V and 150 V respectively and the rms. value of current is 5 A. Calculate (i) the impedance of the circuit and (ii) the phase angle between the voltage and the current.

Ans.



Voltage applied $V = \sqrt{V_L^2 + V_R^2} = \sqrt{(200)^2 + (150)^2} = 250V$

Impedance of circuit, $Z = \frac{V}{I} = \frac{250}{5} = 50\ \Omega$

Phase angle between voltage and current

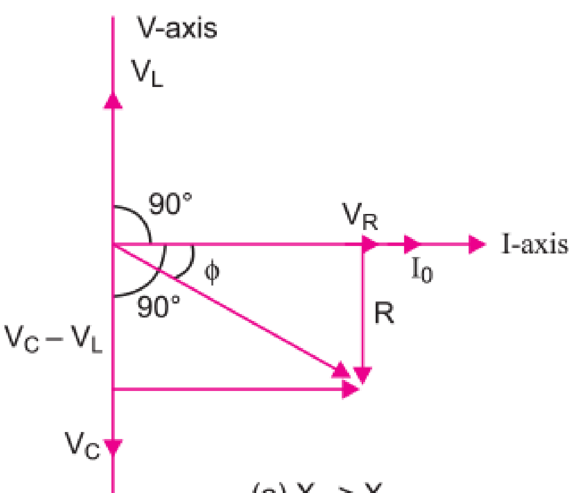
$$\tan\varphi = \frac{X_L}{R} = \frac{V_L}{V_R} = \frac{200}{150} = \frac{4}{3}$$

$$\varphi = \tan^{-1}\left(\frac{4}{3}\right) = 53^o$$

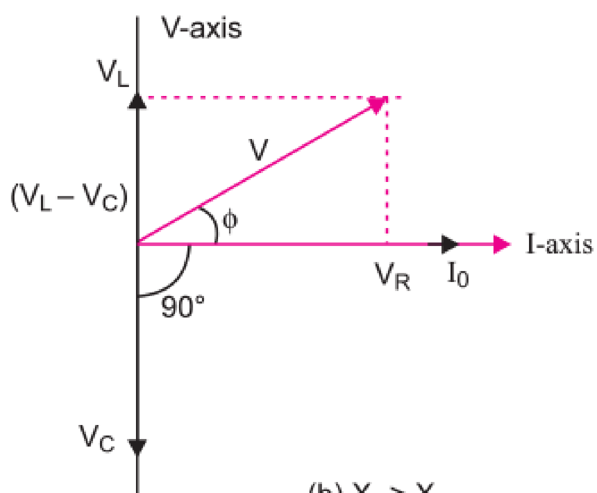
Q. 4. An AC source of voltage V = Vm sin ωt is applied across a series LCR circuit. Draw the phasor diagrams for this circuit, when

(i) Capacitive impedance exceeds the inductive impedance.
(ii) Inductive impedance exceeds capacitive impedance.

Ans. (i) When $X_C > X_L$; the phasor diagram is shown in fig. (a).



(a) $X_C > X_L$



(b) $X_L > X_C$

(ii) When $X_L > X_C$; the phasor diagram is shown in fig. (b).

Q. 5. In a series LCR circuit, R=1k Ω, C = 2μ F and voltage across R is 100 V. The resonant frequency of the circuit ω is 200 rad s^{-1}. Calculate the value of voltage across L at resonance.

Ans.

Current flowing in the circuit is

$$I = \frac{V_R}{R} = \frac{100}{1000} = 0.1\ A$$

$\therefore$ Also at resonance, $\omega L = \frac{1}{\omega C}$

$$\omega L = \frac{1}{200 \times 2 \times 10^{-6}} = 2500\ \Omega$$

$\therefore$ Voltage across, $L = I\,(\omega L) = 0.1 \times 2500 = \mathbf{250V}$

Q. 6. What is the power dissipated in an ac circuit in which voltage and current are given by $V = 230 \sin\left(\omega t + \frac{\pi}{2}\right)$ and $I = 10 \sin \omega t$?

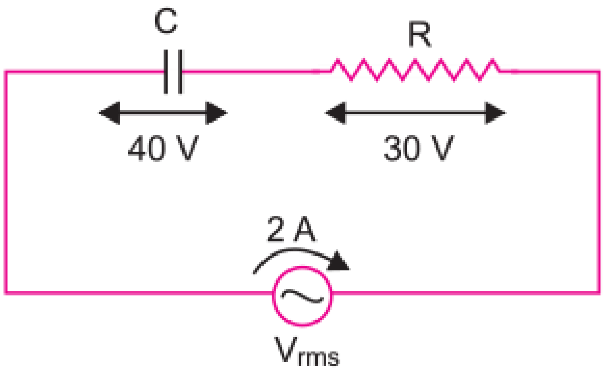
Ans.

Power dissipated $P = \frac{1}{2} V_0\ I_0 \cos\varphi$

Here $V_0 = 230V,\ I_0 = 10\ A,\ \varphi = \frac{\pi}{2}$

$\therefore \qquad P = \frac{1}{2} \times 230 \times 10 \cos\frac{\pi}{2} = 0.$

Q. 7. Calculate the following:



(i) Impedance of the given ac circuit.
(ii) Wattless current of the given ac circuit.

Ans. (i)

Potential difference across capacitance, $V_C = X_C I$

∴ Capacitive reactance,

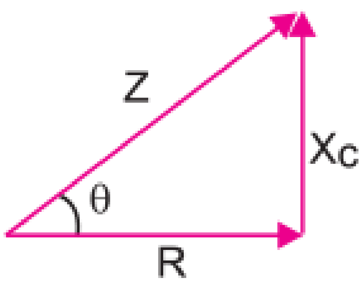
$$X_C = \frac{V_C}{I} = \frac{40}{2} = 20\Omega$$

Resistance, $R = \frac{V_R}{I} = \frac{30}{2} = 15\Omega$

Impedance, $Z = \sqrt{R^2 + X_C^2} = \sqrt{(15)^2 + (20)^2}$

$$= \sqrt{225 + 400} = \sqrt{625}\Omega = 25\ \Omega$$

(ii)



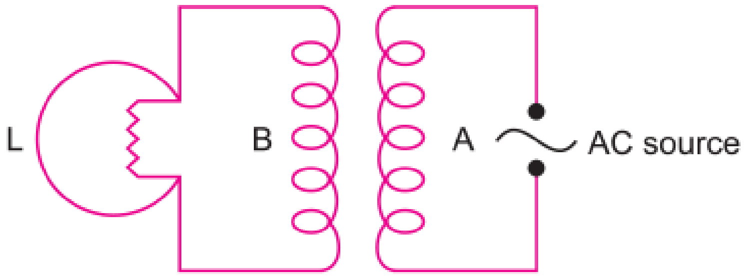
The phase lead (φ) of current over applied voltage is

$$\varphi = \frac{X_C}{R}$$

Wattless Current, $I_{\text{wattless}} = I \sin\ \varphi = I.\left(\frac{X_C}{Z}\right)$

$$= 2 \times \frac{20}{25} A = 1.6\ A$$

Q. 8. In the given diagram, a coil B is connected to low voltage bulb L and placed parallel to another coil A as shown. Explain the following observations:



(i) Bulb lights and
(ii) Bulb gets dimmer if the coil B moves upwards

Ans. (i) Bulb lights up due to induced current in B because of change in flux linked with it as a consequence of continuous variation of magnitude of alternative current flowing in A.

(ii) When coil B moves upward, the magnetic flux linked with B decreases and hence lesser current is induced in B.

Q. 9. In a series LCR circuit, the voltage across an inductor, a capacitor and a resistor are 30 V, 30 V and 60 V respectively. What is the phase difference between the applied voltage and current in the circuit?

Ans.

Given $V_L = 30\text{V},\ V_C = 30\text{V},\ V_R = 60\text{V}$

Phase difference (φ) in series LCR circuit is given by

$$\tan\varphi = \frac{X_C - X_L}{R} = \frac{V_C - V_L}{V_R} = \frac{30-30}{60} = 0$$

$$\Rightarrow \quad \varphi = 0 \ (\text{zero})$$

Q. 10. When a capacitor is connected in series LR circuit, the alternating current flowing in the circuit increases. Explain why.

Ans. Impedance of series LR circuit

$$Z_1 = \sqrt{R^2 + X_L^2}$$

When capacitor is also connected in circuit,

Then impedance

$$Z_L = \sqrt{R^2 + (X_L - X_C)^2}$$

Clearly impedance of circuit decreases ($Z_2 < Z_1$), so the value of current I = $\frac{V}{Z} \propto \frac{1}{Z}$ in the circuit increases.

Q. 11. In India, Domestic power supply is at 220 V, 50 Hz; while in USA it is 110 V, 50 Hz. Give one advantage and one disadvantage of 220 V supply over 110 V supply.

Ans. Advantage: Line loss is low.

Disadvantage: High voltage is dangerous.

Q. 12. Distinguish between the terms 'effective value' and peak value of alternating current.

Ans. Alternating current changes in magnitude as well as direction. The maximum value of the alternating current is called the peak value. It is denoted by I_0. The square root of mean square value of current is called the 'effective value' or 'rms value' of current. The two are related by

$$\text{Effective value, } E_{\text{eff}} = \frac{I_0}{\sqrt{2}}$$

Q. 13. What is the power dissipated by an ideal inductor in ac circuit? Explain.

Ans. The power P = $V_{rms} I_{rms} \cos \varphi$

An ideal inductor is the one whose resistive component is zero.

$$\text{where } \cos\varphi = \frac{R}{Z}; \qquad \text{For ideal inductor } R = 0,$$

$$\therefore \cos\varphi = \frac{R}{Z} = 0$$

∴ P = $V_{rms} I_{rms} \cos \varphi$ =0, i.e., power dissipated by an ideal inductor in ac circuit is zero.

Q. 14. Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current? [NCERT Exemplar]

Ans. An ac current changes direction with the source frequency and the attractive force would average to zero. Thus, the ac ampere must be defined in terms of some property that is independent of the direction of current. Joule's heating effect is such property and hence it is used to define rms value of ac.

Q. 15. A 60W load is connected to the secondary of a transformer whose primary draws line voltage. If a current of 0.54 A flows in the load, what is the current in

the primary coil? Comment on the type of transformer being used. [NCERT Exemplar]

Ans.

Here $P_L = 60$ W, $I_L = 0.54$ A

$$V_L = \frac{60}{0.54} = 110\ V$$

The transformer is step-down and have $\frac{1}{2}$ input voltage. Hence

$$i_p = \frac{1}{2} \times I_L = \frac{1}{2} \times 0.54 = 0.27\ A.$$

Q. 16. Explain why the reactance provided by a capacitor to an alternating current decreases with increasing frequency. [NCERT Exemplar]

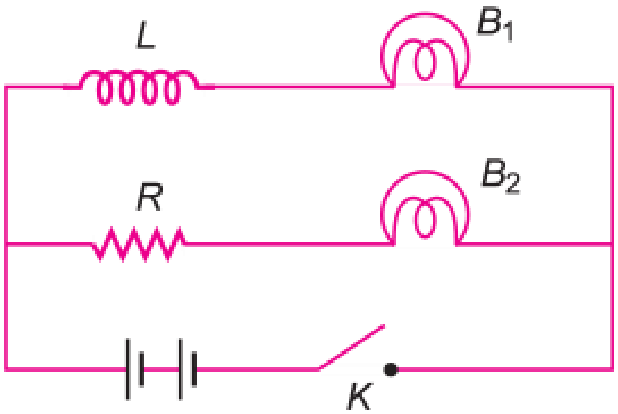
Ans. A capacitor does not allow flow of direct current through it as the resistance across the gap is infinite. When an alternating voltage is applied across the capacitor plates, the plates are alternately charged and discharged. The current through the capacitor is a result of this changing voltage (or charge). Thus, a capacitor will pass more current through it if the voltage is changing at a faster rate, i.e., if the frequency of supply is higher. This implies that the reactance offered by a capacitor is less with increasing frequency; it is given by $1/\omega C$.

Q. 17. Explain why the reactance offered by an inductor increases with increasing frequency of an alternating voltage. [NCERT Exemplar]

Ans. An inductor opposes flow of current through it by developing an induced emf according to Lenz's law. The induced voltage has a polarity so as to maintain the current at its present value. If the current is decreasing, the polarity of the induced emf will be so as to increase the current and vice versa. Since the induced emf is proportional to the rate of change of current, it will provide greater reactance to the flow of current if the rate of change is faster, i.e., if the frequency is higher. The reactance of an inductor, therefore, is proportional to the frequency, being given by ωL.

Q. 18. In the given circuit, the value of resistance effect of the coil L is exactly equal to the resistance R. Bulbs B_1 and B_2 are exactly identical.

Answer the following questions based on above information:



(i) Which one of the two bulbs lights up earlier, when key K is closed and why?
(ii) What will be the comparative brightness of the two bulbs after sometime if the key K is kept closed and why?

Ans. (i) Bulb B_2 lights up earlier. The self-induction effect due to coil L in bulb B1 arm does not allow the current to attain maximum value immediately on closing the circuit.

(ii) Since the resistance effect of the coil L is equal to R and the self-induction effect in coil L will disappear after sometime, the current in both the arms will be equal. Hence, both the bulbs will glow with equal brightness after sometime.

Short Answer Questions – II (PYQ)

Q. 1. Show that the current leads the voltage in phase by π/2 in an ac circuit containing an ideal capacitor. [CBSE (F) 2014]

Ans. The instantaneous voltage,

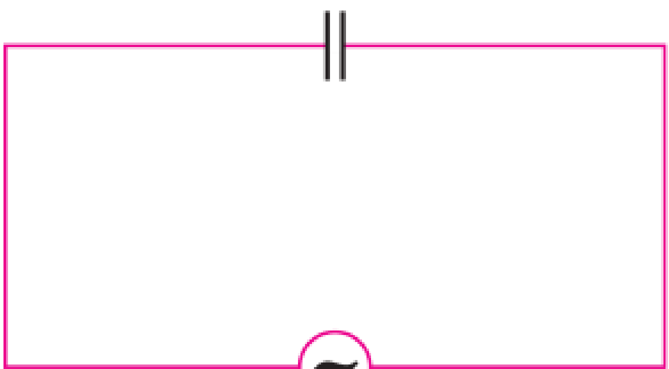
$V = V_0 \sin \omega t$ … (i)

Let q be the charge on capacitor and I, the current in the circuit at any instant, then instantaneous potential difference,

$$V = \frac{q}{c} \qquad \ldots (ii)$$

From (i) and (ii)

$$\frac{q}{C} = V_0 \sin \omega t \quad \Rightarrow \quad q = CV_0 \sin \omega t$$



$V = V_0$ Sin ωt

The instantaneous current,

$$I = \frac{dq}{dt} = (CV_0 \sin \omega t) = CV_0 \frac{d}{dt}(\sin \omega t) = CV_0 \omega \cos \omega t$$

$$I = \frac{V_0}{1/\omega C} \cos \omega t$$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Hence, the current leads the applied voltage in phase by π/2

Q. 2. In a series LCR circuit, obtain the conditions under which [CBSE (F) 2014]

(i) The impedance of the circuit is minimum, and
(ii) Wattless current flows in the circuit.

Ans. (i) Impedance of series LCR circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

For the impedance, Z to be minimum

$$X_L = X_C$$

(ii)

Power $P = V_{rms} I_{rms} \cos \varphi$

When $\varphi = \frac{\pi}{2}$

Power $= V_{\text{rms}} I_{\text{rms}} \cos \frac{\pi}{2} = 0$

Therefore, wattless current flows when the impedance of the circuit is purely inductive or purely capacitive.

In another way we can say,

For wattless current to flow, circuit should not have any ohmic resistance (R= 0)

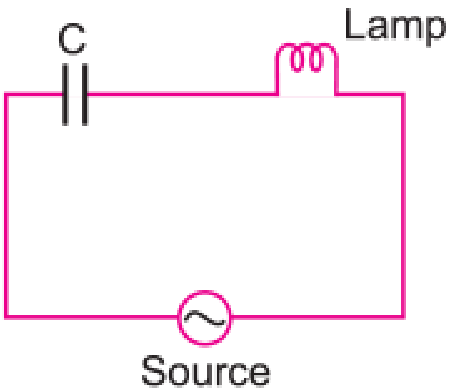
Q. 3. State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distances done with the use of transformers?
[CBSE (AI) 2012]

Ans. The principle of transformer is based upon the principle of mutual induction which states that due to continuous change in the current in the primary coil an emf gets induced across the secondary coil. At the power generating station, the step up transformers step up the output voltage which reduces the current through the cables and hence reduce resistive power loss. Then, at the consumer end, a step down transformer steps down the voltage.

Hence, the large scale transmission of electric energy over long distances is done by stepping up the voltage at the generating station to minimise the power loss in the transmission cables.

Q. 4. An electric lamp connected in series with a capacitor and an ac source is glowing with of certain brightness. How does the brightness of the lamp change on reducing the

(i) Capacitance and



(ii) Frequency? [CBSE Delhi 2010, (North) 2016]

Ans. (i)

When capacitance is reduced, capacitive reactance $X_C = \frac{1}{\omega C}$ increases, hence impedance of circuit

$$Z = \sqrt{R^2 + X_C^2}$$

increases and so current $I = \frac{V}{Z}$ decreases. As a result the **brightness of the bulb is reduced.**

(ii) When frequency decreases; capacitive reactance $X_c = \frac{1}{2\pi v c}$ increases and hence impedance of circuit increases, so current decreases. As a result brightness of bulb is reduced.

Q. 5. State the principle of working of a transformer. Can a transformer be used to step up or step down a d.c. voltage? Justify your answer. [CBSE (AI) 2011]

Ans. Working of a transformer is based on the principle of mutual induction. Transformer cannot step up or step down a dc voltage.

Reason: No change in magnetic flux.

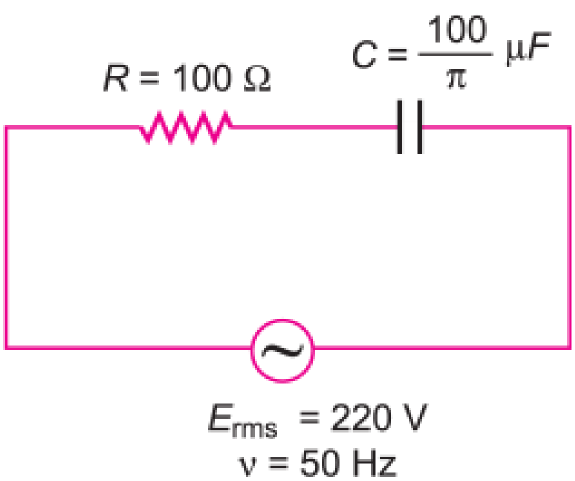
Explanation: When dc voltage source is applied across a primary coil of a transformer, the current in primary coil remains same, so there is no change in magnetic flux associated with it and hence no voltage is induced across the secondary coil.

Q. 6. A resistor of 100 Ω and a capacitor of 100/π μF are connected in series to a 220 V, 50 Hz a.c. supply.

(i) Calculate the current in the circuit.

(ii) Calculate the (rms) voltage across the resistor and the capacitor. Do you find the algebraic sum of these voltages more than the source voltage? If yes, how do you resolve the paradox? [CBSE Chennai 2015]

Ans. (i)



$$\text{Capacitive reactance } X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

$$= \frac{1}{2\pi \times 50 \times \frac{100}{\pi}\mu F} = 100\Omega$$

$$\text{Impedance of the circuit, } Z = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}$$

$$\text{Current in the circuit } I_{rms} = \frac{E_{rms}}{Z} = \frac{220}{100\sqrt{2}} = 1.55\ A$$

(ii) Voltage across resistor, VR = I_{rms} R

=1.55 × 100 = 155V

Voltage across capacitor, VC = I_{rms} × C =1.55 × 100V = 155 V

The algebraic sum of voltages across the combination is

$V_{rms} = V_R + V_C$ $=155V + 155V = 310V$

While Vrms of the source is 220 V. Yes, the voltages across the combination is more than the voltage of the source. The voltage across the resistor and capacitor are not in phase.

This paradox can be resolved as when the current passes through the capacitor, it leads the voltage V_C by phase $\frac{\pi}{2}$ So, voltage of the source can be given as

$$V_{\text{rms}} = \sqrt{V_R^2 + V_C^2}$$

$$= \sqrt{(155)^2 + (155)^2}$$

$$= 155\sqrt{2} = 220\ V$$

Q. 7. A capacitor of unknown capacitance, a resistor of 100 Ω and an inductor of self-inductance L = ($\frac{4}{\pi^2}$) henry are connected in series to an ac source of 200 V and 50 Hz. Calculate the value of the capacitance and impedance of the circuit when the current is in phase with the voltage. Calculate the power dissipated in the circuit.
[CBSE South 2016]

Ans.

$$\text{Capacitance,}\ \ C = \frac{1}{L\omega^2}$$

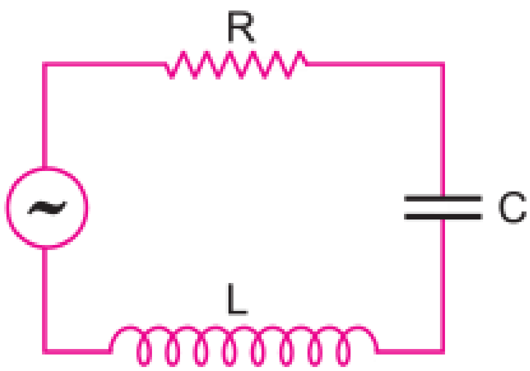
$$= \frac{1}{\frac{4}{\pi^2}(2\pi\times 50)^2}F = \frac{1}{40000}F = 2.5\ \times\ 10^{-5}F$$

Since V and I are in same phase

Impedance = Resistance = 100Ω

$$\text{Power dissipated}\ = \frac{E_{\text{rms}}^2}{2} = \frac{(200)^2}{100}W = 400W$$

Q. 8. The figure shows a series LCR circuit with L = 5.0 H, C = 80 μF, R = 40 Ω connected to a variable frequency 240V source. Calculate.



(i) The angular frequency of the source which drives the circuit at resonance.
(ii) The current at the resonating frequency.
(iii) The rms potential drop across the capacitor at resonance. [CBSE Delhi 2012]

Ans. (i) We know

$$\omega_r = \text{Angular frequency at resonance} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\times 80\times 10^{-6}}} = 50 \text{ rad}/s$$

(ii)

$$\text{Current at resonance, } I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{240}{40} = 6 \ A$$

(iii) V_{rms} across capacitor

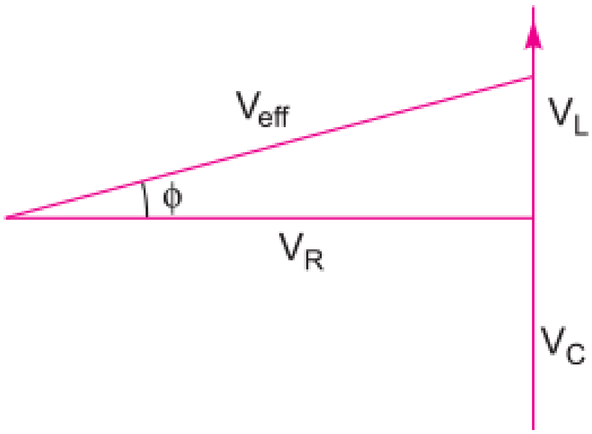
$$I_{\text{rms}} = I_{\text{rms}} X_C = 6 \times \frac{1}{50\times 80\times 10^{-6}} = \frac{6\times 10^6}{4\times 10^3} = 1500 \ V$$

Q. 9. Given the value of the resistance of R is 40Ω, calculate the current in the circuit.
[CBSE (F) 2013]

Ans. Given R = 40Ω, so current in the LCR circuit.

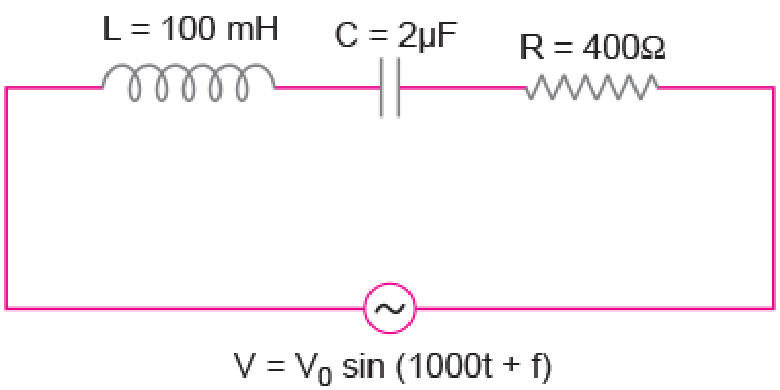
$$I_{\text{eff}} = \frac{V_{\text{eff}}}{R} \qquad [X_L = X_C \quad \text{or} \quad Z = R]$$

$$= \frac{200}{40} = 5A$$



Q. 10. (i) Find the value of the phase difference between the current and the voltage in the series LCR circuit shown below. Which one leads in phase: current or voltage?

(ii) Without making any other change, find the value of the additional capacitor, C_1, to be connected in parallel with the capacitor C, in order to make the power factor of the circuit unity. [CBSE Delhi 2017]



Ans.

i. Inductive reactance,

$X_L = \omega L = (1000 \times 100 \times 10^{-3})\Omega = 100\Omega$

Capacitive reactance,

$X_C = \frac{1}{\omega C} = \left(\frac{1}{1000 \times 2 \times 10^{-6}}\right)\Omega = 500\Omega$

Phase angle,

$\tan\varphi = \frac{X_L - X_C}{R}$

$\tan\varphi = \frac{100 - 500}{400} = -1$

$\varphi = -\frac{\pi}{4}$ As $X_C > X_L$ (phase angle is negative), hence current leads voltage.

ii. To make power factor unity

$X_C = X_L$ (where C = net capacitance of parallel combination)

$\frac{1}{\omega C'} = 100$

$C' = 10 \times 10^{-6}$ F

$\therefore$ $C' = 10\mu F$

$\because$ $C' = C + C_1$

$\Rightarrow$ $10 = 2 + C_1$ $\Rightarrow C_1 = 8\mu F$

Q. 11. Answer the following question :

(i) For a given ac, $i = i_m \sin \omega t$, show that the average power dissipated in a resistor R over a complete cycle is $\frac{1}{2} i_m^2 R.$

(ii) A light bulb is rated at 100 W for a 220 V ac supply. Calculate the resistance of the bulb. [CBSE (AI) 2013]

Ans. (i) Average power consumed in resistor R over a complete cycle

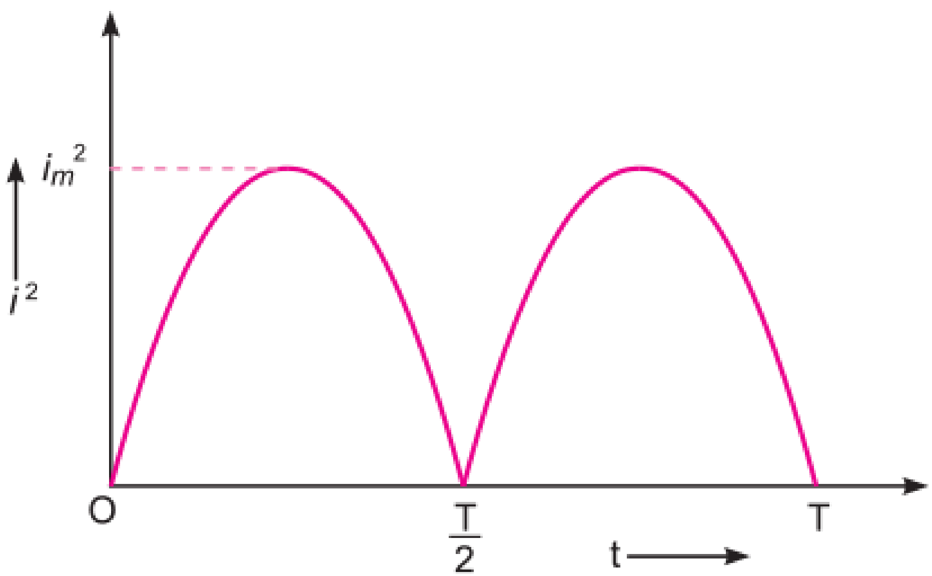
$$P_{av} = \frac{1}{\int_0^T dt} \cdot \int_0^T i^2 R \ dt$$

$$= \frac{i_m^2 R}{T} \int_0^T \sin^2 \omega t \ dt \quad \text{...}(i)$$

$$= \frac{i_m^2 R}{2T} \int_0^T (1 - \cos 2\omega t) \ dt$$

$$= \frac{i_m^2 R}{2T} \left[\int_0^T dt - \int_0^T \cos 2\omega t \ dt\right] \quad \text{...}(ii)$$

$$= \frac{i_m^2 R}{2T} [T - 0] = \frac{i_m^2 R}{2}$$



(ii)

In case of ac

$$P_{av} = \frac{V_{rms}^2}{R} = \frac{V_{eff}^2}{R}$$

$$R = \frac{V_{rms}^2}{P} = \frac{220 \times 220}{100} = 484\,\Omega$$

Q. 12. Determine the current quality factor at resonance for a series LCR circuit with L = 1.00 mH, C= 1.00 nF and R=100Ω connected to an ac source having peak voltage of 100 V. [CBSE (F) 2011]

Ans.

$I_V = ?, \; Q = ?$

$L = 1.00 \text{ mH} = 1 \times 10^{-3} \text{ H}, \; C = 1.00 \text{ nF} = 1 \times 10^{-9} \text{ F}, \; R = 100\Omega, \; E_0 = 100 \text{ V}$

$$I_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{E_0}{Z} \qquad \left\{ \begin{array}{l} \text{at resonance } \omega L = \frac{1}{\omega c} \\ \text{Hence } Z = R \end{array} \right\}$$

$$\therefore \qquad I = \frac{V}{R} = \frac{100}{100}$$

$$I_\nu = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{2} = \frac{1.44}{2} = 0.707 \; A \qquad [\therefore I_0 = 1\text{A}]$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{100}\sqrt{\frac{1.0\times10^{-3}}{1.0\times10^{-9}}} = \frac{1}{100} \times 10^3 = 10$$

Q. 13. A circuit is set up by connecting inductance L = 100 mH, resistor R = 100 Ω and a capacitor of reactance 200 Ω in series. An alternating emf of 150 $\sqrt{2}$ V, 500/π Hz is applied across this series combination. Calculate the power dissipated in the resistor.
[CBSE (F) 2014]

Ans.

Here, $L = 100 \times 10^{-3}$ H, $R = 100\ \Omega$, $X_C = 200\ \Omega$, $V_{rms} = 150\sqrt{2}$ **V**

$\nu = \frac{500}{\pi}$ Hz .

Inductive reactance $X_L = \omega L = 2\pi\nu L$

$= 2\pi\frac{500}{\pi} \times 100 \times 10^{-3} = 100.$

Impedance of circuit

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

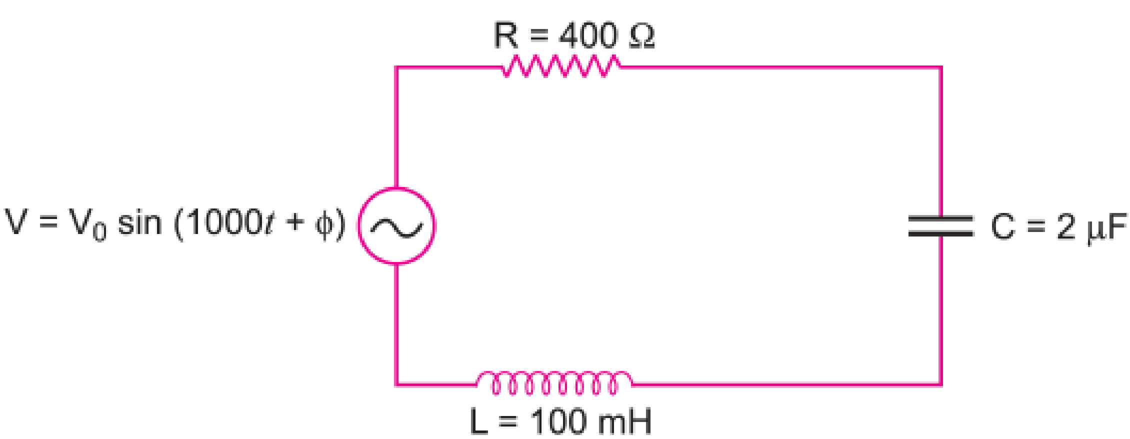
$$= \sqrt{(100)^2 + (200-100)^2} = \sqrt{20000} = 100\sqrt{2}$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{150\sqrt{2}}{100\sqrt{2}} = \frac{3}{2}$$

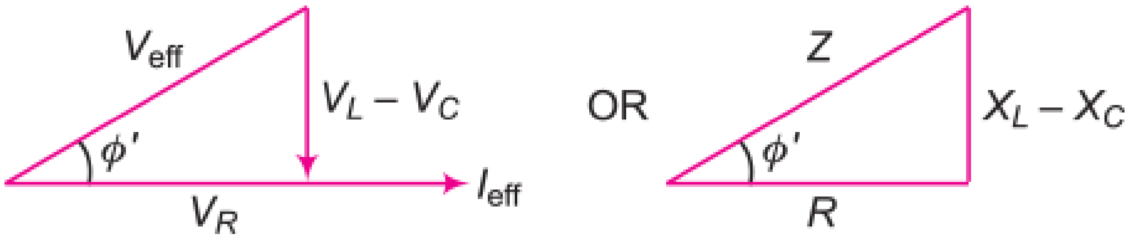
Power dissipated $(I_{rms})^2\ R = \frac{9}{4} \times 100 = 225W$

Q. 14. (a) Determine the value of phase difference between the current and the voltage in the given series LCR circuit.

(b) Calculate the value of the additional capacitor which may be joined suitably to the capacitor C that would make the power factor of the circuit unity.
[CBSE Allahabad 2015]



Ans. (a) From phasor diagrams,



Phase angle, $\tan\varphi = \frac{X_L - X_C}{R}$

$X_L = \omega L = 1000 \times 100 \times 10^{-3} = 100$ ohm

$X_C = \frac{1}{\omega C} = \frac{1}{1000 \times 2 \times 10^{-6}} = \frac{1000}{2} = 500$ ohm

$\therefore \quad \tan\varphi' = \frac{500 - 100}{400} = 1 \quad \Rightarrow \quad \varphi' = 45^o$

The phase angle between the current and applied voltage is

$45^o \left(= \frac{\pi}{4}\right)$

b. If power factor of the circuit is unity. It means the series LCR would be in resonance. It is possible, if another capacitor C is used in the circuit.

So, $\quad X_{C'} = X_L \quad \Rightarrow \frac{1}{\omega C'} = \omega L$

$\Rightarrow \quad C' = \frac{1}{\omega^2 L} = \frac{1}{(1000)^2 \times 100 \times 10^{-3}} = 10^{-5} F \quad \Rightarrow C' = 10\ \mu F$

Since $C' > C$, so an additional capacitor of 8 μF would be connected in parallel to the capacitor of $C = 2$ μF

Q. 15. The primary coil of an ideal step up transformer has 100 turns and transformation ratio is also 100. The input voltage and power are 220 V and 1100 W respectively. Calculate

(a) The number of turns in the secondary coil.

(b) The current in the primary coil.

(c) The voltage across the secondary coil.

(d) The current in the secondary coil.

(e) The power in the secondary coil. [CBSE Delhi 2016]

Ans.

a. Transformation ratio $r = \frac{\text{Number of turns in secondary coil } (N_s)}{\text{Number of turns in primary coil } (N_P)}$

Given $N_P = 100$, $r = 100$

$\therefore$ Number of turns in secondary coil, $N_S = rN_P = 100 \times 100 = 10{,}000$

b. Input voltage $V_P = 220$ V, Input power $P_{in} = 1100$ W

Current in primary coil $I_p = \frac{P_{in}}{V_P} = \frac{1100}{220} = 5A$

c. Voltage across secondary coil (V_S) is given by

$r = \frac{V_s}{V_P}$

$\Rightarrow V_S = rV_p = 100 \times 220 = 22{,}000$ V = **22kV**

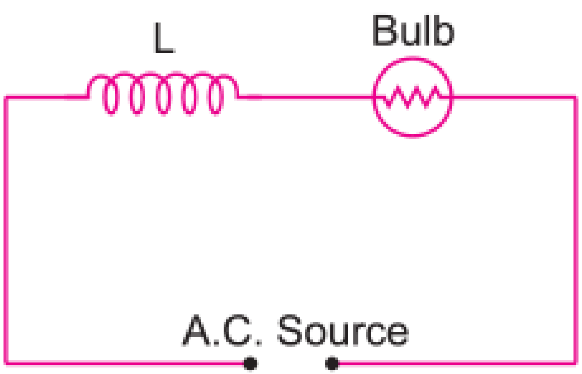
d. Current in secondary coil is given by

$r = \frac{I_P}{I_S} \quad \Rightarrow \quad I_S = \frac{I_P}{r} = \frac{5}{100} = 0.05\,A$

e. Power in secondary coil, $P_{out} = V_S\, I_S = 22 \times 10^3 \times 0.05 =$ **1100 W**

Obviously power in secondary coil is same as power in primary. This means that the transformer is ideal, *i.e.* there are no energy losses.

Q. 16. An inductor L of reactance X_L is connected in series with a bulb B to an ac source as shown in figure. Explain briefly how does the brightness of the bulb change when (i) number of turns of the inductor is reduced (ii) an iron rod is inserted in the inductor and (iii) a capacitor of reactance $X_C = X_L$ is included in the circuit.
[CBSE Delhi 2014, 2015]



Ans. Brightness of the bulb depends on square of the Irms (i.e., I^2_{rms})

Impedance of the circuit, $Z = \sqrt{R^2 + (\omega L)^2}$

and Current in the circuit, $I = \frac{V}{Z}$

(i) When the number of turns in the inductor is reduced, the self-inductance of the coil decreases; so impedance of circuit reduces and so current in the circuit $\left(I = \frac{E}{Z}\right)$ increases. Thus the brightness of the bulb increases.

(ii) When iron (being a ferromagnetic substance) rod is inserted in the coil, its inductance increases and in turn, impedance of the circuit increases. As a result, a larger fraction of the applied ac voltage appears across the inductor, leaving less voltage across the bulb. Hence, brightness of the bulb decreases.

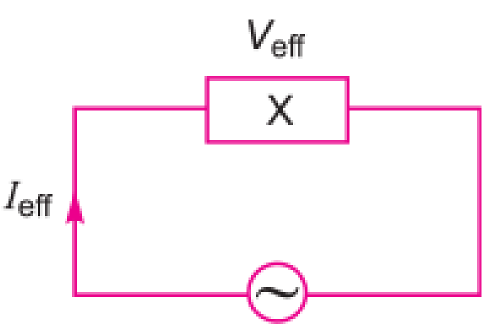
(iii) When capacitor of reactance $X_C = X_L$ is introduced, the net reactance of circuit becomes zero, so impedance of circuit decreases; it becomes Z = R; so current in circuit increases; hence brightness of bulb increases. Thus brightness of bulb in both cases **increases.**

Q. 17. You are given three circuit elements X, Y and Z. When the element X is connected across an a.c. source of a given voltage, the current and the voltage are in the same phase. When the element Y is connected in series with X across the source, voltage is ahead of the current in phase by π/4. But the current is ahead of the voltage in phase by π/4 when Z is connected in series with X across the source. Identify the circuit elements X, Y and Z.

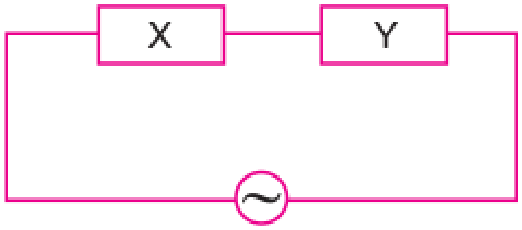
When all the three elements are connected in series across the same source, determine the impedance of the circuit.

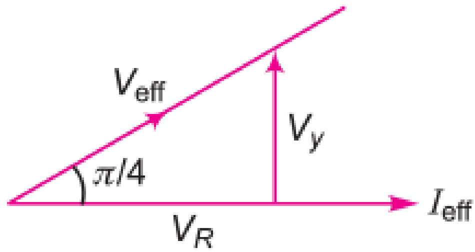
Draw a plot of the current versus the frequency of applied source and mention the significance of this plot. [CBSE Panchkula 2015]

Ans.

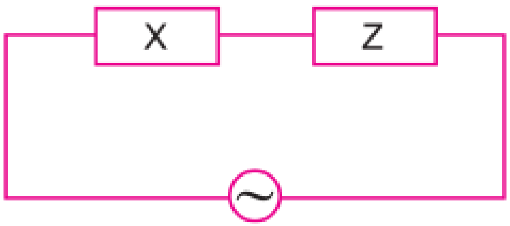


Since the phase angle between the current. I_{eff} and voltage drop V_{eff} is zero. Hence, the element X is a resistor.

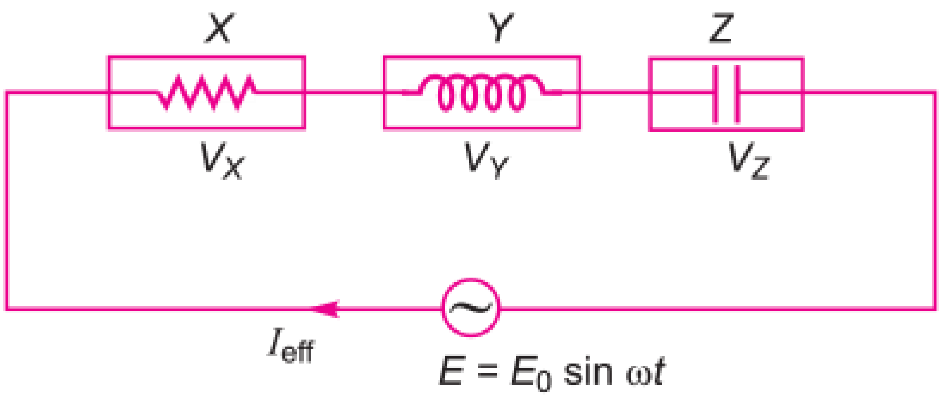




The voltage drop across the combination X and Y is ahead of the current flow. Hence, the element Y is an inductor.



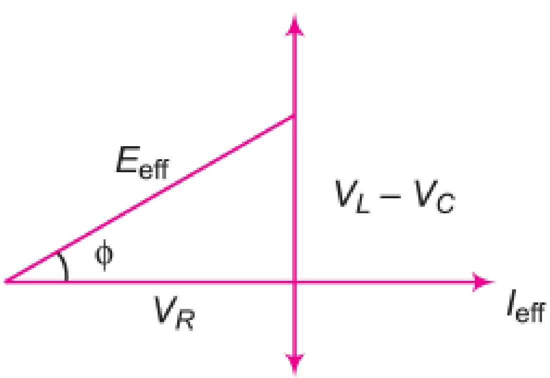
The voltage drop across the combination X and Z lags behind the current flow. Hence, the element Z is a capacitor.



Let I_{eff} be the current flows through each element X, Y and Z. The voltage V_X, V_Y and V_Z drop across the elements. However,

$V_X + V_Y + V_Z > E_{eff}$.

It is not possible. So, on plotting phasor diagram, we have



$$|E_{eff}|^2 = v_R^2 + (V_L - V_C)^2 \qquad \ldots(1)$$

$$E_{eff} = I_{eff}\sqrt{R^2 + (X_L - X_C)^2}$$

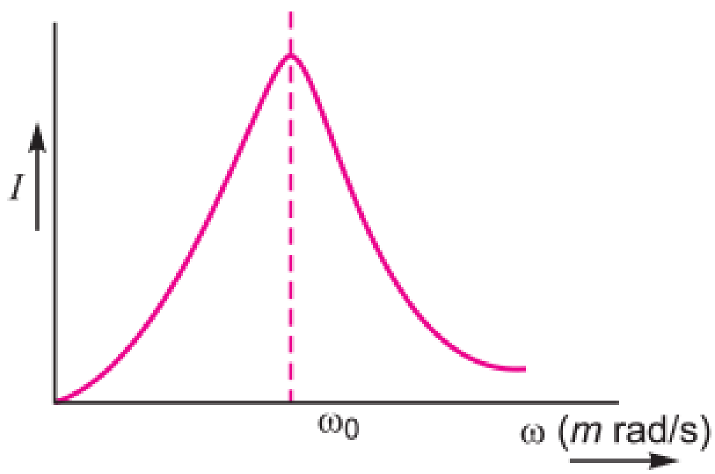
Since $E_{eff} = I_{eff} Z$ $\qquad \ldots(2)$

The impedance of the circuit can be given

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The flow of the current in series LCR circuit varies as a function of frequency of a.c source is shown in figure.

Current amplitude is maximum at resonant frequency ω_0 and can be given as $I_{max} = \frac{V_{eff}}{R}$.
It is possible at $Z = R\left(\text{or } \omega_0 = \frac{1}{\sqrt{LC}}\right)$



Significance:

The resonance phenomenon is exhibit by the circuit only when both L and C are present in the circuit. When voltages across L and C cancel each other (on being out of phase).

The current in the circuit reaches to its maximum value and can be given as $\frac{V_{eff}}{R}$ So, we cannot have resonance in a R.L. or R.C. circuits.

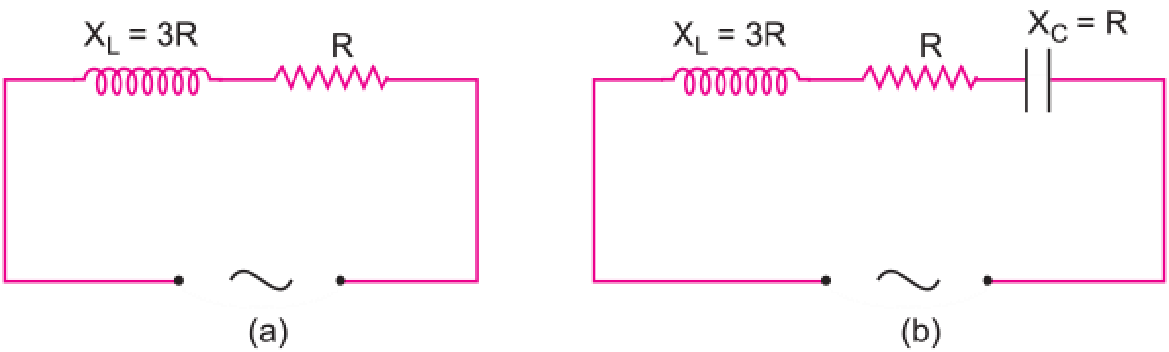
Short Answer Questions –II (OIQ)

Q. 1. Given below are two electrical circuits A and B. Calculate the ratio of power factor of circuit B to the power factor of circuit A.

Ans.

Power factor, $\cos\varphi = \frac{R}{Z}$

Impedance of circuit $A, Z_A = \sqrt{R^2 + X_L^2}$



Impedance of circuit $B, Z_B = \sqrt{R^2 + (X_L - X_C)^2}$

Ratio of power factor of circuit B to that of A is

$$\frac{(\cos\varphi)_B}{(\cos\varphi)_A} = \frac{R/Z_B}{R/Z_A} = \frac{Z_A}{Z_B} = \frac{\sqrt{R^2 + X_L^2}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{\sqrt{R^2+(3R)^2}}{\sqrt{\left[R^2+\sqrt{(3R)-(R)^2}\right]}} = \frac{\sqrt{10}}{\sqrt{5}} = \sqrt{2}$$

Q. 2. Prove that the power dissipated in an ideal resistor connected to an ac source is $V_{\text{eff}}^2/R.$

Ans.

Power in ac circuit, $P = V_{rms}\, i_{rms} \cos\varphi$

As rms values of current and voltage are also called effective values *i.e.*

$$P = V_{eff}\, I_{eff} \cos\varphi \qquad \text{... (i)}$$

But $\qquad \cos\varphi = \text{power factor} = \frac{R}{Z}$

In a purely resistive circuit $Z = R$, $\cos\varphi = 1$

and $\qquad i_{\text{eff}} = \frac{V_{\text{eff}}}{Z} = \frac{V_{\text{eff}}}{R}$

Substituting these values in (i), we get

$$P = V_{\text{eff}} . \frac{V_{\text{eff}}}{R} \times 1 = \frac{V_{\text{eff}}^2}{R}.$$

Q. 3. An inductor 200 mH, a capacitor 100 μF and a resistor 10Ω are connected in series to an a.c. source of 100 V, having variable frequency.

(i) At what frequency of the applied voltage will the power factor of the circuit be 1?
(ii) What will be the current amplitude at this frequency?
(iii) Calculate the Q-factor of the circuit.

Ans. (i) Since the power factor φ =1 it means the phase difference between voltage and the current is zero.

This is possible when

$$\text{wL} = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{\text{LC}} \Rightarrow \nu = \frac{1}{2\pi\sqrt{\text{LC}}} = 35.58 \ \text{Hz}$$

(ii)

$$\text{Current Amplitude, } I = \frac{V_{\text{eff}}}{Z} = \frac{V_{\text{eff}}}{R} = \frac{100}{10} = 10\ A \qquad (\because Z = R)$$

(iii)

$$\text{Quality factor, } Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 4.47$$

Q. 4. An AC voltage V = V$_m$ is applied across a:

(i) Series RC circuit in which capacitive reactance is 'a' times the resistance in the circuit.
(ii) Series RL circuit in which inductive reactance is 'b' times the resistance in the circuit.

Find the value of power factor of the circuit in each case.

Ans.

$$\text{Power factor } \cos\varphi = \frac{R}{Z}, \quad \text{when} \quad Z = \sqrt{R^2 + X^2}$$

i. $X = X_C = aR$,

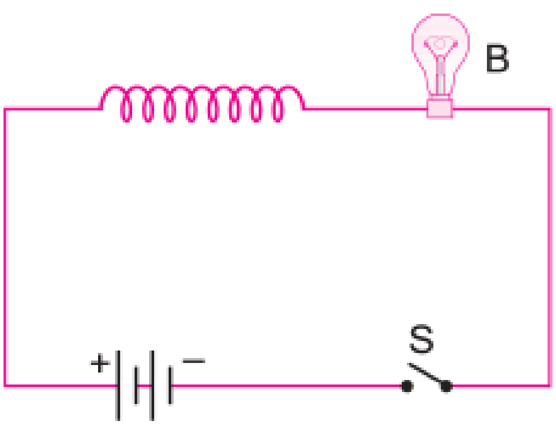
$$\therefore \quad Z = \sqrt{R^2 + (\mathrm{aR})^2} = R\sqrt{1 + a^2}$$

$$\therefore \quad \cos\varphi = \frac{R}{R\sqrt{1+a^2}} = \frac{1}{\sqrt{1+a^2}}$$

ii. $X = X_L = bR$, $\therefore$ $Z = \sqrt{R^2 + (\mathrm{bR})^2} = R\sqrt{1 + b^2}$

$$\therefore \quad \cos\varphi = \frac{R}{R\sqrt{1+b^2}} = \frac{1}{\sqrt{1+b^2}}$$

Q. 5. Figure shows a light bulb (B) and iron cored inductor connected to a dc battery through a switch (S).



(i) What will one observe when switch (S) is closed?

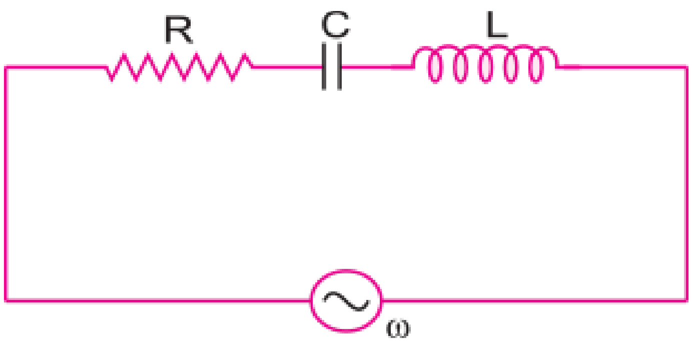
(ii) How will the glow of the bulb change when the battery is replaced by an ac source of rms voltage equal to the voltage of dc battery? Justify your answer in each case.

Ans. When switch S is closed, the bulb will give full brightness slowly, because inductor opposes the rise of current in the circuit depending on the value of ratio $\frac{L}{R}$.

(L = inductance, R = resistance of bulb).

(ii) When battery is replaced by an ac source, the inductor offers reactance (ωL) so impedance of circuit increases and the bulb will glow with less brightness.

Q. 6. In the circuit shown below R represents an electric bulb. If the frequency v = ($\frac{\omega}{2\pi}$) of the supply is doubled, how should the values of C and L be changed, so that the glow of bulb remains unchanged?



Ans. For same current value, the total impedance

$$\sqrt{R^2 + \left(\omega L + \frac{1}{\omega C}\right)}$$

must remain same.

Therefore, $\omega L - \frac{1}{\omega C}$ must remain same. As frequency (ω) is doubled, L and C must both be halved simultaneously.

Q. 7. When a circuit element 'X' is connected across an a.c. source, a current of $\sqrt{2}$ A flows through it and this current is in phase with the applied voltage. When another element 'Y' is connected across the same a.c. source, the same current flows in the circuit but it leads the voltage by $\frac{\pi}{2}$ radians.

(i) Name the circuit element X and Y.
(ii) Find the current that flows in the circuit when the series combination of X and Y is connected across the same a.c. voltage.
(iii) Net impedance

Ans. (i) When circuit element is X, the current is in phase with the applied emf, this implies that X is **pure resistance.**

When circuit element Y is connected, the current leads the voltage by $\frac{\pi}{2}$ so **Y** is **pure capacitance.**

(ii)

Resistance of $X = R$; $I = \frac{V}{R}$ or $\sqrt{2} = \frac{V}{R}$ $\quad (i)$

Reactance of Y, $X_C = \frac{1}{\omega C}$, $I = \frac{V}{X_C} \Rightarrow \sqrt{2C} = \frac{V}{X_C}$ $\quad (ii)$

This implies $X_C = R$

When R and C are connected in series across same voltage source, then

Impedance $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + R^2} = \sqrt{2}R$ $\quad (iii)$

$\therefore$ Current in circuit $I = \frac{V}{Z} = \frac{V}{\sqrt{2}R}$

From (1), $\frac{V}{R} = \sqrt{2}, \therefore I = \frac{1}{\sqrt{2}} \times \sqrt{2} = 1A$

(iii)

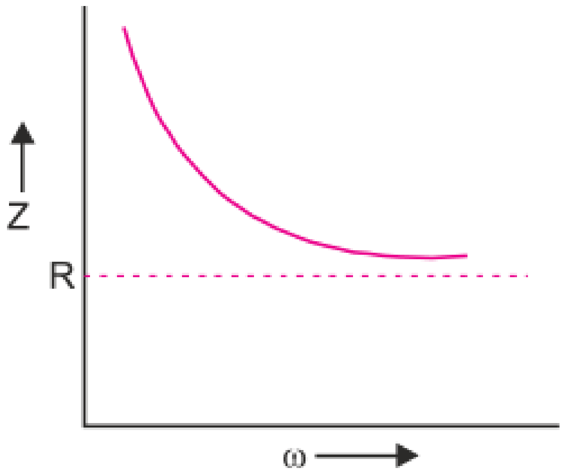
$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

i.e., impedance decreases with increase of angular frequency ω.

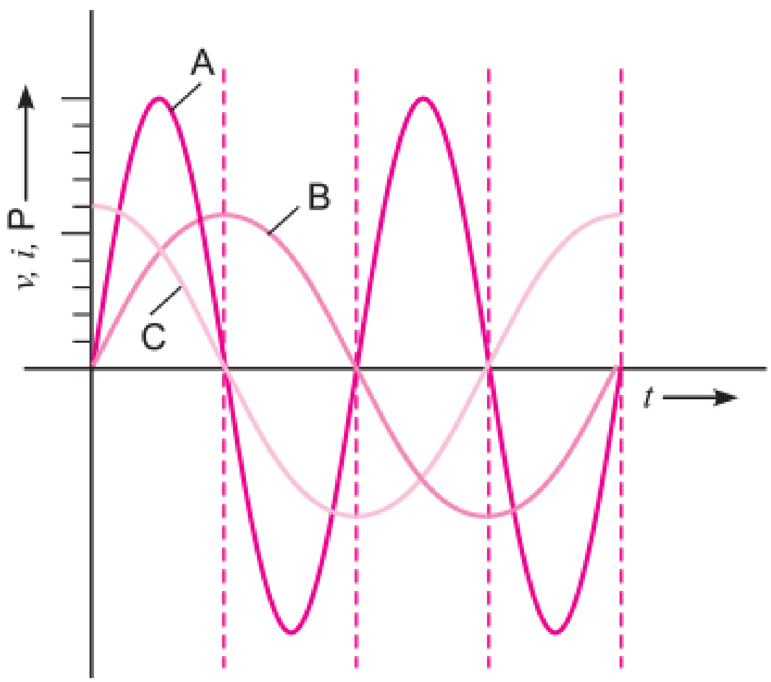
When ω = 0, Z = ∞

When ω = ∞, Z = R.

The graph of impedance Z versus ω is shown in fig.



Q. 8. A device ‘X’ is connected to an a.c. source. The variation of voltage, current and power in one complete cycle is shown in the figure.



(i) Which curve shows power consumption over a full cycle?

(ii) What is the average power consumption over a cycle?

(iii) Identify the device ‘X’. **[NCERT Exemplar]**

Ans. (i) A

(ii) Zero

(iii) L or C or LC

Q. 9. Answer the following question:

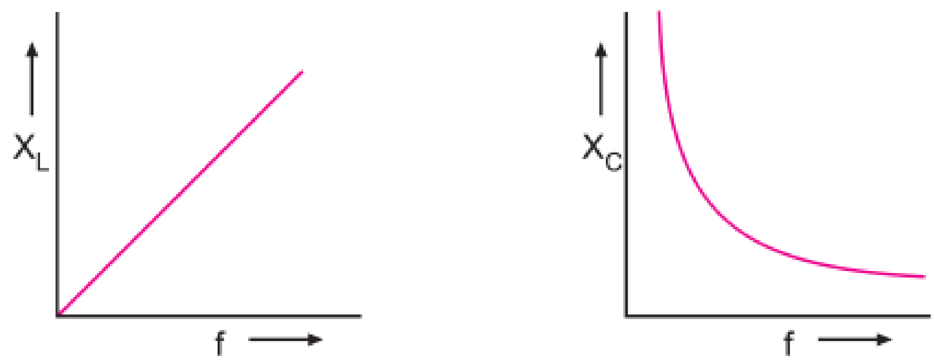
(i) Draw the graphs showing variation of inductive reactance and capacitive reactance with frequency of applied a.c. source.

(ii) Can the voltage drop across the inductor or the capacitor in a series LCR circuit be greater than the applied voltage of the a.c. source? Justify your answer.

Ans. (i)

a. $X_L = w_L = 2\pi VL$; graph X_L of f and f is a straight line

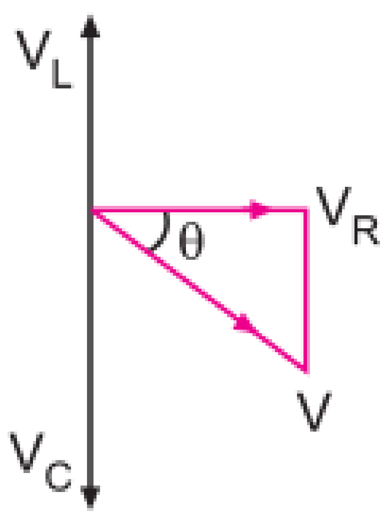
b. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi v C}$, graph of XC and f is a rectangular hyperbola as shown in figs.



(a) (b)

(ii) Yes; because

As V_C and V_L have opposite faces, V_C or V_L may be greater than V. The situation may be as shown in figure where $V_C > V$



Long Answer Questions

Q. 1. Explain the term inductive reactance. Show graphically the variation of inductive reactance with frequency of the applied alternating voltage.

An ac voltage $V = V_0 \sin \omega t$ is applied across a pure inductor of inductance L. Find an expression for the current i, flowing in the circuit and show mathematically that the current flowing through it lags behind the applied voltage by a phase angle of $\frac{\pi}{2}$ Also draw (i) phasor diagram (ii) graphs of V and i versus ωt for the circuit.
[CBSE East 2016]

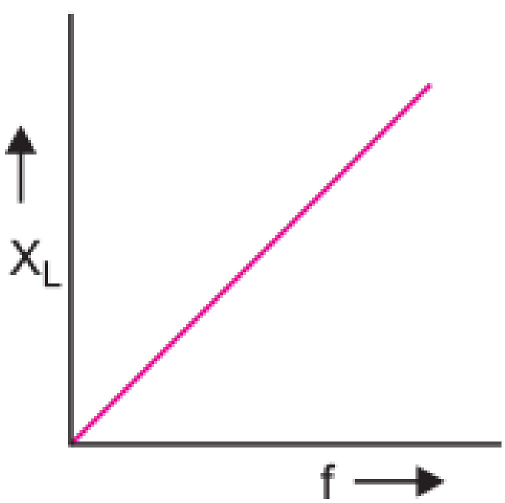
Ans. Inductive Reactance: The opposition offered by an inductor to the flow of alternating current through it is called the inductive reactance. It is denoted by X_L. Its value is X_L. = $\omega L = 2\pi f L$

Where L is inductance and f is the frequency of the applied voltage.

Obviously $X_L \propto f$

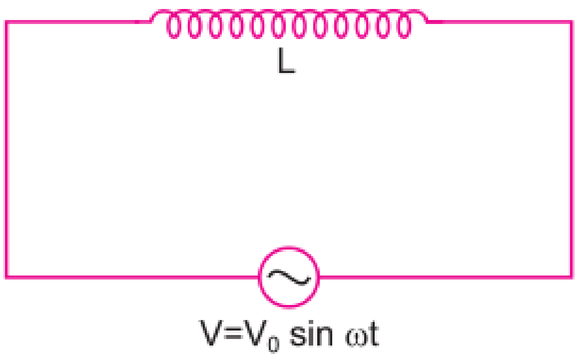
That the graph between X_L and frequency (f) is linear (as shown in fig.).

Phase Difference between Current and Applied Voltage in Purely Inductive circuit:



AC circuit containing pure inductance: Consider a coil of self-inductance L and negligible ohmic resistance. An alternating potential difference is applied across its ends. The magnitude and direction of AC changes periodically, due to which there is a continual change in magnetic flux linked with the coil. Therefore according to Faraday's law, an induced emf is produced in the coil, which opposes the applied voltage. As a result the current in the circuit is reduced. That is **inductance acts like a resistance in ac circuit.** The instantaneous value of alternating voltage applied

$V = V_0 \sin \omega t$ ...(i)



(a)

If i is the instantaneous current in the circuit and $\frac{di}{dt}$ the rate of change of current in the circuit at that instant, then instantaneous induced emf

$$\varepsilon = -L\frac{\mathrm{di}}{\mathrm{dt}}$$

According to Kirchhoff's loop rule

$$V + e = 0 \;\Rightarrow\; V - L\frac{\mathrm{di}}{\mathrm{dt}} = 0$$

or $$V = L\frac{\mathrm{di}}{\mathrm{dt}} \quad \text{or} \quad \frac{\mathrm{di}}{\mathrm{dt}} = \frac{V}{L}$$

or $$\frac{\mathrm{di}}{\mathrm{dt}} = \frac{V_0 \sin \omega t}{L} \quad \text{or} \quad \mathrm{di} = \frac{V_0 \sin \omega\, t}{L}\mathrm{dt}$$

Integrating with respect to time 't',

$$i = \frac{V_0}{L} \int \sin \omega t \,\mathrm{dt} = \frac{V_0}{L}\left\{-\frac{\cos \omega t}{\omega}\right\} = -\frac{V_0}{\omega L}\cos \omega t = -\frac{V_0}{\omega L} \sin \left(\frac{\pi}{2} - \omega t\right)$$

or $$i = \frac{V_0}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{...}(ii)$$

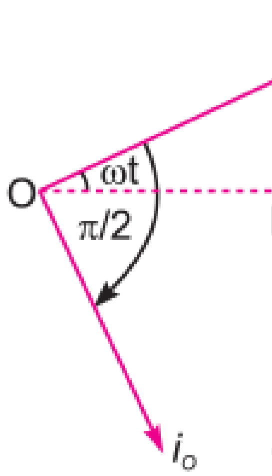
This is required expression for current

or $$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{...}(iii)$$

where $$i_0 = \frac{V_0}{\omega L} \quad \text{...}(iv)$$

is the peak value of alternating current

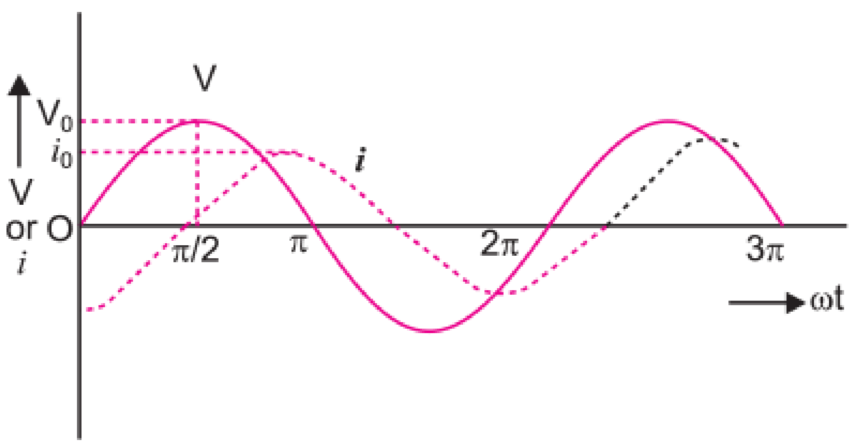
Also comparing (i) and (iii), we note that current lags behind the applied voltage by an angle $\frac{\pi}{2}$ (Fig. b).



(b) Phasor diagram

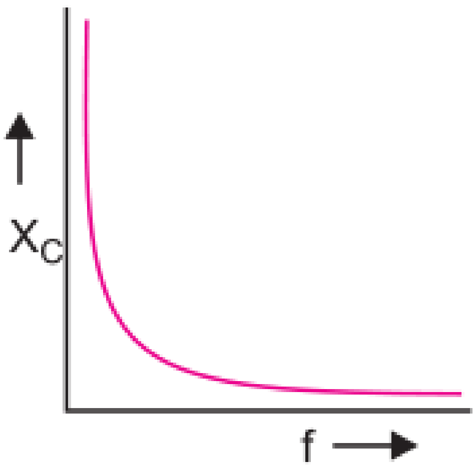
Phasor diagram: The phasor diagram of circuit containing pure inductance is shown in Fig. (b).

Graphs of V and I versus ωt for this circuit is shown in Fig. (c).



Q. 2. Define the term capacitive reactance. Show graphically the variation of capacitive reactance with frequency of applied alternating voltage.

An ac voltage $V = V_0 \sin \omega t$ is applied across a pure capacitor of capacitance C. Find an expression for current flowing through it. Show mathematically the current flowing through it leads the applied voltage by angle π /2.

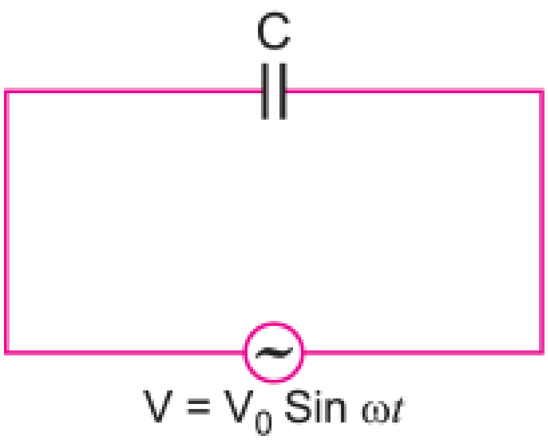


Ans. Capacitive Reactance: The opposition offered by a capacitor alone to the flow of alternating current through it is called the capacitive reactance.

It is denoted by X_C. Its value is $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

The graph of variation of capacitive reactance with frequency is shown in figure.

Phase Difference between Current and Applied voltage in Purely Capacitive Circuit:



(a)

Circuit Containing Pure Capacitance: Consider a capacitor of capacitance C; connected to an alternating voltage source as shown.

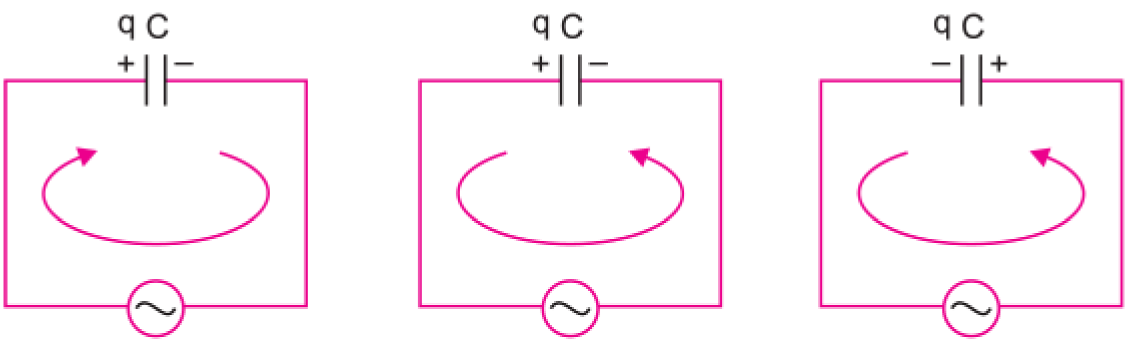
As ac voltage changes in magnitude and direction periodically with a definite frequency; therefore the plates of capacitor get charged, discharged and charged in opposite direction, discharged continuously (Fig. b). Therefore the flow of alternating current in the circuit is maintained. The instantaneous voltage,

$V = V_0 \sin \omega t$ ...(i)

Let q be the charge on capacitor and i , the current in the circuit at any instant, then instantaneous potential difference,

$V = \frac{q}{C}$ ...(ii)

Or $q = CV0 \sin \omega t$



Charge (q) increasing

Charge (q) decreasing

Charge (q) increasing in opposite direction

The instantaneous current,

$$i = \frac{\mathrm{dq}}{\mathrm{dt}} = \frac{d}{\mathrm{dt}}(C\,V_0 \sin \omega t) = C\,V_0 \frac{d}{\mathrm{dt}}(\sin \omega t) = C\,V_0\,\omega \cos \omega t$$

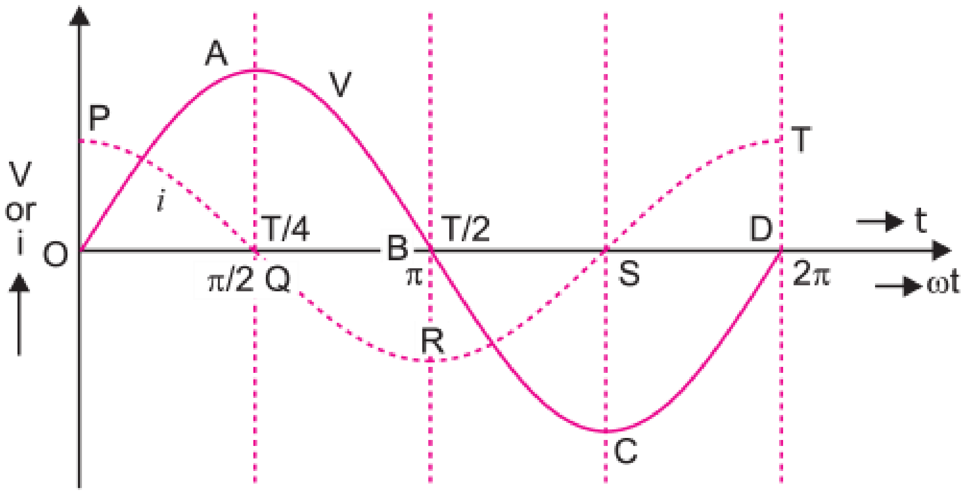
or $$i = \frac{V_0}{(1/\omega C)} \cos \omega t = \frac{V_0}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

or $$i = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \quad ...(iii)$$

where $i_0 = \frac{V_0}{(1/\omega C)}$ = peak value of A.C. ...(iv)

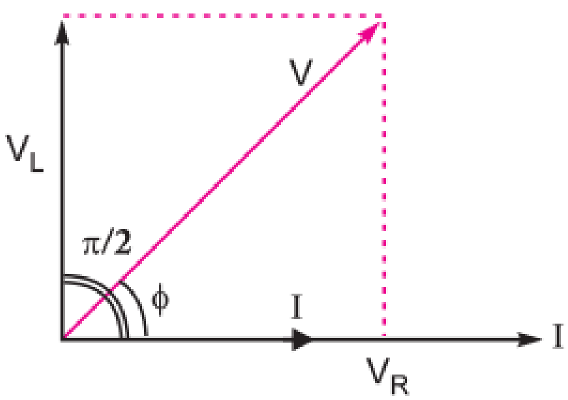
Also comparing (i) and (iii), we note that the current leads the applied emf by an angle $\frac{\pi}{2}$

This is shown graphically in fig. (*c*).



Q. 3. Derive an expression for impedance of an a.c. circuit consisting of an inductor and a resistor. [CBSE Delhi 2008]

Ans. Let a circuit contain a resistor of resistance R and an inductor of inductance L connected in series. The applied voltage is $V = V_0 \sin \omega t$. Suppose the voltage across resistor VR and that across inductor is VL. The voltage V_R and current I are in the same phase, while the voltage V_L leads the current by an angle $\frac{\pi}{2}$. Thus, V_R and V_L are mutually perpendicular. The resultant of VR and VL is the applied voltage i.e.,



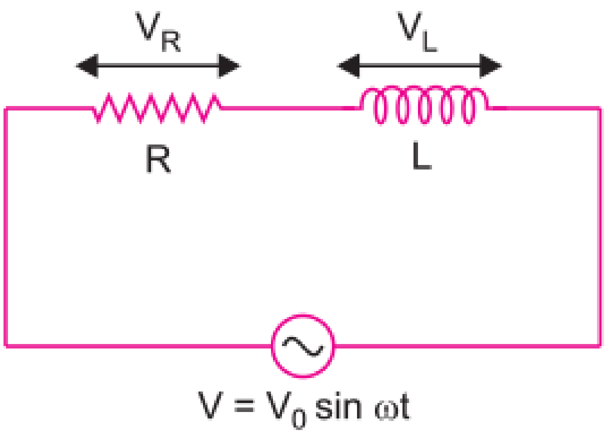
$$V = \sqrt{V_R^2 + V_L^2}$$

But $V_R = Ri, \quad V_L = X_L \; i = \omega Li$

∴ where $X_L = \omega L$ is inductive reactance

∴ $V = \sqrt{(\mathrm{Ri})^2 + (X_L i)^2}$

∴ Impedance, $Z = \frac{V}{i} = \sqrt{R^2 + X_L^2} \Rightarrow Z = \sqrt{R^2 + (\omega L)^2}$



Q. 4. (a) What is impedance?

(b) A series LCR circuit is connected to an ac source having voltage $V = V_0 \sin \omega t$. Derive expression for the impedance, instantaneous current and its phase relationship to the applied voltage. Find the expression for resonant frequency. [CBSE Delhi 2010]

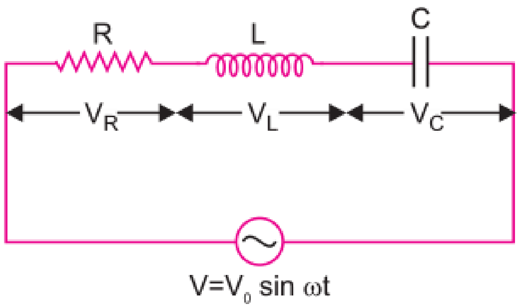
OR

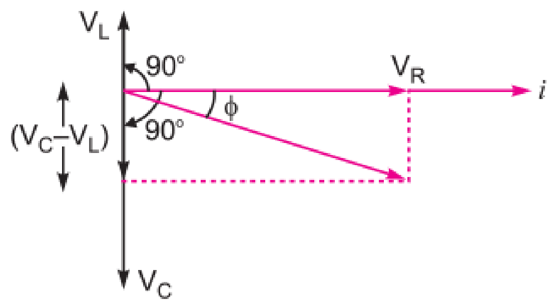
An ac source of voltage $V = V_0 \sin \omega t$ is connected to a series combination of L, C and R. Use the phasor diagram to obtain expressions for impedance of the circuit and phase angle between voltage and current. Find the condition when current will be in phase with the voltage. What is the circuit in this condition called?

In a series L_R circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance C such that XL = XC is put in series, the power factor becomes P_2. Calculate $\frac{P1}{P2}$. [CBSE Delhi 2016]

Ans. Impedance: The opposition offered by the combination of a resistor and reactive component to the flow of ac is called impedance. Mathematically it is the ratio of rms voltage applied and rms current produced in circuit i.e., $Z = \frac{V}{1}$.

Expression for Impedance in LCR series circuit: Suppose resistance R, inductance L and capacitance C are connected in series and an alternating source of voltage $V = V_0 \sin \omega t$ is applied across it. (fig. a) On account of being in series, the current (i) flowing through all of them is the same.





Suppose the voltage across resistance R is V_R voltage across inductance L is V_L and voltage across capacitance C is VC. The voltage VR and current i are in the same phase, the voltage V_L will lead the current by angle 90° while the voltage VC will lag behind the current by angle 90° (fig. b). Clearly VC and VL are in opposite directions, therefore their resultant potential difference = $V_C – V_L$ (if $V_C > V_L$)

Thus VR and ($V_C – V_L$) are mutually perpendicular and the phase difference between them is 90°. As applied voltage across the circuit is V, the resultant of VR and ($V_C – V_L$) will also be V. From fig.

$$V^2 = V_R^2 + (V_C - V_L)^2 \quad \Rightarrow \quad V = \sqrt{V_R^2 + (V_C - V_L)^2} \qquad ...(i)$$

But $V_R = R\,i$, $V_C = X_C\,i$ and $V_L = X_L\,i$ ...(ii)

where $X_C = \frac{1}{\omega C}$ = capacitance reactance and $X_L = \omega L$= inductive reactance

$$V = \sqrt{(\mathrm{Ri})^2 + (X_C i - X_L i)^2}$$

Impedance of circuit, $Z = \frac{V}{i} = \sqrt{R^2 + (X_C - X_L)^2}$

i.e., $Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}$

Instantaneous current $I = \dfrac{V_0 \sin(\omega t + \varphi)}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \mathrm{wL}\right)^2}}$

The phase difference (φ) between current and voltage φ is given by tan $\varphi = \frac{X_C - X_L}{R}$

Resonant Frequency: For resonance φ= 0, so $X_C - X_L = 0$

$$\frac{1}{\omega C} = \omega L \Rightarrow \omega^2 = \frac{1}{\mathrm{LC}}$$

∴ Resonant frequency $\omega_r = \frac{1}{\sqrt{\mathrm{LC}}}$

a. Phase difference (φ) in series LCR circuit is given by

$$\tan\varphi = \frac{V_C - V_L}{V_R} = \frac{i_m(X_C - X_L)}{i_m R} = \frac{(X_C - X_L)}{R}$$

When current and voltage are in phase

$\varphi = 0 \quad \Rightarrow X_C - X_L = 0 \quad \Rightarrow X_C = X_L$

This condition is called resonance and the circuit is called resonant circuit.

b. **Case I:** $X_L = R$

∴ $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + R^2} = \sqrt{2}R$

Power factor, $P_1 = \cos\varphi = \frac{R}{Z} = \frac{R}{\sqrt{2}R} = \frac{1}{\sqrt{2}}$

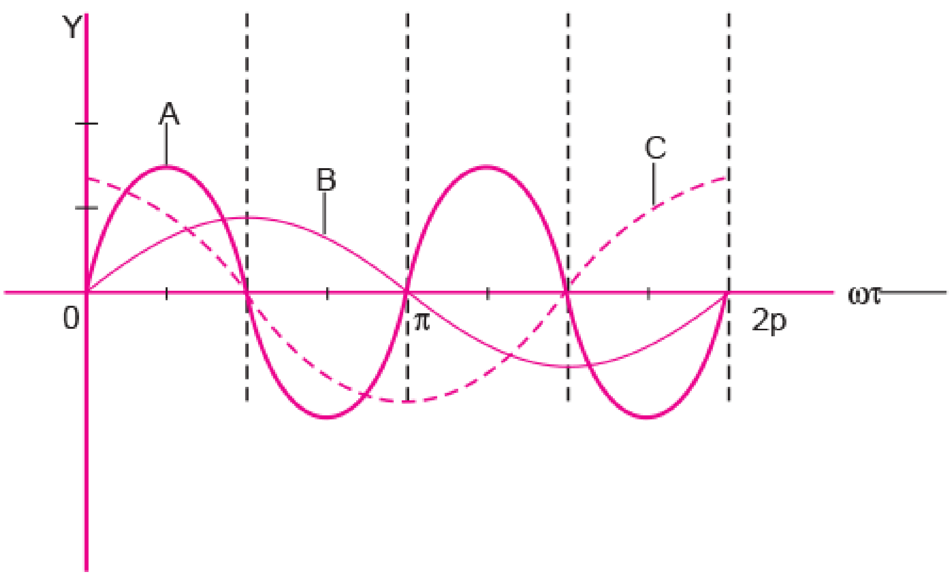
Case II: $X_L = X_C$

∴ $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2} = R$

Power factor, $P_2 = \frac{R}{Z} = \frac{R}{R} = 1$

∴ $\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$

Q. 5. A device 'X' is connected to an ac source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is show in the following graph:



(i) Identify the device 'X'.

(ii) Which of the curves, A, B and C represent the voltage, current and the power consumed in the circuit? Justify your answer.

(iii) How does its impedance vary with frequency of the ac source? Show graphically.

(iv) Obtain an expression for the current in the circuit and its phase relation with ac voltage.

Ans. (i) The device 'X' is a capacitor.

(ii) Curve B: Voltage

Curve C: Current

Curve A: Power consumed in the circuit

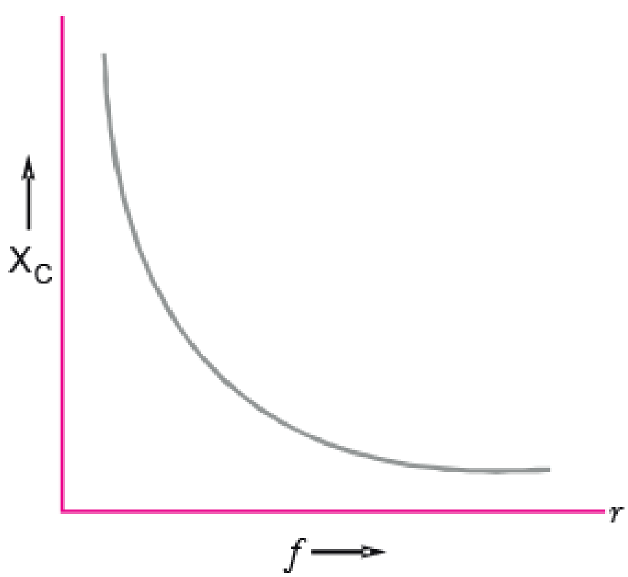
Reason: This is because current leads the voltage in phase by $\frac{\pi}{2}$ for a capacitor.

(iii) Impedance:

XC = $\frac{1}{\omega C}$ = $\frac{1}{2\pi}$ = C

$$\Rightarrow \qquad X_C \propto \frac{1}{f}$$

(iv)



Voltage applied to the circuit is

$V = V_0 \sin \omega t$

Due to this voltage, a charge will be produced which will charge the plates of the capacitor with positive and negative charges.

$$V = \frac{Q}{C} \qquad \Rightarrow \qquad Q = \text{CV}$$

Therefore, the instantaneous value of the current in the circuit is

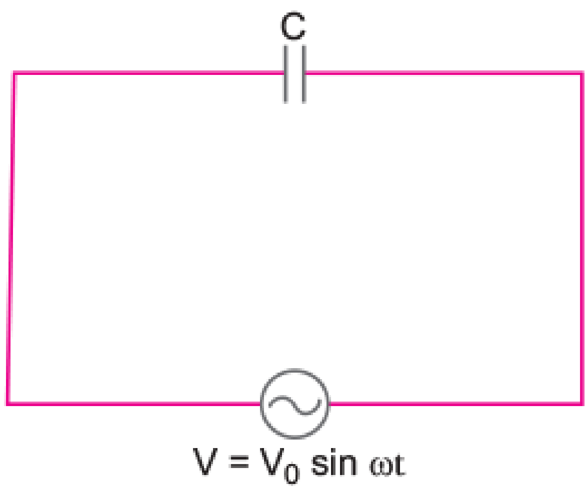
$$I = \frac{\text{dQ}}{\text{dt}} = \frac{d(\text{CV})}{\text{dt}} = \frac{d}{\text{dt}}(\text{CV}_0 \ \sin\ \omega t)$$

$$I = \omega\,\text{CV}_0 \ \cos\ \omega t = \frac{V_0}{\frac{1}{\omega C}}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$I = I_0 \ \sin\ \left(\omega t + \frac{\pi}{2}\right)$$

where, $I_0 = \frac{V_0}{\frac{1}{\omega C}}$ = Peak value of current

Hence, current leads the voltage in phase by $\frac{\pi}{2}$.



Q. 6. (a) State the condition for resonance to occur in series LCR a.c. circuit and derive an expression for resonant frequency. [CBSE Delhi 2010]

(b) Draw a plot showing the variation of the peak current (i_m) with frequency of the a.c. source used. Define the quality factor Q of the circuit.

Ans. (a) Condition for resonance to occur in series LCR ac circuit:

For resonance the current produced in the circuit and emf applied must always be in the same phase.

Phase difference (φ) in series LCR circuit is given by

$$\tan \varphi = \frac{X_C - X_L}{R}$$

For resonance $\varphi = 0 \Rightarrow X_C - X_L = 0$

or $X_C = X_L$

If ω_r is resonant frequency, then $X_C = \frac{1}{\omega_r C}$

and $X_C = \omega_r L$

$$\frac{1}{\omega_r C} = \omega_r L \quad \Rightarrow \quad \omega_r = \frac{1}{\sqrt{LC}}$$

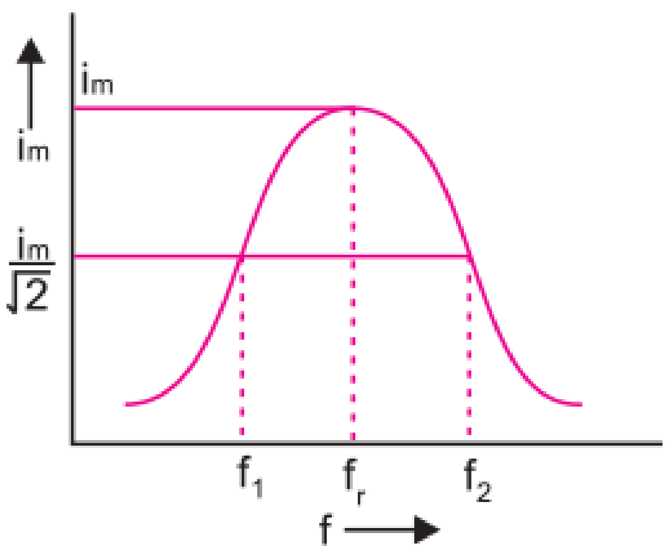
Linear resonant frequency, $f_r = \frac{\omega_r}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$

(b) The graph of variation of peak current i_m with frequency is shown in fig.

Half power frequencies are the frequencies on either side of resonant frequency for which current reduces to half of its maximum value. In fig. f_1 and f_2 are half power frequencies.

Quality Factor (Q): The quality factor is defined as the ratio of resonant frequency to the width of half power frequencies.

$$i.e., \quad Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{f_r}{f_2 - f_1} = \frac{\omega_r L}{R}$$

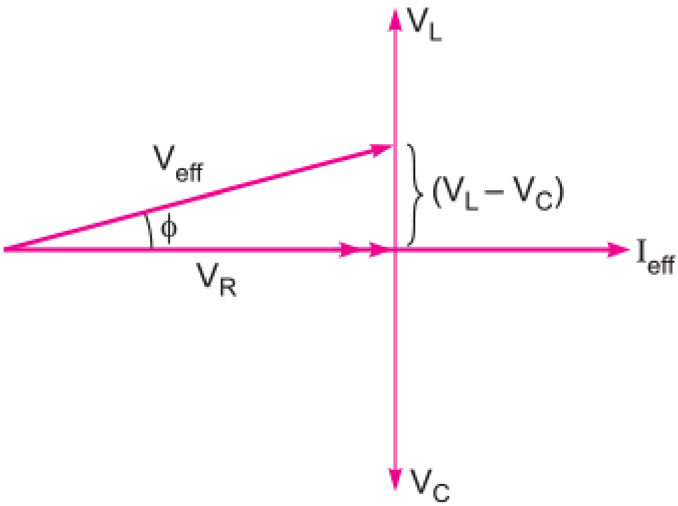


Q. 7. Using phasor diagram for a series LCR circuit connected to an ac source of voltage v = v_0 sin ωt, derive the relation for the current flowing in the circuit and the phase angle between the voltage across the resistor and the net voltage in the circuit.

Draw a plot showing the variation of the current I as a function of angular frequency 'ω' of the applied ac source for the two cases of a series combination of (i) inductance L_1, capacitance C_1 and resistance R1 and (ii) inductance L_2, capacitance C_2 and resistance R_2 where $R_2 > R_1$.

Write the relation between L_1, C_1 and L_2,C_2 at resonance. Which one, of the two, would be better suited for fine tuning in a receiver set? Give reason. [CBSE (F) 2013]

Ans. For I_{eff} flow of current through each element R, L and C, effective voltage across the combination can be given as.



$\Rightarrow \qquad \overrightarrow{V_{\text{eff}}} = \widehat{\mathrm{i}}\mathrm{V}_R + \hat{j}(V_L - V_C)$

$\Rightarrow \qquad |V_{\text{eff}}| = \sqrt{|V_R|^2 + (V_L - V_C)^2}$

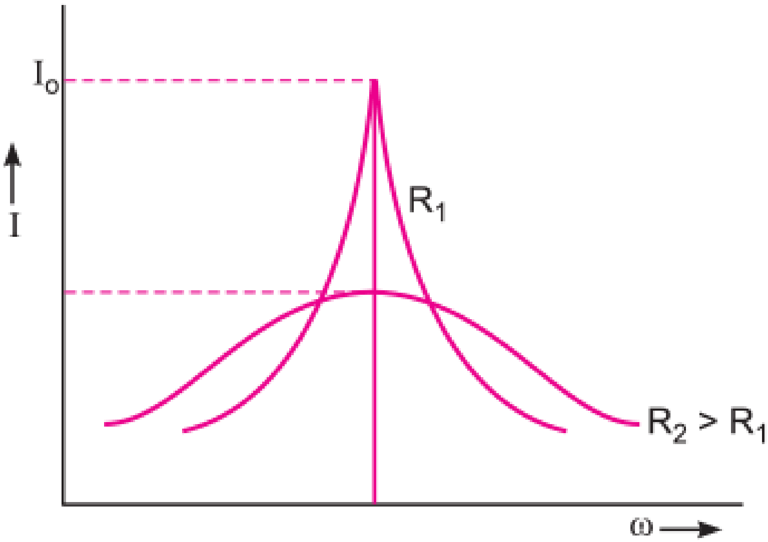
$\Rightarrow \qquad I_{\text{eff}}\ Z = \sqrt{(I_{\text{eff}}R)^2 + (I_{\text{eff}}X_L - I_{\text{eff}}X_C)^2}$

$\Rightarrow \qquad |Z| = \sqrt{R^2 + (X_L - X_C)^2}$

Effective current flow $I_{\text{eff}} = \frac{E_{\text{eff}}}{Z} = \frac{E_{\text{eff}}}{\sqrt{R^2 + (X_L - X_C)^2}}$

Phase angle between V_R and V_{eff} is

$$\cos\varphi = \frac{V_R}{V_{\text{eff}}} = \frac{V_R}{\sqrt{V_R^2 + (V_L - V_C)^2}}$$

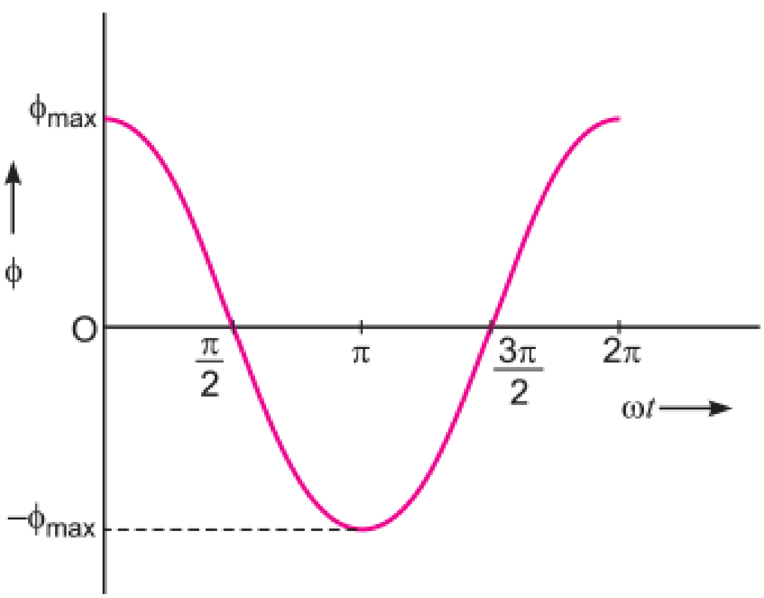


(i) $I = I_0 \sin(\omega t - \varphi)$ For $V_L > V_C$ or $X_L > X_C$

(ii) $I = I_0 \sin(\omega t + \varphi)$ For $V_L < V_C$ or $X_L < X_C$

Variation of the current I as a function of angular frequency ω.

At resonance, when maximum current flows through the circuit.

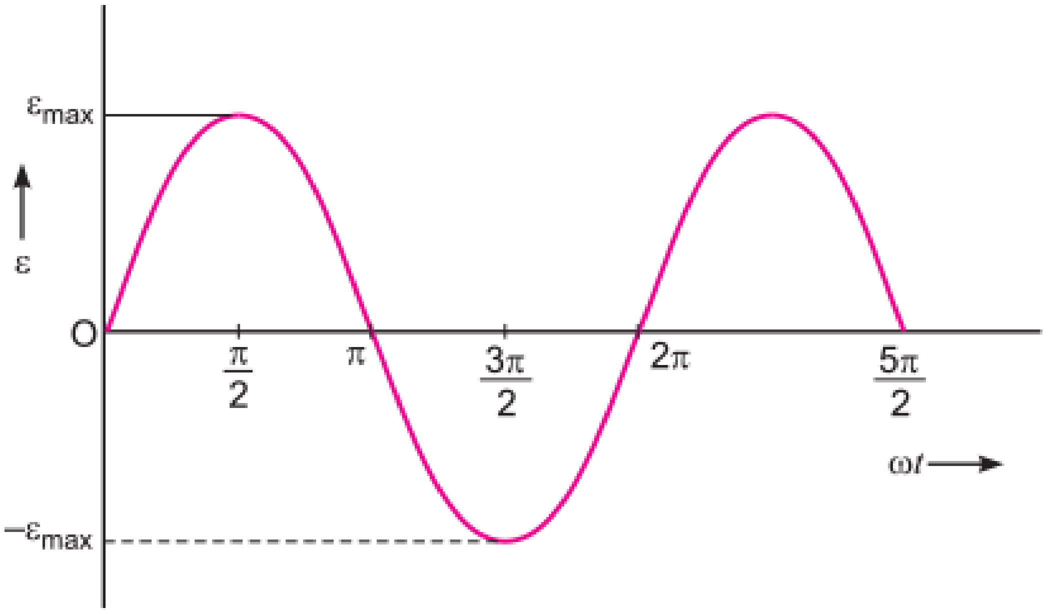


$$\omega_r = \frac{1}{\sqrt{L_1C_1}} = \frac{1}{\sqrt{L_2C_2}}$$

$$L_1C_1 = L_2C_2 \Rightarrow \frac{L_1}{L_2} = \frac{C_2}{C_1}$$

For fine tuning in the receiver set, combination L_1C_1 and R_1 is better because maximum current flows through the circuit.

From Lenz's law, induced emf



$$\varepsilon = -\frac{d\Phi}{dt}$$

$$= -|B|\,|A|\,\frac{d}{dt}\cos\omega t$$

$$= |B|\,A\,\omega\sin\omega t$$

Q. 8. Answer the following question :

(i) An alternating voltage V = V_m Sin ωt (applied to a series LCR circuit drives a current given by i = i_m sin (ωt +φ) . Deduce an expression for the average power dissipated over a cycle.

(ii) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain. [CBSE (F) 2011]

Ans. (i) $V = V_m \sin \omega t$ and $i = i_m (\omega t + \varphi)$

And instantaneous power, $P = V_i$

$= V_m \sin \omega t.\ i_0 \sin (\omega t + \varphi)$

$= V_m\, i_m \sin \omega t \sin (\omega t + \varphi)$

$$= \frac{1}{2} V_m i_m 2 \sin \omega t . \sin (\omega t + \varphi)$$

From trigonometric formula

$$2 \sin A \sin B = \cos (A - B) - \cos (A + B)$$

$\therefore$ Instantaneous power, $P = \frac{1}{2} V_m i_m [\cos (\omega t + \varphi - \omega t) - \cos (\omega t + \varphi + \omega t)]$

$$= \frac{1}{2} V_m i_m [\cos \varphi - \cos (2\omega t + \varphi)] \qquad \ldots (i)$$

Average power for complete cycle $\overrightarrow{P} = \frac{1}{2} V_m i_m [\cos \varphi - \cos (2\overline{\omega t} + \varphi)]$

where $\cos (\overline{\omega t} + \varphi)$ is the mean value of $\cos (2\omega t + \varphi)$ over complete cycle. But for a complete cycle, $\cos (2\omega t + \varphi) = 0$

$\therefore$ Average power, $\overrightarrow{P} = \frac{1}{2} V_m i_m \cos \varphi = \frac{V_0}{\sqrt{2}} \frac{i_0}{\sqrt{2}} \cos \varphi$

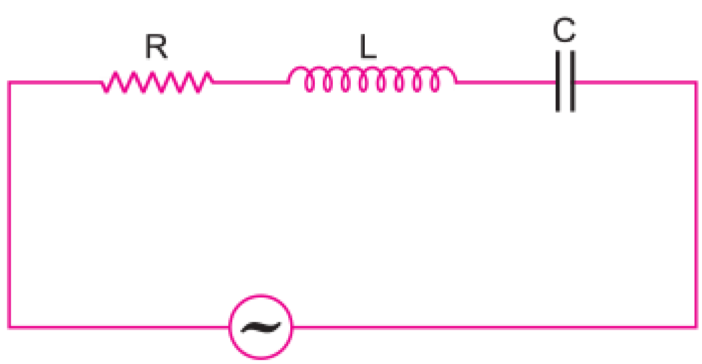
$$\overrightarrow{P} = V_{\text{rms}} i_{\text{rms}} \cos \varphi$$

(ii) The power is P = V_{rms} I_{rms} cos φ. If cos φ is small, then current considerably increases when voltage is constant. Power loss, we know is I_2R. Hence, power loss increases.

Q. 9. A voltage V = V_0 sin ωt is applied to a series LCR circuit. Derive the expression for the average power dissipated over a cycle.

Under what condition is (i) no power dissipated even though the current flows through the circuit, (ii) maximum power dissipated in the circuit? [CBSE (AI) 2014]

Ans.



The voltage V = V0 sin ωt is applied across the series LCR circuit. However due to impedance of the circuit, either current lags or leads the voltage by phase opposite so the current in the circuit is given by

$I = I_0 \sin(\omega t - \varphi)$

Instantaneous power dissipation in the circuit

P = VI

$$V_0 \sin\omega t \times I_0 \sin(\omega t - \varphi) = \frac{V_0 I_0}{2} \times 2\sin\omega t . \sin(\omega t - \varphi)$$

$$= \frac{V_0 I_0}{2}(\cos\varphi - \cos(2\omega t - \varphi)) \qquad [\cos(A-B) - \cos(A+B) = 2\sin A \sin B]$$

Average power loss over one complete cycle

$$\bar{P} = \frac{1}{T}\int_0^T \mathrm{Pdt} = \frac{V_0 I_0}{2T}\left[\int_0^T \cos\varphi\,\mathrm{dt} - \int_0^T \cos(2\omega t - \varphi)\,\mathrm{dt}\right]$$

$$\text{However,} \qquad \int_0^T \cos(2\omega t - \varphi)\mathrm{dt} = 0 = \frac{V_0 I_0}{2T} . \cos\varphi \int_0^T \mathrm{dt} = \frac{V_0 I_0}{2}\cos\varphi$$

$$P_{\mathrm{av}} = \frac{V_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos\varphi$$

$$P_{av} = V_{eff} \cdot I_{eff} \cos \varphi$$

(i) If phase angle φ =90° (resistance R is used in the circuit) then no power dissipated.

(ii) If phase angle φ =0° or circuit is pure resistive (or XL= XC) at resonance then

$$\text{Max power } P = V_{eff} \times I_{eff} = \frac{V_0 I_0}{2}$$

Q. 10. Explain with the help of a labelled diagram, the principle and working of an ac generator? Write the expression for the emf generated in the coil in terms of speed of rotation. Can the current produced by an ac generator be measured with a moving coil galvanometer?

OR

Describe briefly, with the help of a labelled diagram, the basic elements of an ac generator. State its underlying principle. Show diagrammatically how an alternating emf is generated by a loop of wire rotating in a magnetic field. Write

the expression for the instantaneous value of the emf induced in the rotating loop.
[CBSE Delhi 2010]

OR

State the working of ac generator with the help of a labelled diagram.

The coil of an ac generator having N turns, each of area A, is rotated with a constant angular velocity ω. Deduce the expression for the alternating emf generated in the coil.

What is the source of energy generation in this device? [CBSE (AI) 2011]

Ans. AC generator: A dynamo or generator is a device which converts mechanical energy into electrical energy.

Principle: It works on the principle of electromagnetic induction. When a coil rotates continuously in a magnetic field, the effective area of the coil linked normally with the magnetic field lines, changes continuously with time. This variation of magnetic flux with time results in the production of an alternating emf in the coil.

Construction: It consists of the four main parts:

(i) Field Magnet: It produces the magnetic field. In the case of a low power dynamo, the magnetic field is generated by a permanent magnet, while in the case of large power dynamo, the magnetic field is produced by an electromagnet.

(ii) Armature: It consists of a large number of turns of insulated wire in the soft iron drum or ring. It can revolve round an axle between the two poles of the field magnet. The drum or ring serves the two purposes: (i) It serves as a support to coils and (ii) It increases the magnetic field due to air core being replaced by an iron core.

(iii) Slip Rings: The slip rings R1 and R2 are the two metal rings to which the ends of armature coil are connected. These rings are fixed to the shaft which rotates the armature coil so that the rings also rotate along with the armature.

(iv) Brushes: These are two flexible metal plates or carbon rods (B1 and B2) which are fixed and constantly touch the revolving rings. The output current in external load RL is taken through these brushes.

Working: When the armature coil is rotated in the strong magnetic field, the magnetic flux linked with the coil changes and the current is induced in the coil, its direction being given by Fleming's right hand rule. Considering the armature to be in vertical position and as it rotates in anticlockwise direction, the wire ab moves upward and cd downward, so that the direction of induced current is shown in fig. In the external circuit, the current flows along B_1 R_LB_2. The direction of current remains unchanged during the first half turn of armature. During the second half revolution, the wire ab moves downward and cd upward, so the direction of current is reversed and in external circuit it flows along $B_2 R_L B_1$. Thus the direction of induced emf and current changes in the external circuit after each half revolution.

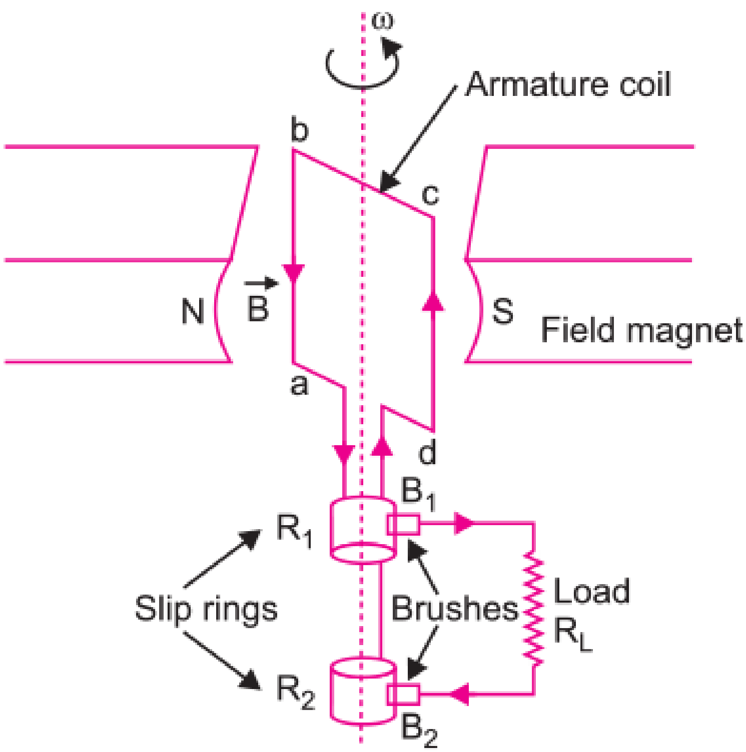
Expression for Induced emf: When the coil is rotated with a constant angular speed ω , the angle θ between the magnetic field vector B and the area vector A of the coil at any instant t is θ = ωt

(Assuming θ = 0° at t = 0). As a result, the effective area of the coil exposed to the magnetic field lines changes with time, the flux at any time t is

Φ_B= BA cos θ = BA cos ωt

From Faraday's law, the induced emf for the rotating coil of N turns is then,

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NBA\,\frac{d}{dt}(\cos \omega t)$$



Thus, the instantaneous value of the emf is

ε = NBA ω sin ωt

Where NBAω=2π vNBA is the maximum value of the emf, which occurs when sin ωt = ±1. If we denote NBAω as ε0, then

$\varepsilon = \varepsilon_0 \sin \omega t \quad \Rightarrow \varepsilon = \varepsilon_0 \sin 2\pi v t$

Where v is the frequency of revolution of the generator's coil.

Obviously, the emf produced is alternating and hence the current is also alternating.

Current produced by an ac generator cannot be measured by moving coil ammeter; because the average value of ac over full cycle is zero.

The source of energy generation is the mechanical energy of rotation of armature coil.

Q. 11. (a) Describe briefly, with the help of a labelled diagram, the working of a step up transformer.

(b) Write any two sources of energy loss in a transformer. [CBSE (F) 2012]

(c) A step up transformer converts a low voltage into high voltage. Does it not violate the principle of conservation of energy? Explain. [CBSE Delhi 2011, 2009]

OR

Draw a schematic diagram of a step-up transformer. Explain its working principle. Deduce the expression for the secondary to primary voltage in terms of the number of turns in the two coils. In an ideal transformer, how is this ratio related to the currents in the two coils?

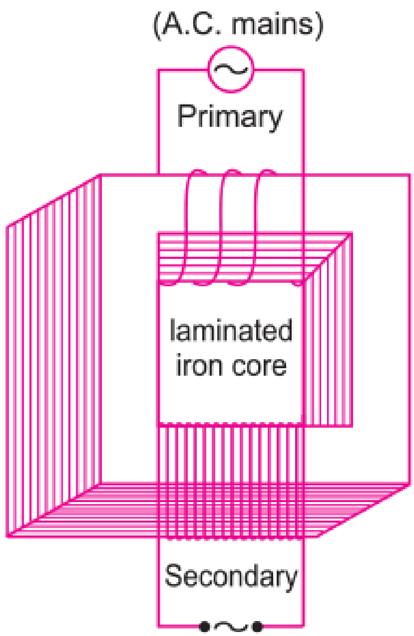
How is the transformer used in large scale transmission and distribution of electrical energy over long distances? [CBSE (AI) 2010, (East) 2016]

Ans. (a) **Transformer:** A transformer converts low voltage into high voltage ac and vice-versa.

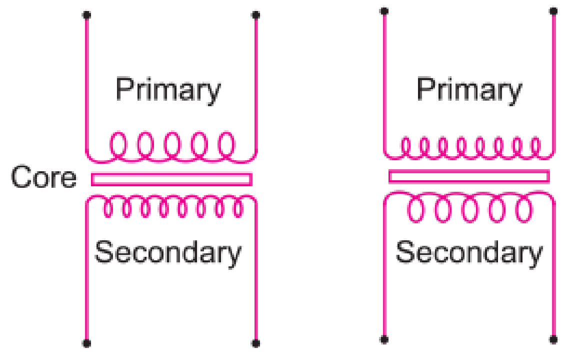
Construction: It consists of laminated core of soft iron, on which two coils of insulated copper wire are separately wound. These coils are kept insulated from each other and from the iron-core, but are coupled through mutual induction. The number of turns in these coils are different. Out of these coils one coil is called primary coil and other is called the secondary coil. The terminals of primary coils are connected to AC mains and the terminals of the secondary coil are connected to external circuit in which alternating current of desired voltage is required. Transformers are of two types:

1. Step up Transformer: It transforms the alternating low voltage to alternating high voltage and in this the number of turns in secondary coil is more than that in primary coil. (i.e., $N_S > N_p$)

2. Step down Transformer: It transforms the alternating high voltage to alternating low voltage and in this the number of turns in secondary coil is less than that in primary coil (i.e., $N_S < N_p$).



Transformer



(a) Step up (b) Step down

Working: When alternating current source is connected to the ends of primary coil, the current changes continuously in the primary coil; due to which the magnetic flux linked with the secondary coil changes continuously, therefore the alternating emf of same frequency is developed across the secondary.

Let N_P be the number of turns in primary coil, NS the number of turns in secondary coil and φ the magnetic flux linked with each turn. **We assume that there is no leakage of flux so that the flux linked with each turn of primary coil and secondary coil is the same.** According to Faraday's laws the emf induced in the primary coil

$$\varepsilon_p = - N_p \frac{\Delta\varphi}{\Delta t} \qquad \ldots(i)$$

and emf induced in the secondary coil

$$\varepsilon_S = - N_S \frac{\Delta\varphi}{\Delta t} \qquad \ldots(ii)$$

From (i) and (ii)

$$\frac{\varepsilon_S}{\varepsilon_p} = \frac{N_S}{N_p} \qquad \ldots(iii)$$

If the resistance of primary coil is negligible, the emf (ε_p) induced in the primary coil, will be equal to the applied potential difference (V_P) across its ends. Similarly if the secondary circuit is open, then the potential difference VS across its ends will be equal to the emf (ε_S) induced in it; therefore

$$\frac{V_S}{V_p} = \frac{\varepsilon_S}{\varepsilon_p} = \frac{N_S}{N_p} = r \text{ (say)} \quad \ldots(iv)$$

where $r = \frac{N_S}{V_P}$ is called the transformation ratio. If i_P and i_S are the instantaneous currents in primary and secondary coils and there is no loss of energy; then

For about 100% efficiency, Power in primary = Power in secondary

$V_P\, i_P = V_S\, i_S$

$$\frac{i_s}{i_p} = \frac{V_p}{V_S} = \frac{N_p}{N_S} = \frac{1}{r} \quad \ldots(v)$$

In step up transformer, $N_S > N_P \rightarrow r > 1$;

So $V_S > V_P$ and $i_S < i_p$

i.e. step up transformer increases the voltage.

In step down transformer, $N_S < N_P \rightarrow r < 1$

So $V_S < V_P$ and $i_S > i_p$

i.e. step down transformer decreases the voltage, but increases the current.

Laminated core: The core of a transformer is laminated to reduce the energy losses due to eddy currents, so that its efficiency may remain nearly 100%.

In a transformer with 100% efficiency (say),

Input power = output power $V_P\, I_P = V_S\, I_S$

(b) The sources of energy loss in a transformer are (i) eddy current losses due to iron core (ii) flux leakage losses. (iii) Copper losses due to heating up of copper wires (iv) Hysteresis losses due to magnetisation and demagnetisation of core.

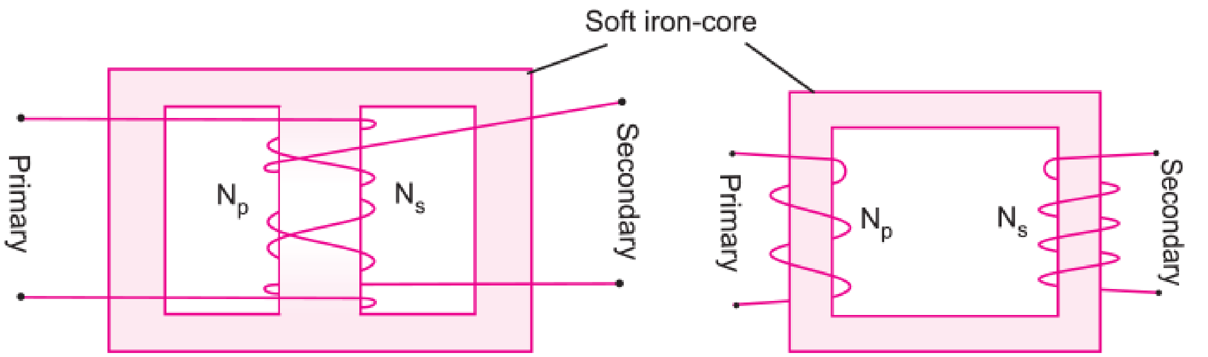
(c) When output voltage increases, the output current automatically decreases to keep the power same. Thus, there is no violation of conservation of energy in a step up transformer.

Q. 12. Show diagrammatically two different arrangements used for winding the primary and secondary coils in a transformer. Assuming the transformer to be an ideal one, write the expression for the ratio of its

(i) Output voltage to input voltage
(ii) Output current to input current.

Mention two reasons for energy losses in an actual transformer. [CBSE (F) 2012]

Ans. Arrangements of winding of primary and secondary coil in a transformer are shown in fig. (a) and (b).



(a) Two coils on top of each other (b)Two coils on separate limbs of the core

(i) Ratio of output voltage to input voltage

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

(ii) Ratio of output current to input current

$$\frac{I_s}{I_p} = \frac{N_p}{N_s}$$

Reasons for energy losses in a transformer

(i) Joule Heating: Energy is lost due to heating of primary and secondary windings as heat (I_2Rt).

(ii) Flux Leakage: Energy is lost due to coupling of primary and secondary coils not being perfect, i.e., whole of magnetic flux generated in primary coil is not linked with the secondary coil.

Q. 13. A 2 μF capacitor, 100 W resistor and 8 H inductor are connected in series with an AC source.

(i) What should be the frequency of the source such that current drawn in the circuit is maximum? What is this frequency called?

(ii) If the peak value of emf of the source is 200 V, find the maximum current.

(iii) Draw a graph showing variation of amplitude of circuit current with changing frequency of applied voltage in a series LRC circuit for two different values of resistance R_1 and R_2 ($R_1 > R_2$).

(iv) Define the term 'Sharpness of Resonance'. Under what condition, does a circuit become more selective? [CBSE (F) 2016]

Ans. (i)

For maximum frequency

$$\omega L = \frac{1}{\omega C}$$

$$\Rightarrow \quad 2\pi\nu 8 = \frac{1}{2\pi\nu \times 10^{-6} \times 2} \quad \Rightarrow \quad (2\pi\nu)^2 = \frac{1}{16 \times 10^{-6}}$$

$$\Rightarrow \quad 2\pi\nu = \frac{1}{4 \times 10^{-3}} \quad \Rightarrow \quad 2\pi\nu = \frac{10^3}{4}$$

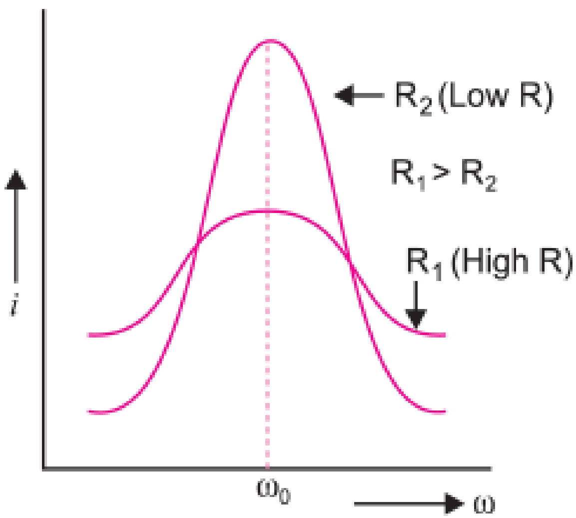
$$\Rightarrow \quad \nu = \frac{250}{2\pi} = 39.77 \ s^{-1}$$

This frequency is called resonance frequency.

(ii)

Maximum current, $I_0 = \frac{E_0}{R} = \frac{200}{100} = 2A \ [E_0 \text{ maximum emf}]$

(iii)



(iv) $\frac{\omega_0}{2\Delta\omega}$ is measure of sharpness of resonance, where ω_0 is the resonant frequency and $2\Delta\omega$ is the bandwidth.

Circuit is more selective if it has greater value of sharpness. The circuit should have smaller bandwidth $\Delta\omega$.

Q. 14. (i) Draw a labelled diagram of AC generator. Derive the expression for the instantaneous value of the emf induced in the coil.

(ii) A circular coil of cross-sectional area 200 cm^2 and 20 turns is rotated about the vertical diameter with angular speed of 50 rad s^{-1} in a uniform magnetic field of magnitude 3.0 × 10^{-2} T. Calculate the maximum value of the current in the coil. [CBSE Delhi 2017]

Ans. Given, N = 20

$A = 200\ \text{cm}^2$

$= 200 \times 10^{-4}\text{m}^2$

$B = 3.0 \times 10^{-2}\ \text{T}$

$\omega = 50\ \text{rad s}^{-1}$

EMF induced in the coil

$\varepsilon = NBA\omega \sin \omega t$

Maximum emf induced

$\varepsilon_{max} = NBA$

$= 20 \times 3.0 \times 10^{-2} \times 200 \times 10^{-4} \times 50$

$= 600\ \text{mV}$

Maximum value of current induced

$$I_{max} = \frac{\varepsilon_{\max}}{R} = \frac{600}{R}\text{mA}$$

Q. 15. (i) Draw a labelled diagram of a step-up transformer. Obtain the ratio of secondary to primary voltage in terms of number of turns and currents in the two coils.

(ii) A power transmission line feeds input power at 2200 V to a step-down transformer with its primary windings having 3000 turns. Find the number of turns in the secondary to get the power output at 220 V. [CBSE Delhi 2017]

Ans.

ii. Given, $V_p = 2200$ V

$N_p = 3000$ turns

$V_s = 220$ V

We have, $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

$$N_s = \frac{V_s}{V_p} \times N_p$$

$$= \frac{220}{2200} \times 300$$

$N_s = 300$ turns