25. Product of Three Vectors

Exercise 25A

Q. 1

Prove that

i. $[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$
ii. $[\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = [\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = [\hat{\jmath} \quad \hat{\imath} \quad \hat{k}] = -1$

Answer :

i. $[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$

Let, $\hat{\imath}, \hat{\jmath}, \hat{k}$ be unit vectors in the direction of positive X-axis, Y-axis, Z-axis respectively.

Hence,

$Magnitude\ of\ \hat{\imath}\ is\ 1 \Rightarrow |\hat{\imath}| = 1$

$Magnitude\ of\ \hat{\jmath}\ is\ 1 \Rightarrow |\hat{\jmath}| = 1$

$Magnitude\ of\ \hat{k}\ is\ 1 \Rightarrow |\hat{k}| = 1$

To Prove :

$[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$

Formulae :

a) Dot Products :

i) $\hat{\imath} . \hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

ii) $\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

b) Cross Products :

i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$

iii) $\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}, \hat{\imath} \times \hat{k} = -\hat{\jmath}$

c) Scalar Triple Product :

$[\bar{a} \quad \bar{b} \quad \bar{c}] = \bar{a} . (\bar{b} \times \bar{c})$

Now,

(i) $[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = \hat{\imath} \,.(\hat{\jmath} \times \hat{k})$

$= \hat{\imath} \,.\, \hat{\imath}$ ………… $(\because \hat{\jmath} \times \hat{k} = \hat{\imath})$

= 1 ………… $(\because \hat{\imath} \,.\, \hat{\imath} = 1)$

$\therefore \; [\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = 1$ ………… eq(1)

(ii) $[\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = \hat{\jmath} \,.(\hat{k} \times \hat{\imath})$

$= \hat{\jmath} \,.\, \hat{\jmath}$ ………… $(\because \hat{k} \times \hat{\imath} = \hat{\jmath})$

= 1 ………… $(\because \hat{\jmath} \,.\, \hat{\jmath} = 1)$

$\therefore \; [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = 1$ ………… eq(2)

(iii) $[\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = \hat{k} \,.(\hat{\imath} \times \hat{\jmath})$

$= \hat{k} \,.\, \hat{k}$ ………… $(\because \hat{\imath} \times \hat{\jmath} = \hat{k})$

= 1 ………… $(\because \hat{k} \,.\, \hat{k} = 1)$

$\therefore \; [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$ ………… eq(3)

From eq(1), eq(2) and eq(3),

$[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = [\hat{\jmath} \quad \hat{k} \quad \hat{\imath}] = [\hat{k} \quad \hat{\imath} \quad \hat{\jmath}] = 1$

Hence Proved.

Notes :

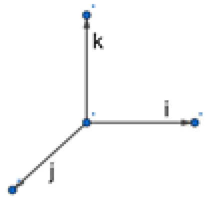
1. A cyclic change of vectors in a scalar triple product does not change its value i.e.

$[\bar{a} \quad \bar{b} \quad \bar{c}] = [\bar{b} \quad \bar{c} \quad \bar{a}] = [\bar{c} \quad \bar{a} \quad \bar{b}]$

2. Scalar triple product of unit vectors taken in a clockwise direction is 1, and that of unit vectors taken in anticlockwise direction is -1

$[\hat{\imath} \quad \hat{\jmath} \quad \hat{k}] = 1$

$[\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = -1$



ii. $[\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = [\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = [\hat{\jmath} \quad \hat{\imath} \quad \hat{k}] = -1$

Let, $\hat{\imath}, \hat{\jmath}, \hat{k}$ be unit vectors in the direction of positive X-axis, Y-axis, Z-axis respectively.

Hence,

$Magnitude\ of\ \hat{\imath}\ is\ 1 \Rightarrow |\hat{\imath}| = 1$

$Magnitude\ of\ \hat{\jmath}\ is\ 1 \Rightarrow |\hat{\jmath}| = 1$

$Magnitude\ of\ \hat{k}\ is\ 1 \Rightarrow |\hat{k}| = 1$

To Prove :

$[\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = [\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = [\hat{\jmath} \quad \hat{\imath} \quad \hat{k}] = -1$

Formulae :

a) Dot Products :

i) $\hat{\imath} . \hat{\imath} = \hat{\jmath} . \hat{\jmath} = \hat{k} . \hat{k} = 1$

ii) $\hat{\imath} . \hat{\jmath} = \hat{\jmath} . \hat{k} = \hat{k} . \hat{\imath} = 0$

b) Cross Products :

i) $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$

ii) $\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath}$

iii) $\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}, \hat{\imath} \times \hat{k} = -\hat{\jmath}$

c) Scalar Triple Product :

$[\bar{a} \quad \bar{b} \quad \bar{c}] = \bar{a} . (\bar{b} \times \bar{c})$

Answer :

(i) $[\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = \hat{\imath} . (\hat{k} \times \hat{\jmath})$

$= \hat{\imath} . (-\hat{\imath})$ ………… $(\because \hat{k} \times \hat{\jmath} = -\hat{\imath})$

$= -\hat{\imath} . \hat{\imath}$

= -1 ………… $(\because \hat{\imath} . \hat{\imath} = 1)$

$\therefore [\hat{\imath} \quad \hat{k} \quad \hat{\jmath}] = -1$ ………… eq(1)

(ii) $[\hat{k} \quad \hat{\jmath} \quad \hat{\imath}] = \hat{k} . (\hat{\jmath} \times \hat{\imath})$

$= \hat{k}.(-\hat{k})$ ………… $(\because \hat{j} \times \hat{\imath} = -\hat{k})$

$= -\hat{k}.\hat{k}$

= -1 ………… $(\because \hat{k}.\hat{k} = 1)$

$\therefore\ [\hat{k}\quad \hat{j}\quad \hat{\imath}] = -1$ ………… eq(2)

(iii) $[\hat{j}\quad \hat{\imath}\quad \hat{k}] = \hat{j}.(\hat{\imath} \times \hat{k})$

$= \hat{j}.(-\hat{j})$ ………… $(\because \hat{\imath} \times \hat{k} = -\hat{j})$

$= -\hat{j}.\hat{j}$

= -1 ………… $(\because \hat{j}.\hat{j} = 1)$

$\therefore\ [\hat{j}\quad \hat{\imath}\quad \hat{k}] = -1$ ………… eq(3)

From eq(1), eq(2) and eq(3),

$[\hat{\imath}\quad \hat{k}\quad \hat{j}] = [\hat{k}\quad \hat{j}\quad \hat{\imath}] = [\hat{j}\quad \hat{\imath}\quad \hat{k}] = -1$

Hence Proved.

Notes :

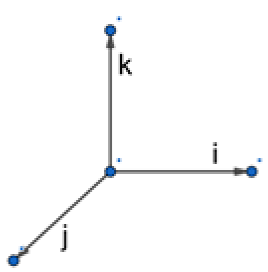
1. A cyclic change of vectors in a scalar triple product does not change its value i.e.

$[\bar{a}\quad \bar{b}\quad \bar{c}] = [\bar{b}\quad \bar{c}\quad \bar{a}] = [\bar{c}\quad \bar{a}\quad \bar{b}]$

2. Scalar triple product of unit vectors taken in a clockwise direction is 1, and that of unit vectors taken in anticlockwise direction is -1

$[\hat{\imath}\quad \hat{j}\quad \hat{k}] = 1$

$[\hat{k}\quad \hat{j}\quad \hat{\imath}] = -1$



Q. 2

Find $\left[\vec{a}\ \vec{b}\ \vec{c}\right]$, **when**

i. $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ **and .** $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$ **.**

ii. $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ **and** $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

iii. $\vec{a} = 2\hat{i} - 3\hat{j}, \vec{b} = \hat{i} + \hat{j} - \hat{k}$ **and** $\vec{c} = 3\hat{i} - \hat{k}$

Answer :

i. $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$

Given Vectors :

1) $\bar{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

2) $\bar{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

3) $\bar{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

$\bar{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\bar{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & 1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

$= 2(2 \times 2 - 1 \times 1) - 1((-1) \times 2 - 3 \times 1) + 3((-1) \times 1 - 3 \times 2)$

$= 2(3) - 1(-5) + 3(-7)$

= 6 + 5 – 21

= - 10

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -10$$

ii. $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

Given Vectors :

1) $\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

2) $\bar{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

3) $\bar{c} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\bar{c} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$= 2(2 \times 2 - (-1) \times (-1)) - (-3)(1 \times 2 - 3 \times (-1)) + 4(1 \times (-1) - 3 \times 2)$

$= 2(3) + 3(5) + 4(-7)$

= 6 + 15 – 28

= - 7

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -7$$

iii. $\vec{a} = 2\hat{i} - 3\hat{j}$, $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{k}$

Given Vectors :

1) $\bar{a} = 2\hat{\imath} - 3\hat{\jmath}$

2) $\bar{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$

3) $\bar{c} = 3\hat{\imath} - \hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 0\hat{k}$

$\bar{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = 3\hat{\imath} + 0\hat{\jmath} - \hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$$

$= 2(1 \times (-1) - (-1) \times 0) - (-3)(1 \times (-1) - 3 \times (-1)) + 0(1 \times 0 - 3 \times 1)$

$= 2(-1) + 3(2) + 0$

= - 2 + 6

= 4

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 4$

Q. 3

Find the volume of the parallelepiped whose conterminous edges are represented by the vectors

i. $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$

ii. $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}, \vec{b} = -5\hat{i} + 7\hat{j} - 3\hat{k},\ \vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$

iii. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$

iv. $\bar{a} = 6\hat{\imath}, \bar{b} = 2\hat{\jmath}, \bar{c} = 5\hat{k}$

Answer :

i. $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}-\hat{j}+\hat{k}, \vec{c}=\hat{i}+2\hat{j}-\hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{c} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

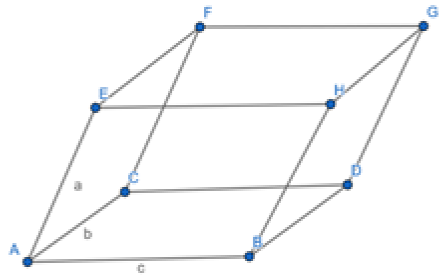
Answer :

Volume of parallelopiped with coterminous edges

$\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{c} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$$= 1((-1) \times (-1) - 2 \times 1) - 1(1 \times (-1) - 1 \times 1) + 1(1 \times 2 - 1 \times (-1))$$

= 1(-1) -1(-2) + 1(3)

= -1+2+3

= 4

Therefore,

Volume of parallelepiped = 4 cubic unit

ii. $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}, \vec{b} = -5\hat{i} + 7\hat{j} - 3\hat{k}, \vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$

$\bar{b} = -5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$

$\bar{c} = 7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

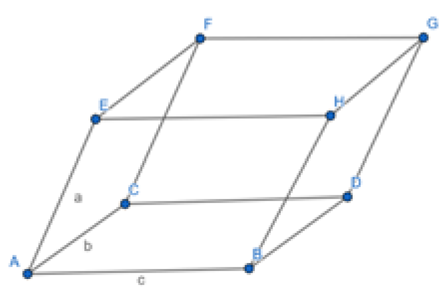
Answer :

Volume of parallelopiped with coterminous edges

$$\bar{a} = -3\hat{\imath} + 7\hat{\jmath} + 5\hat{k}$$

$$\bar{b} = -5\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$$

$$\bar{c} = 7\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & 7 & 5 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix}$$

$$= -3\big(7 \times (-3) - (-5) \times (-3)\big) - 7\big((-5) \times (-3) - 7 \times (-3)\big) + 5((-5) \times (-5) - 7 \times 7)$$

= -3(-36) -7(36) + 5(-24)

= 108 – 252 – 120

= -264

As volume is never negative

Therefore,

Volume of parallelepiped = 264 cubic unit

iii. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \hat{\jmath} + \hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

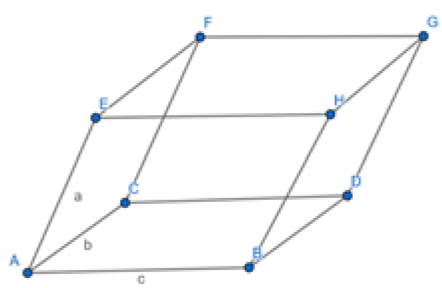
Answer :

Volume of parallelopiped with coterminous edges

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = 0\hat{\imath} + \hat{\jmath} + \hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 1(1 \times 1 - 1 \times (-1)) - (-2)(2 \times 1 - 0 \times (-1)) + 3(2 \times 1 - 0 \times 1)$$

= 1(2) +2(2) + 3(2)

= 2 + 4 + 6

= 12

Therefore,

Volume of parallelepiped = 12 cubic unit

iv. $\bar{a} = 6\hat{\imath}, \bar{b} = 2\hat{\jmath}, \bar{c} = 5\hat{k}$

Given :

Coterminous edges of parallelopiped are $\bar{a}, \bar{b}, \bar{c}$ where,

$\bar{a} = 6\hat{\imath}$

$\bar{b} = 2\hat{\jmath}$

$\bar{c} = 5\hat{k}$

To Find : Volume of parallelepiped

Formulae :

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

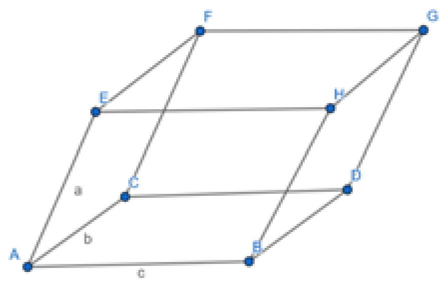
Answer :

Volume of parallelopiped with coterminous edges

$\bar{a} = 6\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

$\bar{b} = 0\hat{\imath} + 2\hat{\jmath} + 0\hat{k}$

$\bar{c} = 0\hat{\imath} + 0\hat{\jmath} + 5\hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} 6 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{vmatrix}$$

$= 6(2 \times 5 - 0 \times 0) - 0(0 \times 5 - 0 \times 0) + 0(0 \times 0 - 0 \times 2)$

= 6(10) + 0 + 0

= 60

Therefore,

Volume of parallelepiped = 60 cubic unit

Q. 4

Show that the vectors $\vec{a}, \vec{b}, \vec{c}$ **are coplanar, when**

i. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ **and** $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

ii. $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ **and** $\vec{c} = 7\hat{j} + 3\hat{k}$

iii. $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ **and** $\vec{c} = 3\hat{i} - 4\hat{j} + 7\hat{k}$

Answer :

i. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 5\hat{k}$

Given Vectors :

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = -2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

$\bar{c} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

i.e. $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = -2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

$\bar{c} = \hat{\imath} - 3\hat{\jmath} + 5\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix}$$

$$= 1(3 \times 5 - (-3) \times (-4)) - (-2)((-2) \times 5 - 1 \times (-4)) + 3((-2) \times (-3) - 3 \times 1)$$

= 1(3) + 2(-6) + 3(3)

= 3 – 12 +9

= 0

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

Hence, the vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Note : For coplanar vectors $\bar{a}, \bar{b}, \bar{c}$,

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

ii. $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{c} = 7\hat{j} + 3\hat{k}$

Given Vectors :

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

i.e. $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$$

$$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$$

$$\bar{c} = 7\hat{\jmath} + 3\hat{k}$$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & 7 & 3 \end{vmatrix}$$

$$= 1((-1) \times 3 - 7 \times (-1)) - 3(2 \times 3 - 0 \times (-1)) + 1(2 \times 7 - 0 \times (-1))$$

= 1(4) – 3(6) + 1(14)

= 4 – 18 + 14

= 0

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

Hence, the vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Note : For coplanar vectors $\bar{a}, \bar{b}, \bar{c}$,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

iii. $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - 4\hat{j} + 7\hat{k}$

Given Vectors :

$$\bar{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$$

$$\bar{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$$

$$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

i.e. $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Formulae :

1) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$$\bar{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$$

$$\bar{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$$

$$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -1 & 2 \\ 1 & 2 & -3 \\ 3 & -4 & 7 \end{vmatrix}$$

$$= 2(2 \times 7 - (-3) \times (-4)) - (-1)(1 \times 7 - 3 \times (-3)) + 2(1 \times (-4) - 3 \times 2)$$

= 2(2) + 1(16) + 2(-10)

= 4 + 16 -20

= 0

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

Hence, the vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Note : For coplanar vectors $\bar{a}, \bar{b}, \bar{c}$,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$$

Q. 5

Find the value of λ for which the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, when

i. $\vec{a} = (2\hat{i} - \hat{j} + \hat{k}), \vec{b} = (\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{c} = (3\hat{i} + \lambda\hat{j} + 5\hat{k})$

ii. $\vec{a} = \lambda\hat{i} - 10\hat{j} - 5\hat{k}, \vec{b} = -7\hat{i} - 5\hat{j}$ and $\vec{c} = \hat{i} - 4\hat{j} - 3\hat{k}$

iii. $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

Answer :

i. . $\vec{a} = (2\hat{i} - \hat{j} + \hat{k}), \vec{b} = (\hat{i} + 2\hat{j} + 3\hat{k})$. and $\vec{c} = (3\hat{i} + \lambda\hat{j} + 5\hat{k})$

Given : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Where,

$$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$$

$$\bar{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\bar{c} = 3\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$$

To Find : value of λ

Formulae :

1) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{c} = 3\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 3 \\ 3 & \lambda & 5 \end{vmatrix}$$

$= 2(2 \times 5 - 3 \times \lambda) - (-1)(1 \times 5 - 3 \times 3) + 1(1 \times \lambda - 3 \times 2)$

$= 2(10 - 3\lambda) - 4 + 1(\lambda - 6)$

$= 20 - 6\lambda - 4 + \lambda - 6$

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 10 - 5\lambda$ ....eq(2)

From eq(1) and eq(2),

$10 - 5\lambda = 0$

$\therefore 5\lambda = 10$

$\boxed{\therefore \lambda = 2}$

ii. $\vec{a} = \lambda\hat{i} - 10\hat{j} - 5\hat{k}, \vec{b} = -7\hat{i} - 5\hat{j}$ and $\vec{c} = \hat{i} - 4\hat{j} - 3\hat{k}$

Given : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Where,

$\bar{a} = \lambda\hat{\imath} - 10\hat{\jmath} - 5\hat{k}$

$\bar{b} = -7\hat{\imath} - 5\hat{\jmath}$

$\bar{c} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

To Find : value of λ

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = \lambda\hat{\imath} - 10\hat{\jmath} - 5\hat{k}$

$\bar{b} = -7\hat{\imath} - 5\hat{\jmath} + 0\hat{k}$

$\bar{c} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} \lambda & -10 & -5 \\ -7 & -5 & 0 \\ 1 & -4 & -3 \end{vmatrix}$$

$$= \lambda((-5) \times (-3) - 0 \times (-4)) - (-10)\big((-7) \times (-3) - 0 \times 1\big) + (-5)((-7) \times (-4) - 1 \times (-5))$$

$$= \lambda(15) + 10(21) - 5(33)$$

$= 15\lambda + 45$

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 15\lambda + 45$ …………eq(2)

From eq(1) and eq(2),

$15\lambda + 45 = 0$

$\therefore 15\lambda = 45$

$\therefore \lambda = -3$

iii. . $\vec{a} = \hat{i} - \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. and $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$

Given : Vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar.

Where,

$\bar{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \lambda\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$

To Find : value of λ

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \lambda\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix}$$

$= 1(1 \times \lambda - (-1) \times (-1)) - (-1)(2 \times \lambda - (-1) \times \lambda) + 1(2 \times (-1) - \lambda \times 1)$

$= 1(\lambda - 1) + 1(3\lambda) + 1(-\lambda - 2)$

$= \lambda - 1 + 3\lambda - 2 - \lambda$

$= 3\lambda - 3$

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 3\lambda - 3$ ....eq(2)

From eq(1) and eq(2),

$3\lambda - 3 = 0$

$\therefore 3\lambda = 3$

$\boxed{\therefore \lambda = 1}$

Q. 6

If $\vec{a} = (2\hat{i} - \hat{j} + \hat{k}), \vec{b} = (\hat{i} - 3\hat{j} - 5\hat{k})$ **and** $\vec{c} = (3\hat{i} - 4\hat{j} - \hat{k})$, **find** $[\vec{a}\ \vec{b}\ \vec{c}]$ **and interpret the result.**

Answer :

Given Vectors :

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} - \hat{k}$

To Find : $[\bar{a} \quad \bar{b} \quad \bar{c}]$

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given vectors,

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\bar{b} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

$\bar{c} = 3\hat{\imath} - 4\hat{\jmath} - \hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 2 & -1 & 1 \\ 1 & -3 & -5 \\ 3 & -4 & -1 \end{vmatrix}$$

$$= 2((-3) \times (-1) - (-4) \times (-5)) - (-1)((-1) \times 1 - 3 \times (-5)) + 1((-4) \times 1 - 3 \times (-3))$$

$$= 2(-17) + 1(14) + 1(5)$$

= - 34 + 14 + 5

= - 15

$$\therefore [\bar{a} \quad \bar{b} \quad \bar{c}] = -15$$

Q. 7

The volume of the parallelepiped whose edges are $\left(-12\hat{i}+\lambda\hat{k}\right), \left(3\hat{j}-\hat{k}\right)$ **and** $\left(2\hat{i}+\hat{j}-15\hat{k}\right)$ **is 546 cubic units. Find the value of λ.**

Answer :

Given :

1) Coterminous edges of parallelepiped are

$$\bar{a} = -12\hat{\imath} + \lambda\hat{k}$$

$$\bar{b} = 3\hat{\jmath} - \hat{k}$$

$$\bar{c} = 2\hat{\imath} + \hat{\jmath} - 15\hat{k}$$

2) Volume of parallelepiped,

V = 546 cubic unit

To Find : value of λ

1) Volume of parallelepiped :

If $\bar{a}, \bar{b}, \bar{c}$ are coterminous edges of parallelepiped,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then, volume of parallelepiped V is given by,

$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

Given volume of parallelepiped,

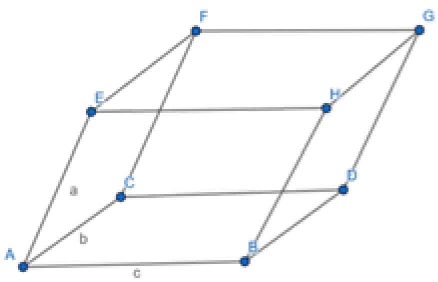
V = 546 cubic unit ....eq(1)

Volume of parallelopiped with coterminous edges

$\bar{a} = -12\hat{\imath} + \lambda\hat{k}$

$\bar{b} = 3\hat{\jmath} - \hat{k}$

$\bar{c} = 2\hat{\imath} + \hat{\jmath} - 15\hat{k}$



$$V = [\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} -12 & 0 & \lambda \\ 0 & 3 & -1 \\ 2 & 1 & -15 \end{vmatrix}$$

$$= -12(3 \times (-15) - 1 \times (-1)) - 0 + \lambda(0 \times 1 - 3 \times 2)$$

$$= 528 - 0 - 6\,\lambda$$

$$= 528 - 6\,\lambda$$

$\therefore V = (528 - 6\,\lambda)$ cubic unit ....eq(2)

From eq(1) and eq(2)

$$528 - 6\,\lambda = 546$$

$$\therefore -6\,\lambda = 18$$

$$\therefore \lambda = -3$$

Q. 8

Show that the vectors $\vec{a} = \left(\hat{i} + 3\hat{j} + \hat{k}\right), \vec{b} = \left(2\hat{i} - \hat{j} - \hat{k}\right)$ **and** $\vec{c} = \left(7\hat{j} + 3\hat{k}\right)$ **are parallel to the same plane.**

{HINT: Show that $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 0$ **}**

Answer :

Given Vectors :

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

To Prove : Vectors $\bar{a}, \bar{b}, \bar{c}$ are parallel to same plane.

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

Vectors will be parallel to the same plane if they are coplanar.

For vectors $\bar{a}, \bar{b}, \bar{c}$ to be coplanar, $[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Now, for given vectors,

$\bar{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = 7\hat{\jmath} + 3\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & 7 & 3 \end{vmatrix}$$

$= 1(3 \times (-1) - 7 \times (-1)) - \ 3(2 \times 3 - 0 \times (-1)) + \ 1(2 \times 7 - 0 \times (-1))$

= 1(4) – 3(6) + 1(14)

= 4 – 18 + 14

= 0

$\therefore \ [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Hence, given vectors are parallel to the same plane.

Q. 9

If the vectors $\left(a\hat{i} + a\hat{j} + c\hat{k}\right), \left(\hat{i} + \hat{k}\right)$ **and** $\left(c\hat{i} + c\hat{j} + b\hat{k}\right)$ **be coplanar, show that c^2 = ab.**

Answer :

Given : vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar. Where

$\bar{a} = a\hat{\imath} + a\hat{\jmath} + c\hat{k}$

$\bar{b} = \hat{\imath} + \hat{k}$

$\bar{c} = c\hat{\imath} + c\hat{\jmath} + b\hat{k}$

To Prove : c^2 = ab

Formulae :

1) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

2) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

As vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar

$\therefore\ [\bar{a} \quad \bar{b} \quad \bar{c}] = 0$ ....eq(1)

For given vectors,

$\bar{a} = a\hat{\imath} + a\hat{\jmath} + c\hat{k}$

$\bar{b} = \hat{\imath} + \hat{k}$

$\bar{c} = c\hat{\imath} + c\hat{\jmath} + b\hat{k}$

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix}$$

$= a(0 \times b - c \times 1) - \ a(1 \times b - 1 \times c) + \ c(1 \times c - 0 \times c)$

= a.(- c) – a.(b - c) + c(c)

= – ac – ab + ac + c^2

= - ab + c^2

$\therefore\ [\bar{a} \quad \bar{b} \quad \bar{c}] = -ab + c^2$ ....eq(2)

From eq(1) and eq(2),

- ab + c^2 = 0

Therefore,

c^2 = ab

Hence proved.

Note : Three vectors $\bar{a}, \bar{b}\ \&\ \bar{c}$ are coplanar if and only if

$[\bar{a} \quad \bar{b} \quad \bar{c}] = 0$

Q. 10

Show that the four points with position vectors $\left(4\hat{i}+8\hat{j}+12\hat{k}\right), \left(2\hat{i}+4\hat{j}+6\hat{k}\right),$ $\left(3\hat{i}+5\hat{j}+4\hat{k}\right)$ **and** $\left(5\hat{i}+8\hat{j}+5\hat{k}\right)$ **are coplanar.**

Answer :

Given :

Let A, B, C & D be four points with position vectors $\bar{a}, \bar{b}, \bar{c}$ & $\bar{d}$.

Therefore,

$\bar{a} = 4\hat{\imath} + 8\hat{\jmath} + 12\hat{k}$

$\bar{b} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

$\bar{c} = 3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$

$\bar{d} = 5\hat{\imath} + 8\hat{\jmath} + 5\hat{k}$

To Prove : Points A, B, C & D are coplanar.

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then vector $\overline{AB}$ is given by,

$\overline{AB} = \bar{b} - \bar{a}$

$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

2) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$\bar{a} = 4\hat{\imath} + 8\hat{\jmath} + 12\hat{k}$

$\bar{b} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

$\bar{c} = 3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$

$\bar{d} = 5\hat{\imath} + 8\hat{\jmath} + 5\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (4-2)\hat{\imath} + (8-4)\hat{\jmath} + (12-6)\hat{k}$

$\therefore\ \overline{BA} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (4-3)\hat{\imath} + (8-5)\hat{\jmath} + (12-4)\hat{k}$

$\therefore\ \overline{CA} = \hat{\imath} + 3\hat{\jmath} + 8\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (4-5)\hat{\imath} + (8-8)\hat{\jmath} + (12-5)\hat{k}$

$\therefore\ \overline{DA} = -\hat{\imath} + 0\hat{\jmath} + 7\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

$\overline{CA} = \hat{\imath} + 3\hat{\jmath} + 8\hat{k}$

$\overline{DA} = -\hat{\imath} + 0\hat{\jmath} + 7\hat{k}$

$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} 2 & 4 & 6 \\ 1 & 3 & 8 \\ -1 & 0 & 7 \end{vmatrix}$

$= 2(3 \times 7 - 0 \times 8) - 4(1 \times 7 - (-1) \times 8) + 6(1 \times 0 - (-1) \times 3)$

= 2(21) – 4(15) + 6(3)

= 42 – 60 + 18

= 0

$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Hence, vectors $\overline{BA}, \overline{CA} \,\&\, \overline{DA}$ are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if $[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Q. 11

Show that the four points with position vectors $\left(6\hat{i} - 7\hat{j}\right), \left(16\hat{i} - 19\hat{j} - 4\hat{k}\right), \left(3\hat{j} - 6\hat{k}\right)$ **and** $\left(2\hat{i} - 5\hat{j} + 10\hat{k}\right)$ **are coplanar.**

Answer :

Given :

Let A, B, C & D be four points with position vectors $\bar{a}, \bar{b}, \bar{c} \,\&\, \bar{d}$.

Therefore,

$\bar{a} = 6\hat{\imath} - 7\hat{\jmath}$

$\bar{b} = 16\hat{\imath} - 19\hat{\jmath} - 4\hat{k}$

$\bar{c} = 3\hat{\jmath} - 6\hat{k}$

$\bar{d} = 2\hat{\imath} - 5\hat{\jmath} + 10\hat{k}$

To Prove : Points A, B, C & D are coplanar.

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then vector $\overline{AB}$ is given by,

$$\overline{AB} = \bar{b} - \bar{a}$$

$$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$$

2) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$$\bar{a} = 6\hat{\imath} - 7\hat{\jmath}$$

$$\bar{b} = 16\hat{\imath} - 19\hat{\jmath} - 4\hat{k}$$

$$\bar{c} = 3\hat{\jmath} - 6\hat{k}$$

$$\bar{d} = 2\hat{\imath} - 5\hat{\jmath} + 10\hat{k}$$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$$\overline{BA} = \bar{a} - \bar{b}$$

$$= (6 - 16)\hat{\imath} + (-7 + 19)\hat{\jmath} + (0 + 4)\hat{k}$$

$\therefore\ \overline{BA} = -10\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (6-0)\hat{\imath} + (-7-3)\hat{\jmath} + (0+6)\hat{k}$

$\therefore\ \overline{CA} = 6\hat{\imath} - 10\hat{\jmath} + 6\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (6-2)\hat{\imath} + (-7+5)\hat{\jmath} + (0-10)\hat{k}$

$\therefore\ \overline{DA} = 4\hat{\imath} - 2\hat{\jmath} - 10\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -10\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$

$\overline{CA} = 6\hat{\imath} - 10\hat{\jmath} + 6\hat{k}$

$\overline{DA} = 4\hat{\imath} - 2\hat{\jmath} - 10\hat{k}$

$$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = \begin{vmatrix} -10 & 12 & 4 \\ 6 & -10 & 6 \\ 4 & -2 & -10 \end{vmatrix}$$

$$= -10\big((-10)\times(-10) - (-2)\times 6\big) - 12(6\times(-10) - 4\times 6) + 4(6\times(-2) - (-10)\times 4)$$

= -10(112) – 12(-84) + 4(28)

= -1120 + 1008 + 112

= 0

$\therefore [\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$

Hence, vectors $\overline{BA}, \overline{CA}\ \&\ \overline{DA}$ are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if $[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$

Q. 12

Find the value of λ for which the four points with position vectors $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$, $\left(3\hat{i}-\hat{j}+2\hat{k}\right), \left(-2\hat{i}+\lambda\hat{j}+\hat{k}\right)$ **and** $\left(6\hat{i}-4\hat{j}+2\hat{k}\right)$ **are coplanar.**

Ans. λ = 3

Answer :

Given :

Let, A, B, C & D be four points with given position vectors

$$\bar{a} = 1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\bar{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$$

$$\bar{c} = -2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$$

$$\bar{d} = 6\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$$

To Find : value of λ

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then vector $\overline{AB}$ is given by,

$$\overline{AB} = \bar{b} - \bar{a}$$

$$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$$

2) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$\bar{a} = 1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\bar{c} = -2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$

$\bar{d} = 6\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (1-3)\hat{\imath} + (2+1)\hat{\jmath} + (3-2)\hat{k}$

$\therefore\ \overline{BA} = -2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (1+2)\hat{\imath} + (2-\lambda)\hat{\jmath} + (3-1)\hat{k}$

$\therefore\ \overline{CA} = 3\hat{\imath} + (2-\lambda)\hat{\jmath} + 2\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (1-6)\hat{\imath} + (2+4)\hat{\jmath} + (3-2)\hat{k}$

$\therefore\ \overline{DA} = -5\hat{\imath} + 6\hat{\jmath} + \hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -2\hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\overline{CA} = 3\hat{\imath} + (2-\lambda)\hat{\jmath} + 2\hat{k}$

$\overline{DA} = -5\hat{\imath} + 6\hat{\jmath} + \hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} -2 & 3 & 1 \\ 3 & (2-\lambda) & 2 \\ -5 & 6 & 1 \end{vmatrix}$$

$$= -2\big((2-\lambda)\times 1 - 2\times 6\big) - 3(3\times 1 - 2\times(-5)) + 1(6\times 3 - (2-\lambda)\times(-5))$$

= -2(- λ – 10) – 3(13) + 1(28 – 5λ)

= 2λ + 20 – 39 + 28 – 5λ

= 9 – 3λ

$\therefore [\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 9 - 3\lambda$ ....eq(4)

Four points A, B, C & D are coplanar if and only if

$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$ ....eq(5)

From eq(4) and eq(5)

9 – 3λ = 0

3λ = 9

λ = 3

Q. 13

Find the value of λ for which the four points with position vectors $\left(-\hat{j}+\hat{k}\right),$ $\left(2\hat{i}-\hat{j}-\hat{k}\right),\left(\hat{i}+\lambda\hat{j}+\hat{k}\right)$ **and** $\left(3\hat{j}+3\hat{k}\right)$ **are coplanar.**

Answer :

Given :

Let, A, B, C & D be four points with given position vectors

$\bar{a} = -\hat{j} + \hat{k}$

$\bar{b} = 2\hat{i} - \hat{j} - \hat{k}$

$\bar{c} = \hat{i} + \lambda\hat{j} + \hat{k}$

$\bar{d} = 3\hat{j} + 3\hat{k}$

To Find : value of λ

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then vector $\overline{AB}$ is given by,

$\overline{AB} = \bar{b} - \bar{a}$

$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

2) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$\bar{a} = -\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = \hat{\imath} + \lambda\hat{\jmath} + \hat{k}$

$\bar{d} = 3\hat{\jmath} + 3\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (0 - 2)\hat{\imath} + (-1 + 1)\hat{\jmath} + (1 + 1)\hat{k}$

$\therefore\ \overline{BA} = -2\hat{\imath} + 0\hat{\jmath} + 2\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (0-1)\hat{\imath} + (-1-\lambda)\hat{\jmath} + (1-1)\hat{k}$

$\therefore\ \overline{CA} = -\hat{\imath} + (-1-\lambda)\hat{\jmath} + 0\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (0-0)\hat{\imath} + (-1-3)\hat{\jmath} + (1-3)\hat{k}$

$\therefore\ \overline{DA} = 0\hat{\imath} - 4\hat{\jmath} - 2\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -2\hat{\imath} + 0\hat{\jmath} + 2\hat{k}$

$\overline{CA} = -\hat{\imath} + (-1-\lambda)\hat{\jmath} + 0\hat{k}$

$\overline{DA} = 0\hat{\imath} - 4\hat{\jmath} - 2\hat{k}$

$$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = \begin{vmatrix} -2 & 0 & 2 \\ -1 & (-1-\lambda) & 0 \\ 0 & -4 & -2 \end{vmatrix}$$

$$= -2\big((-1-\lambda)\times(-2) - (-4)\times 0\big) - 0((-1)\times(-2) - 0\times 0) + 2((-1)\times(-4) - (-1-\lambda)\times 0)$$

= -2(2 + 2λ) – 0 + 2(4)

= - 4 - 4λ + 8

= 4 – 4λ

$\therefore\ [\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 4 - 4\lambda$ ....eq(4)

Four points A, B, C & D are coplanar if and only if

$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$ ....eq(5)

From eq(4) and eq(5)

4 – 4λ = 0

4λ = 4

λ = 1

Q. 14

Using vector method, show that the points A(4, 5, 1), B(0, -1, -1), C(3, 9, 4) and D(-4, 4, 4) are coplanar.

Answer :

Given Points :

A ≡ (4, 5, 1)

B ≡ (0, -1, -1)

C ≡ (3, 9, 4)

D ≡ (-4, 4, 4)

To Prove : Points A, B, C & D are coplanar.

Formulae :

4) Position Vectors :

If A is a point with co-ordinates (a_1, a_2, a_3)

then its position vector is given by,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

5) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then vector $\overline{AB}$ is given by,

$$\overline{AB} = \bar{b} - \bar{a}$$

$$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$$

6) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

7) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given points,

A ≡ (4, 5, 1)

B ≡ (0, -1, -1)

C ≡ (3, 9, 4)

D ≡ (-4, 4, 4)

Position vectors of above points are,

$$\bar{a} = 4\hat{\imath} + 5\hat{\jmath} + \hat{k}$$

$$\bar{b} = 0\hat{\imath} - \hat{\jmath} - \hat{k}$$

$$\bar{c} = 3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}$$

$$\bar{d} = -4\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$$\overline{BA} = \bar{a} - \bar{b}$$

$$= (4 - 0)\hat{\imath} + (5 + 1)\hat{\jmath} + (1 + 1)\hat{k}$$

$\therefore \ \overline{BA} = 4\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$ ....eq(1)

$$\overline{CA} = \bar{a} - \bar{c}$$

$$= (4 - 3)\hat{\imath} + (5 - 9)\hat{\jmath} + (1 - 4)\hat{k}$$

$\therefore \ \overline{CA} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$ ....eq(2)

$$\overline{DA} = \bar{a} - \bar{d}$$

$$= (4 + 4)\hat{\imath} + (5 - 4)\hat{\jmath} + (1 - 4)\hat{k}$$

$\therefore \ \overline{DA} = 8\hat{\imath} + 1\hat{\jmath} - 3\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = 4\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$

$\overline{CA} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

$\overline{DA} = 8\hat{\imath} + 1\hat{\jmath} - 3\hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} 4 & 6 & 2 \\ 1 & -4 & -3 \\ 8 & 1 & -3 \end{vmatrix}$$

$$= 4((-4) \times (-3) - 1 \times (-3)) - 6(1 \times (-3) - (-3) \times 8) + 2(1 \times 1 - (-4) \times 8)$$

= 4(15) – 6(21) + 2(33)

= 60 – 126 + 66

= 0

$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Hence, vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if $[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Q. 15

Find the value of λ for which the points A(3, 2, 1), B(4, λ, 5), C(4, 2, -2) and D(6, 5, -1) are coplanar.

Ans. λ = 5

Answer :

Given :

Points A, B, C & D are coplanar where,

A ≡ (3, 2, 1)

B ≡ (4, λ, 5)

C ≡ (4, 2, -2)

D ≡ (6, 5, -1)

To Find : value of λ

Formulae :

1) Position Vectors :

If A is a point with co-ordinates (a_1, a_2, a_3)

then its position vector is given by,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

2) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then vector $\overline{AB}$ is given by,

$$\overline{AB} = \bar{b} - \bar{a}$$

$$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$$

3) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

4) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given points,

A ≡ (3, 2, 1)

B ≡ (4, λ, 5)

≡ (4, 2, -2)

D ≡ (6, 5, -1)

Position vectors of above points are,

$\bar{a} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\bar{b} = 4\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$

$\bar{c} = 4\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

$\bar{d} = 6\hat{\imath} + 5\hat{\jmath} - \hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (3 - 4)\hat{\imath} + (2 - \lambda)\hat{\jmath} + (1 - 5)\hat{k}$

$\therefore\ \overline{BA} = -\hat{\imath} + (2 - \lambda)\hat{\jmath} - 4\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (3 - 4)\hat{\imath} + (2 - 2)\hat{\jmath} + (1 + 2)\hat{k}$

$\therefore\ \overline{CA} = -\hat{\imath} + 0\hat{\jmath} + 3\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (3 - 6)\hat{\imath} + (2 - 5)\hat{\jmath} + (1 + 1)\hat{k}$

$\therefore\ \overline{DA} = -3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -\hat{\imath} + (2 - \lambda)\hat{\jmath} - 4\hat{k}$

$\overline{CA} = -\hat{\imath} + 0\hat{\jmath} + 3\hat{k}$

$\overline{DA} = -3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} -1 & (2-\lambda) & -4 \\ -1 & 0 & 3 \\ -3 & -3 & 2 \end{vmatrix}$$

$$= -1\big(0 \times 2 - 3 \times (-3)\big) - (2 - \lambda)(2 \times (-1) - (-3) \times 3) - 4((-1) \times (-3) - (-3) \times 0)$$

= - 1(9) – (2 - λ).(7) – 4(3)

= - 9 – 14 + 7λ – 12

= 7λ − 35

$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 7\lambda - 35$ ………… eq(4)

But points A, B, C & D are coplanar if and only if

$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$ ………… eq(5)

From eq(4) and eq(5)

7λ − 35 = 0

$\therefore 7\lambda = 35$

Exercise 25B

Q. 1

If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors the x + y + z = ?

Answer :

$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$

$\vec{a} = x\hat{\imath} + 2\hat{\jmath} - z\hat{k}$

$\vec{b} = 3\hat{\imath} - y\hat{\jmath} + \hat{k}$

Since, these two vectors are equal, therefore comparing these two vectors we get,

x = 3 , - y = 2 , - z = 1

⇒x = 3,y = - 2,z = - 1

∴x + y + z = 3 + (- 2) + (- 1) = 3 - 2 - 1 = 0

Ans:x + y + z = 0

Q. 2

Write a unit vector in the direction of the sum of the vectors $\vec{a} = \left(2\hat{i} + 2\hat{j} - 5\hat{k}\right)$ and $\vec{b} = \left(2\hat{i} + \hat{j} - 7\hat{k}\right)$.

Answer :

Let $\vec{s}$ be the sum of the vectors $\vec{a}$ *and* $\vec{b}$

$\Rightarrow \vec{s} = \vec{a} + \vec{b}$

$\Rightarrow \vec{s} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k} + 2\hat{\imath} + \hat{\jmath} - 7\hat{k}$

$\Rightarrow \vec{s} = 4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}$

$|\vec{s}| = (4^2 + 3^2 + (-12)^2)^{1/2}$

$\Rightarrow |\vec{s}| = (16 + 9 + 144)^{1/2} = (169)^{1/2} = 13$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{s} = \frac{\vec{s}}{|\vec{s}|} = \frac{4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}}{13}$$

Ans: $\hat{s} = \frac{4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}}{13}$

Q. 3

Write the value of λ so that the vectors $\vec{a} = (2\hat{i} + \lambda\hat{j} + \hat{k})$ **and** $\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k})$ **are perpendicular to each other.**

Answer :

$\vec{a} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$

$\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

Since these two vectors are perpendicular the dot product of these two vectors is zero.

i.e.: $\vec{a}.\vec{b} = 0$

$\Rightarrow (2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}).(\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) = 0$

$\Rightarrow 2 + \lambda \times (-2) + 3 = 0$

$\Rightarrow 5 = 2\lambda$

$\Rightarrow \lambda = 5/2$

Ans: $\lambda = 5/2$

Q. 4

Find the value of p for which the vectors $\vec{a} = (3\hat{i} + 2\hat{j} + 9\hat{k})$ **and** $\vec{b} = (\hat{i} - 2p\hat{j} + 3\hat{k})$ **are parallel.**

Answer :

$\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$

$\vec{b} = \hat{\imath} - 2p\hat{\jmath} + 3\hat{k}$

Since these two vectors are parallel

$\therefore \frac{3}{1} = \frac{2}{-2p} = \frac{9}{3}$

$\Rightarrow \frac{3}{1} = \frac{1}{-p}$

$\Rightarrow p = \frac{-1}{3}$

Ans: $p = \frac{-1}{3}$

Q. 5

Find the value of λ when the projection of $\vec{a} = \left(\lambda\hat{i} + \hat{j} + 4\hat{k}\right)$ **on** $\vec{b} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right)$ **is 4 units.**

Answer :

$\vec{a} = \lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}$

$\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$

projection of a on b is given by: $\vec{a}.\hat{b}$

$|\vec{b}| = (2^2 + 6^2 + 3^2)^{1/2}$

$\Rightarrow |\vec{b}| = (4 + 36 + 9)^{1/2} = (49)^{1/2} = 7$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{b} = \frac{b}{|\vec{b}|} = \frac{2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}}{7}$$

Now it is given that: $\vec{a}.\hat{b} = 4$

$\Rightarrow (\lambda\hat{\imath} + \hat{\jmath} + 4\hat{k}).\left(\frac{2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}}{7}\right) = 4$

$\Rightarrow 2\lambda + 6 + (3\times4) = 28$

$\Rightarrow \lambda = (28 - 12 - 6)/2$

$\Rightarrow \lambda = 10/2 = 5$

Ans: $\lambda = 5$

Q. 6

If $\vec{a}$ and $\vec{b}$ are perpendicular vectors such that $|\vec{a}+\vec{b}|=13$ and $|\vec{a}|=5,$ find the value of $|\vec{b}|.$

Answer :

Since a and b vectors are perpendicular .

$\Rightarrow \theta = \frac{\pi}{2}$

Now,

$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 13^2 = 5^2 + |\vec{b}|^2 + 0$ ...($\cos\theta = \cos\frac{\pi}{2} = 0$)

$\Rightarrow |\vec{b}|^2 = 169 - 25 = 144$

$\Rightarrow |\vec{b}| = 12$

Ans: $|\vec{b}| = 12$

Q. 7

If $\vec{a}$ is a unit vector such that $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=15,$ find $|\vec{x}|.$

Answer :

$(\vec{x} - \vec{a})(\vec{x} + \vec{a}) = 15$

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 15$

$\Rightarrow |\vec{x}|^2 = |\vec{a}|^2 + 15$

Now , a is a unit vector,

$\Rightarrow |\vec{a}| = 1$

$\Rightarrow |\vec{x}|^2 = 1^2 + 15$

$\Rightarrow |\vec{x}|^2 = 16$

$\Rightarrow |\vec{x}| = 4$

Ans: $|\vec{x}| = 4$

Q. 8

Find the sum of the vectors $\vec{a} = (\hat{i} - 3\hat{k}),\ \vec{b} = (2\hat{j} - \hat{k})$ **and** $\vec{c} = (2\hat{i} - 3\hat{j} + 2\hat{k})$.

Answer :

$\vec{a} = \hat{\imath} - 3\hat{k}$

$\vec{b} = 2\hat{\jmath} - \hat{k}$

$\vec{c} = 2\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

Now ,

$\vec{a} + \vec{b} + \vec{c} = \hat{\imath} - 3\hat{\jmath} + 2\hat{\jmath} - \hat{k} + 2\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$

Ans: $\vec{a} + \vec{b} + \vec{c} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$

Q. 9

Find the sum of the vectors $\vec{a} = (\hat{i} - 2\hat{j}),\ \vec{b} = (2\hat{i} - 3\hat{j})$ **and** $\vec{c} = (2\hat{i} + 3\hat{k})$.

Answer :

$\vec{a} = \hat{\imath} - 2\hat{\jmath}$

$\vec{b} = 2\hat{\imath} - 3\hat{\jmath}$

$\vec{c} = 2\hat{\imath} + 3\hat{k}$

Now ,

$\vec{a} + \vec{b} + \vec{c} = \hat{\imath} - 2\hat{\jmath} + 2\hat{\imath} - 3\hat{\jmath} + 2\hat{\imath} + 3\hat{k}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 5\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$

Ans: $\vec{a} + \vec{b} + \vec{c} = 5\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$

Q. 10

Write the projection of the vector $(\hat{i} + \hat{j} + \hat{k})$ **along the vector** $\hat{j}$.

Answer :

projection of a on b is given by: $\vec{a}.\hat{b}$

∴ the projection of the vector $\left(\hat{i}+\hat{j}+\hat{k}\right)$ along the vector $\hat{j}$. is :

$\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right).\hat{\jmath} = 0 + 1 + 0 = 1$

Ans: the projection of the vector $\left(\hat{i}+\hat{j}+\hat{k}\right)$ along the vector $\hat{j}$. is:1

Q. 11

Write the projection of the vector $\left(7\hat{i}+\hat{j}-4\hat{k}\right)$ **on the vector** $\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$.

Answer :

$\vec{a} = 7\hat{\imath} + \hat{\jmath} - 4\hat{k}$

$\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$

projection of a on b is given by: $\vec{a}.\hat{b}$

$|\vec{b}| = (2^2 + 6^2 + 3^2)^{1/2}$

$\Rightarrow |\vec{b}| = (4 + 36 + 9)^{1/2} = (49)^{1/2} = 7$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}}{7}$$

$$\vec{a}.\hat{b} = \left(7\hat{\imath} + \hat{\jmath} - 4\hat{k}\right).\left(\frac{2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}}{7}\right) = \frac{(7\times 2) + (1\times 6) - (4\times 3)}{7}$$
$$= \frac{14 + 6 - 12}{7} = \frac{8}{7}$$

Ans: the projection of the vector $\left(7\hat{i}+\hat{j}-4\hat{k}\right)$ on the vector $\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$.

Q. 12

Find $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$ **when** $\vec{a}=\left(2\hat{i}+\hat{j}+3\hat{k}\right)$, $\vec{b}=\left(-\hat{i}+2\hat{j}+\hat{k}\right)$ **and** $\vec{c}=\left(3\hat{i}+\hat{j}+2\hat{k}\right)$.

Answer :

$\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

$\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\vec{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$\vec{b} \times \vec{c} = \left(-\hat{\imath} + 2\hat{\jmath} + \hat{k}\right) \times \left(3\hat{\imath} + \hat{\jmath} + 2\hat{k}\right) = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

$\begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} = \hat{\imath}(4-1) - \hat{\jmath}(-2-3) + \hat{k}(-1-6) = 3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$

$\therefore \vec{b} \times \vec{c} = 3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$

$\therefore \vec{a}.\left(\vec{b} \times \vec{c}\right) = \left(2\hat{\imath} + \hat{\jmath} + 3\hat{k}\right).\left(3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}\right)$ = (2×3) + (1×5) + (3× - 7)

= 6 + 5 - 21 = - 10

Ans: - 10

Q. 13

Find a vector in the direction of $\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$ **which has magnitude 21 units.**

Answer :

$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$

$|\vec{a}| = (2^2 + (-3)^2 + 6^2)^{1/2}$

$\Rightarrow |\vec{a}| = (4 + 9 + 36)^{1/2} = (49)^{1/2} = 7$

a unit vector in the direction of the sum of the vectors is given by:

$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}}{7}$

a vector in the direction of $\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$ which has magnitude 21 units.

$= 21\hat{a} = 21 \times \frac{2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}}{7} = 3\left(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}\right) = 6\hat{\imath} - 9\hat{\jmath} + 18\hat{k}$

Ans: $6\hat{\imath} - 9\hat{\jmath} + 18\hat{k}$

Q. 14

If $\vec{a}=\left(2\hat{i}+2\hat{j}+3\hat{k}\right), \vec{b}=\left(-\hat{i}+2\hat{j}+\hat{k}\right)$ **and** $\vec{c}=\left(3\hat{i}+\hat{j}\right)$ **are such that** $\left(\vec{a}+\lambda\vec{b}\right)$ **is perpendicular to** $\vec{c}$ **then find the value of λ.**

Answer :

$\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\vec{c} = 3\hat{\imath} + \hat{\jmath}$

$\vec{a} + \lambda\vec{b} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda\left(-\hat{\imath} + 2\hat{\jmath} + \hat{k}\right)$

$\Rightarrow \vec{a} + \lambda\vec{b} = (2-\lambda)\hat{\imath} + (2 + 2\lambda)\hat{\jmath} + (3 + \lambda)\hat{k}$

Since $\vec{a} + \lambda\vec{b}$ *is perpendicular to* $\vec{c}$

$\Rightarrow \overrightarrow{(a + \lambda b)}.\vec{c} = 0$

$\Rightarrow \left((2-\lambda)\hat{\imath} + (2 + 2\lambda)\hat{\jmath} + (3 + \lambda)\hat{k}\right).(3\hat{\imath} + \hat{\jmath}) = 0$

$\Rightarrow (2 - \lambda)\times3 + (2 + 2\lambda)\times1 = 0$

$\Rightarrow 6 + 2 - 3\lambda + 2\lambda = 0$

$\Rightarrow \lambda = 8$

Ans: λ = 8

Q. 15

Write the vector of magnitude 15 units in the direction of vector $\left(\hat{i}-2\hat{j}+2\hat{k}\right).$

Answer :

$\vec{a} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$|\vec{a}| = (1^2 + (-2)^2 + 2^2)^{1/2}$

$\Rightarrow |\vec{a}| = (1 + 4 + 4)^{1/2} = (9)^{1/2} = 3$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3}$$

a vector in the direction of $\left(\hat{i}-2\hat{j}+2\hat{k}\right).$ which has magnitude 15 units.

= $15\hat{a} = 15 \times \frac{\hat{\imath}-2\hat{\jmath}+2\hat{k}}{3} = 5(\hat{\imath}-2\hat{\jmath}+2\hat{k}) = 5\hat{\imath}-10\hat{\jmath}+10\hat{k}.$

Ans: $5\hat{\imath}-10\hat{\jmath}+10\hat{k}.$

Q. 16

If $\vec{a}=\left(\hat{i}+\hat{j}+\hat{k}\right), \vec{b}=\left(4\hat{i}-2\hat{j}+3\hat{k}\right)$ and $\vec{c}=\left(\hat{i}-2\hat{j}+\hat{k}\right),$ find a vector of magnitude 6 units which is parallel to the vector $\left(2\vec{a}-\vec{b}+3\vec{c}\right).$

Answer :

$\vec{a} = \hat{\imath}+\hat{\jmath}+\hat{k}$

$\vec{b} = 4\hat{\imath}-2\hat{\jmath}+3\hat{k}$

$\vec{c} = \hat{\imath}-2\hat{\jmath}+\hat{k}$

$\therefore (2\vec{a}-\vec{b}+3\vec{c}) = 2(\hat{\imath}+\hat{\jmath}+\hat{k})-(4\hat{\imath}-2\hat{\jmath}+3\hat{k})+3(\hat{\imath}-2\hat{\jmath}+\hat{k})$

$\Rightarrow (2\vec{a}-\vec{b}+3\vec{c}) = \hat{\imath}-2\hat{\jmath}+2\hat{k}$

LET, $(2\vec{a}-\vec{b}+3\vec{c}) = \vec{s}$

$\vec{s} = \hat{\imath}-2\hat{\jmath}+2\hat{k}$

$|\vec{s}| = (1^2 + (-2)^2 + 2^2)^{1/2}$

$\Rightarrow |\vec{s}| = (1 + 4 + 4)^{1/2} = (9)^{1/2} = 3$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{s} = \frac{\vec{s}}{|\vec{s}|} = \frac{\hat{\imath}-2\hat{\jmath}+2\hat{k}}{3}$$

a vector of magnitude 6 units which is parallel to the vector $\left(2\vec{a}-\vec{b}+3\vec{c}\right).$ is:

$$6\hat{s} = 6 \times \frac{\hat{\imath}-2\hat{\jmath}+2\hat{k}}{3} = 2(\hat{\imath}-2\hat{\jmath}+2\hat{k}) = 2\hat{\imath}-4\hat{\jmath}+4\hat{k}.$$

Ans: $2\hat{\imath}-4\hat{\jmath}+4\hat{k}$

Q. 17

Write the projection of the vector $\left(\hat{i}-\hat{j}\right)$ on the vector $\left(\hat{i}+\hat{j}\right)$.

Answer :

$\vec{a} = \hat{\imath} - \hat{\jmath}$

$\vec{b} = \hat{\imath} + \hat{\jmath}$

projection of a on b is given by: $\vec{a}.\hat{b}$

$|\vec{b}| = (1^2 + 1^2 + 0^2)^{1/2}$

$\Rightarrow |\vec{b}| = (1 + 1)^{1/2} = (2)^{1/2}$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}$$

$$\vec{a}.\hat{b} = (\hat{\imath} - \hat{\jmath}).\left(\frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}\right) = \frac{(1 \times 1) + (-1 \times 1)}{\sqrt{2}} = \frac{0}{\sqrt{2}} = 0$$

Ans: the projection of the vector $\left(7\hat{i}+\hat{j}-4\hat{k}\right)$ on the vector $\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$.

Q. 18

Write the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a}\cdot\vec{b}=\sqrt{6}$.

Answer :

$|\vec{a}| = \sqrt{3}$

$|\vec{b}| = 2$

Since, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|cos\theta$

Substituting the given values we get:

$\Rightarrow \sqrt{6} = \sqrt{3} \times 2 \times cos\theta$

$\Rightarrow cos\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \cos^{-1}\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 45^\circ = \frac{\pi}{4}$

Ans: $\theta = 45^\circ = \frac{\pi}{4}$

Q. 19

If $\vec{a} = (\hat{i} - 7\hat{j} + 7\hat{k})$ **and** $\vec{b} = (3\hat{i} - 2\hat{j} + 2\hat{k})$ **then find** $|\vec{a} \times \vec{b}|.$

Answer :

$\vec{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k}$

$\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$$\vec{a} \times \vec{b} = (\hat{\imath} - 7\hat{\jmath} + 7\hat{k}) \times (3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}) = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix} = \hat{\imath}(-14 - (-14)) - \hat{\jmath}(2 - 21) + \hat{k}(-2 - (-21))$$
$$= 0\hat{\imath} + 19\hat{\jmath} + 19\hat{k}$$

$\therefore \vec{a} \times \vec{b} = 0\hat{\imath} + 19\hat{\jmath} + 19\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = (0^2 + 19^2 + 19^2)^{1/2} = (2 \times 19^2)^{1/2} = 19\sqrt{2}$

Ans: $\therefore |\vec{a} \times \vec{b}| = 19\sqrt{2}$

Q. 20

Find the angle between two vectors $\vec{a}$ **and** $\vec{b}$ **with magnitudes 1 and 2 respectively, when** $|\vec{a} \times \vec{b}| = \sqrt{3}.$

Answer :

$|\vec{a}| = 1$

$|\vec{b}| = 2$

Since, $\overline{|a \times b|} = |\vec{a}||\vec{b}|sin\theta$

Substituting the given values we get:

$\Rightarrow \sqrt{3} = 1 \times 2 \times sin\theta$

$\Rightarrow sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = \sin^{-1}\frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 60^{\circ} = \frac{\pi}{3}$

Ans: $\theta = 60^{\circ} = \frac{\pi}{3}$

Q. 21

What conclusion can you draw about vectors $\vec{a}$ **and** $\vec{b}$ **when** $\vec{a} \times \vec{b} = \vec{0}$ **and** $\vec{a} \cdot \vec{b} = 0?$

Answer :

It is given that:

$\vec{a} \times \vec{b} = \vec{0}\ and\ \vec{a}.\vec{b} = \vec{0}$

$\Rightarrow |\vec{a}||\vec{b}|sin\theta = |\vec{a}||\vec{b}|cos\theta = \vec{0}$

Since sinθ and cosθ cannot be 0 simultaneously $\therefore |\vec{a}| = |\vec{b}| = 0$

Conclusion: when $\vec{a} \times \vec{b} = \vec{0}\ and\ \vec{a}.\vec{b} = \vec{0}$

Then $|\vec{a}| = |\vec{b}| = 0$

Q. 22

Find the value of λ when the vectors $\vec{a} = (\hat{i} + \lambda\hat{j} + 3\hat{k})$ **and** $\vec{b} = (3\hat{i} + 2\hat{j} + 9\hat{k})$ **are parallel.**

Answer :

$\vec{a} = \hat{\imath} + \lambda\hat{\jmath} + 3\hat{k}$

$\vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$

It is given that $\vec{a} \parallel \vec{b}$

$\Rightarrow \frac{1}{3} = \frac{\lambda}{2} = \frac{3}{9}$

$\Rightarrow \frac{1}{3} = \frac{\lambda}{2}$

$\Rightarrow \lambda = 2 \times \frac{1}{3} = \frac{2}{3}$

Ans: λ = 2/3

Q. 23

Write the value of

$$\hat{i} \cdot \left(\hat{j} \times \hat{k}\right) + \hat{j} \cdot \left(\hat{i} \times \hat{k}\right) + \hat{k} \cdot \left(\hat{i} \times \hat{j}\right).$$

Answer :

We know that:

$\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath},$

$\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}, \hat{\imath} \times \hat{k} = -\hat{\jmath}$

$\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

$\therefore \hat{\imath}.\left(\hat{\jmath} \times \hat{k}\right) + \hat{\jmath}.\left(\hat{\imath} \times \hat{k}\right) + \hat{k}.(\hat{\imath} \times \hat{\jmath}) = \hat{\imath}.\hat{\imath} + \hat{\jmath}.(-\hat{\jmath}) + \hat{k}.\hat{k} = 1 - 1 + 1 = 1$

Ans: $\hat{\imath}.\left(\hat{\jmath} \times \hat{k}\right) + \hat{\jmath}.\left(\hat{\imath} \times \hat{k}\right) + \hat{k}.(\hat{\imath} \times \hat{\jmath}) = 1$

Q. 24

Find the volume of the parallelepiped whose edges are represented by the vectors $\vec{a} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right), \vec{b} = \left(\hat{i} + 2\hat{j} - \hat{k}\right)$ **and** $\vec{c} = \left(3\hat{i} - 2\hat{j} + 2\hat{k}\right).$

Answer :

Scalar triple product geometrically represents the volume of the parallelepiped whose three coterminous edges are represented by $\vec{a}, \vec{b}, \vec{c}$.i.e. V = $[\vec{a}\vec{b}\vec{c}]$

$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\vec{c} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$\therefore$ V = $[\vec{a}\vec{b}\vec{c}] = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -2 & 2 \end{bmatrix}$ = 2(4 - 2) - (- 3)(2 - (- 3)) + 4(- 2 - 6) = 4 + 15 - 32 = | - 13| = 13 cubic units.

Ans:13 cubic units.

Q. 25

If $\vec{a} = \left(-2\hat{i} - 2\hat{j} + 4\hat{k}\right), \vec{b} = \left(-2\hat{j} + 4\hat{j} - 2\hat{k}\right)$ **and** $\vec{c} = \left(4\hat{i} - 2\hat{j} - 2\hat{k}\right)$ **then prove that** $\vec{a}, \vec{b}$ **and** $\vec{c}$ **are coplanar.**

Answer :

$\vec{a} = -2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$

$\vec{b} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

$\vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$

If $\vec{a}, \vec{b}, \vec{c}$ are coplanar then $[\vec{a}\vec{b}\vec{c}]$ = 0

L.H.S = $\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & -2 \\ 4 & -2 & -2 \end{bmatrix} =$ - 2(- 8 - 4) + 2(4 + 8) + 4(4 - 16) = 24 + 24 - 48 = 0 = R.H.S

$\therefore$L.H.S = R.H.S

Hence proved that the vectors $\vec{a} = -2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$

$\vec{b} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

$\vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$

Are coplanar.

Q. 26

If $\vec{a} = \left(2\hat{i} + 6\hat{j} + 27\hat{k}\right)$ **and** $\vec{b} = \left(\hat{i} + \lambda\hat{j} + \mu\hat{k}\right)$ **are such that** $\vec{a} \times \vec{b} = \vec{0}$ **then find the values of λ and μ.**

Answer :

$\vec{a} = 2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}$

$\vec{b} = \hat{\imath} + \lambda\hat{\jmath} + \mu\hat{k}$

It is given that $\vec{a} \times \vec{b} = \vec{0}$

$\Rightarrow \left(2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}\right) \times \left(\hat{\imath} + \lambda\hat{\jmath} + \mu\hat{k}\right) = 0$

$\Rightarrow \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{bmatrix} = 0 = \hat{\imath}(6\mu - 27\lambda) - \hat{\jmath}(2\mu - 27) + \hat{k}(2\lambda - 6)$

$\Rightarrow 2\lambda - 6 = 0$

$\Rightarrow \lambda = 6/2 = 3$

$\Rightarrow 2\mu - 27 = 0$

$\Rightarrow \mu = 27/2$

Ans: $\lambda = 3$, $\mu = 27/2$

Q. 27

If θ is the angle between $\vec{a}$ and $\vec{b}$, and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ then what is the value of θ?

Answer :

It is given that:

$|\vec{a} \times \vec{b}| = |\vec{a}.\vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}| sin\theta = |\vec{a}||\vec{b}| cos\theta$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = \tan^{-1} 1 = \frac{\pi}{4}$

Ans: $\theta = \frac{\pi}{4}$

Q. 28

When does $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$ hold?

Answer :

When the two vectors are parallel or collinear, they can be added in a scalar way because the angle between them is zero degrees,they are I the same or opposite direction.

Therefore when two vectors $\vec{a}$ *and* $\vec{b}$ are either parallel or collinear then

$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$

Q. 29

Find the direction cosines of a vector which is equally inclined to the x - axis, y - axis and z - axis.

Answer :

Let the inclination with:

x - axis = α

y - axis = β

z - axis = γ

The vector is equally inclined to the three axes.

$\Rightarrow \alpha = \beta = \gamma$

Direction cosines: $cos\alpha, cos\beta, cos\gamma$

We know that: $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$\Rightarrow \cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$ ...($\alpha = \beta = \gamma$)

$\Rightarrow 3 \cos^2 \alpha = 1$

$$cos\alpha = \frac{1}{\sqrt{3}}$$

$$\Rightarrow cos\alpha = \frac{1}{\sqrt{3}}$$

$$cos\beta = \frac{1}{\sqrt{3}}$$

$$cos\gamma = \frac{1}{\sqrt{3}}$$

Ans: $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Q. 30

If P(1, 5, 4) and Q(4, 1, - 2) be the position vectors of two points P and Q, find the direction ratios of $\overrightarrow{PQ}$.

Answer :

Let $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ be the two points then Direction ratios of line joining P and Q i.e. PQ are $x_2 - x_1, y_2 - y_1, z_2 - z$

Here, P is(1, 5, 4) and Q is (4, 1, - 2)

Direction ratios of PQ are:(4 - 1),(1 - 5),(- 2 - 4) = 3, - 4, - 6

Ans: the direction ratios of $\overrightarrow{PQ}$. are: 3, - 4, - 6

Q. 31

Find the direction cosines of the vector $\vec{a} = (\hat{i} + 2\hat{j} + 3\hat{k})$

Answer :

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

Let the inclination with:

x - axis = α

y - axis = β

z - axis = γ

Direction cosines: $\cos\alpha, \cos\beta, \cos\gamma = l, m, n$

For a vector $\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}, l = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

$$\therefore l = \frac{1}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{1}{\sqrt{1 + 4 + 9}} = \frac{1}{\sqrt{14}}.$$

$$\therefore m = \frac{2}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{2}{\sqrt{1 + 4 + 9}} = \frac{2}{\sqrt{14}}$$

$$\therefore n = \frac{3}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{3}{\sqrt{1 + 4 + 9}} = \frac{3}{\sqrt{14}}$$

Ans: $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

Q. 32

If $\hat{a}$ and $\hat{b}$ are unit vectors such that $\left(\hat{a}+\hat{b}\right)$ is a unit vector, what is the angle between $\hat{a}$ and $\hat{b}$?

Answer :

It is given that $\hat{a}$ *and* $\hat{b}$ are unit vectors ,as well as $(\hat{a} + \hat{b})$ is also a unit vector

$\Rightarrow |\hat{a}| = |\hat{b}| = |\hat{a} + \hat{b}| = 1$

Since the modulus of a unit vector is unity.

Now,

$|\hat{a} + \hat{b}|^2 = |\hat{a}|^2 + |\hat{b}|^2 + 2|\hat{a}||\hat{b}|\cos\theta$

$\Rightarrow 1^2 = 1^2 + 1^2 + 2\times1\times1\times \cos\theta$

$\Rightarrow \cos\theta = (1 - 1 - 1)/2$

$\Rightarrow \cos\theta = \frac{-1}{2}$

$\Rightarrow \theta = \cos^{-1}\frac{-1}{2} = \frac{2\pi}{3}$

Ans: $\frac{2\pi}{3}$

Objective Questions

Q. 1

Mark (√) against the correct answer in each of the following:

A unit vector in the direction of the vector $\vec{a} = \left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$ is

A. $\left(\hat{i} - \frac{3}{2}\hat{j} + 3\hat{k}\right)$

B. $\left(\frac{2}{5}\hat{i} - \frac{3}{5}\hat{j} + \frac{6}{5}\hat{k}\right)$

C. $\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right)$

D. none of these

Answer :

Tip – A vector in the direction of another vector $a\hat{i}+b\hat{j}+c\hat{k}$ is given by $\lambda(a\hat{i}+b\hat{j}+c\hat{k})$ and the unit vector is given by $\frac{\lambda(a\hat{i}+b\hat{j}+c\hat{k})}{\sqrt{(a\lambda)^2+(b\lambda)^2+(c\lambda)^2}}$

So, a vector parallel to $\vec{a}=2\hat{i}-3\hat{j}+6\hat{k}$ is given by $\lambda(2\hat{i}-3\hat{j}+6\hat{k})$ where λ is an arbitrary constant.

Now, $|\vec{a}|=\sqrt{2^2+3^2+6^2}=7$

Hence, the required unit vector

$$=\frac{\lambda\left(2\hat{i}-3\hat{j}+6\hat{k}\right)}{\sqrt{(2\lambda)^2+(3\lambda)^2+(6\lambda)^2}}$$

$$=\frac{\lambda\left(2\hat{i}-3\hat{j}+6\hat{k}\right)}{\lambda\sqrt{2^2+3^2+6^2}}$$

$$=\frac{2}{7}\hat{i}-\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$$

Q. 2

Mark (√) against the correct answer in each of the following:

The direction cosines of the vector $\vec{a}=\left(-2\hat{i}+\hat{j}-5\hat{k}\right)$ **are**

A. -2, 1, -5

B. $\frac{1}{3},\frac{-1}{6},\frac{-5}{6}$

C. $\frac{2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{5}{\sqrt{30}}$

D. $\frac{-2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{-5}{\sqrt{30}}$

Answer :

Formula to be used – The direction cosines of a vector $a\hat{i}+b\hat{j}+c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}},\frac{b}{\sqrt{a^2+b^2+c^2}},\frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $-2\hat{i}+\hat{j}-5\hat{k}$ is given by

$$\left(\frac{-2}{\sqrt{2^2+1^2+5^2}},\frac{1}{\sqrt{2^2+1^2+5^2}},\frac{-5}{\sqrt{2^2+1^2+5^2}}\right)$$

$$=\frac{-2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{-5}{\sqrt{30}}$$

Q. 3

Mark (√) against the correct answer in each of the following:

If A(1, 2, -3) and B(-1, -2, 1) are the end points of a vector $\overrightarrow{AB}$ then the direction cosines of $\overrightarrow{AB}$ are

A. -2, -4, 4

B. $\frac{-1}{2},-1,1$

C. $\frac{-1}{3},\frac{-2}{3},\frac{2}{3}$

D. none of these

Answer :

Given - A(1, 2, -3) and B(-1, -2, 1) are the end points of a vector $\overrightarrow{AB}$

Tip – If P(a_1,b_1,c_1) and Q(a_2,b_2,c_2) be two points then the vector $\overrightarrow{PQ}$ is represented by $(a_2-a_1)\hat{i}+(b_2-b_1)\hat{j}+(c_2-c_1)\hat{k}$

Hence, $\overrightarrow{AB}=(-1-1)\hat{i}+(-2-2)\hat{j}+(1+3)\hat{k}=-2\hat{i}-4\hat{j}+4\hat{k}$

Formula to be used – The direction cosines of a vector $a\hat{i}+b\hat{j}+c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}},\frac{b}{\sqrt{a^2+b^2+c^2}},\frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $-2\hat{i}-4\hat{j}+4\hat{k}$ is given by

$$\left(\frac{-2}{\sqrt{2^2+4^2+4^2}},\frac{-4}{\sqrt{2^2+4^2+4^2}},\frac{4}{\sqrt{2^2+4^2+4^2}}\right)$$

$= \left(\frac{-2}{6}, \frac{-4}{6}, \frac{4}{6}\right)$

$= \frac{-1}{3}, \frac{-2}{3}, \frac{2}{3}$

Q. 4

Mark (√) against the correct answer in each of the following:

If a vector makes angle α, β and γ with the x-axis, y-axis and z-axis respectively then the value of ($\sin^2\alpha + \sin^2\beta + \sin^2\gamma$) is
A. 1

B. 2
C. 0
D. 3

Answer :

Given - A vector makes angle α, β and γ with the x-axis, y-axis and z-axis respectively.

To Find - ($\sin^2\alpha + \sin^2\beta + \sin^2\gamma$)

Formula to be used - $\cos^2\alpha + \cos^2\beta + \cos^2\gamma=1$

Hence,

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$=(1-\cos^2\alpha) +(1-\cos^2\beta) +(1-\cos^2\gamma)$

$= 3-(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$=3-1$

$=2$

Q. 5

Mark (√) against the correct answer in each of the following:

The vector $(\cos\alpha\cos\beta)\hat{i} + (\cos\alpha\cos\beta)\hat{j} + (\sin\alpha)\hat{k}$ **is a**
A. null vector

B. unit vector
C. a constant vector
D. none of these

Answer :

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

A unit vector is a vector whose magnitude = 1.

Formula to be used - $\sin^2\theta + \cos^2\theta = 1$

Hence, magnitude of $(\cos\alpha\cos\beta)\hat{\imath} + (\cos\alpha\sin\beta)\hat{\jmath} + (\sin\alpha)\hat{k}$ will be given by $\sqrt{(\cos\alpha\cos\beta)^2 + (\cos\alpha\sin\beta)^2 + (\sin\alpha)^2}$

$= \sqrt{\cos^2\alpha(\cos^2\beta + \sin^2\beta) + \sin^2\alpha}$

$= \sqrt{\cos^2\alpha + \sin^2\alpha}$

= 1 i.e a unit vector

Q. 6

Mark (√) against the correct answer in each of the following:

What is the angle which the vector $\left(\hat{i} + \hat{j} + \sqrt{2}\hat{k}\right)$ makes with the z-axis?

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{2\pi}{3}$

Answer :

Formula to be used – The direction cosines of a vector $a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ is given by $\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$.

Hence, the direction cosines of the vector $\hat{\imath} + \hat{\jmath} + \sqrt{2}\hat{k}$ is given by

$$\left(\frac{1}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}, \frac{1}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}, \frac{\sqrt{2}}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}}\right)$$

$= \frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}$

$= \frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$

The direction cosine of z-axis = $\frac{1}{\sqrt{2}}$ i.e. $\cos\theta = \frac{1}{\sqrt{2}}$ where θ is the angle the vector makes with the z-axis.

$\therefore \theta = \cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$

Q. 7

Mark (√) against the correct answer in each of the following:

f $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = \sqrt{3}, \ |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{2\pi}{3}$

Answer :

Given - $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = \sqrt{6}$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Hence, $\sqrt{6} = 2\sqrt{3}\cos\theta$ i.e. $\cos\theta = \frac{1}{\sqrt{2}}$ $\quad\therefore \theta = \frac{\pi}{4}$

Q. 8

Mark (√) against the correct answer in each of the following:

If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a} \cdot \vec{b} = -1$ then the angle between

$\vec{a}$ **and** $\vec{b}$ **is**

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{2\pi}{3}$

Answer :

Given - $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}| = |\vec{b}| = \sqrt{2}$ and $\vec{a}.\vec{b} = -1$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Hence, $-1 = \sqrt{2}\sqrt{2}\cos\theta$ i.e. $\cos\theta = \frac{1}{2}$ $\therefore \theta = \frac{\pi}{3}$

Q. 9

Mark (√) against the correct answer in each of the following:

The angle between the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ **and** $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$ **is**

A. $\cos^{-1}\frac{5}{7}$

B. $\cos^{-1}\frac{3}{5}$

C. $\cos^{-1}\frac{3}{\sqrt{14}}$

D. none of these

Answer :

Given - $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

To find – Angle between $\vec{a}$ and $\vec{b}$.

Formula to be used - $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $\vec{a}.\vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k}).(3\hat{i} - 2\hat{j} + \hat{k})$=3+4+3=10

$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$|\vec{b}| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$

Hence, $10 = \sqrt{14}\sqrt{14}\cos\theta$ i.e. $\cos\theta = \frac{10}{14} = \frac{5}{7}$

$\therefore \theta = \cos^{-1}\frac{5}{7}$

Q. 10

Mark (√) against the correct answer in each of the following:

If $\vec{a} = (\hat{i} + 2\hat{j} - 3\hat{k})$ **and** $\vec{b} = (3\hat{i} - \hat{j} + 2\hat{k})$ **then the angle between** $(\vec{a} + \vec{b})$ **and** $(\vec{a} - \vec{b})$ **is**

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer :

Given - $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

To find – Angle between $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $\vec{a} + \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k}) = 4\hat{i} + \hat{j} - \hat{k}$

and $\vec{a} - \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) - (3\hat{i} - \hat{j} + 2\hat{k}) = -2\hat{i} + 3\hat{j} - 5\hat{k}$

$\therefore (\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = (4\hat{i} + \hat{j} - \hat{k}).(-2\hat{i} + 3\hat{j} - 5\hat{k}) = -8 + 3 + 5 = 0$

$|\vec{a} + \vec{b}| = \sqrt{4^2 + 1^2 + 1^2} = \sqrt{18}$

$|\vec{a} - \vec{b}| = \sqrt{2^2 + 3^2 + 5^2} = \sqrt{38}$

Hence, $0 = \sqrt{18}\sqrt{38}\cos\theta$ i.e. $\cos\theta = 0$

$\therefore \theta = \frac{\pi}{2}$

Q. 11

Mark (√) against the correct answer in each of the following:

If $\vec{a} = (\hat{i} + 2\hat{j} - 3\hat{k})$ **and** $\vec{b} = (3\hat{i} - \hat{j} + 2\hat{k})$ **then the angle between** $(2\vec{a} + \vec{b})$ **and** $(\vec{a} + 2\vec{b})$ **is**

A. $\cos^{-1}\left(\frac{21}{40}\right)$

B. $\cos^{-1}\left(\frac{31}{50}\right)$

C. $\cos^{-1}\left(\frac{11}{30}\right)$

D. none of these

Answer :

Given - $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$

To find – Angle between $2\vec{a} + \vec{b}$ and $\vec{a} + 2\vec{b}$.

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – Magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Here, $2\vec{a} + \vec{b} = 2(\hat{i} + 2\hat{j} - 3\hat{k}) + (3\hat{i} - \hat{j} + 2\hat{k}) = 5\hat{i} + 3\hat{j} - 4\hat{k}$

and $\vec{a}+2\vec{b}=(\hat{i}+2\hat{j}-3\hat{k})+2(3\hat{i}-\hat{j}+2\hat{k})=7\hat{i}+\hat{k}$

$\therefore (2\vec{a}+\vec{b}).(\vec{a}-2\vec{b})=(5\hat{i}+3\hat{j}-4\hat{k}).(7\hat{i}+\hat{k})=35-4=31$

$|2\vec{a}+\vec{b}|=\sqrt{5^2+3^2+4^2}=\sqrt{50}$

$|\vec{a}-2\vec{b}|=\sqrt{7^2+1^2}=\sqrt{50}$

Hence, $31=\sqrt{50}\sqrt{50}\cos\theta$ i.e. $\cos\theta=\frac{31}{50}$

$\therefore \theta=\cos^{-1}\frac{31}{50}$

Q. 12

Mark (√) against the correct answer in each of the following:

If $\vec{a}=(2\hat{i}+4\hat{j}-\hat{k})$ **and** $\vec{b}=(3\hat{i}-2\hat{j}+\lambda\hat{k})$ **be such that** $\vec{a}\perp\vec{b}$ **then λ = ?**
A. 2

B. -2
C. 3
D. -3

Answer :

Given - $\vec{a}=2\hat{i}+4\hat{j}-\hat{k},\ \vec{b}=3\hat{i}-2\hat{j}+\lambda\hat{k}$ and $\vec{a}\perp\vec{b}$

To find – Value of λ

Formula to be used - $\vec{p}.\vec{q}=|\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – For perpendicular vectors, $\theta=\frac{\pi}{2}$ i.e. $\cos\theta=0$ i.e. the dot product=0

Hence, $\vec{a}.\vec{b}=0$

$\therefore (2\hat{i}+4\hat{j}-\hat{k}).(3\hat{i}-2\hat{j}+\lambda\hat{k})=0$

⇨ $6-8-\lambda=0$

i.e. $\lambda=-2$

Q. 13

Mark (√) against the correct answer in each of the following:

What is the projection of $\vec{a}=\left(2\hat{i}-\hat{j}+\hat{k}\right)$ **on** $\vec{b}=\left(\hat{i}-2\hat{j}+\hat{k}\right)$ **?**

A. $\frac{2}{\sqrt{3}}$

B. $\frac{4}{\sqrt{5}}$

C. $\frac{5}{\sqrt{6}}$

D. none of these

Answer :

Given - $\vec{a}=2\hat{\imath}-\hat{\jmath}+\hat{k},\ \vec{b}=\hat{\imath}-2\hat{\jmath}+\hat{k}$

To find – Projection of $\vec{a}$ on $\vec{b}$ i.e. $\vec{a}\cos\theta$

Formula to be used - $\vec{p}.\vec{q}=|\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – If $\vec{p}$ and $\vec{q}$ are two vectors, then the projection of $\vec{p}$ on $\vec{q}$ is defined as $\vec{p}\cos\theta$

Magnitude of a vector $\vec{p}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$ is given by $|\vec{p}|=\sqrt{x^2+y^2+z^2}$

So,

$\vec{a}.\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

⇨ $(2\hat{\imath}-\hat{\jmath}+\hat{k}).(\hat{\imath}-2\hat{\jmath}+\hat{k})=\sqrt{1^2+2^2+1^2}|\vec{a}|\cos\theta$

⇨ $|\vec{a}|\cos\theta=\frac{2+2+1}{\sqrt{6}}$

⇨ $|\vec{a}|\cos\theta=\frac{5}{\sqrt{6}}$

Q. 14

Mark (√) against the correct answer in each of the following:

If $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|,$ **then**

A. $\left|\vec{a}\right|=\left|\vec{b}\right|$

B. $\vec{a}\parallel\vec{b}$

C. $\vec{a} \perp \vec{b}$

D. none of these

Answer :

Given - $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

Tip – If $\vec{a}$ and $\vec{b}$ are two vectors then $|\vec{a} \pm \vec{b}| = \sqrt{a^2 + b^2 \pm 2ab\cos\theta}$

Hence,

$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

⇨ $\sqrt{a^2 + b^2 + 2ab\cos\theta} = \sqrt{a^2 + b^2 - 2ab\cos\theta}$

⇨ $a^2 + b^2 + 2ab\cos\theta = a^2 + b^2 - 2ab\cos\theta$

⇨ $4ab\cos\theta = 0$

⇨ $\cos\theta = 0$

i.e. $\theta = \frac{\pi}{2}$

So, $\vec{a} \perp \vec{b}$

Q. 15

Mark (√) against the correct answer in each of the following:

If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors then $\left(3\vec{a}+2\vec{b}\right)\cdot\left(5\vec{a}-6\vec{b}\right) = ?$

A. 3

B. 5

C. 6

D. 12

Answer :

Given - $\vec{a}$ and $\vec{b}$ are two mutually perpendicular unit vectors i.e. $|\vec{a}| = |\vec{b}| = 1$

To Find – $(3\vec{a}+2\vec{b}).(5\vec{a}-6\vec{b})$

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip - $\vec{a} \perp \vec{b}$

∴ $|\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\cos\frac{\pi}{2} = 0$

$\therefore \vec{a}.\vec{b} = \vec{b}.\vec{a} = 0$

Hence,

$\left(3\vec{a} + 2\vec{b}\right).\left(5\vec{a} - 6\vec{b}\right)$

$= 15|\vec{a}|^2 + 10\vec{b}.\vec{a} - 18\vec{a}.\vec{b} - 12\left|\vec{b}\right|^2$

$= 15 - 12$

$= 3$

Q. 16

Mark (√) against the correct answer in each of the following:

If the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ **and** $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ **are perpendicular to each other then λ = ?**

A. -3

B. -6

C. -9

D. -1

Answer :

Given - $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{a} \perp \vec{b}$

To find – Value of λ

Formula to be used - $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ and $\vec{q}$ are two vectors

Tip – For perpendicular vectors, $\theta = \frac{\pi}{2}$ i.e. $\cos\theta = 0$ i.e. the dot product=0

Hence, $\vec{a}.\vec{b} = 0$

$\therefore \left(3\hat{i} + \hat{j} - 2\hat{k}\right).\left(\hat{i} + \lambda\hat{j} - 3\hat{k}\right) = 0$

⇨ $3 + \lambda + 6 = 0$

i.e. $\lambda = -9$

Q. 17

Mark (√) against the correct answer in each of the following:

If θ is the angle between two unit vectors $\hat{a}$ **and** $\hat{b}$ **then** $\frac{1}{2}\left|\hat{a} - \hat{b}\right| = ?$

A. $\cos\frac{\theta}{2}$

B. $\sin\frac{\theta}{2}$

C. $\tan\frac{\theta}{2}$

D. none of these

Answer :

Given - $\hat{a}$ and $\hat{b}$ are two unit vectors with an angle θ between them

To find - $\frac{1}{2}|\hat{a}-\hat{b}|$

Formula used - If $\vec{a}$ and $\vec{b}$ are two vectors then $|\vec{a}\pm\vec{b}| = \sqrt{a^2 + b^2 \pm 2ab\cos\theta}$

$\cos2\theta = 1 - 2\sin^2\theta$

Tip - $|\hat{a}|^2 = |\hat{b}|^2 = 1$ & $\hat{a}.\hat{b} = 1$

Hence,

$\frac{1}{2}|\hat{a}-\hat{b}|$

$= \frac{1}{2}\sqrt{|\hat{a}|^2 + |\hat{b}|^2 + 2ab\cos\theta}$

$= \frac{1}{2}\sqrt{2 + 2\cos\theta}$

$= \frac{1}{\sqrt{2}}\sqrt{1 + \cos\theta}$

$= \frac{1}{\sqrt{2}} \times \sqrt{2\sin^2\frac{\theta}{2}}$

$= \sin\frac{\theta}{2}$

Q. 18

Mark (√) against the correct answer in each of the following:

If $\vec{a}=\left(\hat{i}-\hat{j}+2\hat{k}\right)$ **and** $\vec{b}=\left(2\hat{i}+3\hat{j}-4\hat{k}\right)$ **then** $\left|\vec{a}\times\vec{b}\right|=?$

A. $\sqrt{174}$

B. $\sqrt{87}$

C. $\sqrt{93}$

D. none of these

Answer :

Given - $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}-4\hat{k}$ are two vectors.

To find - $|\vec{a}\times\vec{b}|$

Formula to be used - $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$ where $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

Tip – Magnitude of a vector $\vec{p}=x\hat{i}+y\hat{j}+z\hat{k}$ is given by $|\vec{p}|=\sqrt{x^2+y^2+z^2}$

So,

$\vec{a}\times\vec{b}$

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ 2 & 3 & -4\end{vmatrix}$

$=\hat{i}(4-6)+\hat{j}\,(4+4)+\hat{k}(3+2)$

$=-2\hat{i}+8\hat{j}+5\hat{k}$

$\therefore\ \left|\vec{a}\times\vec{b}\right|=\sqrt{2^2+8^2+5^2}=\sqrt{93}$

Q. 19

Mark (√) against the correct answer in each of the following:

If $\vec{a}=\left(\hat{i}-2\hat{j}+3\hat{k}\right)$ **and** $\vec{b}=\left(\hat{i}-3\hat{k}\right)$ **then** $\left|\vec{b}\times 2\vec{a}\right|=?$

A. $10\sqrt{3}$

B. $5\sqrt{17}$

C. $4\sqrt{19}$

D. $2\sqrt{23}$

Answer :

Given - $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{k}$ are two vectors.

To find - $|\vec{b} \times 2\vec{a}|$

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

So,

$\vec{b} \times 2\vec{a}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -3 \\ 2 & -4 & 6 \end{vmatrix}$

$= \hat{i}(12) + \hat{j}\,(-6 - 6) + \hat{k}(-4)$

$= 12\hat{i} - 12\hat{j} - 4\hat{k}$

$\therefore \left|\vec{b} \times 2\vec{a}\right| = \sqrt{12^2 + 12^2 + 4^2} = \sqrt{304} = 4\sqrt{19}$

Q. 20

Mark (√) against the correct answer in each of the following:

If $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 7$ **and** $\left(\vec{a} \times \vec{b}\right) = \left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ **then the angle between** $\vec{a}$ **and** $\vec{b}$ **is**

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{3\pi}{4}$

Answer :

Given - $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

To find – Angle between $\vec{a}$ and $\vec{b}$

Formula to be used - $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$

Tip – $|\vec{p} \times \vec{q}| = ||\vec{p}||\vec{q}|\sin\theta\hat{n}| = |\vec{p}||\vec{q}|\sin\theta$ & magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence, $|\vec{a} \times \vec{b}| = |3\hat{i} + 2\hat{j} + 6\hat{k}| = \sqrt{3^2 + 2^2 + 6^2} = 7$

$\therefore 7 = 2 \times 7\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

Q. 21

Mark (√) against the correct answer in each of the following:

If $|\vec{a}| = \sqrt{26}, |\vec{b}| = 7$ **and** $|\vec{a} \times \vec{b}| = 35$ **then** $\vec{a} \cdot \vec{b} = ?$

A. 5

B. 7

C. 13

D. 12

Answer :

Given – $|\vec{a}| = \sqrt{26}, |\vec{b}| = 7$ and $|\vec{a} \times \vec{b}| = 35$

To find - $\vec{a}.\vec{b}$

Formula to be used - $\vec{p} \times \vec{q} = |\vec{p}||\vec{q}|\sin\theta\hat{n}$ & $\vec{p}.\vec{q} = |\vec{p}||\vec{q}|\cos\theta$ where $\vec{p}$ & $\vec{q}$ are any two vectors

Tip – $|\vec{p} \times \vec{q}| = ||\vec{p}||\vec{q}|\sin\theta\hat{n}| = |\vec{p}||\vec{q}|\sin\theta$

So,

$|\vec{a} \times \vec{b}| = 35$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = 35$

$\Rightarrow \sin\theta = \frac{35}{7\sqrt{26}} = \frac{5}{\sqrt{26}}$

$\therefore \cos\theta = \sqrt{1 - \left(\frac{5}{\sqrt{26}}\right)^2} = \frac{1}{\sqrt{26}}$

$\therefore \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta = \sqrt{26} \times 7 \times \frac{1}{\sqrt{26}} = 7$

Q. 22

Mark (√) against the correct answer in each of the following:

Two adjacent sides of a || gm are represented by the vectors $\vec{a} = \left(3\hat{i} + \hat{j} + 4\hat{k}\right)$ **and** $\vec{b} = \left(\hat{i} - \hat{j} + \hat{k}\right)$. **The area of the || gm is**

A. $\sqrt{42}$ **sq units**

B. 6 sq units

C. $\sqrt{35}$ **sq units**

D. none of these

Answer :

Given - Two adjacent sides of a || gm are represented by the vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

To find – Area of the parallelogram

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

Tip – Area of ||gm $= |\vec{a} \times \vec{b}|$ and magnitude of a vector $\vec{p} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 1 & 4 \\ 1 & -1 & 1 \end{vmatrix}$

$= \hat{i}(-4-1) + \hat{j}(4-3) + \hat{k}(-3-1)$

$= -5\hat{i} + \hat{j} - 4\hat{k}$

$\therefore \left|\vec{a} \times \vec{b}\right| = \sqrt{5^2 + 1^2 + 4^2} = \sqrt{42}$

i.e. the area of the parallelogram = $\sqrt{42}$ sq. units

Q. 23

Mark (√) against the correct answer in each of the following:

The diagonals of a || gm are represented by the vectors $\overrightarrow{d_1} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right)$ **and** $\overrightarrow{d_2} = \left(\hat{i} - 3\hat{j} + 4\hat{k}\right).$ **The area of the || gm is**

A. $7\sqrt{3}$ **sq units**

B. $5\sqrt{3}$ **sq units**

C. $3\sqrt{5}$ **sq units**

D. none of these

Answer :

Given - Two diagonals of a || gm are represented by the vectors $\overrightarrow{d_1} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{d_2} = \hat{i} - 3\hat{j} + 4\hat{k}$

To find – Area of the parallelogram

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Area of ||gm $= \frac{1}{2}\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right|$ and magnitude of a vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$\overrightarrow{d_1} \times \overrightarrow{d_2}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$

$= \hat{i}(4-6) + \hat{j}(-2-12) + \hat{k}(-9-1)$

$= -2\hat{i} - 14\hat{j} - 10\hat{k}$

$\therefore \left|\vec{d_1} \times \vec{d_2}\right| = \sqrt{2^2 + 14^2 + 10^2} = \sqrt{300}$

i.e. the area of the parallelogram = $\frac{1}{2} \times \sqrt{300} = 5\sqrt{3}$ sq. units

Q. 24

Mark (√) against the correct answer in each of the following:

Two adjacent sides of a triangle are represented by the vectors $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$. The area of the triangle is

A. 41 sq units

B. 37 sq units

C. $\frac{41}{2}$ sq units

D. none of these

Answer :

Given - Two adjacent sides of a triangle are represented by the vectors $\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$

To find – Area of the triangle

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – Area of triangle $= \frac{1}{2}\left|\vec{a} \times \vec{b}\right|$ and magnitude of a vector $\vec{p} = x\hat{i} + y\hat{j} + z\hat{k}$ is given by $|\vec{p}| = \sqrt{x^2 + y^2 + z^2}$

Hence,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ -5 & 7 & 0 \end{vmatrix}$

$= \hat{k}(21 + 20)$

$= 41\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{41^2} = 41$

i.e. the area of the parallelogram = $\frac{41}{2}$ sq. units

Q. 25

Mark (√) against the correct answer in each of the following:

The unit vector normal to the plane containing $\vec{a} = (\hat{i} - \hat{j} - \hat{k})$ **and** $\vec{b} = (\hat{i} + \hat{j} + \hat{k})$ **is**

A. $(\hat{j} - \hat{k})$

B. $(-\hat{j} + \hat{k})$

C. $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

D. $\frac{1}{\sqrt{2}}(-\hat{i} + \hat{k})$

Answer :

Given - $\vec{a} = \hat{i} - \hat{j} - \hat{k}$ & $\vec{b} = \hat{i} + \hat{j} + \hat{k}$

To find – A unit vector perpendicular to the two given vectors.

Formula to be used - $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – A vector perpendicular to two given vectors is their cross product.

The unit vector of any vector $a\hat{i} + b\hat{j} + c\hat{k}$ is given by $\frac{(a\hat{i}+b\hat{j}+c\hat{k})}{\sqrt{a^2+b^2+c^2}}$

Hence,

$\vec{a} \times \vec{b}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$

$= -2\hat{j} + 2\hat{k}$, which the vector perpendicular to the two given vectors.

The required unit vector $= \frac{-2\hat{j}+2\hat{k}}{\sqrt{2^2+2^2}} = \frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

Q. 26

Mark (√) against the correct answer in each of the following:

If $\vec{a}, \vec{b}$ **and** $\vec{c}$ **are unit vectors such that** $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ **then** $\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)=?$

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. $\frac{3}{2}$

D. $\frac{-3}{2}$

Answer :

Given - $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors and $(\vec{a}+\vec{b}+\vec{c}) = 0$

To find - $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$

Tip – $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

So,

$(\vec{a}+\vec{b}+\vec{c})^2 = 0$

⇨ $|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$

⇨ $3+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$

⇨ $(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = \frac{-3}{2}$

Q. 27

Mark (√) against the correct answer in each of the following:

If $\vec{a}, \vec{b}$ **and** $\vec{c}$ **are mutually perpendicular unit vectors then** $\left[\vec{a}+\vec{b}+\vec{c}\right]=?$

A. 1

B. $\sqrt{2}$

C. $\sqrt{3}$
D. 2

Answer :

Given - $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors

To find - $[\vec{a} + \vec{b} + \vec{c}]$

Tip – $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ & $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

So,

$(\vec{a} + \vec{b} + \vec{c})^2$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$

=3

$\therefore [\vec{a} + \vec{b} + \vec{c}] = \sqrt{3}$

Q. 28

Mark (√) against the correct answer in each of the following:

$\left[\hat{i}\ \hat{j}\ \hat{k}\right] = ?$

A. 0

B. 1
C. 2
D. 3

Answer :

To find - $[\hat{i}\ \ \hat{j}\ \ \hat{k}]$

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c})$

$\therefore [\hat{i}\ \ \hat{j}\ \ \hat{k}]$

$= \hat{i}\,.(\hat{j} \times \hat{k})$

$= \hat{i}.\hat{i}$

$= |\hat{i}|^2$

$= 1$

Q. 29

Mark (√) against the correct answer in each of the following:

If $\vec{a} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right), \vec{b} = \left(\hat{i} + 2\hat{j} - \hat{k}\right)$ **and** $\vec{c} = \left(3\hat{i} - \hat{j} - 2\hat{k}\right)$ **be the coterminous edges of a parallelepiped then its volume is**

A. 21 cubic units

B. 14 cubic units

C. 7 cubic units

D. none of these

Answer :

Given – The three coterminous edges of a parallelepiped are $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k},$

$\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ & $\vec{c} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$

To find – The volume of the parallelepiped

Formula to be used - $[\hat{a} \;\; \hat{b} \;\; \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

Tip - The volume of the parallelepiped $= |\,[\hat{a} \;\; \hat{b} \;\; \hat{c}]|$

Hence,

$[\hat{a} \;\; \hat{b} \;\; \hat{c}]$

$= \hat{a}\,.(\hat{b} \times \hat{c})$

$= (2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}).\{(\hat{\imath} + 2\hat{\jmath} - \hat{k}) \times (3\hat{\imath} - \hat{\jmath} - 2\hat{k})\}$

$= (2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}).\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & -2 \end{vmatrix}$

$= (2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}).(-5\hat{\imath} - \hat{\jmath} - 7\hat{k})$

$= -10 + 3 - 28$

$= -35$

The volume = 35 sq units

Q. 30

Mark (√) against the correct answer in each of the following:

If the volume of a parallelepiped having $\vec{a}=\left(5\hat{i}-4\hat{j}+\hat{k}\right), \vec{b}=\left(4\hat{i}+3\hat{j}+\lambda\hat{k}\right)$ **and** $\vec{c}=\left(\hat{i}-2\hat{j}+7\hat{k}\right)$ **as conterminous edges, is 216 cubic units then the value of λ is**

A. $\frac{5}{3}$

B. $\frac{4}{3}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer :

Given – The three coterminous edges of a parallelepiped are $\vec{a}=5\hat{i}-4\hat{j}+\hat{k}$,

$\vec{b}=4\hat{i}+3\hat{j}+\lambda\hat{k}$ & $\vec{c}=\hat{i}-2\hat{j}+7\hat{k}$

To find – The value of λ

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}]=\hat{a}.(\hat{b}\times\hat{c})$

$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$ where $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

Tip - The volume of the parallelepiped = $|\ [\hat{a}\ \ \hat{b}\ \ \hat{c}]|$

Hence,

$[\hat{a}\ \ \hat{b}\ \ \hat{c}]$

$=\hat{a}.(\hat{b}\times\hat{c})$

$=(5\hat{i}-4\hat{j}+\hat{k}).\{(4\hat{i}+3\hat{j}+\lambda\hat{k})\times(\hat{i}-2\hat{j}+7\hat{k})\}$

$=(5\hat{i}-4\hat{j}+\hat{k}).\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 3 & \lambda\\ 1 & -2 & 7\end{vmatrix}$

$= \left(5\hat{i} - 4\hat{j} + \hat{k}\right).\left((21 + 2\lambda)\hat{i} + (\lambda - 28)\hat{j} - 11\hat{k}\right)$

=5(21+2λ)-4(λ-28)-11

=206+6λ

The volume =206+6λ

But, the volume = 216 sq units

So, 206+6λ=216 ⇨λ= $\frac{10}{6} = \frac{5}{3}$

Q. 31

Mark (√) against the correct answer in each of the following:

It is given that the vectors $\vec{a} = \left(2\hat{i} - 2\hat{k}\right),\ \vec{b} = \hat{i} + (\lambda + 1)\hat{j}$ **and** $\vec{c} = \left(4\hat{i} + 2\hat{k}\right)$ **are coplanar. Then, the value of λ is**

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. 2

D. 1

Answer :

Given – The vectors $\vec{a} = 2\hat{i} - 2\hat{k}, \vec{b} = \hat{i} + (\lambda + 1)\hat{j}\ \&\ \vec{c} = 4\hat{i} + 2\hat{k}$ are coplanar

To find – The value of λ

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.(\hat{b} \times \hat{c})$

$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ where $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Tip – For vectors to be coplanar, $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = 0$

Hence,

$[\hat{a}\ \ \hat{b}\ \ \hat{c}] = 0$

⇨ $\hat{a}\,.\left(\hat{b} \times \hat{c}\right) = 0$

⇨ $\left(2\hat{i} - 2\hat{k}\right).\left\{(\hat{i} + (\lambda + 1)\hat{j}) \times \left(4\hat{i} + 2\hat{k}\right)\right\} = 0$

$\Rightarrow \left(2\hat{i}-2\hat{k}\right).\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \lambda+1 & 0\\ 4 & 0 & 2\end{vmatrix}=0$

$\Rightarrow \left(2\hat{i}-2\hat{k}\right).\left(2(\lambda+1)\hat{i}-2\hat{j}-4(\lambda+1)\hat{k}\right)=0$

⇨ 4(λ-1)+8(λ-1)=0

⇨ 12(λ-1)=0 i.e. λ= 1

Q. 32

Mark (√) against the correct answer in each of the following:

Which of the following is meaningless?

A. $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$

B. $\vec{a}\times\left(\vec{b}\cdot\vec{c}\right)$

C. $\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}$

D. none of these

Answer :

Tip - $[\hat{a}\ \ \hat{b}\ \ \hat{c}]=\hat{a}.\left(\hat{b}\times\hat{c}\right)=\hat{b}.\left(\hat{c}\times\hat{a}\right)=\hat{c}.\left(\hat{a}\times\hat{b}\right)=\left(\hat{a}\times\hat{b}\right).\hat{c}$ since, dot product is commutative

Hence, $\hat{a}\times\left(\hat{b}.\hat{c}\right)$ is meaningless.

Q. 33

Mark (√) against the correct answer in each of the following:

$\vec{a}\cdot\left(\vec{a}\times\vec{b}\right)=?$

A. 0

B. 1

C. a^2b

D. meaningless

Answer :

Tip – The cross product of two vectors is the vector perpendicular to both the vectors.

∴ $\vec{a}\times\vec{b}$ gives a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

Now,

$\vec{a}.\left(\vec{a}\times\vec{b}\right)$

$= |\vec{a}|\left|\vec{b}\right|\cos\theta$

$= |\vec{a}|\left|\vec{b}\right|\cos\frac{\pi}{2}$

$= 0$

Q. 34

Mark (√) against the correct answer in each of the following:

For any three vectors $\vec{a}, \vec{b}, \vec{c}$ **the value of** $\left[\vec{a}-\vec{b}\ \vec{b}-\vec{c}\ \vec{c}-\vec{a}\right]$ **is**

A. $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

B. 1
C. 0
D. none of these

Answer :

Formula to be used - $[\hat{a}\ \ \hat{b}\ \ \hat{c}] = \hat{a}\,.\left(\hat{b}\times\hat{c}\right) = \hat{b}.\left(\hat{c}\times\hat{a}\right)$ for any three arbitrary vectors

$\therefore \left[\hat{a}-\hat{b}\ \ \hat{b}-\hat{c}\ \ \hat{c}-\hat{a}\right]$

$= \left(\hat{a}-\hat{b}\right).\left\{\left(\hat{b}-\hat{c}\right)\times\left(\hat{c}-\hat{a}\right)\right\}$

$= \left(\hat{a}-\hat{b}\right).\left(\hat{b}\times\hat{c}-\hat{c}\times\hat{c}-\hat{b}\times\hat{a}+\hat{c}\times\hat{a}\right)$

$= \left(\hat{a}-\hat{b}\right).\left(\hat{b}\times\hat{c}-\hat{b}\times\hat{a}+\hat{c}\times\hat{a}\right)$

$= \left[\hat{a}.\left(\hat{b}\times\hat{c}\right)-\hat{b}\left(\hat{b}\times\hat{c}\right)-\hat{a}.\left(\hat{b}\times\hat{a}\right)+\hat{b}\left(\hat{b}\times\hat{a}\right)+\hat{a}.\left(\hat{c}\times\hat{a}\right)-\hat{b}.\left(\hat{c}\times\hat{a}\right)\right]$

$= \left[\hat{a}\ \hat{b}\ \hat{c}\right]-\left[\hat{a}\ \hat{b}\ \hat{c}\right] = 0$