Very Short Answer Type Questions

[1 Mark]

Que 1. If a point P is 17 cm from the centre of a circle of radius 8 cm, then find the length of the tangent drawn to the circle from point P.

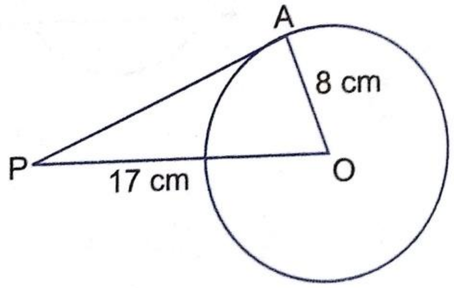

Fig. 8.4

Sol. OA ⊥ PA (∵ radius is ⊥ to tangent at point of contact.)

∴ In ΔOAP, we have

$PO^2 = PA^2 + AO^2 \Rightarrow (17)^2 = (PA)^2 + (8)^2$

$(PA)^2 = 289 - 64 = 225 \Rightarrow PA = \sqrt{225} = 15$

Hence, the length of the tangent from point P is 15 cm.

Que 2. The lenght of the tangent to a circle from a point P, which is 25 cm away from the centre, is 24 cm. What is the radius of the circle?

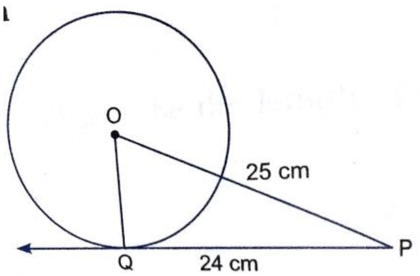

Fig. 8.5

Sol. $\because OQ \perp PQ$

$\therefore PQ^2 + QO^2 = OP^2$

$\Rightarrow 25^2 = OQ^2 + 24^2$

or $OQ = \sqrt{625 - 576}$

$= \sqrt{49} = 7\ cm$

Que 3. In Fig. 8.6, ABCD is a cyclic quadrilateral. If ∠BAC = 50° and ∠DBC = 60° then find ∠BCD.

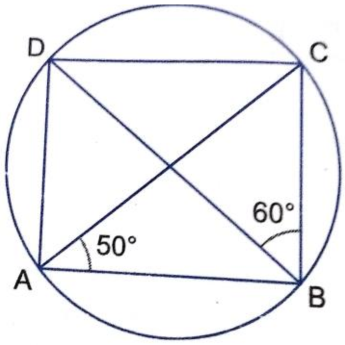

Fig. 8.6

Sol. Here $\angle BDC = \angle BAC = 50°$ (angles in same segment are equal)

In $\Delta BCD, we\ have$

$$\angle BCD = 180° - (\angle BDC + \angle DBC)$$
$$= 180°(50° + 60°) = 70°$$

Que 4. In Fig. 8.7, the quadrilateral ABCD circumscribes a circle with centre O. If$\angle AOB = 115°$, then find ∠COD.

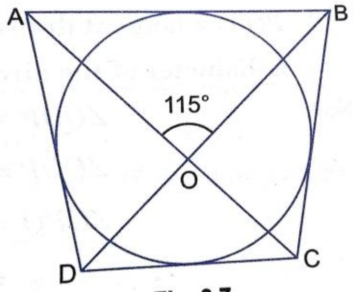

Fig. 8.7

Sol. ∵ $\angle AOB = \angle COD$ (Vertically opposite angle)

∴ $\angle COD = 115°$

Que 5. In Fig. 8.8, ΔABC is circumscribing a circle. Find the length of BC.

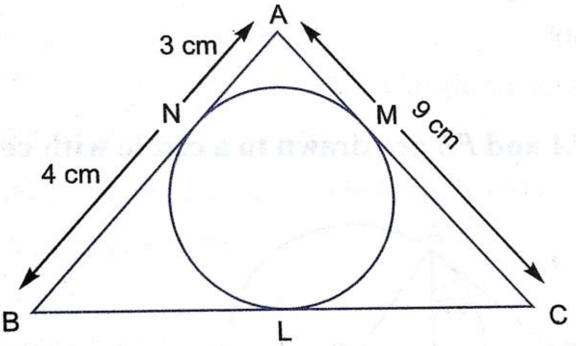

Fig. 8.8

Sol. AN = AM = 3 cm [Tangents drawn from an external point]

BN = BL = 4 cm [Tangents drawn from an external point]

CL = CM = AC – AM = 9 – 3 = 6 cm

$\Rightarrow\ BC = BL + CL = 4 + 6 = 10\ cm.$

Que 6. In Fig. 8.9, O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find $\angle POQ$.

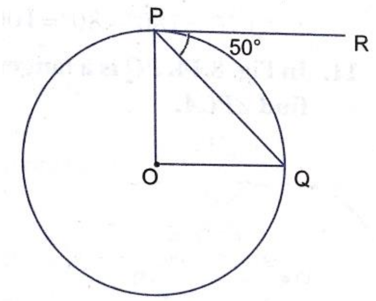

Fig. 8.9

Sol. $\angle OPQ = 90° - 50° = 40°$

OP = OQ [Radii of a circle]

$\Rightarrow$ $\angle OPQ = \angle OQP = 40°$

(Equal opposite sides have equal opposite angles)

$\therefore$ $\angle POQ = 180° - 40° - 40° = 100°$

Que 7. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

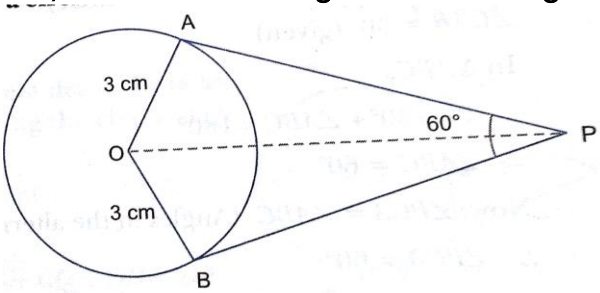

Fig. 8.10

Sol. In Fig. 8.10

$\Delta AOP \cong \Delta BOP$ (By SSS congruence criterion)

$\Rightarrow\quad \angle APO = \angle BPO = \frac{60°}{2} = 30°$

In $\Delta AOP, OA \perp AP$

$\therefore\quad \tan 30° = \frac{OA}{AP} = \frac{1}{\sqrt{3}} = \frac{3}{AP}$

$\Rightarrow$ AP = $3\sqrt{3}\ cm$

Short Answer Type Questions – I

[2 marks]

State true or false for each of the following and justify your answer (Q.1 to 3)

Que 1. AB is a diameter of a circle and AC is its chord such that $\angle BAC = 30°$ If the tangent at C intersects AB extended at D, then BC = BD.

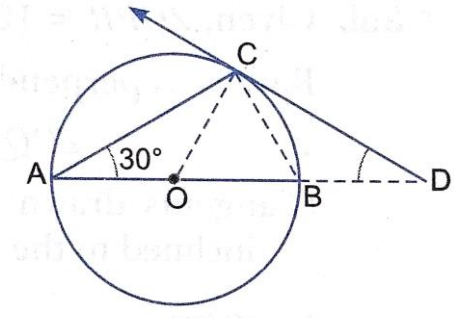

Fig. 8.16

Sol. True, Join OC,

$\angle ACB = 90°$ (Angle in semi-circle)

$\therefore \angle OBC = 180° - (90° + 30°) = 60°$

Since, OB = OC = radii of same circle [Fig. 8.16]

$\therefore \angle OBC = \angle OCB = 60°$

Also, $\angle OCD = 90°$

$\Rightarrow \angle BCD = 90° - 60° = 30°$

Now, $\angle OBC = \angle BCD + \angle BDC$ (Exterior angle property)

$\Rightarrow 60° = 30° + \angle BDC$ $\Rightarrow \angle BDC = 30°$

$\because \angle BCD = \angle BDC = 30°$ $\therefore BC = BD$

Que 2. The length of tangent from an external point P on a circle with centre O is always less than OP.

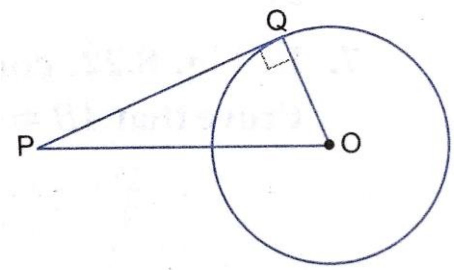

Fig. 8.17

Sol. True. Let PQ be the tangent from the external point P.

Then ΔPQO is always a right angled triangle with OP as the hypotenuse.

So, PQ is always less than OP.

Que 3. If angle between two tangents drawn from a point to a circle of radius 'a' and centre O is 90°, then OP = $a\sqrt{2}$.

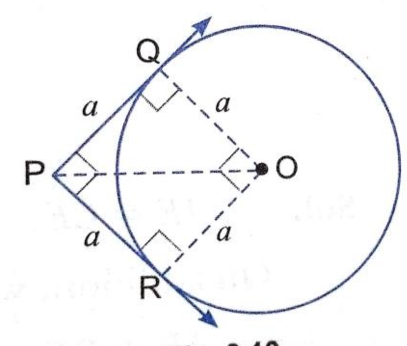

Fig. 8.18

Sol. True, let PQ and PR be the tangents

since $\angle P = 90°, so\ \angle QOR = 90°$

Also, OR = OQ = a

∴ $PQOR$ is a square

$\Rightarrow\ OP = \sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$

Que 4. In Fig. 8.19, PA and PB are tangents to the circle drawn from an external point P. CD is the third tangent touching the circle at Q. If PA = 15 cm, find the perimeter of ΔPCD.

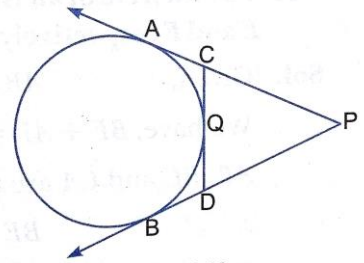

Fig. 8.19

Sol. ∵ PA and PB are tangent from same external point

∴ PA = PB = 15 cm

Now, perimeter of $\Delta PCD = PC + CD + DP = PC + CQ + QD + DP$

$= PC + CA + DB + DP$

$= PA + PB = 15 + 15 = 30\ cm$

Que 5. Prove that the line segment joining the points of contact of two parallel tangent of a circle, passes through its centre.

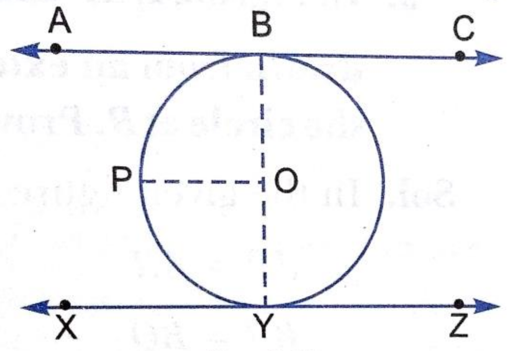

Fig. 8.20

Sol. Let the tangent to a circle with centre O be ABC and XYZ.

Construction: Join OB and OY.

Draw OP || AC

Since AB || PO

$\angle ABO + \angle POB = 180°$ (Adjacent interior angles)

$\angle ABO = 90°$ (A tangent to a circle is perpendicular to the radius through the point of contact)

$\Rightarrow \quad 90° + \angle POB = 180° \Rightarrow \quad \angle POB = 90°$

Similarly $\quad \angle POY = 90°$

$\therefore \quad \angle POB + \angle POY = 90° + 90° = 180°$

Hence, BOY is a straight line passing through the centre of the circle.

Que 6. If from an external point P of a circle with centre O, two tangents PQ and PR are drawn such that ∠QPR = 120°, prove that 2 PQ = PO.

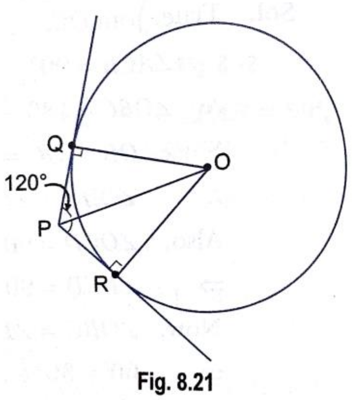

Fig. 8.21

Sol. Given, $\angle QPR = 120°$

Radius is perpendicular to the tangent at the point of contact.

$\therefore \quad \angle OQP = 90° \quad \Rightarrow \quad \angle QPO = 60°$

(Tangent drawn to a circle from an external point are equally inclined to the segment, joining the centre to that point)

In ΔQPO, cos 60° $= \frac{PQ}{PO}$ ⇒ $\frac{1}{2} = \frac{PQ}{PO}$

⇒ 2 PQ = PO

Que 7. In Fig. 8.22, common tangent AB and CD to two circles with centres O_1 and O_2 intersect at E. Prove that AB = CD.

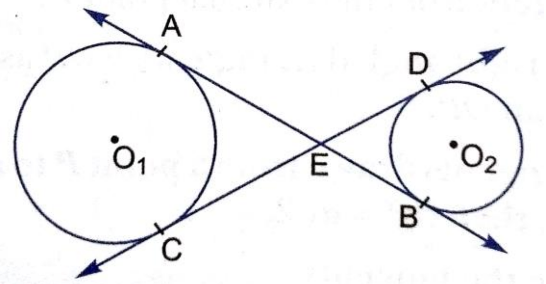

Fig. 8.22

Sol. AE = CE and BE = ED [Tangents drawn from an external point are equal]

On addition, we get
AE + BE = CE + ED ⇒ AB = CD

Que 8. The incircle of an isosceles triangle ABC, in which AB = AC, touches the sides BC, CA and AB at D, E and F respectively. Prove that BD = DC.

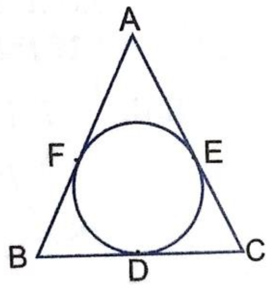

Fig. 8.23

Sol. Given, AB = AC
We have, BF + AF = AE + CE ….(i)
AB, BC and CA are tangent to the circle at F, D and E respectively.
∴ BF = BD and CE = CD ….(ii)

From (i) and (ii)
BD + AE = AE + CD(∵ AF = AE)
⇒ BD = CD

Short Answer Type Questions – II

[3 marks]

Que 1. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Find the length of PQ.

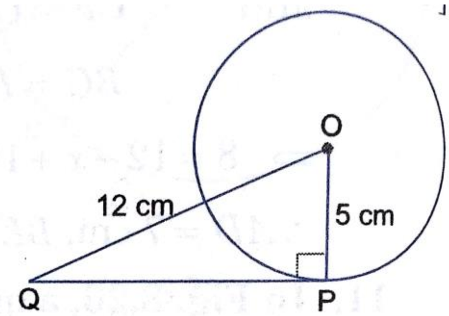

Fig. 8.30

Sol. We have, $\angle OPQ = 90°$

OQ = 12 cm and OP = 5 cm

∴ By Pythagoras Theorem

$OQ^2 = OP^2 + QP^2 \Rightarrow 12^2 = 5^2 + QP^2$

$\Rightarrow QP^2 = 144 - 25 = 119 \Rightarrow QP = \sqrt{119}\, cm$

Que 2. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle.

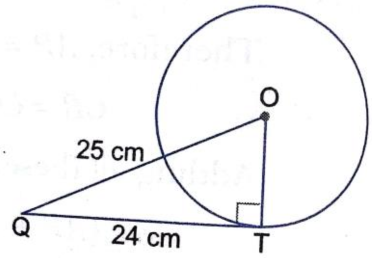

Fig. 8.31

Sol. Let QT be the tangent and OT be the radius of circle. Therefore

$OT \perp QT \quad i.e., \angle OTQ = 90°$

and OQ = 25 cm and QT = 24 cm

Now, by Pythagoras Theorem, we have

$OQ^2 = QT^2 + OT^2 \Rightarrow 25^2 = 24^2 + OT^2$

$\Rightarrow OT^2 = 25^2 - 24^2 = 625 - 576$

$OT^2 = 49 \quad \therefore \quad OT = 7\, cm$

Que 3. In Fig. 8.32, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then find $\angle PTQ$.

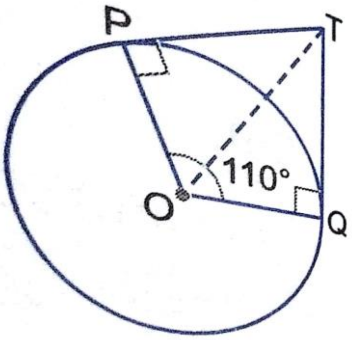

Fig. 8.32

Sol. Since TP and TQ are the tangents to the circle with centre O

So, OP ⊥ PT and OQ ⊥ QT

$\Rightarrow \angle OPT = 90°, \angle OQT = 90° \text{ and } \angle POQ = 110°$

So, in quadrilateral OPTQ, we have

$\angle POQ + \angle OPT + \angle PTQ + \angle TQO = 360°$

$\Rightarrow 110° + 90° + \angle PTQ + 90° = 360° \Rightarrow \angle PTQ + 290° = 360°$

$\therefore \angle PTQ = 360° - 290° \Rightarrow \angle PTQ = 70°$

Que 4. Prove that the tangent drawn at the ends of a diameter of a circle are parallel.

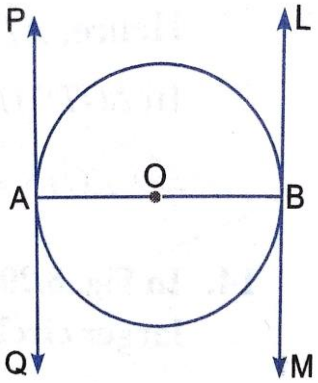

Fig. 8.33

Sol. Let AB be the diameter of the given circle with centre O, and two tangents PQ and LM are drawn at the end of diameter AB respectively.

Now, since the tangent at a point to a circle is perpendicular to the radius through the point of contact.

Therefore, $OA \perp PQ \text{ and } OB \perp LM$

i.e., $AB \perp PQ \text{ and also } AB \perp LM$

$\Rightarrow \angle BAQ = \angle ABL \ (each\ 90°)$

$\therefore$ PQ || LM (∵ $\angle BAQ$ and $\angle ABL$ are alternate angles)

Que 5. If tangent PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then find $\angle POA$.

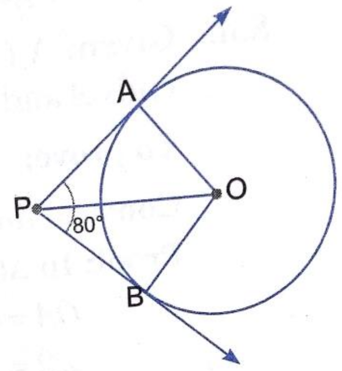

Fig. 8.34

Sol. ∵ PA and PB are tangents to a circle with centre O.

$\therefore OA \perp AP \text{ and } OB \perp PB$

i.e., $\angle APB = 80°, \angle OAP = 90° \text{ and } \angle OBP = 90°$

Now, in quadrilateral OAPB, we have

$\angle APB + \angle PBO + \angle OAP = 360°$

⇒ $80° + 90° + \angle BOA + 90° = 360°$

⇒ $260° + \angle BOA = 360°$

∴ $\angle BOA = 360° - 260°$ ⇒ $\angle BOA = 110°$

Now, in $\Delta POA \text{ and } \Delta POB, we\ have$

OP = OP (Common)

OA = OB (Radii of the same circle)

$\angle OAP = \angle OBP = 90°$

∴ $\Delta POA \cong \Delta POB$ (RHS congruence condition)

⇒ $\angle POA = \angle POB$ (CPCT)

Now, $\angle POA = \frac{1}{2} \times \angle BOA = \frac{1}{2} \times 100 = 50°$

Que 6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm.

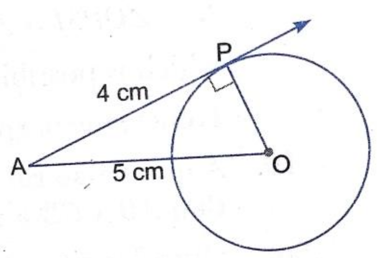

Fig. 8.35

Sol. Let O be the centre and P be the point of contact.

Since tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore \quad \angle OPA = 90°$

Now, in right $\Delta OPA,$ we have

$OA^2 = OP^2 + PA^2$ [By Pythagoras Theorem]

$5^2 = OP^2 + 4^2$ $\Rightarrow \; 25 = OP^2 + 16$

$\Rightarrow \; OP^2 = 25 - 16 = 9$ $\therefore \; OP = 3\ cm$

Hence, the radius of the circle is 3 cm.

Que 7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

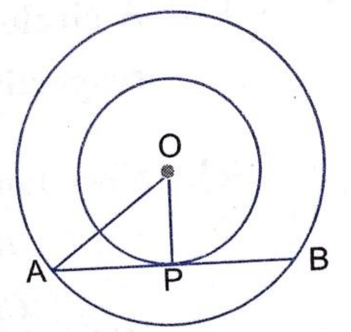

Fig. 8.36

Sol. Let O be the common centre of two concentric circles and let AB be a chord of larger circle touching the smaller circle at P. Join OP.

Since OP is the radius of the smaller circle and AB is tangent to this circle at P.

$\therefore \quad OP \perp AB$

We know that the perpendicular drawn from the centre of a circle to any chord of the circle bisects the chord.

Therefore, AP = BP

In right $\Delta APO, we\ have$

$OA^2 = AP^2 + OP^2$

$\Rightarrow \quad 5^2 = AP^2 + 3^2$ $\Rightarrow \; 25 - 9 = AP^2$

$\Rightarrow \quad AP^2 = 16$ $\Rightarrow \; AP = 4$

Now, AB = 2. AP = 2 × 4 = 8 [$\because \; AP = PB$]

Hence, the length of the chord of the larger circle which touches the smaller circle is 8 cm.

Long Answer Type Questions

[4 MARKS]

Que 1. Prove that the tangent to a circle is perpendicular to the radius through the point of contact.

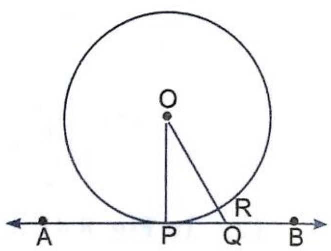

Fig. 8.43

Sol. Given: A circle C (O, r) and a tangent AB at a point P.
To prove: $OP \perp AB$.
Construction: Take any point Q, other than P, on the tangent AB. Join OQ. Suppose OQ meets the circle at R.
Proof: We know that among all line segment joining the point O to point on AB, the shortest one is perpendicular to AB. So, to prove that OP ⊥ AB, it is sufficient to prove that OP is shorter than any other segment joining O to any point of AB.

Clearly, OP = OR [Radii of the same circle]
Now, OQ = OR + RQ
⇒ OQ > OR
⇒ OQ > OP [$\because OP = OR$]
Thus, OP is shorter than any other segment joining O to any point on AB.
Hence, $OP \perp AB$.

Que 2. Prove that the length of two tangent drawn from an external point to a circle are equal.

Sol. Given: AP and AQ are two tangent from a point A to a circle C (O, r).
To prove: AP = AQ
Construction: Join OP, OQ and OA.
Proof: In order to prove that AP = AQ, we shall first prove that $\Delta OPA \cong \Delta OQA$.
Since a tangent at any point of a circle is perpendicular to the radius through the point of contact.

∴ $OP \perp AP \text{ and } OQ \perp AQ$.
⇒ $\angle OPA = \angle OQA = 90°$ …(i)
Now, in right triangles OPA and OQA, we have
OP = OQ [Radii of a circle]
$\angle OPA = \angle OQA$ [Each 90°]
and OA = OA [Common]

So, by RHS-criterion of congruence, we get

$$\Delta OPA \cong \Delta OQA \quad \Rightarrow \quad AP = AQ \qquad \text{[CPCT]}$$

Hence, lengths of two tangents from an external point are equal.

Que 3. Prove that the parallelogram circumscribing a circle is a rhombus.

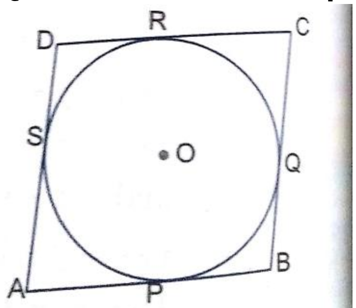

Fig. 8.45

Sol. Let ABCD be a parallelogram such that its sides touch a circle with centre O.
We know that the tangent to a circle from an exterior point are equal in length.
Therefore, we have

AP = AS [Tangents from A] …(i)
BP = BQ [Tangents from B] …(ii)
CR = CQ [Tangents from C] …(iii)
And DR = DS [Tangents from D] …(iv)

Adding (i), (ii), (iii) and (iv), we have

$$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

⇒ AB + CD + = AD + BC
⇒ AB + AB = BC + BC [$\because$ $ABCD\ is\ a\ paralleogram\ \therefore\ AB = CD, BC = DA$]
⇒ 2AB = 2BC ⇒ AB = BC
Thus, AB = BC = CD = AD
Hence, ABCD is a rhombus.

Que 4. In Fig.8.46, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T. Find the length of TP.

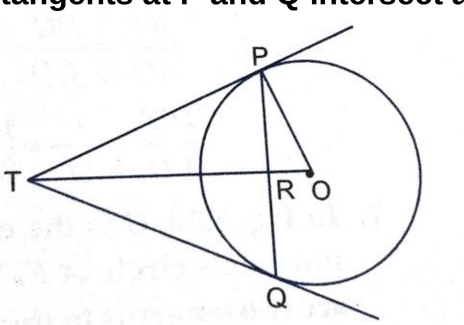

Fig. 8.46

Sol. Given: PQ = 16 cm

PO = 10 cm

To find: TP

$PR = RQ = \frac{16}{2} = 8cm$ [Perpendicular from the center bisects the chord]

$In\ \Delta OPR$

$$OR = \sqrt{OP^2 - PR^2}$$
$$= \sqrt{10^2 - 8^2} = \sqrt{100 - 64}$$
$$= \sqrt{36} = 6\ cm$$

Let $\angle POR\ be\ \theta$

In ΔPOR, $\tan\theta = \frac{PR}{RO} = \frac{8}{6}$

$$\tan\theta = \frac{4}{3}$$

We know, OP ⊥ TP (Point of contact of a tangent is perpendicular to the line from the centre)

In ΔOTP, $\tan\theta = \frac{OP}{TP}$ $\Rightarrow \frac{4}{3} = \frac{10}{TP}$

$$TP = \frac{10\times 3}{4} = \frac{15}{2} = 7.5\ cm.$$

HOTS (Higher Order Thinking Skills)

Que 1. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

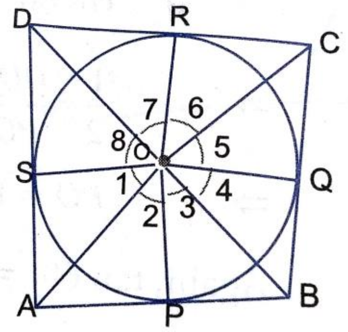

Fig. 8.51

Sol. Let a circle with centre O touches the sides AB, BC, CD and DA of a quadrilateral ABCD

at the points P, Q, R and S respectively. Then, we have to prove that

$\angle AOB + \angle COD = 180°$ and $\angle AOD + \angle BOC = 180°$

Now, Join OP, OQ, OR and OS.

Since the two tangents drawn from an external point to a circle subtend equal angles at

the centre.

∴ $\angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6 \ and \ \angle 7 = \angle 8$ ....(i)

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ ....(ii)

[sum of all the angles subtended at a point is 360 °]

⇒ $2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$

⇒ $2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 180°$

$\angle AOB + \angle COD = 180°$

again $2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360°$ [from (i) and (ii)]

∴ $(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$

⇒ $\angle AOD + \angle BOC = 180°$

Que 2. A triangle ABC [Fig.8.52] is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.

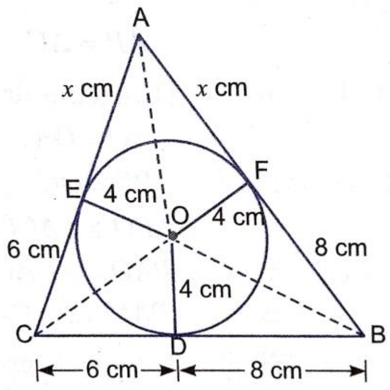

Fig. 8.52

Sol. Let ΔABC be drawn to circumscribe a circle with centre O and radius 4 cm and circle

touches the sides BC, CA and AB at D, E and F respectively.

We have given that CD = 6 cm and BD = 8 cm

∴ $BF = BD = 8\ cm$ and $CE = CD = 6\ cm$

{Length of two tangents drawn from an external point of circle are equal}

Now, let AF = AE = x cm

Then, AB = c = (x + 8) cm, BC = a = 14 cm, CA = b = (x + 6) cm

∴ 2s = (x + 8) + 14 + (x + 6)

⇒ 2s = 2x + 28 or s = x + 14

∴ s – a = (x + 14) – 14 = x

s – b = (x + 14) – (x + 6) = 8

s – c = (x + 14) – (x + 8) = 6

∴ $area\ of\ \Delta ABC = \sqrt{s(s-a)\ (s-b)\ (s-c)}$

$= \sqrt{(x+14)\ (x)\ (8)\ (6)} = \sqrt{48x(x+14)}$

Also, $area\ (\Delta ABC) = Area\ (\Delta OBC) + area(\Delta OCA) + area(\Delta OAB)$

$= \frac{1}{2} \times BC \times OD + \frac{1}{2} \times CA \times OE + \frac{1}{2} \times AB \times OF$

$= \frac{1}{2} \times 14 \times 4 + \frac{1}{2} \times (x+6) \times 4 + \frac{1}{2} \times (x+8) \times 4$

$= 28 + 2x + 12 + 2x + 16 = 4x + 56$

∴ $\sqrt{48x\ (x+14)} = 4x + 56 \quad \Rightarrow \quad \sqrt{48x\ (x+14)} = 4\ (x+14)$

Squaring both sides, we have

$48x\ (x+14) = 16\ (x+14)^2 \quad \Rightarrow \quad 48x\ (x+14) - 16\ (x+14)^2 = 0$

⇒ $16\ (x+14)\ [3x - (x+14)] = 0$

⇒ $16(x+14)\ (2x-14) = 0$

either 16 (x + 14) = 0 or 2x – 14 = 0

⇒ x = – 14 or 2x = 14

⇒ x = – 14 or x = 7

But x cannot be negative so x ≠ – 14

∴ x = 7 cm

Hence, the sides $AB = x + 8 = 7 + 8 = 15\ cm$

$AC = x + 6 = 7 + 6 = 13\ cm.$

Que 3. In Fig. 8.53, XY and X' Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X' Y' at B. Prove that $\angle AOB = 90°$

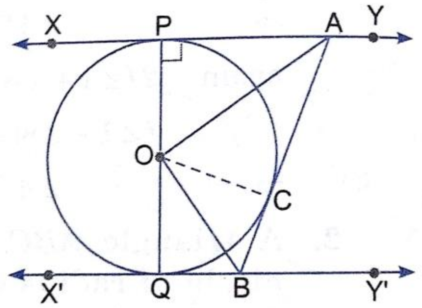

Fig. 8.53

Sol. Join OC. In ΔAPO and $\Delta ACO,$ we have

AP = AC

(Tangents drawn from external point A)

AO = OA (Common)

PO = OC (Radii of the same circle)

∴ $\Delta APO \cong \Delta ACO$ (By SSS criterion of congruence)

∴ $\angle PAO = \angle CAO$ (CPCT)

⇒ $\angle PAC = 2\angle CAO$

Similarly, we can prove that

$\Delta OQB \cong \Delta OCB$

∴ $\angle QBO = \angle CBO \Rightarrow \angle CBQ = 2\angle CBO$

Now, $\angle PAC + \angle CBQ = 180°$

[Sum of interior angle on the same side of transversal is 180°]

⇒ $2\angle CAO + 2\angle CBO = 180°$

⇒ $\angle CAO + \angle CBO = 90°$

⇒ $180° - \angle AOB = 90°$ [$\because \angle CAO + \angle CBO + \angle AOB = 180°$]

⇒ $180° - 90° = \angle AOB \Rightarrow \angle AOB = 90°$

Que 4. Let A be one point of intersection of two intersecting circles with centres O and Q. The tangents at A to the two circles meet the circles again at B and C respectively. Let the point P be located so that AOPQ is a parallelogram. Prove that P is the circumcentre of the triangle ABC.

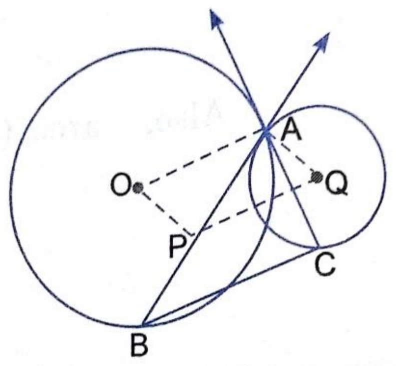

Fig. 8.54

Sol. In order to prove that P is the circumcentre of $\Delta ABC,$ it is sufficient to show that P is the point of intersection of perpendicular bisectors of the sides ofΔABC**,** i.e., OP and PQ are perpendicular bisectors of sides AB and AC respectively, Now, AC is tangent at A to the circle with centre at O and OA is its radius.

∴ OA ⊥ AC

⇒ PQ ⊥ AC $[\because\ OAQP\ is\ a\ parallelogram\ \therefore\ OA||PQ]$

Also, Q is the centre of the circle

∴ QP bisects AC

[Perpendicular from the centre to the chord bisects the chord]

⇒ PQ is the perpendicular bisector of AC.

Similarly, BA is the tangent to the circle at A and AQ is its radius through A.

∴ BA ⊥ AQ

∴ BA ⊥ OP $\begin{bmatrix}\because\ AQPO\ is\ parallelogram \\ \therefore\ OP||AQ\end{bmatrix}$

Also, OP bisects AB $[\because\ O\ is\ the\ centre\ of\ the\ circle]$

⇒ OP is the perpendicular bisector of AB.

Thus, P is the point of intersection of perpendicular bisects PQ and PO of sides AC and AB respectively. Hence, P is the circumcentre of ΔABC.

Value Based Questions

Que 1. Puneet prepared two posters on 'National Integration' for decoration on Independence Day on triangular sheets (say ABC and DEF). The sides AB and AC and the perimeter P_1 of ΔABC are respectively three times the corresponding sides DE and DF and the perimeter P^2 of ΔDEF. Are the two triangular sheets similar? If yes, find $\frac{ar(\Delta ABC)}{ar(\Delta DEF)}$.

What values can be inculcated through celebration of national festivals?

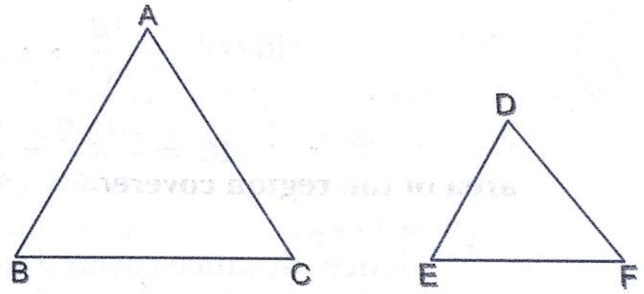

Fig. 4

Sol. In ΔABC and DEF

AB = 3 DE, AC = 3DF and $P_1 = 3p_2$

∴ $\frac{AB}{DE} = 3;\ \frac{AC}{DF} = 3$

And $P_1 = 3p_2 \Rightarrow$ BC = 3EF

$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = 3$

⇒ ΔABC ~ΔDEF (By SSS similarity)

$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = (3)^2 = 9$

Unity of nation, fraternity, Patriotism.

Que 2. A man steadily goes 4 m due East and then 3 m due North.
(i) Find the distance from initial point to last point.
(ii) Which mathematical concept is used in this problem?
(iii) What is its value?

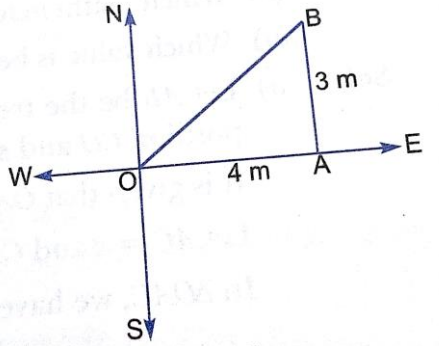

Fig. 5

Sol. (i) Let the initial position of the man be O and his final position be B. Since man goes 4 m due East and then 3 m due North. Therefore, ∆A0B is a right triangle right angled at A such that $OA = 4\, m\ and\ AB = 3m$

By Pythagoras Theorem, we have

$$OB^2 = OA^2 + AB^2$$

$$OB^2 = (4)^2 + (3)^2 = 16 + 9 = 25$$

$$OB = \sqrt{25} = 5\, m.$$

Hence, the man is at a distance of 5 m from the initial position.

(ii) Right-angled triangle, Pythagoras Theorem.

(iii) Knowledge of direction and speed saves the time.

Que 3. Two trees of height x and y are p metres apart.

(i) Prove that the height of the point of intersection of the line joining the top of each tree to the foot of the opposite tree is given by $\frac{xy}{x+y} m$.

(ii) Which mathematical concept is used in this problem?

(iii) What is its value?

Sol. (i) Similar to solution Q. 5, page 161.

(ii) Similarity of triangles.

(iii) Trees are helpful to maintain the balance in the environment. They should be saved at any cost.