# 19. Differential Equations with Variable Separable

## Exercise 19A

### 1. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx}=\left(1+x^2\right)\left(1+y^2\right)$$

**Answer**

$$\frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

Rearranging the terms,we get:

$$\Rightarrow \frac{dy}{1 + y^2} = (1 + x^2)dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{1 + y^2} = \int (1 + x^2)dx + c$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + c \ldots(\int \frac{dy}{1+y^2} = \tan^{-1} y, \int x^n = \frac{x^{n+1}}{n+1})$$

Ans: $\tan^{-1} y = x + \frac{x^3}{3} + c$

### 2 Ans:. Question

Find the general solution of each of the following differential equations:

$$x^4\frac{dy}{dx}=-y^4$$

**Answer**

$$x^4 \frac{dy}{dx} = -y^4$$

$$\Rightarrow \frac{dy}{-y^4} = \frac{dx}{x^4}$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{-y^4} = \int \frac{dx}{x^4} + c'$$

$$\Rightarrow \frac{-y^{-4+1}}{-4 + 1} = \frac{x^{-4+1}}{-4 + 1} + c'$$

$$\Rightarrow \frac{1}{3y^3} = -\frac{1}{3x^3} + c'$$

$$\Rightarrow \frac{1}{y^3} + \frac{1}{x^3} = 3c'$$

$$\Rightarrow \frac{1}{x^3} + \frac{1}{y^3} = c \ldots(3c' = c)$$

### 3. Question

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = 1 + x + y + xy$$

**Answer**

$$\frac{dy}{dx} = 1 + x + y + xy = 1 + y + x(1 + y)$$

$$\Rightarrow \frac{dy}{dx} = (1 + y)(1 + x)$$

Rearranging the terms we get:

$$\Rightarrow \frac{dy}{1 + y} = (1 + x)dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{1 + y} = \int (1 + x)dx + c$$

$$\Rightarrow \log|1 + y| = x + \frac{x^2}{2} + c \ldots (\int \frac{dy}{1+y} = \log|1 + y|)$$

Ans: $\log|1 + y| = x + \frac{x^2}{2} + c$

**4. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = 1 - x + y - xy$$

**Answer**

$$\Rightarrow \frac{dy}{dx} = 1 - x + y - xy = 1 + y - x(1 + y)$$

$$\Rightarrow \frac{dy}{dx} = (1 + y)(1 - x)$$

Rearranging the terms we get:

$$\Rightarrow \frac{dy}{1 + y} = (1 - x)dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{1 + y} = \int (1 - x)dx + c$$

$$\Rightarrow \log|1 + y| = x - \frac{x^2}{2} + c \ldots (\int \frac{dy}{1+y} = \log|1 + y|)$$

Ans: $\log|1 + y| = x - \frac{x^2}{2} + c$

**5. Question**

Find the general solution of each of the following differential equations:

$$(x - 1)\frac{dy}{dx} = 2x^3y$$

**Answer**

$$(x - 1)\frac{dy}{dx} = 2x^3y$$

Separating the variables we get:

$$\Rightarrow \frac{dy}{y} = 2x^3 \frac{dx}{(x-1)}$$

$$\Rightarrow \frac{dy}{y} = \frac{2\left((x-1)(x^2 + x + 1) + 1\right)}{(x-1)} dx$$

$$\Rightarrow \frac{dy}{y} = 2\left(x^2 + x + 1 + \frac{1}{x-1}\right) dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{y} = \int 2\left(x^2 + x + 1 + \frac{1}{x-1}\right) dx + c$$

$$\Rightarrow \log|y| = \frac{2x^3}{3} + \frac{2x^2}{2} + 2x + 2\log|x-1| + c$$

$$\Rightarrow \log|y| = \frac{2x^3}{3} + x^2 + 2x + 2\log|x-1| + c$$

Ans: $\log|y| = \frac{2x^3}{3} + x^2 + 2x + 2\log|x-1| + c$

**6. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = e^{x+y}$$

**Answer**

$$\frac{dy}{dx} = e^x e^y$$

Rearringing the terms we get:

$$\Rightarrow \frac{dy}{e^y} = e^x dx$$

Integrating both the sides we get,

$$\Rightarrow \int \frac{dy}{e^y} = \int e^x dx + c$$

$$\Rightarrow \frac{e^{-y}}{-1} = e^x + c$$

$$\Rightarrow e^x + e^{-y} = c$$

Ans: $e^x + e^{-y} = c$

**7. Question**

Find the general solution of each of the following differential equations:

$$\left(e^x + e^{-x}\right)dy - \left(e^x - e^{-x}\right)dx = 0$$

**Answer**

$(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$

$$\Rightarrow dy = \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$$

Integrating both the sides we get,

$$\Rightarrow \int dy = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx + c$$

$\Rightarrow$y = log|$e^x$ + $e^{-x}$| + c ...$\left(\frac{d}{dx}(e^x + e^{-x}) = e^x - e^{-x}\right)$

Ans:y = log|$e^x$ + $e^{-x}$| + c

**8. Question**

Find the general solution of each of the following differential equations:

**Answer**

Given:$\frac{dy}{dx} = e^x e^{-y} + x^2 e^{-y}$

$$\Rightarrow \frac{dy}{dx} = e^{-y}(e^x + x^2)$$

$$\Rightarrow \frac{dy}{e^{-y}} = (e^x + x^2)dx$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dy}{e^{-y}} = \int (e^x + x^2)dx + c$$

$$\Rightarrow e^y = e^x + \frac{x^3}{3} + c$$

Ans:$e^y = e^x + \frac{x^3}{3} + c$

**9. Question**

Find the general solution of each of the following differential equations:

$$e^{2x-3y}dx + e^{2y-3x}dy = 0$$

**Answer**

$e^{2x}e^{-3y}dx + e^{2y}e^{-3x}dy = 0$

Rearringing the terms we get:

$$\Rightarrow \frac{e^{2x}dx}{e^{-3x}} = -\frac{e^{2y}dy}{e^{-3y}}$$

$\Rightarrow e^{2x+3x}dx = -e^{2y+3y}dy$

$\Rightarrow e^{5x}dx = -e^{5y}dy$

Integrating both the sides we get:

$$\Rightarrow \int e^{5x}dx = -\int e^{5y}dy + c'$$

$$\Rightarrow \frac{e^{5x}}{5} = -\frac{e^{5y}}{5} + c'$$

$\Rightarrow e^{5x} + e^{5y} = 5c' = c$

Ans: $e^{5x} + e^{5y} = c$

**10. Question**

Find the general solution of each of the following differential equations:

$e^x$ tan y dx + (1 – $e^x$) $sec^2$ y dy = 0

**Answer**

Rearranging all the terms we get:

$$\frac{e^x dx}{1-e^x} = -\frac{sec^2 y\, dy}{tany}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{e^x dx}{1-e^x} = -\int \frac{sec^2 y\, dy}{tany} + c$$

$$\Rightarrow \frac{\log|1-e^x|}{-1} = -\log|tany| + logc$$

⇒log|1 - $e^x$| = log|tany| - logc

⇒log|1 - $e^x$| + logc = log|tany|

⇒tany = c(1 - $e^x$)

Ans: tany = c(1 - $e^x$)

**11. Question**

Find the general solution of each of the following differential equations:

$sec^2$x tan y dx + $sec^2$y tan x dy = 0

**Answer**

Rearranging the terms we get:

$$\frac{sec^2 x\, dx}{tanx} = -\frac{sec^2 y\, dy}{tany}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{sec^2 x\, dx}{tanx} = -\int \frac{sec^2 y\, dy}{tany} + c$$

⇒log|tanx| = - log|tany| + logc

⇒ log|tanx| + log|tany| = logc

⇒tanx.tany = c

Ans: tanx.tany = c

**12. Question**

Find the general solution of each of the following differential equations:

cos x(1 + cos y)dx – sin y(1 + sin x)dy = 0

**Answer**

Rearranging the terms we get:

$$\frac{\cos x\, dx}{(1+\sin x)} = \frac{siny\, dy}{(1+cosy)}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{\cos x\, dx}{(1+\sin x)} = \int \frac{siny\, dy}{(1+cosy)} + c$$

⇒log|1 + sinx| = - log|1 + cosy| + logc

⇒log|1 + sinx| + log|1 + cosy| = logc

⇒(1 + sinx)(1 + cosy) = c

Ans: (1 + sinx)(1 + cosy) = c

**13. Question**

For each of the following differential equations, find a particular solution satisfying the given condition :

$.\cos\left(\frac{dy}{dx}\right)=a,$. where a ∈ R and y = 2 when x = 0.

**Answer**

$\cos\left(\frac{dy}{dx}\right) = a$

$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$

⇒dy = $\cos^{-1}$a dx

Integrating both the sides we get:

$\Rightarrow \int dy = \int \cos^{-1} a\ dx + c$

⇒y = x$\cos^{-1}$a + c

when x = 0, y = 2

∴2 = 0 + c

∴c = 2

∴y = x$\cos^{-1}$a + 2

$\Rightarrow \frac{y-2}{x} = \cos^{-1} a$

$\Rightarrow \cos\left(\frac{y-2}{x}\right) = a$

Ans: $\cos\left(\frac{y-2}{x}\right) = a$

**14. Question**

For each of the following differential equations, find a particular solution satisfying the given condition :

$\frac{dy}{dx}=-4xy^2,$ it being given that y = 1 when x = 0.

**Answer**

Rearranging the terms we get:

$\frac{dy}{y^2} = -4xdx$

Integrating both the sides we get:

$\Rightarrow \int \frac{dy}{y^2} = -\int 4xdx + c$

$\Rightarrow \frac{y^{-1}}{-1} = -\frac{4x^2}{2} + c$

$\Rightarrow y^{-1} = 2x^2 + c$

y = 1 when x = 0

$\Rightarrow (1)^{-1} = 2(0)^2 + c$

$\Rightarrow c = 1$

$$\Rightarrow \frac{1}{y} = 2x^2 + 1$$

$$\Rightarrow \frac{1}{2x^2 + 1} = y$$

Ans: $y = \frac{1}{2x^2 + 1}$

**15. Question**

For each of the following differential equations, find a particular solution satisfying the given condition :

x dy = $(2x^2 + 1)$ dx (x ≠ 0), given thaty = 1 when x = 1.

**Answer**

Rearranging the terms we get:

$$dy = \frac{2x^2 + 1}{x} dx$$

$$\Rightarrow dy = 2x\, dx + \frac{1}{x}\, dx$$

Integrating both the sides we get:

$$\Rightarrow \int dy = \int 2x\, dx + \int \frac{1}{x}\, dx + c$$

$\Rightarrow y = x^2 + \log|x| + c$

y = 1 when x = 1

$\therefore 1 = 1^2 + \log 1 + c$

$\therefore 1 - 1 = 0 + c$ ...(log1 = 0)

$\Rightarrow c = 0$

$\therefore y = x^2 + \log|x|$

Ans: $y = x^2 + \log|x|$

**16. Question**

For each of the following differential equations, find a particular solution satisfying the given condition :

$\frac{dy}{dx} = y \tan x$, it being given that y = 1 when x = 0.

**Answer**

Rearranging the terms we get:

$$\frac{dy}{y} = \tan x\, dx$$

$$\Rightarrow \int \frac{dy}{y} = \int \tan x\, dx + c$$

$\Rightarrow \log|y| = \log|\sec x| + \log c$

⇒log|y| - log|secx| = logc

⇒log|y| + log|cosx| = logc

⇒ycosx = c

y = 1 when x = 0

∴1×cos0 = c

∴c = 1

⇒ycosx = 1

⇒y = 1/cosx

⇒y = secx

Ans: y = secx

## Exercise 19B

**1. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \frac{x - 1}{y + 2}$$

**Answer**

(y + 2)dy = (x - 1)dx

Integrating on both sides,

$$\int (y + 2)dy = \int (x - 1)dx$$

$$\frac{y^2}{2} + 2y = \frac{x^2}{2} - x + C$$

$$y^2 + 4y - x^2 + 2x = C$$

**2. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \frac{x}{\left(x^2 + 1\right)}$$

**Answer**

$$dy = \frac{x}{x^2 + 1}dx$$

Multiply and divide 2 in numerator and denominator of RHS,

$$y = \frac{1}{2}.\left(\frac{2x}{x^2 + 1}dx\right)$$

Integrating on both sides

$$y = \frac{1}{2}.\log(x^2 + 1) + C$$

**3. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

**Answer**

$$\frac{1}{1 + y^2}dy = (1 + x)dx$$

Integrating on both sides

$$\int \frac{1}{1 + y^2}dy = \int (1 + x)dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^2}{2} + C$$

**4. Question**

Find the general solution of each of the following differential equations:

$$(1+x^2)\frac{dy}{dx} = xy$$

**Answer**

$$\frac{1}{y}.dy = \frac{x}{x^2 + 1}dx$$

Multiply and divide 2 in numerator and denominator of RHS,

$$\frac{1}{y}.dy = \frac{1}{2}.\left(\frac{2x}{x^2 + 1}dx\right)$$

Integrating on both sides

$$\log y = \frac{1}{2}.\log(1 + x^2) + \log C$$

$$\log y = \log\sqrt{1 + x^2} + \log C$$

$$\Rightarrow y = \sqrt{1 + x^2}.C_1$$

**5. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + y = 1(y \neq 1)$$

**Answer**

$$\frac{dy}{dx} = 1 - y$$

$$\frac{1}{1 - y}dy = dx$$

Integrating on both sides

$$\int \frac{1}{1 - y}dy = \int dx$$

$$\Rightarrow \log|1 - y| = x + C$$

**6. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$$

**Answer**

$$\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$\frac{1}{\sqrt{1-y^2}}dy = -\frac{1}{\sqrt{1-x^2}}dx$$

Integrating on both sides

$$\int \frac{1}{\sqrt{1-y^2}}dy = \int -\frac{1}{\sqrt{1-x^2}}dx$$

$$\sin^{-1} y = \sin^{-1} x + C$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = C$$

**7. Question**

Find the general solution of each of the following differential equations:

$$x\frac{dy}{dx}+y=y^2$$

**Answer**

$$\Longrightarrow x.\tfrac{dy}{dx} + y = y^2$$

$$x.\frac{dy}{dx} = y^2 - y$$

$$\frac{1}{y^2-y}dy = \frac{1}{x}dx$$

$$\frac{1}{y(y-1)}dy = \frac{1}{x}dx$$

Integrating on both the sides,

$$\int \frac{1}{y(y-1)}dy = \int \frac{1}{x}dx$$

LHS:

Let $\frac{1}{y(y-1)}dy = \frac{A}{y} + \frac{B}{(y-1)}$

$$\frac{1}{y(y-1)}dy = \frac{A(y-1)}{y} + \frac{By}{(y-1)}$$

$$1 = A(y-1) + By$$

$$1 = Ay + By - A$$

Comparing coefficients in both the sides,

A = - 1, B = 1

$$\frac{1}{y(y-1)}dy = -\frac{1}{y} + \frac{1}{(y-1)}$$

$$\int \frac{1}{y(y-1)}dy = \int \left[-\frac{1}{y} + \frac{1}{(y-1)}\right]dy$$

$$\int -\frac{1}{y}dy + \int \frac{1}{(y-1)}dy$$

$$-\log y + \log(y-1)$$

$$\Rightarrow \log\left(\frac{y-1}{y}\right)$$

RHS:

$$\int \frac{1}{x}dx$$

$$\int \frac{1}{x}dx = \log x + \log C$$

Therefore the solution of the given differential equation is

$$\log\left(\frac{y-1}{y}\right) = \log x + \log C$$

$$\frac{y-1}{y} = x.C$$

$$y-1 = yxC$$

$$\Rightarrow y = 1 + xyC$$

**8. Question**

Find the general solution of each of the following differential equations:

$x^2$ (y + 1) dx + $y^2$ (x – 1) dy = 0

**Answer**

$$x^2(y+1)dx + y^2(x-1)dy = 0$$

$$x^2(y+1)dx = -y^2(x-1)dy$$

$$x^2(y+1)dx = y^2(1-x)dy$$

$$\frac{x^2}{(1-x)}dx = \frac{y^2}{y+1}dy$$

Add and subtract 1 in numerators of both LHS and RHS,

$$\frac{x^2-1+1}{(1-x)}dx = \frac{y^2-1+1}{y+1}dy$$

$$\frac{(x^2-1)+1}{(1-x)}dx = \frac{(y^2-1)+1}{y+1}dy$$

By the identity,$(a^2-b^2) = (a+b).(a-b)$

$$\frac{(x+1)(x-1)+1}{(1-x)}dx = \frac{(y+1)(y-1)+1}{(y+1)}dy$$

Splitting the terms,

$$-(x + 1)dx + \frac{1}{(1-x)}dx = (y-1)dy + \frac{1}{(y + 1)}dy$$

Integrating,

$$\int -(x + 1)dx + \int \frac{1}{(x-1)}dx = \int (y-1)dy + \int \frac{1}{(y + 1)}dy$$

$$-\left(\frac{x^2}{2} + x\right) + \log|x-1| = \left(\frac{y^2}{2} - y\right) + \log|1 + y| + C$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} + x - y + \log|x-1| + \log|1 + y| = C$$

**9. Question**

Find the general solution of each of the following differential equations:

$$y\left(1-x^2\right)\frac{dy}{dx} = x\left(1+y^2\right)$$

**Answer**

$$\frac{y}{1 + y^2}dy = \frac{x}{1-x^2}dx$$

Multiply 2 in both LHS and RHS,

$$\frac{2y}{1 + y^2}dy = \frac{2x}{1-x^2}dx$$

Integrating on both the sides,

$$\int \frac{2y}{1 + y^2}dy = \int \frac{2x}{1-x^2}dx$$

$$\log(1 + y^2) = -\log(1-x^2) + \log C$$

$$\log(1 + y^2) + \log(1-x^2) = \log C$$

$$= (1 + y^2).(1-x^2) = C$$

**10. Question**

Find the general solution of each of the following differential equations:

y log y dx – x dy = 0

**Answer**

$$y.\log y\, dx = x dx$$

$$\frac{1}{x}dx = \frac{1}{y.\log y}dy$$

Integrating on both the sides,

$$\int \frac{1}{x}dx = \int \frac{1}{y.\log y}dy$$

LHS:

$$\int \frac{1}{x}dx = \log x$$

RHS:

$$\int \frac{1}{y.\log y} dy$$

Let $\log y = t$

So, $\frac{1}{y} dy = dt$

$$\int \frac{1}{y.\log y} dy = \int \frac{1}{t} dt$$

$$= \log t$$

$$= \log(\log y)$$

Therefore the solution of the given differential equation is

$$\log x = \log(\log y) + \log C$$

$$x = \log y . C$$

**11. Question**

Find the general solution of each of the following differential equations:

$x(x^2 - x^2 y^2)\, dy + y(y^2 + x^2y^2)\, dx = 0$

**Answer**

$$x.x^2(1 - y^2)dy + y.y^2(1 + x^2)dx = 0$$

$$x^3(1 - y^2)dy + y^3(1 + x^2)dx = 0$$

$$\frac{1 + x^2}{x^3} dx + \frac{1 - y^2}{y^3} dy = 0$$

$$\frac{1}{x^3} dx + \frac{1}{x} dx + \frac{1}{y^3} dy - \frac{1}{y} dy = 0$$

Integrating ,

$$\int \frac{1}{x^3} dx + \int \frac{1}{x} dx + \int \frac{1}{y^3} dy - \int \frac{1}{y} dy = C$$

$$\frac{x^{-3+1}}{-3+1} + \log x - \log y + \frac{y^{-3+1}}{-3+1} = C$$

$$-\frac{1}{2x^2} + -\frac{1}{2y^2} + \log x - \log y = C$$

$$-\frac{1}{2x^2} + -\frac{1}{2y^2} + \log\left(\frac{x}{y}\right) = C$$

**12. Question**

Find the general solution of each of the following differential equations:

$(1 - x^2)\, dy + xy\, (1 - y)\, dx = 0$

**Answer**

$$(1 - x^2)dy = -xy(1 - y)dx$$

$$(1 - x^2)dy = xy(y - 1)dx$$

$$\frac{1}{y(y-1)} dy = \frac{x}{1 - x^2} dx$$

Integrating on both the sides,

$$\int \frac{1}{y(y-1)} dy = \int \frac{x}{1-x^2} dx$$

LHS:

Let $\frac{1}{y(y-1)} dy = \frac{A}{y} + \frac{B}{(y-1)}$

$$\frac{1}{y(y-1)} dy = \frac{A(y-1)}{y} + \frac{By}{(y-1)}$$

$$1 = A(y-1) + By$$

$$\Rightarrow 1 = Ay + By - A$$

Comparing coefficients in both the sides,

A = - 1, B = 1

$$\frac{1}{y(y-1)} dy = -\frac{1}{y} + \frac{1}{(y-1)}$$

$$\int \frac{1}{y(y-1)} dy = \int \left[-\frac{1}{y} + \frac{1}{(y-1)}\right] dy$$

$$\int -\frac{1}{y} dy + \int \frac{1}{(y-1)} dy$$

$$-\log y + \log(y-1)$$

$$= \log\left(\frac{y-1}{y}\right)$$

RHS:

$$\int \frac{x}{1-x^2} dx$$

Multiply and divide 2

$$\frac{1}{2} . \int \frac{2x}{1-x^2} dx$$

$$-\frac{1}{2} . \log(1-x^2) + \log C$$

$$-\log\sqrt{1-x^2} + \log C$$

Therefore the solution of the given differential equation is

$$\log\left(\frac{y-1}{y}\right) = -\log\sqrt{1-x^2} + \log C$$

$$- \log\left(\frac{y-1}{y}\right) = \log\sqrt{1-x^2} + \log C$$

$$\log\left(\frac{y}{y-1}\right) = \log\sqrt{1-x^2} + \log C$$

$$\frac{y}{y-1} = \sqrt{1-x^2} . C$$

$$= y = (y-1) . \sqrt{1-x^2} . C$$

**13. Question**

Find the general solution of each of the following differential equations:

(1 – $x^2$)(1 – y) dx = xy (1 + y) dy

**Answer**

$$\frac{1-x^2}{x}dx = \frac{y(1+y)}{(1-y)}dy$$

$$\left[\frac{1}{x}-x\right]dx = \left[\frac{y+y^2}{1-y}\right]dy$$

$$\left[\frac{1}{x}-x\right]dx = \left[\frac{y}{1-y}+\frac{y^2}{1-y}\right]dy$$

Integrating on both the sides,

$$\int\left[\frac{1}{x}-x\right]dx = \int\left[\frac{y}{1-y}+\frac{y^2}{1-y}\right]dy$$

LHS:

$$\int\left[\frac{1}{x}-x\right]dx = \log x-\frac{x^2}{2}$$

RHS:

$$\int\frac{y}{1-y}dy = \int\frac{y-1+1}{1-y}dy$$

$$\int\frac{y-`1}{1-y}dy + \int\frac{1}{1-y}dy$$

$$\int -1.dy + \int\frac{1}{1-y}dy$$

$$-y+\log|1-y|$$

$$\int\frac{y^2}{1-y}dy$$

Add and subtract 1 in numerators of both LHS and RHS,

$$\frac{y^2-1+1}{(1-y)}dy$$

$$\frac{(y^2-1)+1}{(1-y)}dy$$

By the identity,$(a^2-b^2) = (a+b).(a-b)$

$$\frac{(y+1)(y-1)+1}{(1-y)}dy$$

Splitting the terms,

$$-(y+1)dy+\frac{1}{(1-y)}dy$$

Integrating,

$$\int -(y+1)dy-\int\frac{1}{(y-1)}dy$$

$$-\left(\frac{y^2}{2}+y\right)+\log|y-1|$$

Therefore the solution of the given differential equation is

$$\log x-\frac{x^2}{2}=-y+\log|1-y|-\left(\frac{y^2}{2}+y\right)+\log|y-1|$$

$$=\log|x.(1-y)^2|=\frac{x^2}{2}-\frac{y^2}{2}-2y+C$$

**14. Question**

Find the general solution of each of the following differential equations:

(y + xy) dx + (x – $xy^2$) dy = 0

**Answer**

$$y(1+x)dx+x(1-y^2)dy=0$$

$$\frac{1+x}{x}dx+\frac{1-y^2}{y}dy=0$$

$$\frac{1}{x}dx+1.dx+\frac{1}{y}dy-ydy=0$$

Integrating ,

$$\int\frac{1}{x}dx+\int 1.dx+\int\frac{1}{y}dy-\int ydy=C$$

$$\log|x|+x+\log|y|-\frac{y^2}{2}=C$$

$$=\log|xy|+x-\frac{y^2}{2}=C$$

**15. Question**

Find the general solution of each of the following differential equations:

($x^2$ – $yx^2$) dy + ($y^2$ + $xy^2$) dx = 0

**Answer**

$$x^2(1-y)dy+y^2(1+x)dx=0$$

$$\frac{1+x}{x^2}dx+\frac{1-y}{y^2}dy=0$$

$$\frac{1}{x^2}dx+\frac{1}{x}dx+\frac{1}{y^2}dy-\frac{1}{y}dy=0$$

Integrating,

$$\int\frac{1}{x^2}dx+\int\frac{1}{x}dx+\int\frac{1}{y^2}dy-\int\frac{1}{y}dy=C$$

$$-\frac{1}{x}+\log|x|-\frac{1}{y}-\log|y|=C$$

$$\log\left|\frac{x}{y}\right|=\frac{1}{x}+\frac{1}{y}+C$$

**16. Question**

Find the general solution of each of the following differential equations:

$$\left(x^2y-x^2\right)dx+\left(xy^2-y^2\right)dy=0$$

**Answer**

$$x^2(y-1)dx + y^2(x-1)dy = 0$$

$$\frac{x^2}{x-1}dx + \frac{y^2}{y-1}dy = 0$$

Add and subtract 1 in numerators ,

$$\frac{x^2-1+1}{(x-1)}dx + \frac{y^2-1+1}{(y-1)}dy$$

$$\frac{(x^2-1)+1}{(x-1)}dx + \frac{(y^2-1)+1}{(y-1)}dy$$

By the identity,$(a^2-b^2) = (a+b).(a-b)$

$$\frac{(x+1)(x-1)+1}{(x-1)}dx + \frac{(y+1)(y-1)+1}{(y-1)}dy$$

Splitting the terms,

$$(x+1)dx + \frac{1}{(x-1)}dx + (y+1)dy + \frac{1}{(y-1)}dy$$

Integrating,

$$\int(x+1)dx + \int\frac{1}{(x-1)}dx + \int(y+1)dy + \int\frac{1}{(y-1)}dy = C$$

$$\frac{x^2}{2} + x + \log|x-1| + \frac{y^2}{2} + y + \log|y-1|$$

$$\frac{1}{2}.(x^2+y^2) + (x+y) + \log|(x-1)(y-1)|$$

**17. Question**

Find the general solution of each of the following differential equations:

$$x\sqrt{1+y^2}dx+y\sqrt{1+x^2}dy=0$$

**Answer**

$$\frac{x}{\sqrt{1+x^2}}dx + \frac{y}{\sqrt{1+y^2}}dy = 0$$

Integrating,

$$\int\frac{x}{\sqrt{1+x^2}}dx + \int\frac{y}{\sqrt{1+y^2}}dy$$

$$= C\ formula:\left\{\frac{d}{dx}\left(\sqrt{1+x^2}\right) = \frac{2x}{2.\sqrt{1+x^2}} = \frac{x}{\sqrt{1+x^2}}\right\}$$

$$\sqrt{1+x^2} + \sqrt{1+y^2} = C$$

**18. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = e^{x+y} + x^2 e^y$$

**Answer**

$$\frac{dy}{dx} = e^x . e^y + x^2 . e^y$$

$$\frac{dy}{dx} = e^y (e^x + x^2)$$

$$\frac{1}{e^y} dy = (e^x + x^2) dx$$

Integrating on both the sides,

$$\int \frac{1}{e^y} dy = \int (e^x + x^2) dx$$

$$-e^{-y} = e^x + \frac{x^3}{3} + C$$

$$e^x + e^{-y} + \frac{x^3}{3} = C$$

**19. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \frac{3e^{2x} + 3d^{4x}}{e^x + e^{-x}}$$

**Answer**

Considering ‘d’ as exponential ’e’

$$\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$$

$$\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + \frac{1}{e^x}}$$

$$\frac{dy}{dx} = \frac{(3e^{2x} + 3e^{4x}). e^x}{e^{2x} + 1}$$

$$\frac{dy}{dx} = \frac{3. e^{2x}(1 + e^{2x}). e^x}{e^{2x} + 1}$$

$$\frac{dy}{dx} = 3. e^{3x}$$

$$dy = 3. e^{3x} dx$$

Integrating on both the sides,

$$\int dy = \int 3. e^{3x} dx$$

$$y = 3. \frac{e^{3x}}{3} + C$$

$$y = e^{3x} + C$$

**31. Question**

Find the general solution of each of the following differential equations:

$$(\cos x)\frac{dy}{dx} + \cos 2x = \cos 3x$$

**Answer**

Given: $\frac{dy}{dx} + \frac{cos2x}{cosx} = \frac{cos3x}{cosx}$

$\Rightarrow \frac{dy}{dx} = \frac{cos(x+2x) - cos2x}{cosx}$

$\Rightarrow \frac{dy}{dx} = \frac{(cosxcos2x - sinxsin2x) - (2cos^2 x - 1)}{cosx}$

$\Rightarrow \frac{dy}{dx} = cos2x - \frac{2sinxcosxsinx}{cosx} - 2cosx + secx$

$\Rightarrow \frac{dy}{dx} = cos2x - 2sin^2 x - 2cosx + secx$

$\Rightarrow y = \int (cos2x - 2sin^2 x - 2cosx + secx)dx$

$\Rightarrow y = \int cos2xdx - \int 2sin^2 xdx - \int 2cosxdx + \int secxdx$

$\Rightarrow y = \int cos2xdx - \int (1 - cos2x)dx - \int 2cosxdx + \int secxdx$

$\Rightarrow y = \frac{sin2x}{2} - 2sinx - x + log|secx + tanx| + c$

**20. Question**

Find the general solution of each of the following differential equations:

$$3e^x \tan y\, dx + (1 - e^x)\sec^2 y\, dy = 0$$

**Answer**

$\Rightarrow 3.e^x.\tan y\, dx = (e^x - 1)\sec^2 y\, dy$

$3.\frac{e^x}{e^x - 1}dx = \frac{\sec^2 y}{\tan y}dy$

$3.\left[\frac{1}{\frac{e^x - 1}{e^x}}\right]dx = \frac{\sec^2 y}{\tan y}dy$

$3.\left[\frac{1}{1 - e^{-x}}\right]dx = \frac{\sec^2 y}{\tan y}dy$

Integrating on both the sides,

$\int 3.\left[\frac{1}{1 - e^{-x}}\right]dx = \int \frac{\sec^2 y}{\tan y}dy$

$3.\log|1 - e^{-x}| = \log|\tan y| + \log C$ formula:$\left\{\frac{d}{dy}\tan y = \frac{1}{\tan y}.\sec^2 y\right\}$

$\log(1 - e^{-x})^3 = \log|\tan y| + \log C$

$\tan y = (1 - e^{-x})^3.C$

**32. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx}+\frac{(1+\cos 2y)}{(1-\cos 2x)}=0$$

**Answer**

Given: $\frac{dy}{dx}+\frac{1+cos2y}{1-cos2x}=0$

$\Rightarrow \frac{dy}{dx}=-\frac{2cos^2y}{2sin^2x}$

$\Rightarrow sec^2y\frac{dy}{dx}=-cosec^2x$

$\Rightarrow \int sec^2ydy=-\int cosec^2xdx$

$\Rightarrow tany=cotx+c$

**21. Question**

Find the general solution of each of the following differential equations:

$$e^y\left(1+x^2\right)dy-\frac{x}{y}dx=0$$

**Answer**

$$e^y(1+x^2)dy=\frac{x}{y}dx$$

$$e^y.y\,dy=\frac{x}{1+x^2}dx$$

Integrating on both the sides,

$$\int e^y.y\,dy=\int\frac{x}{1+x^2}dx$$

LHS:

$$\int e^y.y\,dy$$

By ILATE rule,

$$\int e^y.y\,dy=y.\int e^ydy-\int\left[\frac{d}{dy}(y).\int e^ydy\right]dy$$

$$y.e^y-\int e^y\,dy$$

$$y.e^y-e^y$$

$$e^y(y-1)$$

RHS:

$$\int\frac{x}{1+x^2}dx$$

Multiply and divide by 2

$$\frac{1}{2}\int\frac{2x}{1+x^2}dx$$

$$\frac{1}{2}.\log|1+x^2|$$

$\log\sqrt{1 + x^2}$

Therefore the solution of the given differential equation is

$\Rightarrow e^y(y-1) = \log\sqrt{1 + x^2} + C$

**33. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + \frac{\cos x \sin y}{\cos y} = 0$$

**Answer**

Given:$\frac{dy}{dx} = -\frac{cosxsiny}{cosy}$

$\Rightarrow \frac{dy}{dx} = -cosxtany$

$\Rightarrow \int cotydy = -\int cosxdx$

$\Rightarrow \log|siny| = -sinx + c$

**34. Question**

Find the general solution of each of the following differential equations:

cos x(1 + cos y)dx – sin y(1 + sin x)dy = 0

**Answer**

Given: cosx(1+cosy)dx-siny(1+sinx)dy=0

Dividing the whole equation by (1+sinx)(1+cosy), we get,

$\Rightarrow \frac{\int \text{cosxdx}}{1+\text{sinx}} = \frac{\int \text{sinydy}}{1+\text{cosy}}$

⇒ log|1+sinx|+log|1+cosy|=logc

⇒ (1+sinx)(1+cosy)=c

**22. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = e^{x+y} + e^{x-y}$$

**Answer**

$$\frac{dy}{dx} = e^x.e^y + e^x.e^{-y}$$

$$\frac{dy}{dx} = e^x(e^y + e^{-y})$$

$$\frac{1}{e^y + e^{-y}}dy = e^x dx$$

$$\frac{1}{e^y + \frac{1}{e^y}}dy = e^x dx$$

$$\frac{e^y}{(e^y)^2 + 1}dy = e^x dx$$

Integrating on both the sides,

$\int \frac{e^y}{(e^y)^2+1} dy = \int e^x dx$ formula: $\left\{\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}\right\}$

$= \Rightarrow \tan^{-1} e^{-y} = e^x + C$

**35. Question**

Find the general solution of each of the following differential equations:

$\sin^3 x \, dx - \sin y \, dy = 0$

**Answer**

Using $\sin^3 x = \frac{3sinx - sin3x}{4}$

We have,

$\Rightarrow \frac{3sinx-sin3x}{4} dx - sinydy = 0$

$\Rightarrow \frac{3}{4} sinxdx - \frac{sin3x}{4} dx - sinydy = 0$

$\Rightarrow \int \frac{3}{4} sinxdx - \int \frac{sin3x}{4} dx - \int sinydy = 0$

$\Rightarrow \frac{3}{4}(-cosx) + \frac{1}{12} cos3x + cosy = k$

$\Rightarrow 12cosy + cos3x - 9cosx = c$

**23. Question**

Find the general solution of each of the following differential equations:

$(e^y + 1) \cos x \, dx + e^y \sin x \, dy = 0$

**Answer**

$$\frac{\cos x}{\sin x} dx + \frac{e^y}{e^y + 1} dy = 0$$

$$\cot x \, dx + \frac{e^y}{e^y + 1} dy = 0$$

Integrating,

$$\int \cot x \, dx + \int \frac{e^y}{e^y + 1} dy = C$$

$$\log|\sin x| + \log|e^y + 1| = \log C$$

$$\log|\sin x . (e^y + 1)| = \log C$$

$$\Rightarrow \sin x . (e^y + 1) = C$$

**36. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + \sin(x + y) = \sin(x - y)$$

**Answer**

$$\frac{dy}{dx} + \sin(x + y) = \sin(x - y)$$

$\Rightarrow \frac{dy}{dx} = \sin(x - y) - \sin(x + y)$

$\Rightarrow \frac{dy}{dx} = -2sinycosx$ (Using sin(A+B)-sin(A-B)=2sinBcosA)

$\Rightarrow -cosecydy = cosxdx$

$\Rightarrow -\int cosecydy = \int cosxdx$

$\Rightarrow -\log|cosecy - coty| = sinx + c$

⇒ sinx+log|cosecy-coty|+c=0

**24. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} + \frac{xy + y}{xy + x} = 0$$

**Answer**

$$\frac{dy}{dx} + \frac{y(1 + x)}{x(1 + y)} = 0$$

$$\frac{1 + y}{y} dy + \frac{1 + x}{x} dx = 0$$

$$\frac{1}{y} dy + 1.dy + \frac{1}{x} dx + 1.dx = 0$$

Integrating ,

$$\int \frac{1}{y} dy + \int 1.dy + \int \frac{1}{x} dx + \int 1.dx = C$$

$$\log|y| + y + \log|x| + x = C$$

$$\Rightarrow \log|xy| + x + y = C$$

**37. Question**

Find the general solution of each of the following differential equations:

$$\frac{1}{x}\cos^2 y\, dy + \frac{1}{y}\cos^2 x\, dx = 0$$

**Answer**

Given: $\frac{1}{x}cos^2ydy + \frac{1}{y}cos^2xdx = 0$

$\Rightarrow ycos^2ydy + xcos^2xdx = 0$

$\Rightarrow \frac{y}{2}(1 + cos^2)dy + \frac{x}{2}(1 + cos^2)dx = 0$ (Using, $2cos^2a=1+cos2a$)

$\Rightarrow ydy + ycos^2ydy + xdx + xcos^2xdx = 0$

$\Rightarrow \frac{y^2}{2} + \frac{y}{2}sin^2y - \int \frac{sin^2y}{2}dy$

$\Rightarrow \frac{y^2}{2} + \frac{y}{2}sin^2y + \frac{cos^2}{4} + \frac{x^2}{2} + \frac{x}{2}sin^2x + \frac{sin^2}{4} = c$

**25. Question**

Find the general solution of each of the following differential equations:

$$\sqrt{1-x^4}\,dy = x\ dx$$

**Answer**

$$dy\ =\ \frac{x}{\sqrt{1-x^4}}dx$$

Multiply and divide by 2,

$$dy\ =\ \frac{1}{2}.\frac{2x}{\sqrt{1-x^4}}dx$$

$$dy\ =\ \frac{1}{2}.\frac{2x}{\sqrt{1-(x^2)^2}}dx$$

Integrating on both the sides,

$\int dy\ =\ \frac{1}{2}.\int\ dy\ =\ \frac{1}{2}.\frac{2x}{\sqrt{1-x^4}}dx$ formula: $\left\{\frac{d}{dx}\sin^{-1}x\ =\ \frac{1}{\sqrt{1-x^2}}\right\}$

$\Rightarrow y\ =\ \frac{1}{2}.\sin^{-1}x^2\ +\ c$

**38. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \sin^3 x\cos^2 x + xe^x$$

**Answer**

Here we have, $y = \int(\sin^3 x\cos^2 x + xe^x)\,dx$

$\Rightarrow \int \cos^2 x(1-\cos^2 x)\sin x dx + \int xe^x dx$

*Taking cosx as t we have,*

$\Rightarrow cosx = t,$

$\Rightarrow -sinxdx = dt,$

So we have,

$\Rightarrow y = \int cos^2 xsinxdx - \int cos^4 xsinxdx + \int xe^x dx$

$\Rightarrow y = -\int t^2 dt - \int t^4(-dt) + \int xe^x dx$

$\Rightarrow y = -\frac{cos^3 x}{3} + \frac{cos^5 x}{5} + xe^x - e^x + c$

**26. Question**

Find the general solution of each of the following differential equations:

$$\operatorname{cosec} x \log y \frac{dy}{dx} + x^2 y = 0$$

**Answer**

$$\frac{\log y}{y}dy\ +\ \frac{x^2}{\csc x}dx\ =\ 0$$

$$\frac{\log y}{y}dy\ +\ x^2.\sin x\,dx\ =\ 0$$

Integrating ,

$$\int \frac{\log y}{y} dy + \int x^2.\sin x\, dx = C$$

Consider the integral $\int \frac{\log y}{y} dy$

Let $\log y = t$

So, $\frac{1}{y} dy = dt$

$$\int \frac{\log y}{y} dy = \int t.\, dt$$

$$\frac{t^2}{2}$$

$$\frac{(\log y)^2}{2}$$

Consider the integral $\int x^2.\sin x\, dx$

By ILATE rule,

$$\int x^2.\sin x\, dx = x^2 \int \sin x\, dx - \int \left[\frac{d}{dx}(x^2) \int \sin x\, dx.\right] dx$$

$$-x^2.\cos x - \int \left[2x. \int \sin x\, dx\right] dx$$

$$-x^2 \cos x + 2 \int [x.\cos x]\, dx$$

Again by ILATE rule,

$$-x^2 \cos x + 2\left[x. \int \cos x\, dx - \int \left\{\frac{d}{dx} x. \int \cos x\, dx\right\} dx\right]$$

$$-x^2 \cos x + 2\left[x \sin x - \int \sin x\, dx\right]$$

$$-x^2 \cos x + 2\,[x \sin x + \cos x]$$

$$-x^2 \cos x + 2\,x \sin x + 2 \cos x$$

$$\cos x\,(2 - x^2) + 2x \sin x$$

Therefore the solution of the given differential equation is,

$$\frac{(\log y)^2}{2} + \cos x\,(2 - x^2) + 2x \sin x = C$$

**27. Question**

Find the general solution of each of the following differential equations:

$$y\,dx + \left(1 + x^2\right)\tan^{-1} x\, dy = 0$$

**Answer**

$$\frac{1}{\tan^{-1} x.(1 + x^2)} dx + \frac{1}{y} dy = 0$$

Integrating,

$$\int \frac{1}{\tan^{-1} x.(1 + x^2)} dx + \int \frac{1}{y} dy = C$$

Consider the integral $\int \frac{1}{\tan^{-1} x.(1 + x^2)} dx$

Let $\tan^{-1} x = t$

So, $\frac{1}{1 + x^2} dx = dt$

$$\int \frac{1}{\tan^{-1} x.(1 + x^2)} dx = \int \frac{1}{t} dt$$

$\log t$

$\log(\tan^{-1} x)$

Consider the integral $\int \frac{1}{y} dy$

$\log y$

Therefore the solution of the differential equation is

$\log(\tan^{-1} x) + \log y = \log C$

$\tan^{-1} x.y = C$

**39. Question**

Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that y = 0 when x = 1.

**Answer**

Given:

$\frac{dy}{dx} = (1 + x)(1 + y)$

$\Rightarrow \frac{dy}{1+y} = (1 + x)dx$

$\Rightarrow log|y + 1| = (x + \frac{x^2}{2} + c)$

$\Rightarrow now, for\ y = 0\ and\ x = 1,$

We have,

$\Rightarrow 0 = 1 + \frac{1}{2} + c$

$\Rightarrow c = -\frac{3}{2}$

$\Rightarrow \log|y + 1| = \frac{x^2}{2} + x - \frac{3}{2}$

**28. Question**

Find the general solution of each of the following differential equations:

$\frac{1}{x} . \frac{dy}{dx} = \tan^{-1} x$

**Answer**

$dy = x. \tan^{-1} x\, dx$

Integrating on both the sides,

$$\int dy = \int x.\tan^{-1} x\, dx$$

$$y = \tan^{-1} x \int x\, dx - \int \left[\frac{d}{dx}(\tan^{-1} x). \int x\, dx\right] dx \ \langle by\ ILATE\ rule\rangle$$

$$y = \tan^{-1} x . \frac{x^2}{2} - \int \left[\frac{1}{1 + x^2}\right]. \frac{x^2}{2} dx$$

$$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2}. \int \frac{x^2}{x^2 + 1} dx$$

$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2} \int \left[\frac{x^2 - 1 + 1}{x^2 + 1}\right]$ (adding and subtracting 1)

$$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2} \int \left[1 - \frac{1}{x^2 + 1}\right] dx$$

$$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2} [x - \tan^{-1} x] + C$$

$$y = \tan^{-1} x . \frac{x^2}{2} - \frac{1}{2} x + \frac{\tan^{-1} x}{2} + C$$

$$y = \frac{1}{2}.\tan^{-1} x\, (x^2 + 1) - \frac{1}{2} x + C$$

**40. Question**

Find the particular solution of the differential equation $x(1 + y^2)$ dx-$y(1 + x^2)$ dy = 0, given that y = 1 when x = 0.

**Answer**

$$\frac{2xdx}{1+x^2} - \frac{2ydy}{1+y^2} = 0$$

$$\Rightarrow \frac{\log(1+x^2)}{1+y^2} = 0$$

$$\Rightarrow (1 + x^2) = c(1 + y^2)$$

$$\Rightarrow y = 1, x = 0$$

$$\Rightarrow 1 = c(2)$$

$$\Rightarrow c = \frac{1}{2}$$

$$\Rightarrow 2(1 + x^2) = 1 + y^2$$

$$\Rightarrow 2 + 2x^2 - 1 = y^2$$

$$\Rightarrow 2x^2 + 1 = y^2$$

$$\Rightarrow y = \sqrt{2x^2 + 1}$$

**29. Question**

Find the general solution of each of the following differential equations:

$$e^x \sqrt{1 - y^2} dx + \frac{y}{x} dy = 0$$

**Answer**

$e^x.x\,dx + \frac{y}{\sqrt{1-y^2}}dy = 0$

Integrating,

$\int e^x.x\,dx + \int \frac{y}{\sqrt{1-y^2}}dy = C$

Consider the integral $\int e^x.x\,dx$

By ILATE rule,

$\int e^x.x\,dx = x.\int e^x\,dx - \int \left[\frac{d}{dx}(x).\int e^x\,dx\right]dx$

$x.e^x - \int e^x\,dx$

$x.e^x - e^x$

$e^x(x-1)$

Consider the integral $\int \frac{y}{\sqrt{1-y^2}}dy$

Its value is - $\sqrt{1-y^2}$ as $\frac{d}{dx}\left(\sqrt{1-y^2}\right) = \frac{-2y}{2.\sqrt{1-y^2}} = \frac{-y}{\sqrt{1-y^2}}$

Therefore the solution of the given differential equation is

$e^x(x-1)$ - $\sqrt{1-y^2} = C$

**41. Question**

Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$, given that y = 0 when x = 0.

**Answer**

$\log\left(\frac{dy}{dx}\right) = 3x + 4y$

⇒ $y = 0$

⇒ $x = 0$

⇒ $\frac{dy}{dx} = e^{3x}e^{4y}$

⇒ $e^{-4y}dy = e^{3x}dx$

⇒ $\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c$

⇒ For $y = 0, x = 0$, we have

⇒ $-\frac{1}{4} = \frac{1}{3} + c$

⇒ $c = -\frac{7}{12}$

⇒ $\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$

Hence, the particular solution is:

⇒ $4e^{3x} + 3e^{-4x} = 7$

**30. Question**

Find the general solution of each of the following differential equations:

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

**Answer**

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

$$dy = \frac{1 - \cos x}{1 + \cos x} dx$$

$\cos x$ can be written as $\cos x = \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}$

$$dy = \frac{1 - \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}}{1 + \frac{1 - \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right)}} dx$$

$$dy = \frac{\left[\frac{1 + \tan^2\left(\frac{x}{2}\right) - \left(1 - \tan^2\left(\frac{x}{2}\right)\right)}{1 + \tan^2\left(\frac{x}{2}\right)}\right]}{\frac{1 + \tan^2\left(\frac{x}{2}\right) + \left(1 - \tan^2\left(\frac{x}{2}\right)\right)}{1 + \tan^2\left(\frac{x}{2}\right)}} dx$$

$$dy = \frac{1 + \tan^2\left(\frac{x}{2}\right) - 1 + \tan^2\left(\frac{x}{2}\right)}{1 + \tan^2\left(\frac{x}{2}\right) + 1 - \tan^2\left(\frac{x}{2}\right)} dx$$

$$dy = \frac{2\tan^2\left(\frac{x}{2}\right)}{2} dx$$

$$dy = \tan^2\left(\frac{x}{2}\right) dx$$

Integrating on both the sides,

$$\int dy = \int \tan^2\left(\frac{x}{2}\right) dx$$

$y = \int \left[\sec^2\left(\frac{x}{2}\right) - 1\right] dx$ formula: $\{\sec^2 x - \tan^2 x = 1\}$

$y = 2.\tan\left(\frac{x}{2}\right) - x + C$ formula:$\left\{\frac{d}{dx}\tan\left(\frac{x}{2}\right) = \sec^2\left(\frac{x}{2}\right).\frac{1}{2}\right\}$

**42. Question**

Solve the differential equation $(x^2 - yx^2)dy + (y^2 + x^2y^2)\,dx = 0$, given that $y = 1$ when $x = 1$.

**Answer**

$$x^2(1 - y)dy + y^2(1 + x^2)dx = 0$$

$$\Rightarrow \frac{(1-y)}{y^2}dy + \frac{(1+x^2)}{x^2}dx = 0$$

$$\Rightarrow \int \frac{(1-y)}{y^2}dy + \int \frac{(1+x^2)}{x^2}dx = 0$$

$\Rightarrow -\frac{1}{y} - logy - \frac{1}{x} + x = c$

For y=1,x=1, we have,

$\Rightarrow -1 - 0 - 1 + 1 = c$

$\Rightarrow c = -1$

Hence, the required solution is:

$\Rightarrow \frac{1}{y} + logy + \frac{1}{x} - x = 1$

**43. Question**

Find the particular solution of the differential equation $e^x\sqrt{1-y^2}dx + \frac{y}{x}dy = 0,$ given that y = 1 when x = 0.

**Answer**

Given:$e^x\sqrt{1-y^2dx} + \frac{y}{x}dy = 0$ Separating the variables we get,

$\Rightarrow xe^x dx + \frac{y}{\sqrt{1-y^2}}dy = 0$

$\Rightarrow \int xe^x dx + \int \frac{y}{\sqrt{1-y^2}}dy = 0$ Substituting $\sqrt{1-y^2} = t, 1 - y^2 = t^2, -2ydy = 2tdt,$ we have,

$\Rightarrow xe^x - e^x - \frac{1}{2}\log\left|\sqrt{1-y^2}\right| = c$

For y=1 and x=0, we have,

$\Rightarrow 0 - 1 - 0 = c$

$\Rightarrow c = -1$

⇒ Hence, the particular solution will be:-

$\Rightarrow xe^x - e^x - \frac{1}{2}\log\left|\sqrt{1-y^2}\right| + 1 = 0$

**44. Question**

Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x(2\log x + 1)}{(\sin y + y\cos y)},$ given that $y = \frac{\pi}{2}$ when x = 1.

**Answer**

Given: $\frac{dy}{dx} = \frac{x(2logx+1)}{(siny+ycosy)}$

$\Rightarrow \int sinydy + \int ycosydy = \int 2xlogxdx + \int xdx$

Let $\int ycosydy = I$ Then,

$$\int ycosydy = (\int cosydy)\, y - \int ((\int ycosydy).\frac{d}{dx}y)dy$$

And $\int xlogx = (\int xdx)logx - \int ((\int xdx)\frac{d}{dx}logx)dx$

We have,

$\Rightarrow -cosy + ysiny + cosy = x^2logx - \frac{x^2}{2} + \frac{x^2}{2} + c$

For $y = \frac{\pi}{2}, x = 1$ we have,

$0 + \frac{\pi}{2} + 0 = 0 + c$

$c = \frac{\pi}{2}$

$\Rightarrow ysiny = x^2 logx + \frac{\pi}{2}$

**45. Question**

Solve the differential equation $\frac{dy}{dx} = y\sin 2x,$ given that y(0) = 1.

**Answer**

We have,

$\frac{dy}{dx} = ysin2x$

$\Rightarrow \frac{dy}{y} = sin2xdx$

$\Rightarrow logy = -\frac{cos2x}{2} + c$

For y=1, x=0, we have,

c = $\frac{1}{2}$

$\Rightarrow logy = \frac{1}{2}(1 - cos2x)$

$\Rightarrow logy = sin^2 x$

Thus,

The particular solution is:

$y = e^{sin^2 x}$

**46. Question**

Solve the differential equation $(x+1)\frac{dy}{dx} = 2xy,$ given that y(2) = 3.

**Answer**

Given: $(\ x + 1)\frac{dy}{dx} = 2xy$

$\Rightarrow \frac{dy}{y} = 2\frac{x}{x+1}dx$

$\Rightarrow logy = \int 2 - \frac{2}{x+1}dx$

$\Rightarrow logy = 2x - 2\log(x+1) + c$

For x=2 and y=3, we have,

c = 3log3 – 4

Hence, the particular solution is,

$\Rightarrow$ y(x + 1)$^2$ = 27$e^{2x-4}$

**47. Question**

Solve $\frac{dy}{dx} = x(2\log x + 1),$ given that y = 0 when x = 2.

**Answer**

we have, $\frac{dy}{dx} = 2xlogx + x$, Integrating we get,

$y = \int (2xlogx + x)dx$,

$y = \int 2xlogxdx + xdx$

$y = \left(\int 2xdx)logx - \int [(\int 2xdx)\left(\frac{d}{dx}logx\right)\right] dx + \frac{x^2}{2} + c$

given that y=0 when x=2

$\Rightarrow y = x^2logx - \frac{x^2}{2} + \frac{x^2}{2} + c$

now putting x=2 and y=0,

$\Rightarrow 0 = 4log2 + c$

$\Rightarrow c = -4log2$

Thus, the solution is:

$y = x^2logx - 4log2$

**48. Question**

Solve $\left(x^3 + x^2 + x + 1\right)\frac{dy}{dx} = 2x^2 + x$, given that y = 1 when x = 0.

**Answer**

we have, $\left(x^3 + x^2 + x + 1\right)\frac{dy}{dx} = 2x^2 + x$,

$\left\{ \qquad \right\}$Given that: y=1 when x=0,

$\Rightarrow (x^2 + 1)(x + 1)\frac{dy}{dx} = 2x^2 + x$

$\Rightarrow \frac{dy}{dx} = \frac{x^2 + x + x^2}{(x^2+1)(x+1)} dx$

$\Rightarrow \frac{dy}{dx} = \frac{x(x+1) + x^2 + 1 - 1}{(x^2+1)(x+1)} dx$

$\Rightarrow \frac{dy}{dx} = \frac{x}{x^2+1} dx + \frac{x^2+1-1}{(x^2+1)(x+1)} dx$

$\Rightarrow dy = \frac{xdx}{x^2+1} + \frac{dx}{x+1} - \frac{dx}{(x^2+1)(x+1)}$

$\Rightarrow \int dy = \int \frac{xdx}{x^2+1} + \int \frac{dx}{x+1} - \int \frac{\frac{-1}{2}x + \frac{1}{2}}{x^2+1} dx + \int \frac{\frac{1}{2}}{x+1} dx$

$\Rightarrow y = \frac{1}{2}\log|x^2 + 1| + \log|x + 1| + \frac{1}{4}\log|x^2 + 1| - \frac{1}{2}\tan^{-1} x + \frac{1}{2}\log|x + 1| + c$

$\Rightarrow y = \frac{3}{4}\log|x^2 + 1| + \frac{1}{2}\log|x + 1| - \frac{1}{2}\tan^{-1} x + c$

For y=1, when x=0, we have,

$1 = 0 + 0 - 0 + c$

$\Rightarrow c = 1$

$$y=\frac{1}{2}\left\{\log|x+1|+\frac{3}{2}\log\left(x^2+1\right)-\tan^{-1}x\right\}+1$$

**49. Question**

Solve $\frac{dy}{dx}=y\tan x$, given that y = 1 when x = 0.

**Answer**

we have, $\frac{dy}{dx}=y\tan x$,

given that: y=1 when x=0

$\Rightarrow \frac{dy}{dx}=ytanx$

$\Rightarrow \frac{dy}{y}=tanx\,dx$

$\Rightarrow logy=losecx+c$

$\Rightarrow 0=0+c$

$\Rightarrow ycosx=1$ is the particular solution...

**50. Question**

Solve $\frac{dy}{dx}=y^2\tan 2x$, given that y = 2 when x = 0.

**Answer**

we have: $\frac{dy}{dx}=y^2\tan 2x$,

Given that, y=2 when x=0

$\Rightarrow \frac{dy}{y^2}=tan2xdx$

$\Rightarrow \int\frac{dy}{y^2}=\int tan2xdx$ ...integrating both sides

$\Rightarrow -\frac{1}{y}=\frac{\log(sec2x)}{2}$

$\Rightarrow -\frac{1}{2}=0+c$

$\Rightarrow c=-\frac{1}{2}$

$\Rightarrow y(1+logcos2x)=2$ ...is the particular solution

**51. Question**

Solve $\frac{dy}{dx}=y\cot 2x$, given that y = 2 when $x=\frac{\pi}{4}$.

**Answer**

we have $\frac{dy}{dx}=y\cot 2x$,

Given that, y=2 when x=$\frac{\pi}{2}$

$\Rightarrow \frac{dy}{y} = ycot2x$

$\Rightarrow \frac{dy}{y} = cot2xdx$

$\Rightarrow \int \frac{dy}{y} = \int cot2xdx$

$\Rightarrow logy = -\frac{\log(sin2x)}{2} + c$

$\Rightarrow log2 = 0$+c

$\Rightarrow$ Thus, $c = log2$

The particular solution is :- $log\frac{y}{\sqrt{sin2x}} = log2$

$\therefore y = 2\sqrt{sin2x}$

**52. Question**

Solve (1 + $x^2$) sec2 y dy + 2x tan y dx = 0, given that $y = \frac{\pi}{4}$ when x = 1.

**Answer**

we have, (1 + $x^2$) sec2 y dy + 2x tan y dx = 0,

Given that, $y = \frac{\pi}{4}$ when x=1

$\Rightarrow (1 + x^2)sec2ydy + 2xtanydx = 0$

$\Rightarrow \frac{sec^2y}{tany}dy + \frac{2x}{1+x^2}dx = 0$

$\Rightarrow \int \frac{sec^2y}{tany}dy + \int \frac{2x}{1+x^2}dx = 0$

$\Rightarrow logtany + \log(1 + x^2) = logc$

For $y = \frac{\pi}{4}, x = 1$

We have, $0 + log2 = logc,$

c = 2,

Hence the required particular solution is:-

$\therefore tany(1 + x^2) = 2$

**53. Question**

Find the equation of the curve passing through the point $\left(0, \frac{\pi}{4}\right)$ whose differential equation is sin x cos y dx + cos x sin y dy = 0.

**Answer**

we have, sin x cos y dx + cos x sin y dy = 0

$\Rightarrow sinx\ cosy\ dx + cosx\ siny\ dy = 0$

$\Rightarrow tanx\ dx + tany\ dy = 0$

$\Rightarrow \log secx + \log secy = logc$

$\Rightarrow secx\ secy = c$

Given that, coordinates of point, $(0, \frac{\pi}{4})$

$\Rightarrow c = \sqrt{2}$

$\Rightarrow secy = \sqrt{2}cosx$

$\therefore y = cos^{-1}(\frac{1}{\sqrt{2}}secx)$ ...is the required particular solution

**54. Question**

Find the equation of a curve which passes through the origin and whose differential equation is $\frac{dy}{dx} = e^x \sin x.$

**Answer**

Given, $\frac{dy}{dx} = e^x sinx$

$dy = e^x sinxdx$

$\Rightarrow \int dy = \int e^x sinxdx$

Let $I = \int e^x sinxdx$

$\Rightarrow I = \int e^x dx\ sinx - \int(\int e^x dx).(\frac{d}{dx}sinx)\ dx$

$\Rightarrow I = e^x sinx - \int e^x cosxdx$

$\Rightarrow I = e^x sinx - \int e^x dx\ cosx - \int(\int e^x dx).(\frac{d}{dx}cosx)\ dx$

$\Rightarrow I = e^x sinx - e^x cosx - \int e^x sinxdx$

$\Rightarrow 2I = e^x sinx - e^x cosx$

$\Rightarrow I = \frac{e^x sinx - e^x cosx}{2} + c$

$$\therefore\ y = \frac{e^x sinx - e^x cosx}{2} + c$$

For the curve passes through (0,0)

We have, c = $\frac{1}{2}$

$\therefore\ 2y - e^x sinx + e^x cosx = 1$

**55. Question**

A curve passes through the point (0, -2) and at any point (x, y) of the curve, the product of the slope of its tangent and y-coordinate of the point is equal to the x-coordinate of the point. Find the equation of the curve.

**Answer**

Given that the product of slope of tangent and y coordinate equals the x-coordinate i.e., $y\frac{dy}{dx} = x$

We have, $ydy = xdx$

$\Rightarrow \int ydy = \int xdx$

$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c$

For the curve passes through (0, -2), we get c = 2,

Thus, the required particular solution is:-

$\therefore\ y^2 = x^2 + 4$

**56. Question**

A curve passes through the point (-1, 1) and at any point (x, y) of the curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (-4, -3). Find the equation of the curve.

**Answer**

Given : $\frac{dy}{dx} = \frac{2(y+3)}{x+4}$

$\Rightarrow \frac{dy}{y+3} = \frac{2dx}{x+4}$

$\Rightarrow \int \frac{dy}{y+3} = 2\int \frac{dx}{x+4}$

$\Rightarrow \log(y+3) = 2\log(x+4) + c$

The curve passes through (-2, 1)we have,

c = 0,

$\therefore\ y + 3 = (x+4)^2$

**57. Question**

In a bank, principal increases at the rate fo r% per annum. Find the value of r if ` 100 double itself in 10 years.

(Given $\log_e 2 = 0.6931$)

**Answer**

Given:$\frac{dp}{dt} = (\frac{r}{100}) \times p$

Here, p is the principal, r is the rate of interest per annum and t is the time in years.

Solving the differential equation we get,

$\frac{dp}{p} = \left(\frac{r}{100}\right) dt$

$\Rightarrow \int \frac{dp}{p} = \int \frac{r}{100} dt$

$\Rightarrow logp = \frac{rt}{100} + c$

$\Rightarrow p = e^{\frac{rt}{100}+c}$

As it is given that the principal doubles itself in 10 years, so

Let the initial interest be p1 (for t = 0 ), after 10 years p1 becomes 2p1.

Thus, $p1 = e^c$ for ( t = 0) ...(i)

$p = 2p1 = e^{\frac{r.(10)}{100}}.e^c$ ...(ii)

Substituting (i) in (ii), we get,

$\Rightarrow 2\text{p}1 = e^{\frac{r}{10}}.p1$

$\Rightarrow 2 = e^{\frac{r}{10}}$

$\Rightarrow log2 = \frac{r}{10}$

$\Rightarrow r = 10log2$

$\Rightarrow r = 6.931$

∴ Rate of interest = 6.931

**58. Question**

In a bank, principal increases at the rate of 5% per annum. An amount of ` 1000 is deposited in the bank. How much will it worth after 10 years?

(Given $e^{0.5}$ = 1.648)

**Answer**

Given: rate of interest = 5%

P(initial) = Rs 1000

And,

$$\frac{dp}{dt} = \frac{5}{100} \times p$$

$\Rightarrow \frac{dp}{p} = \frac{5}{100}dt$

$\Rightarrow \int \frac{dp}{p} = \int \frac{5}{100}dt$

$\Rightarrow \log p = \frac{5t}{100} + c$

$\Rightarrow p = e^{\frac{5t}{100}+c}$

For t = 0, we have p = 1000

$1000 = e^c$

For t = 10 years we have, $p = e^{\frac{50}{100}}.1000$

$p = 1000e^{1/2}$

$p = 1648$

Thus, principal is Rs1648 for t = 10 years.

**59. Question**

The volume of a spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of the balloon after t seconds.

**Answer**

Given:

Volume V $= \frac{4\pi r^3}{3}$

$$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2\frac{dr}{dt}$$

$\Rightarrow \frac{dV}{dt} = k$ (constant)

$$4\pi r^2\frac{dr}{dt} = k$$

$\Rightarrow 4\pi r^2 dr = kdt$

$\Rightarrow \int 4\pi r^2 dr = \int kdt$

$\Rightarrow \frac{4\pi r^3}{3} = kt + c$

For t = 0, r = 3 and for t = 3, r = 6, So, we have,

$\Rightarrow \frac{4\pi(3)^3}{3} = 0 + c$

$\Rightarrow c = 36\pi$

$\frac{4\pi(6)^3}{3} = k.(3) + 36\pi$

$\Rightarrow \mathrm{k} = 84\pi$

So after t seconds the radius of the balloon will be,

$\Rightarrow \frac{4\pi r^3}{3} = 84\pi t + 36\pi$

$\Rightarrow 4\pi r^3 = 252\pi t + 108\pi$

$\Rightarrow r^3 = \frac{252\pi t+108\pi}{4\pi}$

$\Rightarrow r^3 = 63t + 27$

$\Rightarrow r = \sqrt[3]{63t + 27}$

Hence, radius of the balloon as a function of time is

$\therefore r = (63t + 27)^{1/3}$

**60. Question**

In a culture the bacteria count is 100000. The number is increased by 10% in 2 hours. In how many hours will the count reach 200000, if the rate of growth of bacteria is proportional to the number present?

**Answer**

Let y be the bacteria count, then, we have,

rate of growth of bacteria is proportional to the number present

So, $\frac{dy}{dt} = cy$

Where c is a constant,

Then, solving the equation we have,

$\frac{dy}{y} = cdt$

$\int \frac{dy}{y} = \int cdt$

$logy = ct + k$

Where k is constant of integration

$y = e^{ct+k}$

And we have for t = 0, y = 10000,

$10000 = e^k$ ...(i)

For t = 2hrs, y is increased by 10% i. e. y = 110000

$110000 = e^{c(2)}.e^k$

$\Rightarrow 110000 = e^{2c}.(100000)$ from (i)

$\Rightarrow e^{2c} = 1.1$

$\Rightarrow e^c = \sqrt{1.1}$

$\Rightarrow c = \frac{1}{2}\log(\frac{11}{10})$

When y = 200000, we have,

$200000 = e^{ct}.100000$

$\Rightarrow e^{ct} = 2$

$\Rightarrow (e^c)^t = 2$

$\Rightarrow tc = log2$

$\Rightarrow t = \frac{2log2}{log\frac{11}{10}}$

Hence, t = $\frac{2\log 2}{\log\frac{11}{10}}$