Q. 1. Write the expression for Bohr's radius in hydrogen atom. [CBSE Delhi 2010]

Ans.

Bohr's radius, 
$$r_1 = rac{arepsilon_0 h^2}{\pi\,{
m me}^2} = 0.529~ imes~10^{-10}m$$

Q. 2. In the Rutherford scattering experiment the distance of closest approach for an  $\alpha$ -particle is d<sub>0</sub>. If  $\alpha$ -particle is replaced by a proton, how much kinetic energy in comparison to  $\alpha$ -particle will it require to have the same distance of closest approach d<sub>0</sub>? [CBSE (F) 2009]

Ans.

$$E_k = \frac{1}{4\pi\varepsilon_0} \frac{(\text{Ze})(2e)}{d_0}$$
 (for  $\alpha$ -particle,  $q = 2e$ )

$$E'_k = \frac{1}{4\pi\varepsilon_0} \frac{(\text{Ze})(e)}{d_0}$$
 (for proton,  $q = e$ )

$$rac{E'_k}{E_k} = rac{1}{2} \quad \Rightarrow \quad E'_k = rac{E_k}{2}$$

That is KE of proton must be half on comparison with KE of  $\alpha$ -particle.

Q. 3. What is the ratio of radii of the orbits corresponding to first excited state and ground state in a hydrogen atom? [CBSE Delhi 2010]

Ans.

$$r_n = rac{arepsilon_0 h^2 n^2}{\pi\,{
m me}^2}~\propto~n^2$$

For I excited state, n = 2

For ground state, n = 1

$$\therefore \qquad \frac{r_2}{r_1} = \frac{4}{1}$$

**Q. 4.** Find the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its:

(i) Second permitted energy level to the first level, and

(ii) The highest permitted energy level to the first permitted level. [CBSE (AI) 2010]

Ans.

$$E_I = \operatorname{Rhc}\left(rac{1}{1^2} - rac{1}{2^2}
ight) = rac{3}{4}\operatorname{Rhc}$$

$$E_{\mathrm{II}} = \mathrm{Rhc}\left(rac{1}{1^2} - rac{1}{\infty^2}
ight) = \mathrm{Rhc}$$

Ratio  $\frac{E_I}{E_{II}} = \frac{3}{4}$ 

### Q. 5. State Bohr's quantisation condition for defining stationary orbits. [CBSE (F) 2010]

**Ans.** Quantum Condition: The stationary orbits are those in which angular momentum of electron is an integral multiple of  $\frac{h}{2\pi}$  i.e.,

$$mvr = n \frac{h}{2\pi}, \quad n = 1, 2, 3, \dots$$

Integer n is called the **principal quantum number.** This equation is called Bohr's quantum condition.

Q. 6. The radius of innermost electron orbit of a hydrogen atom is  $5.1 \times 10-11$  m. What is the radius of orbit in the second excited state? [CBSE Delhi 2010]

Ans.

In ground state, n = 1

In second excited state, n = 3

 $Asr_n \propto n^2$ 

 $\therefore \quad \frac{r_3}{r_1} = \left(\frac{3}{1}\right)^2 = 9$ 

 $r_3 = 9r_1 = 9 \times 5.1 \times 10^{-11} \text{ m} = 3.77 \times 10^{-10} \text{ m}$ 

### Very Short Answer Questions (OIQ)

#### Q. 1. Why do $\alpha$ -particles have high ionising power?

**Ans.**  $\alpha$ -particles are heavier, they move slowly; so possess large momentum. Due to this property they come in contact with large number of particles; so they possess high ionising power.

### Q. 2. The mass of H-atom is less than the sum of the masses of a proton and electron. Why is this so? [NCERT Exemplar] [HOTS]

**Ans.** Einstein's mass-energy equivalence gives  $E = mc^2$ . Thus the mass of an H-atom is  $m_p + m_e - \frac{B}{C^2}$  where B  $\approx$  13.6 eV is the binding energy. It is less than the sum of masses of a proton and an electron.

## Q. 3. When an electron falls from a higher energy to a lower energy level, the difference in the energies appears in the form of electromagnetic radiation. Why cannot it be emitted as other forms of energy? [NCERT Exemplar] [HOTS]

Ans. This is because electrons interact only electromagnetically.

## Q. 4. Would the Bohr formula for the H-atom remain unchanged if proton had a charge (+4/3)e and electron had a charge (-3/4)e, where $e = 1.6 \times 10^{-19}$ C? Give reasons for your answer. [NCERT Exemplar] [HOTS]

Ans. Yes, since the Bohr formula involves only the product of the charges.

Q. 5. Consider two different hydrogen atoms. The electron in each atom is in an excited state. Is it possible for the electrons to have different energies but the same orbital angular momentum according to the Bohr model?[NCERT Exemplar] [HOTS]

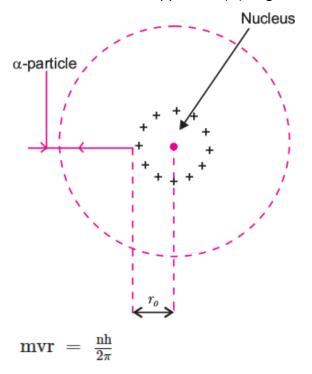
**Ans.** No, because according to Bohr model,  $En = -\frac{13.6}{n^2}$ , and electrons having different energies belong to different levels having different values of n. So, their angular momenta will be different, as  $mvr = \frac{nh}{2\pi}$ .

Q. 1. Define the distance of closest approach. An a-particle of kinetic energy 'K' is bombarded on a thin gold foil. The distance of the closest approach is 'r'. What will be the distance of closest approach for an a-particle of double the kinetic energy?

#### [CBSE Delhi 2017]

**Ans.** Distance of closest approach is the distance of charged particle from the centre of the nucleus, at which the entire initial kinetic energy of the charged particles gets converted into the electric potential energy of the system.

Distance of closest approach (r<sub>o</sub>) is given by



If 'K' is doubled,  $r_o$  becomes  $\mathbf{mvr} = \frac{\mathbf{nh}}{2\pi}$ 

### Q. 2. Write two important limitations of Rutherford nuclear model of the atom. [CBSE Delhi 2017]

Ans. Two important limitations of Rutherford Model are:

(i) According to Rutherford model, electron orbiting around the nucleus, continuously radiates energy due to the acceleration; hence the atom will not remain stable.

(ii) As electron spirals inwards; its angular velocity and frequency change continuously, therefore it should emit a continuous spectrum.

But an atom like hydrogen always emits a discrete line spectrum.

## Q. 3. Define ionization energy. How would the ionization energy change when electron in hydrogen atom is replaced by a particle of mass 200 times than that of the electron but having the same charge? [CBSE Central 2016]

**Ans.** The minimum energy required to free the electron from the ground state of the hydrogen atom is known as ionization energy.

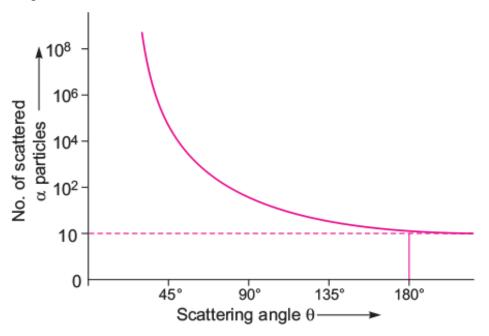
$$E_0 = rac{\mathrm{me}^4}{8 \ \varepsilon^2 h^2}, \ i.e., E_0 \propto m$$

Therefore, ionization energy will become 200 times.

Q. 4. In an experiment on a-particle scattering by a thin foil of gold, draw a plot showing the number of particles scattered versus the scattering angle  $\theta$ .

#### Why is it that a very small fraction of the particles are scattered at $\theta > 90^{\circ}$ ? [CBSE (F) 2013]

**Ans.** A small fraction of the alpha particles scattered at angle  $\theta > 90^\circ$  is due to the reason that if impact parameter 'b' reduces to zero, coulomb force increases, hence alpha particles are scattered at angle  $\theta > 90^\circ$ , and only one alpha particle is scattered at angle 180°.



Q. 5. Find out the wavelength of the electron orbiting in the ground state of hydrogen atom. [CBSE Delhi 2017]

Ans.

Radius of ground state of hydrogen atom,  $E_0 = rac{\mathrm{me}^4}{8 \ arepsilon^2 h^2}$  ,

According to de Broglie relation,  $2\pi r = n\lambda$ 

For ground state, n = 1

 $2 \times 3.14 \times 0.53 \times 10^{-10} = 1 \times \lambda$ 

 $:: \lambda = 3.32 \times 10^{-10} \text{m} = 3.32 \text{ Å}$ 

### Q. 6. When is H $\alpha$ line in the emission spectrum of hydrogen atom obtained? Calculate the frequency of the photon emitted during this transition. [CBSE North 2016]

Ans. The line with the longest wavelength of the Balmer series is called H $\alpha$ .

 $rac{1}{\lambda}=R\left(rac{1}{2^2}-rac{1}{n^2}
ight)$ 

where  $\lambda$  = wavelength

 $R = 1.097 \times 10^7 \text{ m}^{-1}$  (Rydberg constant)

When the electron jumps from the orbit with n = 3 to n = 2,

we have

$$rac{1}{\lambda} = R\left(rac{1}{2^2} - rac{1}{3^2}
ight) \qquad \Rightarrow \qquad rac{1}{\lambda} = rac{5}{36}R$$

The frequency of photon emitted is given by

$$u = rac{c}{\lambda} = c imes rac{5}{36}R$$

$$= 3 imes 10^8 imes rac{5}{36} imes 1.097 imes 10^7 \text{Hz} = 4.57 imes 10^{14} \text{Hz}$$

### Q. 7. Calculate the de-Broglie wavelength of the electron orbiting in the n = 2 state of hydrogen atom. [CBSE Central 2016]

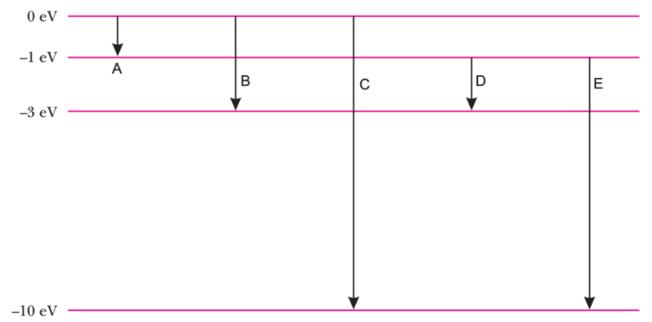
Ans. Kinetic Energy for the second state

$$E_k = rac{13.6 \ {
m eV}}{n^2} = rac{13.6 \ {
m eV}}{2^2} = rac{13.6 \ {
m eV}}{4} = 3.4 imes 1.6 imes 10^{-19} \ J$$

de Broglie wavelength  $\lambda = \frac{h}{\sqrt{2 \,\mathrm{mE}_k}}$ 

$$=rac{6.63 imes 10^{-34}}{\sqrt{2 imes 9.1 imes 10^{-31} imes 3.4 imes 1.6 imes 10^{-19}}}=0.067\,\mathrm{nm}$$





Which of the transitions belong to Lyman and Balmer series? Calculate the ratio of the shortest wavelengths of the Lyman and the Balmer series of the spectra. [CBSE Chennai 2015]

Ans. Transition C and E belong to Lyman series.

**Reason:** In Lyman series, the electron jumps to lowest energy level from any higher energy levels.

Transition B and D belong to Balmer series.

**Reason:** The electron jumps from any higher energy level to the level just above the ground energy level.

The wavelength associated with the transition is given by

$$\lambda = \frac{hc}{\Delta E}$$

Ratio of the shortest wavelength

$$egin{aligned} \lambda_L \, : \lambda_B &= rac{ ext{hc}}{\Delta E_L} \, : rac{ ext{hc}}{\Delta E_B} \ &= rac{1}{0 - (-10)} \, : rac{1}{0 - (-3)} = 3 \, : 10 \end{aligned}$$

### Q. 9. Show that the radius of the orbit in hydrogen atom varies as n2, where n is the principal quantum number of the atom. [CBSE Delhi 2015]

#### Ans. Hydrogen atom

Let *r* be the radius of the orbit of a hydrogen atom. Forces acting on electron are centrifugal force ( $F_c$ ) and electrostatic attraction ( $F_e$ )

At equilibrium,  $F_c = F_e$ 

 $rac{\mathrm{mv}^2}{r} = rac{1}{4\piarepsilon_0} \; rac{e^2}{r^2}$ 

According to Bohr's postulate

Q. 10. When the electron orbiting in hydrogen atom in its ground state moves to the third excited state, show how the de Broglie wavelength associated with it would be affected. [CBSE Ajmer 2015]

Ans.

We know,

de Broglie wavelength,  $\lambda = \frac{h}{p} = \frac{h}{mv}$ 

 $\Rightarrow \lambda \propto \frac{1}{v},$ Also  $v \propto \frac{1}{n}$   $\therefore \lambda \propto n$ 

.. de Broglie wavelength will increase.

## Q. 11. When an electron in hydrogen atom jumps from the third excited state to the ground state, how would the de Broglie wavelength associated with the electron change? Justify your answer. [CBSE Allahabad 2015]

**Ans.** de Broglie wavelength associated with a moving charge particle having a KE 'K' can be given as

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2 \,\mathrm{mK}}} \qquad \qquad \dots (i) \qquad \left[ K = \frac{1}{2} \mathrm{mv}^2 = \frac{p^2}{2m} \right]$$

The kinetic energy of the electron in any orbit of hydrogen atom can be given as

$$K = -E = -\left(\frac{13.6}{n^2} eV\right) = \frac{13.6}{n^2} eV$$
 ...(*ii*)

Let  $K_1$  and  $K_4$  be the KE of the electron in ground state and third excited state, where  $n_1 = 1$  shows ground state and  $n_2 = 4$  shows third excited state.

Using the concept of equation (i) & (ii), we have

$$rac{\lambda_1}{\lambda_4} = \sqrt{rac{K_4}{K_1}} = \sqrt{rac{n_1^2}{n_2^2}}$$
 $rac{\lambda_1}{\lambda_4} = \sqrt{rac{1^2}{4^2}} = rac{1}{4}$ 

$$\Rightarrow \qquad \lambda_1 = \frac{\lambda_4}{4}$$

*i.e.*, the wavelength in the ground state will decrease.

## Q. 12. The kinetic energy of the electron orbiting in the first excited state of hydrogen atom is 3.4 eV. Determine the de Broglie wavelength associated with it. [CBSE (F) 2015]

Ans. Kinetic energy of the electron in the first excited state is 3.4 eV, i.e.,

$$\frac{p^2}{2m} = 3.4 \text{ eV} = 3.4 \times 1.6 \times 10^{-19} \text{ J}$$

$$p^2 = 2 \times 9.1 \times 10^{-31} \times 3.4 \times 1.6 \times 10^{-19}$$

$$p = \sqrt{18.2 \times 3.4 \times 1.6 \times 10^{-50}}$$

$$= \sqrt{99.008} \times 10^{-25} = 9.95 \times 10^{-25}$$

de Broglie wavelength,  $\lambda = \frac{h}{p}$ 

$$= rac{6.63 imes 10^{-34}}{9.95 imes 10^{-25}} = 0.67 imes 10^{-9} = 0.67 ext{ nm}$$

### Short Answer Questions – I (OIQ)

Q. 1. In Bohr's theory of hydrogen atom, calculate the energy of the photon emitted during a transition of the electron from the first excited state to the ground state. Write in which region of the electromagnetic spectrum this transition lies.

**Ans.** The energy levels of hydrogen atom are given by

$$E_n\left(=-rac{\mathrm{Rhc}}{n^2}
ight)=rac{13.6}{n^2}\mathrm{eV}$$

For ground state, n=1

$$E_1 = -13.6 \text{ eV}$$

For first excited state, n = 2

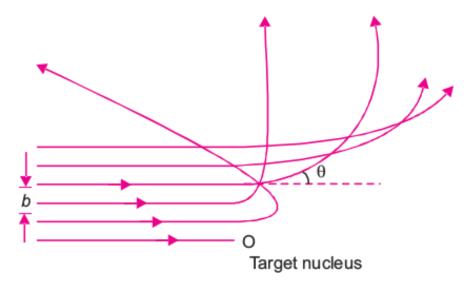
$$E_2 = -\frac{13.6}{4} = -3.4 \text{ eV}$$

.: Energy of photon emitted

 $hV = E_2 - E_1 = -3.4 - (-13.6) \text{ eV} = 10.2 \text{ eV}$ 

As transition from higher state to n=1 corresponds to Lyman series so the corresponding transition belongs to Lyman series.

Q. 2. The trajectories, traced by different  $\alpha$ -particles, in Geiger-Marsden experiment were observed as shown in the figure.



(i) What names are given to the symbols 'b' and ' $\theta$ ' shown here?

(ii) What can we say about the values of b for (i)  $\theta = 0^{\circ}$  (ii)  $\theta = p$  radians? [HOTS]

**Ans. (i)** The symbol 'b' represents impact parameter and ' $\theta$ ' represents the scattering angle.

(ii) (i) When  $\theta = 0^{\circ}$ , the impact parameter will be maximum and represent the atomic size.

(ii) When  $\theta = \pi$  radians, the impact parameter 'b' will be minimum and represent the nuclear size.

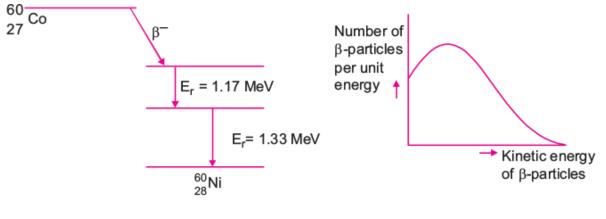
### Q. 3. Which is easier to remove: orbital electron from an atom or a nucleon from a nucleus? [HOTS]

**Ans.** It is easier to remove an orbital electron from an atom. The reason is the binding energy of orbital electron is a few electron-volts while that of nucleon in a nucleus is quite large (nearly 8 MeV). This means that the removal of an orbital electron requires few electron volt energy while the removal of a nucleon from a nucleus requires nearly 8 MeV energy.

### Q. 4. Answer the following questions.

### (i) Draw the energy level diagram showing the emission of b-particles followed by $\gamma$ -rays by a $^{60}_{27}Co$ nucleus.

(ii) Plot of distribution of KE of b-particles is shown in fig. (b).



(a) Energy level diagram

The energy spectrum of b-particles is continuous because an **antineutrino** is simultaneously emitted in  $\beta$ -decay; the total energy released in b-decay is shared by b-particle and the antineutrino so that momentum of system may remain conserved.

### Q. 5. What is the longest wavelength of photon that can ionize a hydrogen atom in its ground state? Specify the type of radiation.

<sup>(</sup>b) Energy distribution of β-particles

#### Ans.

Since, the energy of the incident photon  $= h 
u = rac{ ext{hc}}{\lambda} = 13.6 ext{ eV}$ 

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{13.6 \times 1.6 \times 10^{-19}}$$
$$\lambda = 0.910 \times 10^{-10} \text{ m}$$

This radiation is in ultraviolet region.

Q. 1. (a) Using Bohr's second postulate of quantization of orbital angular momentum show that the circumference of the electron in the n<sup>th</sup> orbital state in hydrogen atom is n times the de Broglie wavelength associated with it. [CBSE (F) 2017]

(b) The electron in hydrogen atom is initially in the third excited state. What is the maximum number of spectral lines which can be emitted when it finally moves to the ground state?

OR

(a) State Bohr's quantization condition for defining stationary orbits. How does de Broglie hypothesis explain the stationary orbits?

(b) Find the relation between the three wavelengths  $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$  from the energy level diagram shown below. [CBSE Delhi 2016]

**Ans. (a)** Only those orbits are stable for which the angular momentum of revolving electron is an integral multiple of  $\left(\frac{h}{2\pi}\right)$  where h is the planck's constant.

According to Bohr's second postulate

 $\mathrm{mvr}_n = n rac{h}{2\pi} \qquad \Rightarrow \qquad 2\pi r_n = rac{\mathrm{nh}}{\mathrm{mv}}$ 

But  $\frac{h}{mv} = \frac{h}{p} = \lambda$  (By de Broglie hypothesis)

$$\therefore \qquad 2\pi r_n = n\lambda$$

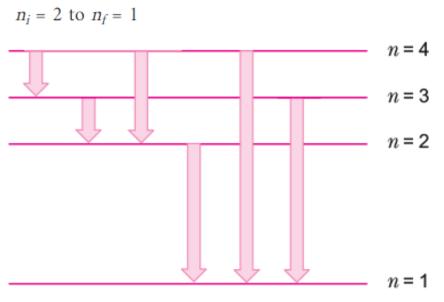
(b) For third excited state, n = 4

For ground state, n = 1

Hence possible transitions are

$$n_i = 4$$
 to  $n_f = 3, 2, 1$ 

 $n_i = 3$  to  $n_f = 2, 1$ 



Total number of transitions = 6

- $E_C E_B = \frac{\mathrm{hc}}{\lambda_1} \qquad \dots (1)$
- $E_B E_A = \frac{\mathrm{hc}}{\lambda_2}$  ...(2)

$$E_C - E_A = \frac{\mathrm{hc}}{\lambda_3} \qquad \dots (3)$$

Adding (1) and (2), we have

$$E_C - E_A = \frac{\mathrm{hc}}{\lambda_1} + \frac{\mathrm{hc}}{\lambda_2} \qquad \dots (4)$$

From (3) and (4), we have

$$rac{\mathrm{hc}}{\lambda_3} = rac{\mathrm{hc}}{\lambda_1} + rac{\mathrm{hc}}{\lambda_2} \quad \Rightarrow \quad rac{1}{\lambda_3} = rac{1}{\lambda_1} + rac{1}{\lambda_2}$$
 $\lambda_3 = rac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$ 

Q. 2. Answer the following questions.

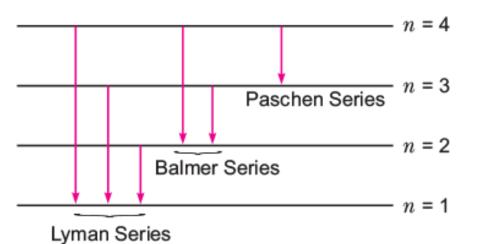
(i) State Bohr postulate of hydrogen atom that gives the relationship for the frequency of emitted photon in a transition.

## (ii) An electron jumps from fourth to first orbit in an atom. How many maximum number of spectral lines can be emitted by the atom? To which series these lines correspond? [CBSE (F) 2016]

**Ans. (i)** Bohr's third postulate: It states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states. The frequency of the emitted photon is given by

 $hv = E_i - E_f$ 

Where  $E_i$  and  $E_f$  are the energies of the initial and final states and  $E_i > E_f$ .



(ii) Electron jumps from fourth to first orbit in an atom

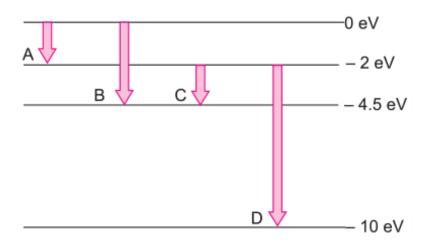
: Maximum number of spectral lines can be  ${}^4C_2 = \frac{4!}{2!2!} = \frac{4 \times 3}{2} = 6$ 

In diagram, possible way in which electron can jump (above).

The line responds to Lyman series ( $e^-$  jumps to 1<sup>st</sup> orbit), Balmer series ( $e^-$  jumps to 2<sup>nd</sup> orbit), Paschen series ( $e^-$  jumps to 3<sup>rd</sup> orbit).

### Q. 3. The energy levels of a hypothetical atom are shown below. Which of the shown transitions will result in the emission of a photon of wavelength 275 nm?

Which of these transitions correspond to emission of radiation of (i) maximum and (ii) minimum wavelength? [CBSE Delhi 2011]



Ans. Energy of photon wavelength 275 nm

$$E = \frac{\text{hc}}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{275 \times 10^{-9} \times 1.6 \times 10^{-19}} \text{ eV} = 4.5 \text{ eV}.$$

This corresponds to transition 'B'.

- i.  $\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}$  For maximum wavelength  $\Delta E$  should be minimum. This corresponds to transition *A*.
- ii. For minimum wavelength  $\Delta E$  should be maximum. This corresponds to transition D.

## Q. 4. The energy level diagram of an element is given below. Identify, by doing necessary calculations, which transition corresponds to the emission of a spectral line of wavelength 102.7 nm. [CBSE Delhi 2008]

Ans.

$$\Delta E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{102.7 \times 10^{-9}} J$$
$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{102.7 \times 10^{-9} \times 1.6 \times 10^{-19}} eV$$
$$= \frac{66 \times 3000}{1027 \times 16} = 12.04 eV$$
Now,  $\Delta E = |-13.6 - (-1.50)|$ 
$$= 12.1 eV$$

Hence, transition shown by arrow D corresponds to emission of  $\lambda$  = 102.7 nm.

Q. 5. Using de Broglie's hypothesis, explain with the help of a suitable diagram, Bohr's second postulate of quantisation of energy levels in a hydrogen atom. [CBSE (AI) 2011, Patna 2015]

#### OR

Show mathematically how Bohr's postulate of quantisation of orbital angular momentum in hydrogen atom is explained by de-Broglie's hypothesis. [CBSE East 2016]

Ans. According to de Broglie's hypothesis,

$$\lambda = \frac{h}{\mathrm{mv}} \qquad \dots (i)$$

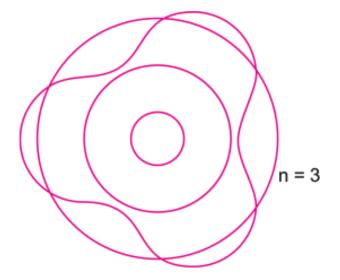
According to de Broglie's condition of stationary orbits, the stationary orbits are those which contain complete de Broglie wavelength.

$$2\pi r = n\lambda$$
 ...(*ii*)

Substituting value of  $\lambda$  from (ii) in (i), we get

$$2\pi r = n \frac{h}{\mathrm{mv}}$$
  
 $\Rightarrow \qquad \mathrm{mvr} = n \frac{h}{2\pi} \qquad \dots (iii)$ 

This is Bohr's postulate of quantisation of energy levels.



Q. 6. Determine the distance of closest approach when an alpha particle of kinetic energy 4.5 MeV strikes a nucleus of Z = 80, stops and reverses its direction. [CBSE Ajmer 2015]

**Ans.** Let r be the centre to centre distance between the alpha particle and the nucleus (Z = 80). When the alpha particle is at the stopping point, then

$$K = \frac{1}{4\pi\varepsilon_0} \frac{(\text{Ze})(2e)}{r}$$
  
or  $r = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\text{Ze}^2}{K}$ 
$$= \frac{9 \times 10^9 \times 2 \times 80 \ e^2}{4.5 \ \text{MeV}} = \frac{9 \times 10^9 \times 2 \times 80 \times (1.6 \times 10^{-19})^2}{4.5 \times 10^6 \times 1.6 \times 10^{-19} \ J}$$
$$= \frac{9 \times 160 \times 1.6}{4.5} \times 10^{-16} = 512 \times 10^{-16} m$$
$$= 5.12 \times 10^{-14} \text{ m}$$

### Q. 7. A 12.3 eV electron beam is used to bombard gaseous hydrogen at room temperature. Upto which energy level the hydrogen atoms would be excited?

Calculate the wavelengths of the second member of Lyman series and second member of Balmer series. [CBSE Delhi 2014]

Ans. The energy of electron in the nth orbit of hydrogen atom is

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

When the incident beam of energy 12.3 eV is absorbed by hydrogen atom. Let the electron jump from n = 1 to n = n level.

$$E = E_n - E_1$$

$$12.3 = -\frac{13.6}{n^2} - \left(-\frac{13.6}{1^2}\right)$$

$$\Rightarrow \quad 12.3 = 13.6 \left[1 - \frac{1}{n^2}\right] \qquad \Rightarrow \qquad \frac{12.3}{13.6} = 1 - \frac{1}{n^2}$$

$$\Rightarrow \quad 0.9 = 1 - \frac{1}{n^2} \qquad \Rightarrow \qquad n^2 = 10 \qquad \Rightarrow \qquad n = 3$$

That is the hydrogen atom would be excited upto second excited state.

#### For Lyman Series

$$\begin{split} &\frac{1}{\lambda} = R \left[ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right] \\ \Rightarrow \quad &\frac{1}{\lambda} = 1.097 \times 10^7 \left[ \frac{1}{1} - \frac{1}{9} \right] \quad \Rightarrow \quad &\frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{8}{9} \\ \Rightarrow \quad &\lambda = \frac{9}{8 \times 1.097 \times 10^7} = 1.025 \times 10^{-7} = 102.5 \,\mathrm{mm} \end{split}$$

For Balmer Series

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left[ \frac{1}{4} - \frac{1}{16} \right] \qquad \Rightarrow \qquad \frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{3}{16}$$
$$\Rightarrow \lambda = 4.86 \times 10^{-7} \text{m} \qquad \Rightarrow \qquad \lambda = 486 \text{ nm}$$

Q. 8. The ground state energy of hydrogen atom is -13.6 eV. If an electron makes a transition from an energy level -1.51 eV to -3.4 eV, calculate the wavelength of the spectral line emitted and name the series of hydrogen spectrum to which it belongs.

[CBSE (AI) 2017]

**Ans.** Energy difference = Energy of emitted photon

 $= E_2 - E_1$ 

 $\begin{aligned} \frac{1}{\lambda} &= 1.097 \times 10^7 \left[ \frac{1}{4} - \frac{1}{16} \right] \implies \frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{3}{16} \\ &= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.89 \times 1.6 \times 10^{-19}} = \frac{19.8}{3.024} \times 10^{-7} \\ &= 6.548 \times 10^{-7} \text{ m} = 6548 \text{ Å} \end{aligned}$ 

This wavelength belongs to Balmer series of hydrogen spectrum.

Q. 9. A hydrogen atom initially in its ground state absorbs a photon and is in the excited state with energy 12.5 eV. Calculate the longest wavelength of the radiation emitted and identify the series to which it belongs.

### [Take Rydberg constant $R = 1.1 \times 10^7 \text{ m}^{-1}$ ] [CBSE East 2016]

**Ans.** Let n<sub>i</sub> and n<sub>f</sub> are the quantum numbers of initial and final states, then we have

$$rac{1}{\lambda_{ ext{max}}} = R\left(rac{1}{n_f^2} - rac{1}{n_i^2}
ight)$$

The energy of the incident photon = 12.5 eV.

Energy of ground state = -13.6 eV

 $\therefore$  Energy after absorption of photon can be -1.1 eV.

This means that electron can go to the excited state  $n_i = 3$ . It emits photon of maximum wavelength on going to  $n_f = 2$ , therefore,

$$\frac{1}{\lambda_{\max}} = \left\{ \frac{1}{2^2} - \frac{1}{3^2} \right\} R$$
$$\lambda_{\max} = \frac{36}{5R} = \frac{36}{5 \times 1.1 \times 10^7} = 6.555 \times 10^{-7} \text{ m} = 6555 \text{ Å}$$

It belongs to Balmer Series.

Q. 10. The short wavelength limit for the Lyman series of the hydrogen spectrum is 913.4 Å. Calculate the short wavelength limit for Balmer series of the hydrogen spectrum. [CBSE (AI) 2017]

Ans.

 $\lambda_{ ext{max}} = rac{36}{5R} = rac{36}{5 imes 1.1 imes 10^7}$ 

 $\lambda_L = \frac{1}{R} = 913.4$  Å

For short wavelength of Lyman series,  $n_1 = 1$ ,  $n_2 = \infty$ 

$$\therefore \qquad rac{1}{\lambda_L} \ = \ R\left(rac{1}{1^2} - rac{1}{\infty}
ight) \ = \ R$$

For short wavelength of Balmer series,  $n_1 = 2$ ,  $n_2 = \infty$ 

$$\lambda_L = rac{1}{R} = 913.4 \mbox{ \AA}$$
  
 $\therefore \quad \lambda_B = rac{4}{R} = 4 imes 913.4 \mbox{ \AA} = 3653.6 \mbox{ \AA}$ 

## Q. 11. A 12.5 eV electron beam is used to excite a gaseous hydrogen atom at room temperature. Determine the wave lengths and the corresponding series of the lines emitted. [CBSE (AI) 2017]

**Ans.** It is given that the energy of the electron beam used to bombard gaseous hydrogen at room temperature is 12.5 eV.

Also, the energy of the gaseous hydrogen in its ground state at room temperature is – 13.6 eV.

When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen becomes -13.6 + 12.5 eV = -1.1 eV.

Orbital energy related to orbit level (n) is

$$E = rac{-13.6}{(n)^2} \mathrm{eV}$$

For n = 3,

$$E = \frac{-13.6}{(3)^2} \,\,\mathrm{eV} = \frac{-13.6}{9} \,\,\mathrm{eV} = -1.5 \,\,\mathrm{eV}$$

This energy is approximately equal to the energy of gaseous hydrogen.

This implies that the electron has jumped from n = 1 to n = 3 level.

During its de-excitation, electrons can jump from n = 3 to n = 1 directly, which forms a line of the Lyman series of the hydrogen spectrum.

Relation for wave number for the Lyman series is

$$rac{1}{\lambda}=R\left[rac{1}{1^2}-rac{1}{n^2}
ight]$$

For first member n = 3

$$\dot{\cdot} \qquad rac{1}{\lambda_1} = R\left[rac{1}{1^2} - rac{1}{(3)^2}
ight] = R\left[rac{1}{1} - rac{1}{9}
ight]$$

 $\frac{1}{\lambda_1} = 1.097 \times 10^7 \left[\frac{9-1}{9}\right] \text{ (where Rydberg constant } R = 1.097 \times 10^7 \text{ m}^{-1}\text{)}$ 

$$\therefore \quad rac{1}{\lambda_1} = 1.097 imes 10^7 imes rac{8}{9} \quad \Rightarrow \quad \lambda_1 = 1.025 imes 10^{-7} \ \mathrm{m}$$

For n = 3,

$$\begin{array}{lll} & \cdot & \frac{1}{\lambda_2} & = & R \left[ \frac{1}{1^2} - \frac{1}{(2)^2} \right] = & R \left[ \frac{1}{1} - \frac{1}{4} \right] \\ & \cdot & \frac{1}{\lambda_2} & = & 1.097 \times 10^7 \left[ \frac{4-1}{4} \right] \text{ (where Rydberg constant } R = & 1.097 \times 10^7 \text{ m}^{-1} \text{)} \\ & \cdot & \frac{1}{\lambda_2} = & 1.097 \times 10^7 \times \frac{3}{4} \qquad \Rightarrow \qquad \lambda_2 = & 1.215 \times 10^7 \text{ m} \end{array}$$

Relation for wave number for the Balmer series is

 $\frac{1}{\lambda} = R \left[ \frac{1}{2^2} - \frac{1}{n^2} \right]$ 

For first member, n = 3

$$\therefore \quad \frac{1}{\lambda_3} = R \left[ \frac{1}{2^2} - \frac{1}{3^2} \right] = 1.097 \times 10^7 \times \left[ \frac{1}{4} - \frac{1}{9} \right]$$

$$\Rightarrow \qquad \lambda_3 = 6.56 \times 10^{-7} \text{ m}$$

### Short Answer Questions – II (OIQ)

Q. 1. Calculate the ratio of energies of photons produced due to transition of electron of hydrogen atom from its

(i) Second permitted energy level to the first level, and(ii) Highest permitted energy level to the second permitted level.

#### Ans. (i)

Energy of electron in permitted level  $E_n = \frac{\text{Rhc}}{n^2}$ 

When an electron jumps from the second to the first permitted energy level,

Energy of photon = 
$$E_{2-1} = \operatorname{Rhc}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}\operatorname{Rhc}$$

(ii) When an electron jumps from the highest permitted level  $(n = \infty)$  to the second permitted level (n=2)

$$E_{\infty - 2} = \operatorname{Rhc}\left(rac{1}{2^2} - rac{1}{\infty}
ight) = rac{\operatorname{Rhc}}{4}$$

:. Ratio 
$$\frac{E_{2-1}}{E_{\infty-2}} = \frac{3 \operatorname{Rhc} / 4}{\operatorname{Rhc} / 4} = \frac{3}{1}$$
; Ratio = 3 : 1

Q. 2. The spectrum of a star in the visible and the ultraviolet region was observed and the wavelength of some of the lines that could be identified were found to be:

824 Å, 970 Å, 1120 Å, 2504 Å, 5173 Å, 6100 Å

Which of these lines cannot belong to hydrogen atom spectrum? (Given Rydberg constant R = 1.03 × 10<sup>7</sup> m<sup>-1</sup> and  $\frac{1}{R}$  = 970 Å. Support your answer with suitable calculations.

**Ans.** For hydrogen atom, the wave number (i.e., reciprocal of wavelength) of the emitted radiation is given by

$$ar{
u} = rac{1}{\lambda} = R\left(rac{1}{n_2^2} - rac{1}{n_1^2}
ight)$$
 $\therefore \qquad \lambda = rac{rac{1}{R}}{\left(rac{1}{n_2^2} - rac{1}{n_1^2}
ight)} = rac{970 {
m \AA}}{\left(rac{1}{n_2^2} - rac{1}{n_1^2}
ight)}$ 

For Lyman series of hydrogen spectrum, we take  $n_2=1$ . Hence the permitted values of  $\lambda$  can be given as:

$$\lambda = \frac{970 \text{\AA}}{3/4}, \frac{970 \text{\AA}}{8/9}, \frac{970 \text{\AA}}{15/16} \dots \frac{970 \text{\AA}}{1} \quad (\text{taking } n_1 = 2, 3, 4, \dots, \infty)$$
$$= 1293.3 \text{\AA}, 1091 \text{\AA}, 1034.6 \text{\AA}, \dots, 970 \text{\AA}$$

For Balmer series of hydrogen spectrum, we take  $n_2 = 2$ . Hence the possible values of  $\lambda$  can be given as:

Hence  $\lambda = 824$  Å, 1120 Å, 2504 Å, 6100 Å, of the given lines, cannot belong to the hydrogen atom spectrum.

Q. 1. Draw a schematic arrangement of Geiger-Marsden experiment for studying a-particle scattering by a thin foil of gold. Describe briefly, by drawing trajectories of the scattered a-particles. How this study can be used to estimate the size of the nucleus?

[CBSE Delhi 2010]

### OR

### Describe Geiger-Marsden experiment. What are its observations and conclusions?

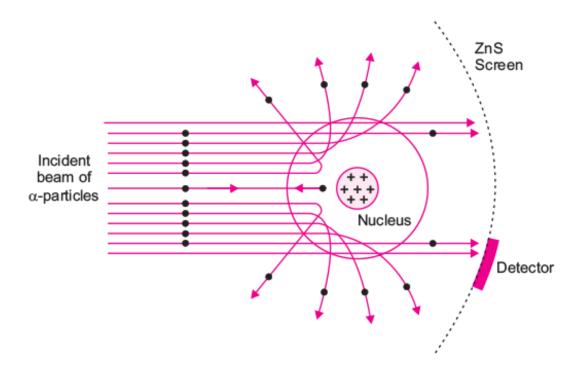
**Ans.** At the suggestion of Rutherford, in 1911, H. Geiger, and E. Marsden performed an important experiment called Geiger-Marsden experiment (or Rutherford's scattering experiment). It consists of

- 1. **Source of** a**-particles:** The radioactive source polonium emits high energetic alpha (a-) particles. Therefore, polonium is used as a source of a-particles. This source is placed in an enclosure containing a hole and a few slits *A*<sub>1</sub>, *A*<sub>2</sub>, ..., etc., placed in front of the hole. This arrangement provides a fine beam of a-particles.
- Thin gold foil: It is a gold foil\* of thickness nearly 10<sup>-6</sup> m, a-particles are scattered by this foil. The foil taken is thin to avoid multiple scattering of a-particles, *i.e.*, to ensure that a-particle be deflected by a single collision with a gold atom.
- 3. Scintillation counter: By this the number of a-particles scattered in a given direction may be counted. The entire apparatus is placed in a vacuum chamber to prevent any energy loss of a-particles due to their collisions with air molecules.

**Method:** When a-particle beam falls on gold foil, the a-particles are scattered due to collision with gold atoms. This scattering takes place in all possible directions. The number of a-particles scattered in any direction is counted by scintillation counter.

### **Observations and Conclusions**

- i. Most of a-particles pass through the gold foil undeflected. This implies that *"most part of the atom is hollow."*
- ii. a-particles are scattered through all angles. Some a-particles (nearly 1 in 2000), suffer scattering through angles more than 90°, while a still smaller number (nearly 1 in 8000) retrace their path. This implies that when fast moving positively charged a-particles come near gold-atom, then a few of them experience such a strong repulsive force that they turn back. On this basis Rutherford concluded that whole of positive charge of atom is concentrated in a small central core, called the nucleus.



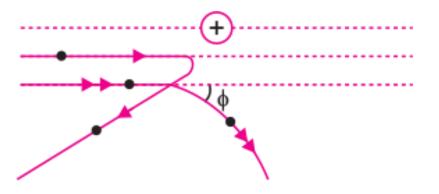
The distance of closest approach of a-particle gives the estimate of nuclear size. If Ze is charge of nucleus,  $E_k$ -kinetic energy of a particle, 2e-charge on a-particle, the size of nucleus  $r_0$  is given by

$$r_0$$
 is given by  $E_k = \frac{1}{4\pi\varepsilon_0} \frac{(\text{Ze})(2e)}{r_0} \qquad \Rightarrow \qquad r_0 = \frac{1}{4\pi\varepsilon_0} \frac{2\,\text{Ze}^2}{E_k}$ 

Calculations show that the size of nucleus is of the order of  $10^{-14}$  m, while size of atom is of the order of  $10^{-10}$ m; therefore the size of nucleus is about

$$\frac{10^{-14}}{10^{-10}} = \frac{1}{10,000}$$
 times the size of atom.

(iii) The negative charges (electrons) do not influence the scattering process. This implies that nearly whole mass of atom is concentrated in nucleus.



Q. 2. Using the postulates of Bohr's model of hydrogen atom, obtain an expression for the frequency of radiation emitted when atom make a transition from the higher energy state with quantum number ni to the lower energy state with quantum number  $n_f (n_f < n_i)$ . [CBSE (AI) 2013, (F) 2012, 2011]

#### OR

Using Bohr's postulates, obtain the expression for the total energy of the electron in the stationary states of the hydrogen atom. Hence draw the energy level diagram showing how the line spectra corresponding to Balmer series occur due to transition between energy levels. [CBSE Delhi 2013, Guwahati 2015]

#### OR

Using Rutherford model of the atom, derive the expression for the total energy of the electron in hydrogen atom. What is the significance of total negative energy possessed by the electron? [CBSE (AI) 2014]

**Ans.** Suppose m be the mass of an electron and v be its speed in nth orbit of radius r. The centripetal force for revolution is produced by electrostatic attraction between electron and nucleus.

 $\frac{\mathrm{m}\mathbf{v}^2}{r} = \frac{1}{4\pi\varepsilon_0} \frac{(Ze)(e)}{r^2} \qquad \qquad \left[ \text{from Rutherford model} \right] \dots (i)$ 

or,  $\mathbf{mv}^2 = \frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{r}$ 

So, Kinetic energy  $[K] = \frac{1}{2}mv^2$ 

$$K = rac{1}{4\pi\,arepsilon_0}\,rac{Z\,e^2}{2r}$$

Potential energy  $= \frac{1}{4\pi \varepsilon_0} \frac{(Ze)(-e)}{r} = -\frac{1}{4\pi \varepsilon_0} \frac{Ze^2}{r}$ 

Total energy, E = KE + PE

$$= \frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{2r} + \left(-\frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{r}\right) = -\frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{r}$$

For *n*th orbit, *E* can be written as  $E_n$ 

so, 
$$E_n = -\frac{1}{4\pi \varepsilon_0} \frac{Z e^2}{2r_n}$$
 ...(*ii*)

Negative sign indicates that the electron remains bound with the nucleus (or electronnucleus form an attractive system)

From Bohr's postulate for quantization of angular momentum.

$$\operatorname{mvr} = rac{n\,h}{2\pi} \quad \Rightarrow \quad v = rac{\mathrm{nh}}{2\pi\,\mathrm{mr}}$$

Substituting this value of v in equation (i), we get

$$\frac{m}{r} \left[ \frac{n h}{2\pi \operatorname{mr}} \right]^2 = \frac{1}{4\pi \varepsilon_0} \frac{Z e^2}{r^2} \quad \text{or} \quad r = \frac{\varepsilon_0 h^2 n^2}{\pi m \operatorname{Ze}^2}$$
or,  $r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m \operatorname{Ze}^2} \quad \dots (iii)$ 

For Bohr's radius, n = 1, *i.e.*, for K shell  $r_B = \frac{\varepsilon_0 h^2}{\pi \, \text{Zme}^2}$ 

Substituting value of  $r_n$  in equation (*ii*), we get

$$\begin{split} E_n =& -\frac{1}{4\pi \varepsilon_0} \frac{\mathrm{Ze}^2}{2\left(\frac{\varepsilon_0 h^2 n^2}{\pi \ \mathrm{mZ} \ e^2}\right)} =& -\frac{m Z^2 e^4}{8 \varepsilon_0 h^2 n^2} \\ \text{or,} \qquad E_n =& -\frac{Z^2 \operatorname{Rhc}}{n^2}, \text{ where } R = \frac{\mathrm{me}^4}{8 \varepsilon_0^2 \operatorname{ch}^3} \end{split}$$

R is called Rydberg constant.

For hydrogen atom Z=1,  $E_n = \frac{-\text{Rhc}}{n^2}$ 

If  $n_i$  and  $n_f$  are the quantum numbers of initial and final states and  $E_i \& E_f$  are energies of electron in H-atom in initial and final state, we have

$$E_i = rac{- ext{Rhc}}{n_i^2} ext{ and } E_f = rac{- ext{Rch}}{n_f^2}$$

If V is the frequency of emitted radiation, we get

$$u = rac{E_i - E_f}{h}$$
 $u = rac{-\operatorname{Rc}}{n_i^2} - \left(rac{-\operatorname{Rc}}{n_f^2}\right) \quad \Rightarrow \quad 
u = \operatorname{Rc}\left[rac{1}{n_f^2} - rac{1}{n_i^2}
ight]$ 

For Balmer series  $n_f = 2$ , while  $n_i = 3, 4, 5, \dots \infty$ .

# Q. 3. Derive the expression for the magnetic field at the site of a point nucleus in a hydrogen atom due to the circular motion of the electron. Assume that the atom is in its ground state and give the answer in terms of fundamental constants. [CBSE Sample Paper 2016]

**Ans.** To keep the electron in its orbit, the centripetal force on the electron must be equal to the electrostatic force of attraction. Therefore,

 $\frac{\mathrm{m}\mathbf{v}^2}{r} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} \qquad \dots (i) \text{ (For H atom, } Z = 1)$ 

From Bohr's quantisation condition

$$\mathrm{mvr} = rac{\mathrm{nh}}{2\pi} \qquad \Rightarrow \qquad v = rac{\mathrm{nh}}{2\pi \mathrm{\,mr}}$$

For K shell, n=1

$$v = \frac{\mathrm{nh}}{2\pi \mathrm{\,mr}}$$
 ...(*ii*)

From (i) and (ii), we have

$$\frac{m}{r}\left(\frac{h}{2\pi\,\mathrm{mr}}
ight)^2 = \frac{1}{4\piarepsilon_0}\,\frac{e^2}{r^2}$$

$$egin{array}{ll} rac{m}{r} rac{h^2}{4\pi^2 m^2 r^2} &= rac{1}{4\piarepsilon_0} rac{e^2}{r^2} &\Rightarrow \pi\,\mathrm{rme}^2 = arepsilon_0 h^2 \ r &= rac{arepsilon_0 h^2}{\pi\,\mathrm{me}^2} & ...(iii) \end{array}$$

From (ii) and (iii), we have

$$v = rac{h imes \pi \, \mathrm{me}^2}{2 \pi m arepsilon_0 h^2} = rac{e^2}{2 arepsilon_0 h}$$

Magnetic field at the centre of a circular loop  $B = \frac{\mu_0 I}{2r}$ 

$$I = rac{\operatorname{Ch} \operatorname{arg} e}{\operatorname{Time}} ext{ and } \operatorname{Time} = rac{2\pi r}{v}$$
  
 $\therefore \qquad I = rac{\operatorname{ev}}{2\pi r}$ 

So,  $B = \frac{\mu_0 \operatorname{ev}}{2r \times 2\pi r} = \frac{\mu_0 \operatorname{ev}}{4\pi r^2}$  ....(*iv*)

From (*ii*), (*iii*) (*iv*), we have

$$B = \frac{\mu_0 e.e^2 \pi^2 m^2 e^4}{2\varepsilon_0 h \times 4\pi \times \varepsilon_0^2 h^4} \qquad \Rightarrow \qquad B = \frac{\mu_0 e^7 \pi m^2}{8\varepsilon_0^3 h^5}$$