

24. Cross, or Vector, Product of Vectors

Exercise 24

1 A. Question

Find $(\vec{a} \times \vec{b})$ and $|\vec{a} \times \vec{b}|$, when

$$\vec{a} = \hat{i} - \hat{j} + 2\hat{k} \text{ and } \vec{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$$

Answer

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

have

$$\vec{a} = \hat{i} - \hat{j} + 2\hat{k} \text{ and } \vec{b} = 2\hat{i} + 3\hat{j} - 4\hat{k}$$

$$\Rightarrow a_1 = 1, a_2 = -1, a_3 = 2 \text{ and } b_1 = 2, b_2 = 3, b_3 = -4$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = ((-1 \times -4) - 3 \times 2)\hat{i} + (2 \times 2 - (-4) \times 1)\hat{j} + (1 \times 3 - 2 \times (-1))\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-2)^2 + 8^2 + 5^2}$$

$$\vec{a} \times \vec{b} = (-2\hat{i} + 8\hat{j} + 5\hat{k}) \text{ and } |\vec{a} \times \vec{b}| = \sqrt{93}$$

1 B. Question

Find $(\vec{a} \times \vec{b})$ and $|\vec{a} \times \vec{b}|$, when

$$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$$

Answer

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = 2\hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$$

$$\Rightarrow a_1 = 2, a_2 = -1, a_3 = 3 \text{ and } b_1 = 3, b_2 = 5, b_3 = -2$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = ((-1 \times -2) - 5 \times 3)\hat{i} + (3 \times 3 - (-2) \times 2)\hat{j} + (2 \times 5 - 3 \times (-1))\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-17)^2 + 13^2 + 7^2} = 13\sqrt{3}$$

$$\Rightarrow \vec{a} \times \vec{b} = (-17)\hat{i} + (13)\hat{j} + (7)\hat{k}$$

1 C. Question

Find $(\vec{a} \times \vec{b})$ and $|\vec{a} \times \vec{b}|$, when

$$\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k} \text{ and } \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

Answer

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = \hat{i} - 7\hat{j} + 7\hat{k} \text{ and } \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$$

$$\Rightarrow a_1 = 1, a_2 = -7, a_3 = 7 \text{ and } b_1 = 3, b_2 = -2, b_3 = 2$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = ((-7 \times 2) - (-2) \times 7)\hat{i} + (7 \times 3 - 1 \times 2)\hat{j} + ((-2) \times 1 - 3 \times (-7))\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(0)^2 + 19^2 + 19^2} = 19\sqrt{2}$$

$$\Rightarrow \vec{a} \times \vec{b} = (0)\hat{i} + (19)\hat{j} + (19)\hat{k}$$

1 D. Question

Find $(\vec{a} \times \vec{b})$ and $|\vec{a} \times \vec{b}|$, when

$$\vec{a} = 4\hat{i} + \hat{j} - 2\hat{k} \text{ and } \vec{b} = 3\hat{i} + \hat{k}$$

Answer

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = 4\hat{i} + \hat{j} - 2\hat{k} \text{ and } \vec{b} = 3\hat{i} + 0\hat{j} + \hat{k}$$

$$\Rightarrow a_1 = 4, a_2 = 1, a_3 = -2 \text{ and } b_1 = 3, b_2 = 0, b_3 = 1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (1 \times 1 - (0) \times (-2))\hat{i} + (-2 \times 3 - 1 \times 4)\hat{j} + (4 \times 0 - 3 \times 1)\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{1^2 + (-10)^2 + (-3)^2} = \sqrt{110}$$

$$\Rightarrow \vec{a} \times \vec{b} = \hat{i} - 10\hat{j} - 3\hat{k}$$

1 E. Question

Find $(\vec{a} \times \vec{b})$ and $|\vec{a} \times \vec{b}|$, when

$$\vec{a} = 3\hat{i} + 4\hat{j} \text{ and } \vec{b} = \hat{i} + \hat{j} + \hat{k}$$

Answer

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

have $\vec{a} = 3\mathbf{i} + 4\mathbf{j} + 0\mathbf{k}$ and $\vec{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

$\Rightarrow a_1 = 3, a_2 = 4, a_3 = 0$ and $b_1 = 1, b_2 = 1, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (4 \times 1 - 1 \times 0)\mathbf{i} + (0 \times 1 - 1 \times 3)\mathbf{j} + (3 \times 1 - 1 \times 4)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{4^2 + (-3)^2 + (-1)^2} = \sqrt{26}$$

$$\Rightarrow \vec{a} \times \vec{b} = 4\mathbf{i} - 3\mathbf{j} - \mathbf{k}$$

2. Question

Find λ if $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$.

Answer

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

have $\vec{a} = 2\mathbf{i} + 6\mathbf{j} + 14\mathbf{k}$ and $\vec{b} = \mathbf{i} - \lambda\mathbf{j} + 7\mathbf{k}$

$\Rightarrow a_1 = 2, a_2 = 6, a_3 = 14$ and $b_1 = 1, b_2 = \lambda, b_3 = 7$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (6 \times 7 - (-\lambda) \times 14)\mathbf{i} + (14 \times 1 - 2 \times 7)\mathbf{j} + (2 \times (-\lambda) - 1 \times 6)\mathbf{k}$$

$$\Rightarrow \vec{a} \times \vec{b} = 0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$$

$$\Rightarrow 42 + 14\lambda = 0,$$

$$\Rightarrow \lambda = -3$$

3. Question

If $\vec{a} = (-3\hat{i} + 4\hat{j} - 7\hat{k})$ and $\vec{b} = (6\hat{i} + 2\hat{j} - 3\hat{k})$, find $(\vec{a} \times \vec{b})$.

Verify that (i) \vec{a} and $(\vec{a} \times \vec{b})$ are perpendicular to each other

and (ii) \vec{b} and $(\vec{a} \times \vec{b})$ are perpendicular to each other.

Answer

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

have $\vec{a} = -3\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$ and $\vec{b} = 6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$

$\Rightarrow a_1 = -3, a_2 = 4, a_3 = -7$ and $b_1 = 6, b_2 = 2, b_3 = -3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (4 \times (-3) - 2 \times (-7))\mathbf{i} + ((-7) \times 6 - (-3) \times (-3))\mathbf{j} + ((-3) \times 2 - 6 \times 4)\mathbf{k}$$

$$\Rightarrow \vec{a} \times \vec{b} = 2\mathbf{i} - 51\mathbf{j} - 30\mathbf{k}$$

If \vec{a} and $\vec{a} \times \vec{b}$ are perpendicular to each other then,

$$\Rightarrow \vec{a} \cdot (\vec{a} \times \vec{b}) = 0$$

i.e.,

$$\vec{a} \cdot (\vec{a} \times \vec{b}) = (-6) - (204) + (210) = 0$$

And in the similar way, we have,

$$\vec{b} \cdot (\vec{a} \times \vec{b}) = (12) - (102) + (90) = 0$$

Hence proved.

4. Question

Find the value of:

$$\text{i. } (\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j} \quad \text{ii. } (\hat{j} \times \hat{k}) \cdot \hat{i} + \hat{j} \cdot \hat{k} \quad \text{iii. } \hat{i} \times (\hat{j} + \hat{k}) + \hat{j} \times (\hat{k} + \hat{i}) + \hat{k} \times (\hat{i} + \hat{j})$$

Answer

i.

The value of $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j}$ is, ... As $\hat{i} \times \hat{j} = \hat{k}$ and $\hat{i} \cdot \hat{j} = 0$

$$\Rightarrow (\hat{k}) \cdot \hat{k} + 0 = 1$$

ii.

The value of $(\hat{j} \times \hat{k}) \cdot \hat{i} + \hat{j} \cdot \hat{k}$ is, ... As $\hat{j} \times \hat{k} = \hat{i}$ and $\hat{j} \cdot \hat{k} = 0$

$$\Rightarrow (\hat{i}) \cdot \hat{i} + 0 = 1$$

iii.

The value of $\hat{i} \times (\hat{j} + \hat{k}) + \hat{j} \times (\hat{k} + \hat{i}) + \hat{k} \times (\hat{i} + \hat{j})$ is, ... As $\hat{i} \times \hat{k} = -\hat{j}$, $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{j} \times \hat{i} = -\hat{k}$, $\hat{k} \times \hat{i} = \hat{j}$, $\hat{k} \times \hat{j} = -\hat{i}$

$$\Rightarrow \hat{k} - \hat{j} + \hat{i} - \hat{k} + \hat{j} - \hat{i} = 0$$

5 A. Question

Find the unit vectors perpendicular to both \vec{a} and \vec{b} when

$$\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k} \quad \text{and} \quad \vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$$

Answer

Let \vec{r} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{r} = k \cdot (\vec{a} \times \vec{b}) \quad \dots \text{where } k \text{ is a scalar}$$

Thus, we have r is a unit vector,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

have $\vec{a} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\vec{b} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

$\Rightarrow a_1 = 3, a_2 = 1, a_3 = -2$ and $b_1 = 2, b_2 = 3, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (1 \times -1 - 3 \times -2)\mathbf{i} + (-2 \times 2 - (-1) \times 3)\mathbf{j} + (3 \times 3 - 2 \times 1)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(5)^2 + (-1)^2 + (7)^2} = 5\sqrt{3}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{5\mathbf{i} - \mathbf{j} + 7\mathbf{k}}{5\sqrt{3}}$$

$$\Rightarrow \vec{r} = \pm \frac{5\mathbf{i} - \mathbf{j} + 7\mathbf{k}}{5\sqrt{3}}$$

5 B. Question

Find the unit vectors perpendicular to both \vec{a} and \vec{b} when

$$\vec{a} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} \text{ and } \vec{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

Answer

Let \vec{r} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{r} = k.(\vec{a} \times \vec{b}) \dots \text{where } k \text{ is a scalar}$$

Thus, we have r is a unit vector,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

have $\vec{a} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\vec{b} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$

$\Rightarrow a_1 = 1, a_2 = -2, a_3 = 3$ and $b_1 = 1, b_2 = 2, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-2 \times -1 - 2 \times 3)\mathbf{i} + (3 \times 1 - (-1) \times 1)\mathbf{j} + (1 \times 2 - (-2) \times 1)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(-4)^2 + (4)^2 + (4)^2} = 4\sqrt{3}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{-4\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}}{4\sqrt{3}}$$

$$\Rightarrow \vec{r} = \pm \frac{-\mathbf{i} + \mathbf{j} + \mathbf{k}}{\sqrt{3}}$$

5 C. Question

Find the unit vectors perpendicular to both \vec{a} and \vec{b} when

$$\vec{a} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}} \text{ and } \vec{b} = -\hat{\mathbf{i}} + 3\hat{\mathbf{k}}$$

Answer

Let \vec{r} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{r} = k.(\vec{a} \times \vec{b}) \dots \text{where } k \text{ is a scalar}$$

Thus, we have r is a unit vector,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} \text{ and } \vec{b} = -\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}$$

$$\Rightarrow a_1 = 1, a_2 = 3, a_3 = -2 \text{ and } b_1 = -1, b_2 = 0, b_3 = 3$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (9 - 0)\mathbf{i} + (2 - 3)\mathbf{j} + (0 - (-3))\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(9)^2 + (-1)^2 + (3)^2} = \sqrt{91}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{9\mathbf{i} - \mathbf{j} + 3\mathbf{k}}{\sqrt{91}}$$

$$\Rightarrow \vec{r} = \pm \frac{9\mathbf{i} - \mathbf{j} + 3\mathbf{k}}{\sqrt{91}}$$

5 D. Question

Find the unit vectors perpendicular to both \vec{a} and \vec{b} when

$$\vec{a} = 4\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}} \text{ and } \vec{b} = \hat{\mathbf{i}} + 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$$

Answer

Let \vec{r} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{r} = k.(\vec{a} \times \vec{b}) \dots \text{where } k \text{ is a scalar}$$

Thus, we have r is a unit vector,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k} \text{ and } \vec{b} = \mathbf{i} + 4\mathbf{j} - \mathbf{k}$$

$$\Rightarrow a_1 = 4, a_2 = 2, a_3 = -1 \text{ and } b_1 = 1, b_2 = 4, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (2 \times -1 - (-1) \times 4)\mathbf{i} + (-1 \times 1 - (-1) \times 4)\mathbf{j} + (4 \times 4 - 1 \times 2)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(2)^2 + (3)^2 + (14)^2} = \sqrt{209}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{2i+3j+14k}{\sqrt{209}}$$

$$\Rightarrow \vec{r} = \pm \frac{2i+3j+14k}{\sqrt{209}}$$

6. Question

Find the unit vectors perpendicular to the plane of the vectors

$$\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k} \text{ and } \vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$$

Answer

Let \vec{r} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{r} = k.(\vec{a} \times \vec{b}) \dots \text{where } k \text{ is a scalar}$$

Thus, we have r is a unit vector,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

$$\text{have } \vec{a} = 2i - 6j - 3k \text{ and } \vec{b} = 4i + 3j - k$$

$$\Rightarrow a_1 = 2, a_2 = -6, a_3 = -3 \text{ and } b_1 = 4, b_2 = 3, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-6 \times (-1) - 3 \times (-3))i + (-3 \times 4 - (-1) \times 2)j + (2 \times 3 - 4 \times (-6))k$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(15)^2 + (-10)^2 + (30)^2} = \sqrt{1225}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{3i-2j+6k}{7}$$

$$\vec{r} = \pm \frac{3i - 2j + 6k}{7}$$

7. Question

Find a vector of magnitude 6 which is perpendicular to both the vectors

$$\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k} \text{ and } \vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}.$$

Answer

Let \vec{r} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{r} = k.(\vec{a} \times \vec{b}) \dots \text{where } k \text{ is a scalar}$$

Thus, we have r is vector of magnitude 6,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

have $\vec{a} = 4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\vec{b} = -2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$

$\Rightarrow a_1 = 4, a_2 = -1, a_3 = 3$ and $b_1 = -2, b_2 = 1, b_3 = -2$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-1 \times (-2) - 1 \times (3))\mathbf{i} + (3 \times (-2) - (-2) \times 4)\mathbf{j} + (4 \times 1 - (-2) \times (-1))\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(-1)^2 + (2)^2 + (2)^2} = 3$$

$$\Rightarrow \hat{\mathbf{a}} \times \hat{\mathbf{b}} = \frac{-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}}{3}$$

$$\vec{r} = \pm k \cdot \frac{-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}}{3}$$

Here, as r is of magnitude 6 thus,

$$k = 6,$$

$$\text{Thus, } \vec{r} = \pm 2(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$$

8. Question

Find a vector of magnitude 5 units, perpendicular to each of the vectors

$$(\vec{a} + \vec{b}) \text{ and } (\vec{a} - \vec{b}), \text{ where } \vec{a} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \text{ and } \vec{b} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

Answer

$$\vec{a} + \vec{b} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} = \vec{l}$$

$$\vec{a} - \vec{b} = 0\mathbf{i} - \mathbf{j} - 2\mathbf{k} = \vec{m}$$

Let \vec{r} be the vector which is perpendicular to \vec{l} & \vec{m} then we have,

$$\vec{r} = k(\vec{l} \times \vec{m}) \dots \text{where } k \text{ is a scalar}$$

Thus, we have r is vector of magnitude 5,

So,

We have,

$$\vec{a} \times \vec{b} = (a_2 b_3 - b_2 a_3)\mathbf{i} + (a_3 b_1 - b_3 a_1)\mathbf{j} + (a_1 b_2 - b_1 a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{l} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} \text{ and } \vec{m} = 0\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

$$\Rightarrow a_1 = 2, a_2 = 3, a_3 = 4 \text{ and } b_1 = 0, b_2 = -1, b_3 = -2$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{l} \times \vec{m} = (-2)\mathbf{i} + (4)\mathbf{j} + (-2)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(-2)^2 + (4)^2 + (-2)^2} = \sqrt{24}$$

$$\Rightarrow \hat{\mathbf{a}} \times \hat{\mathbf{b}} = \frac{-\mathbf{i} + 2\mathbf{j} - \mathbf{k}}{\sqrt{6}}$$

$$\vec{r} = \pm k \cdot \frac{-i + 2j - k}{\sqrt{6}}$$

Here, as r is of magnitude 5 thus,

$$k = 5,$$

$$\text{Thus, } \vec{r} = \pm 5 \left(\frac{-i + 2j - k}{\sqrt{6}} \right)$$

9. Question

Find an angle between two vectors \vec{a} and \vec{b} with magnitudes 1 and 2 respectively and $|\vec{a} \times \vec{b}| = \sqrt{3}$.

Answer

We are given that $|\vec{a}| = 1$ and $|\vec{b}| = 2$.

$$\text{And } |\vec{a} \times \vec{b}| = \sqrt{3}.$$

So we have,

$$|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \sin\theta = \sqrt{3}$$

$$\Rightarrow |\vec{a}| \cdot |\vec{b}| \sin\theta = 1 \times 2 \times \sin\theta$$

$$\Rightarrow 2\sin\theta = \sqrt{3}$$

$$\Rightarrow \theta = \sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

10. Question

If $\vec{a} = (\hat{i} - \hat{j})$, $\vec{b} = (3\hat{j} - \hat{k})$ and $\vec{c} = (7\hat{i} - \hat{k})$, find a vector \vec{d} which is perpendicular to both \vec{a} and \vec{b} and for which $\vec{c} \cdot \vec{d} = 1$.

Answer

Given that

Let \vec{d} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{d} = k \cdot (\vec{a} \times \vec{b}) \dots \text{where } k \text{ is a scalar}$$

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = \hat{i} - \hat{j} \text{ and } \vec{b} = 0\hat{i} + 3\hat{j} - \hat{k}$$

$$\Rightarrow a_1 = 1, a_2 = -1, a_3 = 0 \text{ and } b_1 = 0, b_2 = 3, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (1)\hat{i} + (1)\hat{j} + (3)\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(1)^2 + (1)^2 + (3)^2} = \sqrt{11}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{11}}$$

$$\vec{d} = \pm k \cdot \frac{i + j + 3k}{\sqrt{11}}$$

Given that $\vec{c} \cdot \vec{d} = 1$

$$\vec{c} = 7i - k$$

$$\Rightarrow \vec{c} \cdot \vec{d} = \frac{7k - 3k}{\sqrt{11}} = 1,$$

$$\Rightarrow k = \frac{\sqrt{11}}{4}$$

$$\Rightarrow \vec{d} = \frac{i + j + 3k}{4}$$

11. Question

If $\vec{a} = (4\hat{i} + 5\hat{j} - \hat{k})$, $\vec{b} = (\hat{i} - 4\hat{j} + \hat{k})$ and $\vec{c} = (3\hat{i} + \hat{j} - \hat{k})$, find a vector \vec{d} which is perpendicular to both \vec{a} and \vec{b} and for which $\vec{c} \cdot \vec{d} = 21$.

Answer

Given that

Let \vec{d} be the vector which is perpendicular to \vec{a} & \vec{b} then we have,

$$\vec{d} = k \cdot (\vec{a} \times \vec{b}) \dots \text{where } k \text{ is a scalar}$$

We have,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = 4i + 5j - k \text{ and } \vec{b} = i - 4j + k$$

$$\Rightarrow a_1 = 4, a_2 = 5, a_3 = -1 \text{ and } b_1 = 1, b_2 = -4, b_3 = 1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (1)\hat{i} + (-5)\hat{j} + (-21)\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(1)^2 + (-5)^2 + (-21)^2} = \sqrt{467}$$

$$\Rightarrow \vec{a} \times \vec{b} = \frac{i - 5j - 21k}{\sqrt{467}}$$

$$\vec{d} = \pm k \cdot \frac{i - 5j - 21k}{\sqrt{467}}$$

Given that $\vec{c} \cdot \vec{d} = 21$

$$\vec{c} = 3i + j - k$$

$$\Rightarrow \vec{c} \cdot \vec{d} = \frac{19k}{\sqrt{467}} = 21,$$

$$\Rightarrow k = \frac{\sqrt{467}}{19 \times 21}$$

$$\vec{d} = \frac{i - 5j - 21k}{319} \times \sqrt{467}$$

12. Question

Prove that $|\vec{a} \times \vec{b}| = (\vec{a} \cdot \vec{b}) \tan \theta$, where θ is the angle between \vec{a} and \vec{b} .

Answer

We know that $|\vec{a} \cdot \vec{b}| = |\vec{a}| |\vec{b}| \cos \theta$

And $|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$

So,

$$\tan \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a} \cdot \vec{b}|}$$

Hence, proved.

13. Question

Write the value of p for which $\vec{a} = (3\hat{i} + 2\hat{j} + 9\hat{k})$ and $\vec{b} = (\hat{i} + p\hat{j} + 3\hat{k})$ are parallel vectors.

Answer

As the vectors are parallel vectors so, $\vec{a} \times \vec{b} = \mathbf{0}$

Thus,

We have,

$$\vec{a} \times \vec{b} = (a_2 b_3 - b_2 a_3)\hat{i} + (a_3 b_1 - b_3 a_1)\hat{j} + (a_1 b_2 - b_1 a_2)\hat{k}$$

Here,

We

have $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$

$$\Rightarrow a_1 = 3, a_2 = 2, a_3 = 9 \text{ and } b_1 = 1, b_2 = p, b_3 = 3$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (6 - 9p)\hat{i} + (0)\hat{j} + (3p - 2)\hat{k} = \mathbf{0}$$

$$\Rightarrow 6 - 9p = 0$$

$$\Rightarrow \text{Thus, } p = \frac{2}{3}.$$

14 A. Question

Verify that $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})$, when

$$\vec{a} = \hat{i} - \hat{j} - 3\hat{k}, \vec{b} = 4\hat{i} - 3\hat{j} + \hat{k} \text{ and } \vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$$

Answer

To verify $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})$

We need to prove L.H.S = R.H.S

L.H.S we have,

$$\text{Given, } \vec{a} = \hat{i} - \hat{j} - 3\hat{k} \quad \vec{b} = 4\hat{i} - 3\hat{j} + \hat{k} \quad \vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = (\hat{i} - \hat{j} - 3\hat{k}) \times (6\hat{i} - 4\hat{j} + 3\hat{k})$$

$$\vec{a} \times \vec{b} = (a_2 b_3 - b_2 a_3)\mathbf{i} + (a_3 b_1 - b_3 a_1)\mathbf{j} + (a_1 b_2 - b_1 a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = \mathbf{i} - \mathbf{j} - 3\mathbf{k} \text{ and } \vec{b} + \vec{c} = 6\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$$

$$\Rightarrow a_1 = 1, a_2 = -1, a_3 = -3 \text{ and } b_1 = 6, b_2 = -4, b_3 = 3$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times (\vec{b} + \vec{c}) = (-3 - 12)\mathbf{i} + (3 + 18)\mathbf{j} + (-4 + 6)\mathbf{k}$$

$$\Rightarrow (-15)\mathbf{i} + (21)\mathbf{j} + (2)\mathbf{k}$$

RHS is

$$(\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) = (-10\mathbf{i} + 13\mathbf{j} + \mathbf{k}) + (-5\mathbf{i} + 8\mathbf{j} + \mathbf{k})$$

$$\Rightarrow (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) = (-15)\mathbf{i} + (21)\mathbf{j} + (2)\mathbf{k}$$

Thus, LHS = RHS.

14 B. Question

Verify that $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})$, when

$$\vec{a} = 4\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k} \text{ and } \vec{c} = \hat{i} - \hat{j} + \hat{k}.$$

Answer

To verify $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})$

We need to prove L.H.S = R.H.S

L.H.S we have,

$$\text{Given, } \vec{a} = 4\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{a} \times (\vec{b} + \vec{c}) = (4\hat{i} - \hat{j} + \hat{k}) \times (2\hat{i} + 0\hat{j} + 2\hat{k})$$

$$\vec{a} \times \vec{b} = (a_2 b_3 - b_2 a_3)\mathbf{i} + (a_3 b_1 - b_3 a_1)\mathbf{j} + (a_1 b_2 - b_1 a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{a} = 4\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{b} + \vec{c} = 2\hat{i} + 0\hat{j} + 2\hat{k}$$

$$\Rightarrow a_1 = 4, a_2 = -1, a_3 = 1 \text{ and } b_1 = 2, b_2 = 0, b_3 = 2$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times (\vec{b} + \vec{c}) = (-2)\mathbf{i} + (-2)\mathbf{j} + (2)\mathbf{k}$$

$$\Rightarrow (-2)\mathbf{i} + (-2)\mathbf{j} + (2)\mathbf{k}$$

RHS is

$$(\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) = (-2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}) + (0\mathbf{i} + \mathbf{j} - 3\mathbf{k})$$

$$\Rightarrow (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c}) = (-2)\mathbf{i} + (-2)\mathbf{j} + (2)\mathbf{k}$$

Thus, LHS = RHS.

15 A. Question

Find the area of the parallelogram whose adjacent sides are represented by the vectors:

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{b} = -3\hat{i} - 2\hat{j} + \hat{k}$$

Answer

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = \hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{b} = -3\hat{i} - 2\hat{j} + \hat{k}$$

$$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3 \text{ and } b_1 = -3, b_2 = -2, b_3 = 1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (8)\hat{i} + (-10)\hat{j} + (4)\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(8)^2 + (-10)^2 + (4)^2} = \sqrt{180}$$

$$\Rightarrow \text{area} = 6\sqrt{5} \text{ sq units}$$

15 B. Question

Find the area of the parallelogram whose adjacent sides are represented by the vectors:

$$\vec{a} = (3\hat{i} + \hat{j} + 4\hat{k}) \text{ and } \vec{b} = (\hat{i} - \hat{j} + \hat{k})$$

Answer

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = 3\hat{i} + \hat{j} + 4\hat{k} \text{ and } \vec{b} = \hat{i} - \hat{j} + \hat{k}$$

$$\Rightarrow a_1 = 3, a_2 = 1, a_3 = 4 \text{ and } b_1 = 1, b_2 = -1, b_3 = 1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (5)\hat{i} + (-1)\hat{j} + (-4)\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(5)^2 + (-1)^2 + (-4)^2} = \sqrt{42}$$

$$\Rightarrow \text{area} = \sqrt{42} \text{ sq units}$$

15 C. Question

Find the area of the parallelogram whose adjacent sides are represented by the vectors:

$$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k} \text{ and } \vec{b} = \hat{i} - \hat{j}$$

Answer

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = 2\hat{i} + \hat{j} + 3\hat{k} \text{ and } \vec{b} = \hat{i} - \hat{j} + 0\hat{k}$$

$$\Rightarrow a_1 = 2, a_2 = 1, a_3 = 3 \text{ and } b_1 = 1, b_2 = -1, b_3 = 0$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (3)\hat{i} + (3)\hat{j} + (-3)\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(3)^2 + (3)^2 + (-3)^2} = 3\sqrt{3}$$

$$\Rightarrow \text{area} = 3\sqrt{3} \text{ sq units}$$

15 D. Question

Find the area of the parallelogram whose adjacent sides are represented by the vectors:

$$\vec{a} = 2\hat{i} \text{ and } \vec{b} = 3\hat{j}$$

Answer

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = 2\hat{i} + 0\hat{j} + 0\hat{k} \text{ and } \vec{b} = 0\hat{i} + 3\hat{j} + 0\hat{k}$$

$$\Rightarrow a_1 = 2, a_2 = 0, a_3 = 0 \text{ and } b_1 = 0, b_2 = 3, b_3 = 0$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (6)\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = 6$$

$$\Rightarrow \text{area} = 6 \text{ sq units}$$

16 A. Question

Find the area of the parallelogram whose diagonal are represented by the vectors

$$\vec{d}_1 = 3\hat{i} + \hat{j} - 2\hat{k} \text{ and } \vec{d}_2 = \hat{i} - 3\hat{j} + 4\hat{k}$$

Answer

The diagonals are $\vec{a} + \vec{b} = 3i + j - 2k$ & $\vec{a} - \vec{b} = i - 3j + 4k$

Thus, $\vec{a} = 2i - j + k$, $\vec{b} = i + 2j - 3k$

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

have $\vec{a} = 2i - j + k$ and $\vec{b} = i + 2j - 3k$

$$\Rightarrow a_1 = 2, a_2 = -1, a_3 = 1 \text{ and } b_1 = 1, b_2 = 2, b_3 = -3$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (3 - 2)i + 7j + (5)k$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(1)^2 + (7)^2 + (5)^2} = 5\sqrt{3}$$

\Rightarrow

$$\Rightarrow \text{area} = 5\sqrt{3} \text{ sq units}$$

16 B. Question

Find the area of the parallelogram whose diagonal are represented by the vectors

$$\vec{d}_1 = 2\hat{i} - \hat{j} + \hat{k} \text{ and } \vec{d}_2 = 3\hat{i} + 4\hat{j} - \hat{k}$$

Answer

The diagonals are $\vec{a} + \vec{b} = 2i - j + k$ & $\vec{a} - \vec{b} = 3i + 4j - k$

Thus, $\vec{a} = \frac{5}{2}i + \frac{3}{2}j$, $\vec{b} = -\frac{1}{2}i - \frac{5}{2}j + k$

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

have, $\vec{a} = \frac{5}{2}i + \frac{3}{2}j$, $\vec{b} = -\frac{1}{2}i - \frac{5}{2}j + k$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = \left(\frac{3}{2}\right)i - \frac{5}{2}j + \left(-\frac{11}{2}\right)k$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{5}{2}\right)^2 + \left(-\frac{11}{2}\right)^2} = \frac{1}{2}\sqrt{155}$$

\Rightarrow

$$\Rightarrow \text{area} = \frac{1}{2}\sqrt{155} \text{ sq units}$$

16 C. Question

Find the area of the parallelogram whose diagonal are represented by the vectors

$$\vec{d}_1 = \hat{i} - 3\hat{j} + 2\hat{k} \text{ and } \vec{d}_2 = -\hat{i} + 2\hat{j}.$$

Answer

The diagonals are $\vec{a} + \vec{b} = \hat{i} - 3\hat{j} + 2\hat{k}$ & $\vec{a} - \vec{b} = -\hat{i} + 2\hat{j} + 0\hat{k}$

$$\text{Thus, } \vec{a} = 0\hat{i} - \frac{1}{2}\hat{j} + \hat{k}, \vec{b} = \hat{i} - \frac{5}{2}\hat{j} + \hat{k}$$

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

$$\text{Area} = |\vec{a} \times \vec{b}|$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = 0\hat{i} - \frac{1}{2}\hat{j} + \hat{k} \text{ and } \vec{b} = \hat{i} - \frac{5}{2}\hat{j} + \hat{k}$$

$$\Rightarrow a_1 = 0, a_2 = -\frac{1}{2}, a_3 = 1 \text{ and } b_1 = 1, b_2 = -\frac{5}{2}, b_3 = 1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (2)\hat{i} + 1\hat{j} + \left(\frac{1}{2}\right)\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(2)^2 + (1)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{2}\sqrt{21}$$

\Rightarrow

$$\Rightarrow \text{area} = \frac{\sqrt{21}}{2} \text{ sq units}$$

17 A. Question

Find the area of the triangle whose two adjacent sides are determined by the vectors

$$\vec{a} = -2\hat{i} - 5\hat{k} \text{ and } \vec{b} = \hat{i} - 2\hat{j} - \hat{k}$$

Answer

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

$$\text{Area} = \frac{|\vec{a} \times \vec{b}|}{2}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = -2\hat{i} + 0\hat{j} - 5\hat{k} \text{ and } \vec{b} = \hat{i} - 2\hat{j} - \hat{k}$$

$$\Rightarrow a_1 = -2, a_2 = 0, a_3 = -5 \text{ and } b_1 = 1, b_2 = -2, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (8)\hat{i} + (-10)\hat{j} + (4)\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165}$$

$$\Rightarrow \text{area} = \frac{\sqrt{165}}{2} \text{ sq units}$$

17 B. Question

Find the area of the triangle whose two adjacent sides are determined by the vectors

$$\vec{a} = 3\hat{i} + 4\hat{j} \text{ and } \vec{b} = -5\hat{i} + 7\hat{j}.$$

Answer

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

$$\text{Area} = \frac{|\vec{a} \times \vec{b}|}{2}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

$$\text{have } \vec{a} = 3\hat{i} + 4\hat{j} + 0\hat{k} \text{ and } \vec{b} = -5\hat{i} + 7\hat{j} + 0\hat{k}$$

$$\Rightarrow a_1 = 3, a_2 = 4, a_3 = 0 \text{ and } b_1 = -5, b_2 = 7, b_3 = 0$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (41)\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = 41$$

$$\Rightarrow \text{area} = \frac{41}{2} \text{ sq units}$$

18 A. Question

Using vectors, find the area of ΔABC whose vertices are

$$A(1, 1, 2), B(2, 3, 5) \text{ and } C(1, 5, 5)$$

Answer

Through the vertices we get the adjacent vectors as,

$$\vec{AB} = \hat{i} + 2\hat{j} + 3\hat{k} \text{ and } \vec{AC} = 4\hat{j} + 3\hat{k}$$

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

$$\text{Area} = \frac{|\vec{a} \times \vec{b}|}{2}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\hat{i} + (a_3b_1 - b_3a_1)\hat{j} + (a_1b_2 - b_1a_2)\hat{k}$$

Here,

We

have $\overrightarrow{AB} = i + 2j + 3k$ and $\overrightarrow{AC} = 4j + 3k$

$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3$ and $b_1 = 0, b_2 = 4, b_3 = 3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-6)i + (-3)j + (4)k$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-6)^2 + (-3)^2 + (4)^2} = \sqrt{61}$$

$$\Rightarrow \text{area} = \frac{\sqrt{61}}{2} \text{ sq units}$$

18 B. Question

Using vectors, find the area of ΔABC whose vertices are

$A(1, 2, 3), B(2, -1, 4)$ and $C(4, 5, \Delta 1)$ ((considering $\Delta 1$ as 1))

Answer

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = i - 3j + 1k \text{ and } \overrightarrow{AC} = 3i + 3j - 2k$$

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

$$\text{Area} = \frac{|\vec{a} \times \vec{b}|}{2}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$$

Here,

We

$$\text{have } \overrightarrow{AB} = i - 3j + k \text{ and } \overrightarrow{AC} = 3i + 3j - 2k$$

$$\Rightarrow a_1 = 1, a_2 = -3, a_3 = 1 \text{ and } b_1 = 3, b_2 = 3, b_3 = -2$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (3)i + (5)j + (12)k$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(3)^2 + (5)^2 + (12)^2} = \sqrt{178}$$

$$\Rightarrow \text{area} = \frac{\sqrt{178}}{2} \text{ sq units}$$

18 C. Question

Using vectors, find the area of ΔABC whose vertices are

$A(3, -1, 2), B(1, -1, -3)$ and $C(4, -3, 1)$

Answer

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = -2i + 0j - 5k \text{ and } \overrightarrow{AC} = i - 2j - k$$

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

$$\text{Area} = \frac{|\vec{a} \times \vec{b}|}{2}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{AB} = -2\mathbf{i} - 5\mathbf{k} \text{ and } \vec{AC} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$$

$$\Rightarrow a_1 = -2, a_2 = 0, a_3 = -5 \text{ and } b_1 = 1, b_2 = -2, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-10)\mathbf{i} + (-7)\mathbf{j} + (4)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165}$$

$$\Rightarrow \text{area} = \frac{\sqrt{165}}{2} \text{ sq units}$$

18 D. Question

Using vectors, find the area of ΔABC whose vertices are

$A(1, -1, 2)$, $B(2, 1, -1)$ and $C(3, -1, 2)$.

Answer

Through the vertices we get the adjacent vectors as,

$$\vec{AB} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \text{ and } \vec{AC} = 2\mathbf{i}$$

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

$$\text{Area} = \frac{|\vec{a} \times \vec{b}|}{2}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \vec{AB} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \text{ and } \vec{AC} = 2\mathbf{i}$$

$$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3 \text{ and } b_1 = 0, b_2 = 4, b_3 = 3$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-6)\mathbf{j} + (-4)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = \sqrt{(-6)^2 + (-4)^2} = \sqrt{52}$$

$$\Rightarrow \text{area} = \frac{\sqrt{52}}{2} \text{ sq units}$$

19 A. Question

Using vector method, show that the given points A, B, C are collinear:

$A(3, -5, 1)$, $B(-1, 0, 8)$ and $C(7, -10, -6)$

Answer

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = -4\mathbf{i} + 5\mathbf{j} + 7\mathbf{k} \text{ and } \overrightarrow{AC} = 4\mathbf{i} - 5\mathbf{j} - 7\mathbf{k}$$

To prove that A, B, C are collinear we need to prove that

$$\vec{a} \times \vec{b} = \mathbf{0}.$$

So,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \overrightarrow{AB} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \text{ and } \overrightarrow{AC} = 4\mathbf{j} + 3\mathbf{k}$$

$$\Rightarrow a_1 = -4, a_2 = 5, a_3 = 7 \text{ and } b_1 = 4, b_2 = -5, b_3 = -7$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (0)\mathbf{i} + (0)\mathbf{j} + (0)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = 0$$

19 B. Question

Using vector method, show that the given points A, B, C are collinear:

A(6, -7, -1), B(2, -3, 1) and C(4, -5, 0).

Answer

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = -4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} \text{ and } \overrightarrow{AC} = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$$

To prove that A, B, C are collinear we need to prove that

$$\vec{a} \times \vec{b} = \mathbf{0}.$$

So,

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\mathbf{i} + (a_3b_1 - b_3a_1)\mathbf{j} + (a_1b_2 - b_1a_2)\mathbf{k}$$

Here,

We

$$\text{have } \overrightarrow{AB} = -4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} \text{ and } \overrightarrow{AC} = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$$

$$\Rightarrow a_1 = -4, a_2 = 4, a_3 = 2 \text{ and } b_1 = -2, b_2 = 2, b_3 = 1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (0)\mathbf{i} + (0)\mathbf{j} + (0)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = 0$$

Thus, A, B and C are collinear.

20. Question

Show that the point A, B, C with position vectors $(3\hat{i} - 2\hat{j} + 4\hat{k})$, $(\hat{i} + \hat{j} + \hat{k})$ and $(-\hat{i} + 4\hat{j} - 2\hat{k})$ respectively are collinear.

Answer

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = -2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k} \text{ and } \overrightarrow{AC} = -4\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$$

To prove that A, B, C are collinear we need to prove that

$$\vec{a} \times \vec{b} = \mathbf{0}.$$

So,

$$\vec{a} \times \vec{b} = (a_2 b_3 - b_2 a_3)\mathbf{i} + (a_3 b_1 - b_3 a_1)\mathbf{j} + (a_1 b_2 - b_1 a_2)\mathbf{k}$$

Here,

We

$$\text{have } \overrightarrow{AB} = -2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k} \text{ and } \overrightarrow{AC} = -4\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$$

$$\Rightarrow a_1 = -2, a_2 = 3, a_3 = -3 \text{ and } b_1 = -4, b_2 = 6, b_3 = -6$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (0)\mathbf{i} + (0)\mathbf{j} + (0)\mathbf{k}$$

$$\Rightarrow |\mathbf{a} \times \mathbf{b}| = 0$$

Thus, A, B and C are collinear.

21. Question

Show that the points having position vectors $\vec{a}, \vec{b}, (\vec{c} = 3\vec{a} - 2\vec{b})$ are collinear, whatever be $\vec{a}, \vec{b}, \vec{c}$.

Answer

Through the vertices we get the adjacent vectors as,

$$\overrightarrow{AB} = \vec{b} - \vec{a} \text{ and } \overrightarrow{AC} = \vec{c} - \vec{a} = 2\vec{a} + 2\vec{b}$$

To prove that A, B, C are collinear we need to prove that

$$\overrightarrow{AB} \times \overrightarrow{AC} = \mathbf{0}.$$

So,

Here,

We

$$\text{have } \overrightarrow{AB} = \vec{b} - \vec{a} \text{ and } \overrightarrow{AC} = 2\vec{a} + 2\vec{b}$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = (\vec{b} - \vec{a}) \times (2\vec{a} + 2\vec{b})$$

$$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = \vec{b} \times 2\vec{a} + \mathbf{0} - \mathbf{0} - \vec{a} \times 2\vec{b} = \mathbf{0}$$

Thus, A, B and C are collinear.

22. Question

Show that the points having position vector $(-2\vec{a} + 3\vec{b} + 5\vec{c}), (\vec{a} + 2\vec{b} + 3\vec{c})$ and $(7\vec{a} - \vec{c})$ are collinear, whatever be $\vec{a}, \vec{b}, \vec{c}$.

Answer

We have, $\vec{A} = -2\vec{a} + 3\vec{b} + 5\vec{c}$, $\vec{B} = \vec{a} + 2\vec{b} + 3\vec{c}$, $\vec{C} = 7\vec{a} - \vec{c}$

Through the vertices we get the adjacent vectors as,

$$\vec{AB} = 3\vec{a} - \vec{b} - 2\vec{c} \text{ and } \vec{AC} = 9\vec{a} - 3\vec{b} - 6\vec{c}$$

To prove that A, B, C are collinear we need to prove that

$$\vec{AB} \times \vec{AC} = \vec{0}$$

So,

Here,

We

have

$$\vec{AB} = 3\vec{a} - \vec{b} - 2\vec{c} \text{ and } \vec{AC} = 9\vec{a} - 3\vec{b} - 6\vec{c}$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{AB} \times \vec{AC} = (3\vec{a} - \vec{b} - 2\vec{c}) \times (9\vec{a} - 3\vec{b} - 6\vec{c})$$

$$\Rightarrow \vec{AB} \times \vec{AC} = \vec{0}$$

Thus, A, B and C are collinear.

23. Question

Find a unit vector perpendicular to the plane ABC, where the points A, B, C, are $(3, -1, 2)$, $(1, -1, -3)$ and $(4, -3, 1)$ respectively.

Answer

A unit vector perpendicular to the plane ABC will be,

$$\pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

Through the vertices we get the adjacent vectors as,

$$\vec{AB} = -2\vec{i} + 0\vec{j} - 5\vec{k} \text{ and } \vec{AC} = \vec{i} - 2\vec{j} - \vec{k}$$

$$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)\vec{i} + (a_3b_1 - b_3a_1)\vec{j} + (a_1b_2 - b_1a_2)\vec{k}$$

Here,

We

$$\text{have } \vec{AB} = -2\vec{i} + 0\vec{j} - 5\vec{k} \text{ and } \vec{AC} = \vec{i} - 2\vec{j} - \vec{k}$$

$$\Rightarrow a_1 = -2, a_2 = 0, a_3 = -5 \text{ and } b_1 = 1, b_2 = -2, b_3 = -1$$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$$\Rightarrow \vec{a} \times \vec{b} = (-10)\vec{i} + (-7)\vec{j} + (4)\vec{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165}$$

$$\Rightarrow \text{unit vector} = \frac{-10\vec{i} - 7\vec{j} + 4\vec{k}}{\sqrt{165}}$$

24. Question

If $\vec{a} = (\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{b} = (\hat{i} - 3\hat{k})$ then find $|\vec{b} \times 2\vec{a}|$.

Answer

$\vec{a} = i + 2j + 3k$ and $\vec{b} = i - 3k$

Then, $|\vec{b} \times 2\vec{a}|$,

We have, $\vec{b} \times \vec{a} = (-2a_2 \cdot b_3 + 2b_2 \cdot a_3)i - (a_3 \cdot 2b_1 - 2b_3 \cdot a_1)j - (a_1 \cdot 2b_2 - 2b_1 \cdot a_2)k$

Here,

We

have $\vec{a} = i + 2j + 3k$ and $\vec{b} = i - 3k$

$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3$ and $b_1 = 1, b_2 = 0, b_3 = -3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3 ,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (-12)i + (12)j + (-4)k$

$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(-12)^2 + (12)^2 + (-4)^2} = 4\sqrt{19}$

25. Question

If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, find $\vec{a} \cdot \vec{b}$.

Answer

We have, $|\vec{a}|^2 |\vec{b}|^2 = |\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2$

So, $|\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 - |\vec{a} \times \vec{b}|^2$

$\Rightarrow |\vec{a} \cdot \vec{b}|^2 = 10^2 - 8^2 = 6^2$

$\Rightarrow |\vec{a} \cdot \vec{b}| = 6$

26. Question

If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $(\vec{a} \times \vec{b}) = (3\hat{i} + 2\hat{j} + 6\hat{k})$, find the angle between \vec{a} and \vec{b} .

Answer

We have, $|\vec{a}|^2 |\vec{b}|^2 = |\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2$

$\Rightarrow \vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta$

$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{3^2 + 2^2 + 6^2} = 7$

$\Rightarrow 7 = 7 \times 2\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \sin^{-1} \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$