# 27. Straight Line in Space

## Exercise 27A

### 1. Question

A line passes through the point (3, 4, 5) and is parallel to the vector $\left(2\hat{i}+2\hat{j}-3\hat{k}\right)$. Find the equations of the line in the vector as well as Cartesian forms.

**Answer**

Given: line passes through point (3, 4, 5) and is parallel to $2\hat{i}+2\hat{j}-3\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Here, $\vec{a}=3\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{b}=2\hat{i}+2\hat{j}-3\hat{k}$

Therefore,

Vector form:

$$\vec{r}=3\hat{i}+4\hat{j}+5\hat{k}+\lambda\left(2\hat{i}+2\hat{j}-3\hat{k}\right)$$

Cartesian form:

$$\frac{x-3}{2}=\frac{y-4}{2}=\frac{z-5}{-3}$$

### 2. Question

A line passes through the point (2, 1, -3) and is parallel to the vector $\left(\hat{i}-2\hat{j}+3\hat{k}\right)$. Find the equations of the line in vector and Cartesian forms.

**Answer**

Given: line passes through (2, 1, -3) and is parallel to $\hat{i}-2\hat{j}+3\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Here, $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

Therefore,

Vector form:

$\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

Cartesian form:

$$\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+3}{3}$$

**3. Question**

Find the vector equation of the line passing through the point with position vector $(2\hat{i} + \hat{j} - 5\hat{k})$ and parallel to the vector $(\hat{i} + 3\hat{j} - \hat{k})$. Deduce the Cartesian equations of the line.

**Answer**

Given: line passes through $2\hat{i} + \hat{j} - 5\hat{k}$ and is parallel to $\hat{i} + 3\hat{j} - \hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Here, $\vec{a} = 2\hat{i} + \hat{j} - 5\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - \hat{k}$

Therefore,

Vector form:

$\vec{r} = 2\hat{i} + \hat{j} - 5\hat{k} + \lambda(\hat{i} + 3\hat{j} - \hat{k})$

Cartesian form:

$$\frac{x-2}{1} = \frac{y-1}{3} = \frac{z+5}{-1}$$

**4. Question**

A line is drawn in the direction of $(\hat{i} + \hat{j} - 2\hat{k})$ and it passes through a point with position vector $(2\hat{i} - \hat{j} - 4\hat{k})$. Find the equations of the line in the vector as well as Cartesian forms.

**Answer**

Given: line passes through $2\hat{i} - \hat{j} - 4\hat{k}$ and is drawn in the direction of $\hat{i} + \hat{j} - 2\hat{k}$

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since line is drawn in the direction of $(\hat{i} + \hat{j} - \hat{2k})$, it is parallel to $(\hat{i} + \hat{j} - 2\hat{k})$

Here, $\vec{a} = 2\hat{i} - \hat{j} - 4\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$

Therefore,

Vector form:

$\vec{r} = 2\hat{i} - \hat{j} - 4\hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$

Cartesian form:

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z+4}{-2}$$

**5. Question**

The Cartesian equations of a line are $\frac{x-3}{2} = \frac{y+2}{-5} = \frac{z-6}{4}$. Find the vector equation of the line.

**Answer**

Given: Cartesian equation of line

$$\frac{x-3}{2}=\frac{y+2}{-5}=\frac{z-6}{4}$$

To find: equation of line in vector form

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{\imath}+y_1\hat{\jmath}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$ is a vector parallel to the line.

Explanation:

From the Cartesian equation of the line, we can find $\vec{a}$ and $\vec{b}$

Here, $\vec{a}=3\hat{\imath}-\hat{2}\jmath+6\hat{k}$ and $\vec{b}=\hat{2}\imath-5\hat{\jmath}+4\hat{k}$

Therefore,

Vector form:

$$\vec{r}=3\hat{\imath}-2\hat{\jmath}+6\hat{k}+\lambda\left(2\hat{\imath}-5\hat{\jmath}+4\hat{k}\right)$$

**6. Question**

The Cartesian equations of a line are 3x + 1 = 6y − 2 = 1 − z. Find the fixed point through which it passes, its direction ratios and also its vector equation.

**Answer**

Given: Cartesian equation of line are 3x + 1 = 6y − 2 = 1 - z

To find: fixed point through which the line passes through, its direction ratios and the vector equation.

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{\imath}+y_1\hat{\jmath}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$ is a vector parallel to the line and also its direction ratio.

Explanation:

The Cartesian form of the line can be rewritten as:

$$\frac{x+\frac{1}{3}}{\frac{1}{3}} = \frac{y-\frac{1}{3}}{\frac{1}{6}} = \frac{z-1}{-1} = \lambda$$

$$\Rightarrow \frac{x+\frac{1}{3}}{2} = \frac{y-\frac{1}{3}}{1} = \frac{z-1}{-6} = \lambda$$

Therefore, $\vec{a} = \frac{-1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 6\hat{k}$

So, the line passes through $\left(\frac{-1}{3}, \frac{1}{3}, 1\right)$ and direction ratios of the line are (2, 1, -6) and vector form is:

$$\vec{r} = \frac{-1}{3}\hat{i} + \frac{1}{3}\hat{j} + \hat{k} + \lambda(2\hat{i} + \hat{j} - 6\hat{k})$$

**7. Question**

Find the Cartesian equations of the line which passes through the point (1, 3, -2) and is parallel to the line given by $\frac{x+1}{3} = \frac{y-4}{5} = \frac{z+3}{-6}$. Also, find the vector form of the equations so obtained.

**Answer**

Given: line passes through (1, 3, -2) and is parallel to the line

$$\frac{x+1}{3} = \frac{y-4}{5} = \frac{z+3}{-6}$$

To find: equation of line in vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since the line (say $L_1$) is parallel to another line (say $L_2$), $L_1$ has the same direction ratios as that of $L_2$

Here, $\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$

Since the equation of $L_2$ is

$$\frac{x+1}{3} = \frac{y-4}{5} = \frac{z+3}{-6}$$

$$\vec{b} = 3\hat{i} + 5\hat{j} - 6\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} + 3\hat{j} - 2\hat{k} + \lambda(3\hat{i} + 5\hat{j} - 6\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{3} = \frac{y-3}{5} = \frac{z+2}{-6}$$

**8. Question**

Find the equations of the line passing through the point (1, -2, 3) and parallel to the line $\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}$. Also find the vector form of this equation so obtained.

**Answer**

Given: line passes through (1, -2, 3) and is parallel to the line

$$\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}$$

To find: equation of line in vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since the line (say $L_1$) is parallel to another line (say $L_2$), $L_1$ has the same direction ratios as that of $L_2$

Here, $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

Since the equation of $L_2$ is

$$\frac{x-6}{3} = \frac{y-2}{-4} = \frac{z+7}{5}$$

$$\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} - 4\hat{j} + 5\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{3}=\frac{y+2}{-4}=\frac{z-3}{5}$$

**9. Question**

Find the Cartesian and vector equations of a line which passes through the point (1, 2, 3) and is parallel to the line $\frac{-x-2}{1}=\frac{y+3}{7}=\frac{2z-6}{3}$.

**Answer**

Given: line passes through (1, 2, 3) and is parallel to the line

$$\frac{-x-2}{1}=\frac{y+3}{7}=\frac{2z-6}{3}$$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

Explanation:

Since the line (say $L_1$) is parallel to another line (say $L_2$), $L_1$ has the same direction ratios as that of $L_2$

Here, $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$

Equation of $L_2$ can be rewritten as:

$$\frac{x+2}{-1}=\frac{y+3}{7}=\frac{z-3}{\frac{3}{2}}$$

$$\Rightarrow \frac{x+2}{-2}=\frac{y+3}{14}=\frac{z-3}{3}$$

$$\vec{b}=-2\hat{i}+14\hat{j}+3\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(-2\hat{i}+14\hat{j}+3\hat{k}\right)$$

Cartesian form of the line is:

$$\frac{x-1}{-2}=\frac{y-2}{14}=\frac{z-3}{3}$$

**10. Question**

Find the equations of the line passing through the point (-1, 3, -2) and perpendicular to each of the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$.

**Answer**

Given: line passes through (-1, 3, -2) and is perpendicular to each of the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is a vector parallel to the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

Here, $\vec{a}=-\hat{i}+3\hat{j}-2\hat{k}$

Let the direction ratios of the line be $b_1:b_2:b_3$

Direction ratios of the other two lines are 1 : 2 : 3 and -3 : 2 : 5

Since the other two line are perpendicular to the given line, we have

$b_1 + 2b_2 + 3b_3 = 0$

$-3b_1 + 2b_2 + 5b_3 = 0$

Solving,

$$\frac{b_1}{\begin{vmatrix}2 & 3\\ 2 & 5\end{vmatrix}}=\frac{-b_2}{\begin{vmatrix}1 & 3\\ -3 & 5\end{vmatrix}}=\frac{b_3}{\begin{vmatrix}1 & 2\\ -3 & 2\end{vmatrix}}$$

$$\Rightarrow \frac{b_1}{4}=\frac{b_2}{-14}=\frac{b_3}{8}$$

$$\Rightarrow \frac{b_1}{2}=\frac{b_2}{-7}=\frac{b_3}{4}$$

$\vec{b} = 2\hat{i} - 7\hat{j} + 4\hat{k}$

Therefore,

Vector form of the line is:

$\vec{r} = -\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$

Cartesian form of the line is:

$$\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$$

**11. Question**

Find the Cartesian and vector equations of the line passing through the point (1, 2, -4) and perpendicular to each of the lines $\frac{x-8}{8} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y+29}{8} = \frac{z-5}{-5}$.

**Answer**

Given: line passes through (1, 2, -4) and is perpendicular to each of the lines $\frac{x-8}{8} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y+29}{8} = \frac{z-5}{-5}$

To find: equation of line in Vector and Cartesian form

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is a vector parallel to the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

Here, $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$

Let the direction ratios of the line be $b_1:b_2:b_3$

Direction ratios of other two lines are 8 : -16 : 7 and 3 : 8 : -5

Since the other two line are perpendicular to the given line, we have

$8b_1 - 16b_2 + 7b_3 = 0$

$3b_1 + 8b_2 - 5b_3 = 0$

Solving,

$$\frac{b_1}{\begin{vmatrix} -16 & 7 \\ 8 & -5 \end{vmatrix}} = \frac{-b_2}{\begin{vmatrix} 8 & 7 \\ 3 & -5 \end{vmatrix}} = \frac{b_3}{\begin{vmatrix} 8 & -16 \\ 3 & 8 \end{vmatrix}}$$

$$\Rightarrow \frac{b_1}{24} = \frac{b_2}{61} = \frac{b_3}{112}$$

$$\vec{b} = 24\hat{i} + 61\hat{j} + 112\hat{k}$$

Therefore,

Vector form of the line is:

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(24\hat{i} + 61\hat{j} + 112\hat{k})$$

Cartesian form of the line is:

$$\frac{x-1}{24} = \frac{y-2}{61} = \frac{z+4}{112}$$

**12. Question**

Prove that the lines $\frac{x-4}{1} = \frac{y+3}{4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ intersect each other and find the point of their intersection.

**Answer**

Given: The equations of the two lines are

$$\frac{x-4}{1} = \frac{y+3}{4} = \frac{z+1}{7} \text{ and } \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$$

To Prove: The two lines intersect and to find their point of intersection.

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

Proof:

Let

$$\frac{x-4}{1} = \frac{y+3}{4} = \frac{z+1}{7} = \lambda_1$$

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda_2$$

So a point on the first line is $(\lambda_1 + 4, 4\lambda_1 - 3, 7\lambda_1 - 1)$

A point on the second line is $(2\lambda_2 + 1, -3\lambda_2 - 1, 8\lambda_2 - 10)$

If they intersect they should have a common point.

$\lambda_1 + 4 = 2\lambda_2 + 1 \Rightarrow \lambda_1 - 2\lambda_2 = -3$ … (1)

$4\lambda_1 - 3 = -3\lambda_2 - 1 \Rightarrow 4\lambda_1 + 3\lambda_2 = 2$ … (2)

Solving (1) and (2),

$11\lambda_2 = 14$

$\lambda_2 = \frac{14}{11}$

Therefore, $\lambda_1 = \frac{-5}{11}$

Substituting for the z coordinate, we get

$7\lambda_1 - 1 = \frac{-46}{11}$ and $8\lambda_2 - 10 = \frac{2}{11}$

So, the lines do not intersect.

**13. Question**

Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect each other. Also, find the point of their intersection.

**Answer**

Given: The equations of the two lines are

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \text{ and } \frac{x-4}{5} = \frac{y-1}{2} = z$$

To Prove: The two lines intersect and to find their point of intersection.

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

Proof:

Let

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda_1$$

$$\frac{x-4}{5}=\frac{y-1}{2}=z=\lambda_2$$

So a point on the first line is $(2\lambda_1 + 1, 3\lambda_1 + 2, 4\lambda_1 + 3)$

A point on the second line is $(5\lambda_2 + 4, 2\lambda_2 + 1, \lambda_2)$

If they intersect they should have a common point.

$2\lambda_1 + 1 = 5\lambda_2 + 4 \Rightarrow 2\lambda_1 - 5\lambda_2 = 3$ … (1)

$3\lambda_1 + 2 = 2\lambda_2 + 1 \Rightarrow 3\lambda_1 - 2\lambda_2 = -1$ … (2)

Solving (1) and (2),

$-11\lambda_2 = 11$

$\lambda_2 = -1$

Therefore, $\lambda_1 = -1$

Substituting for the z coordinate, we get

$4\lambda_1 + 3 = -1$ and $\lambda_2 = -1$

So, the lines intersect and their point of intersection is (-1, -1, -1)

**14. Question**

Show that the lines $\frac{x-1}{2}=\frac{y+1}{3}=z$ and $\frac{x+1}{5}=\frac{y-2}{1}, z=2$ do not intersect each other.

**Answer**

Given: The equations of the two lines are

$$\frac{x-1}{2}=\frac{y+1}{3}=z \text{ and } \frac{x+1}{5}=\frac{y-2}{1}, z=2$$

To Prove: the lines do not intersect each other.

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

Proof:

Let

$$\frac{x-1}{2}=\frac{y+1}{3}=z=\lambda_1$$

$$\frac{x+1}{5} = \frac{y-2}{1} = \lambda_2, z = 2$$

So a point on the first line is $(2\lambda_1 + 1, 3\lambda_1 - 1, \lambda_1)$

A point on the second line is $(5\lambda_2 - 1, \lambda_2 + 1, 2)$

If they intersect they should have a common point.

$2\lambda_1 + 1 = 5\lambda_2 - 1 \Rightarrow 2\lambda_1 - 5\lambda_2 = -2$ … (1)

$3\lambda_1 - 1 = \lambda_2 + 1 \Rightarrow 3\lambda_1 - \lambda_2 = 2$ … (2)

Solving (1) and (2),

$-13\lambda_2 = -10$

$$\lambda_2 = \frac{10}{13}$$

Therefore, $\lambda_1 = \frac{33}{65}$

Substituting for the z coordinate, we get

$\lambda_1 = \frac{33}{65}$ and z = 2

So, the lines do not intersect.

**15. Question**

Find the coordinates of the foot of the perpendicular drawn from the point (1, 2, 3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also, find the length of the perpendicular from the given point to the line.

**Answer**

Given: Equation of line is $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$.

To find: coordinates of foot of the perpendicular from (1, 2, 3) to the line. And find the length of the perpendicular.

Formula Used:

1. Equation of a line is

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

2. Distance between two points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Explanation:

Let

$$\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}=\lambda$$

So the foot of the perpendicular is (3λ + 6, 2λ + 7, -2λ + 7)

Direction ratio of the line is 3 : 2 : -2

Direction ratio of the perpendicular is

⇒ (3λ + 6 - 1) : (2λ + 7 - 2) : (-2λ + 7 - 3)

⇒ (3λ + 5) : (2λ + 5) : (-2λ + 4)

Since this is perpendicular to the line,

3(3λ + 5) + 2(2λ + 5) − 2(-2λ + 4) = 0

⇒ 9λ + 15 + 4λ + 10 + 4λ − 8 = 0

⇒ 17λ = -17

⇒ λ = -1

So the foot of the perpendicular is (3, 5, 9)

Distance $=\sqrt{(3-1)^2+(5-2)^2+(9-3)^2}$

$=\sqrt{4+9+36}$

= 7 units

Therefore, the foot of the perpendicular is (3, 5, 9) and length of perpendicular is 7 units.

**16. Question**

Find the length and the foot of the perpendicular drawn from the point (2, -1, 5) to the line $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$.

**Answer**

Given: Equation of line is $\frac{x-11}{10}=\frac{y+2}{-4}=\frac{z+8}{-11}$.

To find: coordinates of foot of the perpendicular from (2, -1, 5) to the line. And find the length of the perpendicular.

Formula Used:

1. Equation of a line is

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $b_1 : b_2 : b_3$ is the direction ratios of the line.

2. Distance between two points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Explanation:

Let

$$\frac{x - 11}{10} = \frac{y + 2}{-4} = \frac{z + 8}{-11} = \lambda$$

So the foot of the perpendicular is (10λ + 11, -4λ - 2, -11λ - 8)

Direction ratio of the line is 10 : -4 : -11

Direction ratio of the perpendicular is

⇒ (10λ + 11 - 2) : (-4λ - 2 + 1) : (-11λ - 8 - 5)

⇒ (10λ + 9) : (-4λ - 1) : (-11λ - 13)

Since this is perpendicular to the line,

10(10λ + 9) - 4(-4λ - 1) - 11(-11λ - 13) = 0

⇒ 100λ + 90 + 16λ + 4 + 121λ + 143 = 0

⇒ 237λ = -237

⇒ λ = -1

So the foot of the perpendicular is (1, 2, 3)

Distance $= \sqrt{(1-2)^2 + (2+1)^2 + (3-5)^2}$

$= \sqrt{1 + 9 + 4}$

= √14 units

Therefore, the foot of the perpendicular is (1, 2, 3) and length of perpendicular is √14 units.

**17. Question**

Find the vector and Cartesian equations of the line passing through the points A(3, 4, -6) and B(5, -2, 7).

**Answer**

Given: line passes through the points (3, 4, -6) and (5, -2, 7)

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = 3\hat{i} + 4\hat{j} - 6\hat{k}$

The direction ratios of the line are (3 - 5) : (4 + 2) : (-6 - 7)

⇒ -2 : 6 : -13

⇒ 2 : -6 : 13

So, $\vec{b} = 2\hat{i} - 6\hat{j} + 13\hat{k}$

Therefore,

Vector form:

$\vec{r} = 3\hat{i} + 4\hat{j} - 6\hat{k} + \lambda(2\hat{i} - 6\hat{j} + 13\hat{k})$

Cartesian form:

$$\frac{x-3}{2} = \frac{y-4}{-6} = \frac{z+6}{13}$$

**18. Question**

Find the vector and Cartesian equations of the line passing through the points A(2, -3, 0) and B(-2, 4, 3).

**Answer**

Given: line passes through the points (2, -3, 0) and (-2, 4, 3)

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = 2\hat{i} - 3\hat{j}$

The direction ratios of the line are (2 + 2) : (-3 - 4) : (0 - 3)

⇒ 4 : -7 : -3

⇒ -4 : 7 : 3

So, $\vec{b} = -4\hat{i} + 7\hat{j} + 3\hat{k}$

Therefore,

Vector form:

$$\vec{r} = 2\hat{i} - 3\hat{j} + \lambda(-4\hat{i} + 7\hat{j} + 3\hat{k})$$

Cartesian form:

$$\frac{x-2}{-4} = \frac{y+3}{7} = \frac{z}{3}$$

**19. Question**

Find the vector and Cartesian equations of the line joining the points whose position vectors are $(\hat{i} - 2\hat{j} + \hat{k})$ and $(\hat{i} + 3\hat{j} - 2\hat{k})$.

**Answer**

Given: line passes through the points whose position vectors are $(\hat{i} - 2\hat{j} + \hat{k})$ and $(\hat{i} + 3\hat{j} - 2\hat{k})$.

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$

The direction ratios of the line are (1 - 1) : (-2 - 3) : (1 + 2)

⇒ 0 : -5 : 3

⇒ 0 : 5 : -3

So, $\vec{b} = -5\hat{j} + 3\hat{k}$

Therefore,

Vector form:

$$\vec{r} = \hat{i} - 2\hat{j} + \hat{k} + \lambda(5\hat{j} - 3\hat{k})$$

Cartesian form:

$$\frac{x-1}{0}=\frac{y+2}{5}=\frac{z-1}{-3}$$

**20. Question**

Find the vector equation of a line passing through the point A(3, -2, 1) and parallel to the line joining the points B(-2, 4, 2) and C(2, 3, 3). Also, find the Cartesian equations of the line.

**Answer**

Given: line passes through the point (3, -2, 1) and is parallel to the line joining points B(-2, 4, 2) and C(2, 3, 3).

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r}=\vec{a}+\lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1}=\frac{y-y_1}{b_2}=\frac{z-z_1}{b_3}=\lambda$

where $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ is a point on the line and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a}=3\hat{i}-2\hat{j}+\hat{k}$

The direction ratios of the line are (-2 - 2) : (4 - 3) : (2 - 3)

⇒ -4 : 1 : -1

⇒ 4 : -1 : 1

So, $\vec{b}=4\hat{i}-\hat{j}+\hat{k}$

Therefore,

Vector form:

$$\vec{r}=3\hat{i}-2\hat{j}+\hat{k}+\lambda(4\hat{i}-\hat{j}+\hat{k})$$

Cartesian form:

$$\frac{x-3}{4}=\frac{y+2}{-1}=\frac{z-1}{1}$$

**21. Question**

Find the vector equation of a line passing through the point having the position vector $(\hat{i}+2\hat{j}-3\hat{k})$ and parallel to the line joining the points with position vectors $(\hat{i}-\hat{j}+5\hat{k})$ and $(2\hat{i}+3\hat{j}-4\hat{k})$.

Also, find the Cartesian equivalents of this equation.

**Answer**

Given: line passes through the point with position vector $\hat{i} + 2\hat{j} - 3\hat{k}$ and parallel to the line joining the points with position vectors $\hat{i} - \hat{j} + 5\hat{k}$ and $2\hat{i} + 3\hat{j} - 4\hat{k}$.

To find: equation of line in vector and Cartesian forms

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

Explanation:

Here, $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$

The direction ratios of the line are (1 - 2) : (-1 - 3) : (5 + 4)

⇒ -1 : -4 : 9

⇒ 1 : 4 : -9

So, $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$

Therefore,

Vector form:

$$\vec{r} = \hat{i} + 2\hat{j} - 3\hat{k} + \lambda(\hat{i} + 4\hat{j} - 9\hat{k})$$

Cartesian form:

$$\frac{x-1}{1} = \frac{y-2}{4} = \frac{z+3}{-9}$$

**22. Question**

Find the coordinates of the foot of the perpendicular drawn from the point A(1, 2, 1) to the line joining the points B(1, 4, 6) and C(5, 4, 4).

**Answer**

Given: perpendicular drawn from point A (1, 2, 1) to line joining points B (1, 4, 6) and C (5, 4, 4)

To find: foot of perpendicular

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

B (1, 4, 6) is a point on the line.

Therefore, $\vec{a} = \hat{i} + 4\hat{j} + 6\hat{k}$

Also direction ratios of the line are (1 - 5) : (4 – 4) : (6 – 4)

⇒ -4 : 0 : 2

⇒ -2 : 0 : 1

So, equation of the line in Cartesian form is

$$\frac{x-1}{-2} = \frac{y-4}{0} = \frac{z-6}{1} = \lambda$$

Any point on the line will be of the form (-2λ + 1, 4, λ + 6)

So the foot of the perpendicular is of the form (-2λ + 1, 4, λ + 6)

The direction ratios of the perpendicular is

(-2λ + 1 – 1) : (4 - 2) : (λ + 6 - 1)

⇒ (-2λ) : 2 : (λ + 5)

From the direction ratio of the line and the direction ratio of its perpendicular, we have

-2(-2λ) + 0 + λ + 5 = 0

⇒ 4λ + λ = -5

⇒ λ = -1

So, the foot of the perpendicular is (3, 4, 5)

**23. Question**

Find the coordinates of the foot of the perpendicular drawn from the point A(1, 8, 4) to the line joining the points B(0, -1, 3) and C(2, -3, -1).

**Answer**

Given: perpendicular drawn from point A (1, 8, 4) to line joining points B (0, -1, 3) and C (2, -3, -1)

To find: foot of perpendicular

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Explanation:

B (0, -1, 3) is a point on the line.

Therefore, $\vec{a} = -\hat{j} + 3\hat{k}$

Also direction ratios of the line are (0 - 2) : (-1 + 3) : (3 + 1)

⇒ -2 : 2 : 4

⇒ -1 : 1 : 2

So, equation of the line in Cartesian form is

$$\frac{x}{-1} = \frac{y+1}{1} = \frac{z-3}{2} = \lambda$$

Any point on the line will be of the form (-λ, λ - 1, 2λ + 3)

So the foot of the perpendicular is of the form (-λ, λ - 1, 2λ + 3)

The direction ratios of the perpendicular is

(-λ - 1) : (λ − 1 - 8) : (2λ + 3 - 4)

⇒ (-λ - 1) : (λ − 9) : (2λ − 1)

From the direction ratio of the line and the direction ratio of its perpendicular, we have

-1(-λ - 1) + λ − 9 + 2(2λ − 1) = 0

⇒ λ + 1 + λ − 9 + 4λ − 2 = 0

⇒ 6λ = 10

$$\Rightarrow \lambda = \frac{5}{3}$$

So, the foot of the perpendicular is $\left(\frac{-5}{3}, \frac{2}{3}, \frac{19}{3}\right)$

**24. Question**

Find the image of the point (0, 2, 3) in the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

**Answer**

Given: Equation of line is $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

To find: image of point (0, 2, 3)

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Mid-point of line segment joining $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

Explanation:

Let

$$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$$

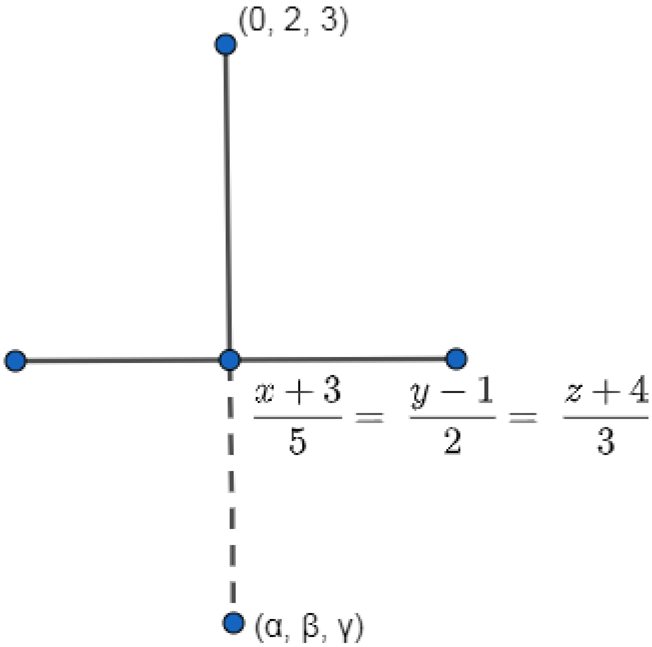
So the foot of the perpendicular is $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

The direction ratios of the perpendicular is

$(5\lambda - 3 - 0) : (2\lambda + 1 - 2) : (3\lambda - 4 - 3)$

$\Rightarrow (5\lambda - 3) : (2\lambda - 1) : (3\lambda - 7)$

Direction ratio of the line is 5 : 2 : 3




From the direction ratio of the line and the direction ratio of its perpendicular, we have

$5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$

$\Rightarrow 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$

$\Rightarrow 38\lambda = 38$

$\Rightarrow \lambda = 1$

So, the foot of the perpendicular is (2, 3, -1)

The foot of the perpendicular is the mid-point of the line joining (0, 2, 3) and (α, β, γ)

So, we have

$$\frac{\alpha+0}{2}=2\Rightarrow\alpha=4$$

$$\frac{\beta+2}{2}=3\Rightarrow\beta=4$$

$$\frac{\gamma+3}{2}=-1\Rightarrow\gamma=\ -5$$

So, the image is (4, 4, -5)

**25. Question**

Find the image of the point (5, 9, 3) in the line $\frac{x-1}{2}=\frac{y=2}{3}=\frac{z-3}{4}$.

**Answer**

Given: Equation of line is $\frac{x-1}{2} = \frac{y=2}{3} = \frac{z-3}{4}$.

To find: image of point (5, 9, 3)

Formula Used: Equation of a line is

Vector form: $\vec{r} = \vec{a} + \lambda\vec{b}$

Cartesian form: $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} = \lambda$

where $\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ is a point on the line and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ with $b_1 : b_2 : b_3$ being the direction ratios of the line.

If 2 lines of direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then $a_1b_1+a_2b_2+a_3b_3 = 0$

Mid-point of line segment joining $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

Explanation:

Let

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

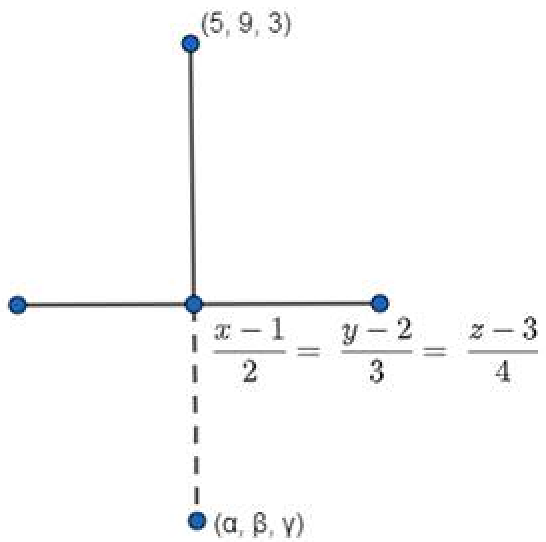
So the foot of the perpendicular is (2λ + 1, 3λ + 2, 4λ + 3)

The direction ratios of the perpendicular is

(2λ + 1 - 5) : (3λ + 2 - 9) : (4λ + 3 - 3)

⇒ (2λ – 4) : (3λ – 7) : (4λ)

Direction ratio of the line is 2 : 3 : 4



From the direction ratio of the line and the direction ratio of its perpendicular, we have

$2(2\lambda - 4) + 3(3\lambda - 7) + 4(4\lambda) = 0$

$\Rightarrow 4\lambda - 8 + 9\lambda - 21 + 16\lambda = 0$

$\Rightarrow 29\lambda = 29$

$\Rightarrow \lambda = 1$

So, the foot of the perpendicular is (3, 5, 7)

The foot of the perpendicular is the mid-point of the line joining (5, 9, 3) and (α, β, γ)

So, we have

$$\frac{\alpha + 5}{2} = 3 \Rightarrow \alpha = 1$$

$$\frac{\beta + 9}{2} = 5 \Rightarrow \beta = 1$$

$$\frac{\gamma + 3}{2} = 7 \Rightarrow \gamma = 11$$

So, the image is (1, 1, 11)

**26. Question**

Find the image of the point (2, -1, 5) in the line

$$\vec{r} = \left(11\hat{i} - 2\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 11\hat{k}\right)$$

**Answer**

Given: Point (2, -1, 5)

Equation of line $= \left(11\,\hat{i} - 2\,\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\,\hat{j} - 11\hat{k}\right)$

The equation of line can be re-arranged as $\frac{x-11}{10} = \frac{x+2}{-4} = \frac{x+8}{-11} = r$

The general point on this line is

(10r + 11, -4r – 2, -11r – 8)

Let N be the foot of the perpendicular drawn from the point P(2, 1, -5) on the given line.

Then, this point is N(10r + 11, -4r – 2, -11r – 8) for some fixed value of r.

D.r.'s of PN are (10r + 9, -4r - 3, -11r - 3)

D.r.'s of the given line is 10, -4, -11.

Since, PN is perpendicular to the given line, we have,

10(10r + 9) – 4(-4r – 3) – 11(-11r – 3) = 0

100r + 90 + 16r + 12 + 121r + 33 = 0

237r = 135

$r = \frac{135}{237}$

Then, the image of the point is

$$\frac{\alpha - 11}{-11} = 0, \frac{\beta + 2}{7} = 1, \frac{\gamma + 8}{9} = 1$$

Therefore, the image is (0, 5, 1).

## Exercise 27B

**1. Question**

Show that the points A(2, 1, 3), B(5, 0, 5) and C(-4, 3, -1) are collinear.

**Answer**

**Given -**

A = (2,1,3)

B = (5,0,5)

C = (-4,3,-1)

**To prove –** A, B and C are collinear

**Formula to be used –** If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((5-2),(0-1),(5-3))

=(3,-1,-2)

Similarly, the direction ratios of the line BC can be given by

((-4-5),(3-0),(-1-5))

=(-9,3,-6)

**Tip –** If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(3,-1,-2)

=(-1/3)X(-9,3,-6)

=(-1/3)Xd.r. of BC

Hence, **A, B and C are collinear**

**2. Question**

Show that the points A(2, 3, -4), B(1, -2, 3) and C(3, 8, -11) are collinear.

**Answer**

**Given -**

A = (2,3,-4)

B = (1,-2,3)

C = (3,8,-11)

**To prove –** A, B and C are collinear

**Formula to be used –** If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((1-2),(-2-3),(3+4))

=(-1,-5,7)

Similarly, the direction ratios of the line BC can be given by

((3-1),(8+2),(-11-3))

=(2,10,-14)

**Tip –** If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(-1,-5,7)

=(-1/2)X(2,10,-14)

=(-1/2)Xd.r. of BC

Hence, **A, B and C are collinear**

**3. Question**

Find the value of λ for which the points A(2, 5, 1), B(1, 2, -1) and C(3, λ, 3) are collinear.

**Answer**

**Given -**

A = (2,5,1)

B = (1,2,-1)

C = (3,λ,3)

**To find –** The value of λ so that A, B and C are collinear

**Formula to be used –** If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((1-2),(2-5),(-1-1))

=(-1,-3,-2)

Similarly, the direction ratios of the line BC can be given by

((3-1),(λ-2),(3+1))

=(2,λ-2,4)

**Tip –** If it is shown that direction ratios of AB=ɑ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(-1,-3,-2)

=(-1/2)X(2,λ-2,4)

=(-1/2)Xd.r. of BC

Since, A, B and C are collinear,

$\therefore -\frac{1}{2}(\lambda - 2) = -3$

⇨ $\lambda - 2 = 6$

⇨ $\boldsymbol{\lambda = 8}$

**4. Question**

Find the values of λ and μ so that the points A(3, 2, -4), B(9, 8, -10) and C(λ, μ -6) are collinear.

**Answer**

**Given -**

A = (3,2,-4)

B = (9,8,-10)

C = (λ,μ,-6)

**To find –** The value of λ and μ so that A, B and C are collinear

**Formula to be used –** If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((9-3),(8-2),(-10+4))

=(6,6,-6)

Similarly, the direction ratios of the line BC can be given by

((λ-9),(μ-8),(-6+10))

=(λ-9,μ-8,4)

**Tip –** If it is shown that direction ratios of AB=ɑ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(6,6,-6)

=(-6/4)X(-4,-4,4)

=(-3/2)Xd.r. of BC

Since, A, B and C are collinear,

$\therefore -\frac{3}{2}(\lambda - 9) = 6$

$\Rightarrow \lambda - 9 = -4$

$\Rightarrow \boldsymbol{\lambda = 5}$

And,

$\therefore -\frac{3}{2}(\mu - 8) = 6$

$\Rightarrow \mu - 8 = -4$

$\Rightarrow \boldsymbol{\lambda = 4}$

**5. Question**

Find the values of λ and μ so that the points A(-1, 4, -2), B(λ, μ 1) and C(0, 2, -1) are collinear.

**Answer**

**Given -**

A = (-1,4,-2)

B = (λ,μ,1)

C = (0,2,-1)

**To find –** The value of λ and μ so that A, B and C are collinear

**Formula to be used –** If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((λ+1),(μ-4),(1+2))

=(λ+1,μ-4,3)

Similarly, the direction ratios of the line BC can be given by

((0-λ),(2-μ),(-1-1))

=(-λ,2-μ,-2)

**Tip –** If it is shown that direction ratios of AB=α times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(λ+1,μ-4,3)

Say, α be an arbitrary constant such that d.r. of AB = α X d.r. of BC

So, 3 = α X (-2)

i.e. α = -3/2

Since, A, B and C are collinear,

$\therefore -\frac{3}{2}(-\lambda) = \lambda + 1$

⇨ $3\lambda = 2\lambda + 2$

⇨ $\boldsymbol{\lambda = 2}$

And,

$\therefore -\frac{3}{2}(2 - \mu) = \mu - 4$

⇨ $-6 + 3\mu = 2\mu - 8$

⇨ $\boldsymbol{\mu = -2}$

**6. Question**

The position vectors of three points A, B and C are $\left(-4\hat{i}+2\hat{j}-3\hat{k}\right),\left(\hat{i}+3\hat{j}-2\hat{k}\right)$ and $\left(-9\hat{i}+\hat{j}-4\hat{k}\right)$ respectively. show that the points A, B and C are collinear.

**Answer**

**Given -**

$\vec{A} = -4\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{B} = \hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{C} = -9\hat{i} + \hat{j} - 4\hat{k}$

It can thus be written as:

A = (-4,2,-3)

B = (1,3,-2)

C = (-9,1,-4)

**To prove –** A, B and C are collinear

**Formula to be used –** If P = (a,b,c) and Q = (a′,b′,c′),then the direction ratios of the line PQ is given by ((a′-a),(b′-b),(c′-c))

The direction ratios of the line AB can be given by

((1+4),(3-2),(-2+3))

=(5,1,1)

Similarly, the direction ratios of the line BC can be given by

((-9-1),(1-3),(-4+2))

=(-10,-2,-2)

**Tip –** If it is shown that direction ratios of AB=λ times that of BC , where λ is any arbitrary constant, then the condition is sufficient to conclude that points A, B and C will be collinear.

So, d.r. of AB

=(5,1,1)

=(-1/2)X(-10,-2,-2)

=(-1/2)Xd.r. of BC

Hence, **A, B and C are collinear**

## Exercise 27C

**1. Question**

Find the angle between each of the following pairs of lines:

$\vec{r} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j} - 5\hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

**Answer**

**Given –** $\overrightarrow{L_1} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$

& $\overrightarrow{L_2} = \left(2\hat{i} - \hat{j} - 5\hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$

**To find –** Angle between the two pair of lines

Direction ratios of $L_1$ = (1,-1,-2)

Direction ratios of $L_2$ = (3,-5,-4)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$=\cos^{-1}\left(\frac{1\times 3+(-1)\times(-5)+(-2)\times(-4)}{\sqrt{1^2+1^2+2^2}\sqrt{3^2+5^2+4^2}}\right)$$

$$=\cos^{-1}\left(\frac{3+5+8}{\sqrt{6}\sqrt{50}}\right)$$

$$=\cos^{-1}\left(\frac{16}{5\sqrt{6}\sqrt{2}}\right)$$

$$=\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$$

**2. Question**

Find the angle between each of the following pairs of lines:

$\vec{r}=\left(3\hat{i}-4\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+3\hat{k}\right)$ and $\vec{r}=5\hat{i}+\mu\left(-\hat{i}+\hat{j}+\hat{k}\right)$

**Answer**

**Given –** $\overrightarrow{L_1}=\left(3\hat{i}-4\hat{j}+2\hat{k}\right)+\lambda\left(\hat{i}+3\hat{k}\right)$

& $\overrightarrow{L_2}=\left(5\hat{i}\right)+\mu\left(-\hat{i}+\hat{j}+\hat{k}\right)$

**To find –** Angle between the two pair of lines

Direction ratios of $L_1$ = (1,0,3)

Direction ratios of $L_2$ = (-1,1,1)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$=\cos^{-1}\left(\frac{1\times(-1)+0\times 1+3\times 1}{\sqrt{1^2+0^2+3^2}\sqrt{1^2+1^2+1^2}}\right)$$

$$=\cos^{-1}\left(\frac{-1+3}{\sqrt{10}\sqrt{3}}\right)$$

$= \cos^{-1}\left(\frac{2}{\sqrt{30}}\right)$

$= \cos^{-1}\left(\frac{\sqrt{30}}{15}\right)$

**3. Question**

Find the angle between each of the following pairs of lines:

$\vec{r} = (\hat{i} - 2\hat{j}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = 3\hat{k} + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$

**Answer**

**Given –** $\overrightarrow{L_1} = (\hat{i} - 2\hat{j}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$

& $\overrightarrow{L_2} = (3\hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$

**To find –** Angle between the two pair of lines

Direction ratios of $L_1$ = (2,-2,1)

Direction ratios of $L_2$ = (1,2,-2)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2} \times \sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the lines

$= \cos^{-1}\left(\frac{2 \times 1 + (-2) \times 2 + 1 \times (-2)}{\sqrt{2^2 + 2^2 + 1^2}\sqrt{1^2 + 2^2 + 2^2}}\right)$

$= \cos^{-1}\left(\frac{2 - 4 - 2}{3 \times 3}\right)$

$= \cos^{-1}\left(-\frac{4}{9}\right)$

**4. Question**

Find the angle between each of the following pairs of lines:

$\frac{x - 1}{1} = \frac{y - 4}{1} = \frac{z - 5}{2}$ and $\frac{x + 3}{3} = \frac{y - 2}{5} = \frac{z + 5}{4}$

**Answer**

**Given –** $\overrightarrow{L_1} = \frac{x-1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

& $\overrightarrow{L_2} = \frac{x+3}{3} = \frac{y-2}{5} = \frac{z+5}{4}$

**To find –** Angle between the two pair of lines

Direction ratios of $L_1$ = (1,1,2)

Direction ratios of $L_2$ = (3,5,4)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times 3+1\times 5+2\times 4}{\sqrt{1^2+1^2+2^2}\sqrt{3^2+5^2+4^2}}\right)$$

$$= \cos^{-1}\left(\frac{3+5+8}{\sqrt{6}\times\sqrt{50}}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)}$$

**5. Question**

Find the angle between each of the following pairs of lines:

$\frac{x-4}{4} = \frac{y+1}{4} = \frac{z-6}{5}$ and $\frac{x-5}{1} = \frac{2y+5}{-2} = \frac{z-3}{1}$

**Answer**

**Given –** $\overrightarrow{L_1} = \frac{x-4}{4} = \frac{y+1}{3} = \frac{z-6}{5}$

& $\overrightarrow{L_2} = \frac{x-5}{1} = \frac{y+5/2}{-1} = \frac{z-3}{1}$

**To find –** Angle between the two pair of lines

Direction ratios of $L_1$ = (4,3,5)

Direction ratios of $L_2$ = (1,-1,1)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$= \cos^{-1}\left(\frac{4\times 1+3\times(-1)+5\times 1}{\sqrt{4^2+3^2+5^2}\sqrt{1^2+1^2+1^2}}\right)$

$= \cos^{-1}\left(\frac{4-3+5}{5\sqrt{2}\times\sqrt{3}}\right)$

$= \cos^{-1}\left(\frac{6}{5\sqrt{6}}\right)$

$= \cos^{-1}\left(\frac{2\sqrt{6}}{15}\right)$

**6. Question**

Find the angle between each of the following pairs of lines:

$\frac{3-x}{-2}=\frac{y+5}{1}=\frac{1-z}{3}$ and $\frac{x}{3}=\frac{1-y}{-2}=\frac{z+2}{-1}$

**Answer**

**Given –** $\overrightarrow{L_1} = \frac{x-3}{2}=\frac{y+5}{1}=\frac{z-1}{-3}$

& $\overrightarrow{L_2} = \frac{x}{3}=\frac{y-1}{2}=\frac{z+2}{-1}$

**To find –** Angle between the two pair of lines

Direction ratios of $L_1$ = (2,1,-3)

Direction ratios of $L_2$ = (3,2,-1)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$= \cos^{-1}\left(\frac{2\times 3+1\times 2+(-3)\times(-1)}{\sqrt{2^2+1^2+3^2}\sqrt{3^2+2^2+1^2}}\right)$

$= \cos^{-1}\left(\frac{6+2+3}{\sqrt{14}\times\sqrt{14}}\right)$

$= \cos^{-1}\left(\frac{11}{14}\right)$

**7. Question**

Find the angle between each of the following pairs of lines:

$$\frac{x}{1}=\frac{z}{-1}, y=0 \text{ and } \frac{x}{3}=\frac{y}{4}=\frac{z}{5}$$

**Answer**

**Given –** $\overrightarrow{L_1} = \frac{x}{1}=\frac{y}{0}=\frac{z}{-1}$

& $\overrightarrow{L_2} = \frac{x}{3}=\frac{y}{4}=\frac{z}{5}$

**To find –** Angle between the two pair of lines

Direction ratios of $L_1$ = (1,0,-1)

Direction ratios of $L_2$ = (3,4,5)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times 3+0\times 4+(-1)\times 5}{\sqrt{1^2+0^2+1^2}\sqrt{3^2+4^2+5^2}}\right)$$

$$= \cos^{-1}\left(\frac{3-5}{5\sqrt{2}\times\sqrt{2}}\right)$$

$$= \cos^{-1}\left(\frac{1}{5}\right)$$

**8. Question**

Find the angle between each of the following pairs of lines:

$$\frac{5-x}{3}=\frac{y+3}{-2}, z=5 \text{ and } \frac{x-1}{1}=\frac{1-y}{3}=\frac{z-5}{2}$$

**Answer**

**Given –** $\overrightarrow{L_1} = \frac{x-5}{-3}=\frac{y+3}{-2}=\frac{z-5}{0}$

& $\overrightarrow{L_2} = \frac{x-1}{1}=\frac{y-1}{-3}=\frac{z-5}{2}$

**To find –** Angle between the two pair of lines

Direction ratios of $L_1$ = (-3,-2,0)

Direction ratios of $L_2$ = (1,-3,2)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{(-3)\times 1+(-2)\times(-3)+0\times 2}{\sqrt{3^2+2^2+0^2}\sqrt{1^2+3^2+2^2}}\right)$$

$$= \cos^{-1}\left(\frac{-3+6}{\sqrt{13}\times\sqrt{14}}\right)$$

$$= \cos^{-1}\left(\frac{3}{\sqrt{182}}\right)$$

**9. Question**

Show that the lines $\frac{x-3}{2}=\frac{y+1}{-3}=\frac{z-2}{4}$ and $\frac{x+2}{2}=\frac{y-4}{4}=\frac{z+5}{2}$ are perpendicular to each other.

**Answer**

**Given –** $\overrightarrow{L_1} = \frac{x-3}{2}=\frac{y+1}{-3}=\frac{z-2}{4}$

& $\overrightarrow{L_2} = \frac{x+2}{2}=\frac{y-4}{4}=\frac{z+5}{2}$

**To prove –** The lines are perpendicular to each other

Direction ratios of $L_1$ = (2,-3,4)

Direction ratios of $L_2$ = (2,4,2)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{2\times 2+(-3)\times 4+4\times 2}{\sqrt{2^2+3^2+4^2}\sqrt{2^2+4^2+2^2}}\right)$$

$$= \cos^{-1}\left(\frac{4-12+8}{\sqrt{29}\times\sqrt{24}}\right)$$

$$= \cos^{-1}(0)$$

$= \frac{\pi}{2}$

Hence, **the lines are perpendicular to each other.**

**10. Question**

If the lines $\frac{x-1}{-3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ and $\frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{6-z}{5}$ are perpendicular to each other then find the value of λ.

**Answer**

**Given –** $\overrightarrow{L_1} = \frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$

& $\overrightarrow{L_2} = \frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{z-6}{-5}$

**To find –** The value of λ

Direction ratios of $L_1$ = (-3,2λ,2)

Direction ratios of $L_2$ = (3λ,1,-5)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

Since the lines are perpendicular to each other,

The angle between the lines

⇨ $\cos^{-1}\left(\frac{(-3)\times 3\lambda+2\lambda\times 1+2\times(-5)}{\sqrt{3^2+(2\lambda)^2+2^2}\sqrt{(3\lambda)^2+1^2+5^2}}\right)=\frac{\pi}{2}$

⇨ $\cos^{-1}\left(\frac{-9\lambda+2\lambda-10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right)=\frac{\pi}{2}$

⇨ $\cos^{-1}\left(\frac{-7\lambda-10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right)=\frac{\pi}{2}$

⇨ $\left(\frac{-7\lambda-10}{\sqrt{13+4\lambda^2}\sqrt{9\lambda^2+26}}\right)=\cos\frac{\pi}{2}=0$

⇨ $-7\lambda-10=0$

⇨ $\lambda=-\frac{10}{7}$

**11. Question**

Show that the lines x = - y = 2z and x + 2 = 2y − 1 = - z + 1 are perpendicular to each other.

**HINT:** The given lines are $\frac{x}{2}=\frac{y}{-2}=\frac{z}{1}$ and $\frac{x+2}{1}=\frac{y-1/2}{1}=\frac{z-1}{-2}$.

**Answer**

**Given –** $\overrightarrow{L_1}=\frac{x}{2}=\frac{y}{-2}=\frac{z}{1}$

& $\overrightarrow{L_2}=\frac{x+2}{2}=\frac{y-1/2}{1}=\frac{z-1}{-2}$

**To prove –** The lines are perpendicular to each other

Direction ratios of $L_1$ = (2,-2,1)

Direction ratios of $L_2$ = (2,1,-2)

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$=\cos^{-1}\left(\frac{2\times 2+(-2)\times 1+1\times(-2)}{\sqrt{2^2+2^2+1^2}\sqrt{1^2+1^2+2^2}}\right)$$

$$=\cos^{-1}\left(\frac{4-2-2}{\sqrt{29}\times\sqrt{24}}\right)$$

$$=\cos^{-1}(0)$$

$$=\frac{\pi}{2}$$

Hence, **the lines are perpendicular to each other.**

**12. Question**

Find the angle between two lines whose direction ratios are

i. 2, 1, 2 and 4, 8, 1

ii. 5, -12, 13 and -3, 4, 5

iii. 1, 1, 2 and $(\sqrt{3}-1),(-\sqrt{3}-1),4$

iv. a, b, c and (b – c), (c – a), (a – b)

**Answer**

**(i): Given –** Direction ratios of $L_1$ = (2,1,2) & Direction ratios of $L_2$ = (4,8,1)

**To find –** Angle between the two pair of lines

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{2\times 4+1\times 8+2\times 1}{\sqrt{2^2+1^2+2^2}\sqrt{4^2+8^2+1^2}}\right)$$

$$= \cos^{-1}\left(\frac{8+8+2}{3\times 9}\right)$$

$$= \cos^{-1}\left(\frac{18}{27}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{2}{3}\right)}$$

**(ii): Given –** Direction ratios of $L_1$ = (5,-12,13) & Direction ratios of $L_2$ = (-3,4,5)

**To find –** Angle between the two pair of lines

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{5\times(-3)+(-12)\times 4+13\times 5}{\sqrt{5^2+12^2+13^2}\sqrt{3^2+4^2+5^2}}\right)$$

$$= \cos^{-1}\left(\frac{-15-48+65}{13\sqrt{2}\times 5\sqrt{2}}\right)$$

$$= \cos^{-1}\left(\frac{2}{130}\right)$$

$$= \mathbf{\cos^{-1}\left(\frac{1}{65}\right)}$$

**(iii) Given –** Direction ratios of $L_1$ = (1,1,2) & Direction ratios of $L_2$ = (√3-1,-√3-1,4)

**To find –** Angle between the two pair of lines

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{1\times(\sqrt{3}-1)+1\times(-\sqrt{3}-1)+2\times 4}{\sqrt{1^2+1^2+2^2}\sqrt{(\sqrt{3}-1)^2+(-\sqrt{3}-1)^2+4^2}}\right)$$

$$= \cos^{-1}\left(\frac{\sqrt{3}-1-\sqrt{3}-1+8}{\sqrt{6}\sqrt{24}}\right)$$

$$= \cos^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{3}$$

**(iv) Given –** Direction ratios of $L_1$ = (a,b,c) & Direction ratios of $L_2$ = ((b-c),(c-a),(a-b))

**To find –** Angle between the two pair of lines

**Tip –** If (a,b,c) be the direction ratios of the first line and (a′,b′,c′) be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{a\times(b-c)+b\times(c-a)+c\times(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right)$$

$$= \cos^{-1}\left(\frac{0}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}\right)$$

$$= \cos^{-1}(0)$$

$$= \frac{\pi}{2}$$

**13. Question**

If A(1, 2, 3), B(4, 5, 7), C(-4, 3, -6) and D(2, 9, 2) are four given points then find the angle between the lines AB and CD.

**Answer**

**Given -**

A = (1,2,3)

B = (4,5,7)

C = (-4,3,-6)

D = (2,9,2)

**Formula to be used –** If P = (a,b,c) and Q = (a',b',c'),then the direction ratios of the line PQ is given by ((a'-a),(b'-b),(c'-c))

The direction ratios of the line AB can be given by

((4-1),(5-2),(7-3))

=(3,3,4)

Similarly, the direction ratios of the line CD can be given by

((2+4),(9-3),(2+6))

=(6,6,8)

**To find –** Angle between the two pair of lines AB and CD

**Tip –** If (a,b,c) be the direction ratios of the first line and (a',b',c') be that of the second, then the angle between these pair of lines is given by $\cos^{-1}\left(\frac{a\times a'+b\times b'+c\times c'}{\sqrt{a^2+b^2+c^2}\times\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the lines

$$= \cos^{-1}\left(\frac{3\times 6+3\times 6+4\times 8}{\sqrt{3^2+3^2+4^2}\sqrt{6^2+6^2+8^2}}\right)$$

$$= \cos^{-1}\left(\frac{18+18+32}{\sqrt{34}\times 2\sqrt{34}}\right)$$

$$= \cos^{-1}\left(\frac{68}{2\times 34}\right)$$

$$= \cos^{-1} 1$$

$$= \mathbf{0}$$

## Exercise 27D

### 1. Question

Find the shortest distance between the given lines.

$$\vec{r} = (\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k}),$$

$$\vec{r} = (2\hat{i}+\hat{j}-\hat{k})+\mu(3\hat{i}-5\hat{j}+2\hat{k}).$$

**Answer**

**<u>Given equations</u> :**

$$\bar{r} = (\hat{i}+\hat{j})+\lambda(2\hat{i}-\hat{j}+\hat{k})$$

$\bar{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$

**<u>To Find</u> :** d

**<u>Formula</u> :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**<u>Answer</u> :**

For given lines,

$\bar{r} = (\hat{i} + \hat{j}) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$

$\bar{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$

Here,

$\overline{a_1} = \hat{i} + \hat{j}$

$\overline{b_1} = 2\hat{i} - \hat{j} + \hat{k}$

$\overline{a_2} = 2\hat{i} + \hat{j} - \hat{k}$

$\overline{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$= \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3)$

$\therefore \overline{b_1} \times \overline{b_2} = 3\hat{i} - \hat{j} - 7\hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{3^2 + (-1)^2 + (-7)^2}$

$= \sqrt{9 + 1 + 49}$

$= \sqrt{59}$

$\overline{a_2} - \overline{a_1} = (2-1)\hat{i} + (1-1)\hat{j} + (-1-0)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} + 0\hat{j} - \hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(3\hat{i} - \hat{j} - 7\hat{k}\right).\left(\hat{i} + 0\hat{j} - \hat{k}\right)$

$= (3 \times 1) + ((-1) \times 0) + ((-7) \times (-1))$

= 3 + 0 + 7

= 10

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{10}{\sqrt{59}}\right|$$

**2. Question**

Find the shortest distance between the given lines.

$\vec{r} = \left(-4\hat{i} + 4\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right),$

$\vec{r} = \left(-3\hat{i} - 8\hat{j} - 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 3\hat{k}\right)$

**Answer**

**Given equations :**

$$\bar{r} = \left(-4\hat{i} + 4\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right)$$

$$\bar{r} = \left(-3\hat{i} - 8\hat{j} - 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 3\hat{k}\right)$$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**Answer :**

For given lines,

$$\bar{r} = \left(-4\hat{i} + 4\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + \hat{j} - \hat{k}\right)$$

$$\bar{r} = \left(-3\hat{i} - 8\hat{j} - 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 3\hat{k}\right)$$

Here,

$\overline{a_1} = -4\hat{i} + 4\hat{j} + \hat{k}$

$\overline{b_1} = \hat{i} + \hat{j} - \hat{k}$

$\overline{a_2} = -3\hat{i} - 8\hat{j} - 3\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + 3\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 2 & 3 & 3 \end{vmatrix}$$

$= \hat{i}(3 + 3) - \hat{j}(3 + 2) + \hat{k}(3 - 2)$

$\therefore \overline{b_1} \times \overline{b_2} = 6\hat{i} - 5\hat{j} + \hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{6^2 + (-5)^2 + 1^2}$

$= \sqrt{36 + 25 + 1}$

$= \sqrt{62}$

$\overline{a_2} - \overline{a_1} = (-3 + 4)\hat{i} + (-8 - 4)\hat{j} + (-3 - 1)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} - 12\hat{j} - 4\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(6\hat{i} - 5\hat{j} + \hat{k}\right).\left(\hat{i} - 12\hat{j} - 4\hat{k}\right)$

$= (6 \times 1) + ((-5) \times (-12)) + (1 \times (-4))$

= 6 + 60 – 4

= 62

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{62}{\sqrt{62}}\right|$$

$d = \sqrt{62}\, units$

**3. Question**

Find the shortest distance between the given lines.

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k}),$$

$$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k}).$$

**Answer**

**Given equations :**

$\bar{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$

$\bar{r} = (4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}) + \mu(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

**Answer :**

For given lines,

$$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$$

$$\bar{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$$

Here,

$$\overline{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{b_1} = \hat{i} - 3\hat{j} + 2\hat{k}$$

$$\overline{a_2} = 4\hat{i} + 5\hat{j} + 6\hat{k}$$

$$\overline{b_2} = 2\hat{i} + 3\hat{j} + \hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \hat{i}(-3 - 6) - \hat{j}(1 - 4) + \hat{k}(3 + 6)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-9)^2 + 3^2 + 9^2}$$

$$= \sqrt{81 + 9 + 81}$$

$$= \sqrt{171}$$

$$\overline{a_2} - \overline{a_1} = (4 - 1)\hat{i} + (5 - 2)\hat{j} + (6 - 3)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = (-9\hat{i} + 3\hat{j} + 9\hat{k}).(3\hat{i} + 3\hat{j} + 3\hat{k})$$

$$= ((-9) \times 3) + (3 \times 3) + (9 \times 3)$$

= - 27 + 9 + 27

= 9

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{9}{\sqrt{171}}\right|$$

$$\therefore d = \frac{9}{\sqrt{19}.\sqrt{9}}$$

$$\therefore d = \frac{3}{\sqrt{19}}$$

$$\therefore d = \frac{3\sqrt{19}}{19}$$

**4. Question**

Find the shortest distance between the given lines.

$$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right),$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right).$$

**Answer**

**Given equations :**

$$\bar{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

**<u>Answer</u> :**

For given lines,

$\bar{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

Here,

$\overline{a_1} = \hat{i} + 2\hat{j} + \hat{k}$

$\overline{b_1} = \hat{i} - \hat{j} + \hat{k}$

$\overline{a_2} = 2\hat{i} - \hat{j} - \hat{k}$

$\overline{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$= \hat{i}(-2 - 1) - \hat{j}(2 - 2) + \hat{k}(1 + 2)$

$\therefore \overline{b_1} \times \overline{b_2} = -3\hat{i} + 0\hat{j} + 3\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-3)^2 + 0^2 + 3^2}$

$= \sqrt{9 + 0 + 9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$\overline{a_2} - \overline{a_1} = (2 - 1)\hat{i} + (-1 - 2)\hat{j} + (-1 - 1)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} - 3\hat{j} - 2\hat{k}$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(-3\hat{i} + 0\hat{j} + 3\hat{k}\right).\left(\hat{i} - 3\hat{j} - 2\hat{k}\right)$$

$$= ((-3) \times 1) + (0 \times (-3)) + (3 \times (-2))$$

= - 3 + 0 − 6

= - 9

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{-9}{3\sqrt{2}}\right|$$

$$\therefore d = \frac{3}{\sqrt{2}}$$

$$\therefore d = \frac{3\sqrt{2}}{2}$$

**5. Question**

Find the shortest distance between the given lines.

$$\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right),$$

$$\vec{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(-2\hat{i} + 3\hat{j} + 8\hat{k}\right).$$

**Answer**

**Given equations :**

$$\bar{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$\bar{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(-2\hat{i} + 3\hat{j} + 8\hat{k}\right)$$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \bar{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

**<u>Answer</u> :**

For given lines,

$$\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\bar{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$$

Here,

$$\bar{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\overline{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\overline{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\overline{b_2} = -2\hat{i} + 3\hat{j} + 8\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix}$$

$$= \hat{i}(24 - 18) - \hat{j}(16 + 12) + \hat{k}(6 - 6)$$

$$\therefore \overline{b_1} \times \overline{b_2} = 6\hat{i} - 28\hat{j} + 0\hat{k}$$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{6^2 + (-28)^2 + 0^2}$

$= \sqrt{36 + 784 + 9}$

$= \sqrt{820}$

$\overline{a_2} - \overline{a_1} = (3-1)\hat{\imath} + (3-2)\hat{\jmath} + (-5+4)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(6\hat{\imath} - 28\hat{\jmath} + 0\hat{k}\right).\left(2\hat{\imath} + \hat{\jmath} - \hat{k}\right)$

$= (6 \times 2) + ((-28) \times 1) + (0 \times (-1))$

= 12 – 28 + 0

= - 16

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{-16}{\sqrt{820}}\right|$$

$$d = \frac{16}{\sqrt{820}}\ units$$

**6. Question**

Find the shortest distance between the given lines.

$$\vec{r} = \left(6\hat{i} + 3\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 4\hat{k}\right),$$

$$\vec{r} = \left(-9\hat{i} + \hat{j} - 10\hat{k}\right) + \mu\left(4\hat{i} + \hat{j} + 6\hat{k}\right).$$

**Answer**

**<u>Given equations</u> :**

$\bar{r} = \left(6\hat{\imath} + 3\hat{k}\right) + \lambda\left(2\hat{\imath} - \hat{\jmath} + 4\hat{k}\right)$

$\bar{r} = \left(-9\hat{\imath} + \hat{\jmath} - 10\hat{k}\right) + \mu\left(4\hat{\imath} + \hat{\jmath} + 6\hat{k}\right)$

**<u>To Find</u> :** d

**<u>Formula</u> :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**<u>Answer</u> :**

For given lines,

$$\bar{r} = \left(6\hat{i} + 3\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 4\hat{k}\right)$$

$$\bar{r} = \left(-9\hat{i} + \hat{j} - 10\hat{k}\right) + \mu\left(4\hat{i} + \hat{j} + 6\hat{k}\right)$$

Here,

$$\overline{a_1} = 6\hat{i} + 3\hat{k}$$

$$\overline{b_1} = 2\hat{i} - \hat{j} + 4\hat{k}$$

$$\overline{a_2} = -9\hat{i} + \hat{j} - 10\hat{k}$$

$$\overline{b_2} = 4\hat{i} + \hat{j} + 6\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 4 \\ 4 & 1 & 6 \end{vmatrix}$$

$$= \hat{i}(-6-4) - \hat{j}(12-16) + \hat{k}(2+4)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -10\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-10)^2 + 4^2 + 6^2}$$

$$= \sqrt{100 + 16 + 36}$$

$$= \sqrt{152}$$

$$\overline{a_2} - \overline{a_1} = (-9-6)\hat{i} + (1-0)\hat{j} + (6-3)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = -15\hat{i} + \hat{j} + 3\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(-10\hat{i} + 4\hat{j} + 6\hat{k}\right).\left(-15\hat{i} + \hat{j} + 3\hat{k}\right)$$

$$= ((-10) \times (-15)) + (4 \times 1) + (6 \times 3)$$

= 150 + 4 + 18

= 172

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{172}{\sqrt{152}}\right|$$

$$\therefore d = \frac{172}{2\sqrt{38}}$$

$$\therefore d = \frac{86}{\sqrt{38}}$$

$$d = \frac{86}{\sqrt{38}}\ units$$

**7. Question**

Find the shortest distance between the given lines.

$$\vec{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k},$$

$$\vec{r} = (1+s)\hat{i} + (3s-7)\hat{j} + (2s-2)\hat{k}.$$

**Answer**

**Given equations :**

$$\bar{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k}$$

$$\bar{r} = (1+s)\hat{i} + (3s-7)\hat{j} + (2s-2)\hat{k}$$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**Answer :**

Given lines,

$$\bar{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k}$$

$\bar{r} = (1 + s)\hat{i} + (3s - 7)\hat{j} + (2s - 2)\hat{k}$

Above equations can be written as

$\bar{r} = \left(3\hat{i} + 4\hat{j} - 2\hat{k}\right) + t\left(-\hat{i} + 2\hat{j} + \hat{k}\right)$

$\bar{r} = \left(\hat{i} - 7\hat{j} - 2\hat{k}\right) + s\left(\hat{i} + 3\hat{j} + 2\hat{k}\right)$

Here,

$\overline{a_1} = 3\hat{i} + 4\hat{j} - 2\hat{k}$

$\overline{b_1} = -\hat{i} + 2\hat{j} + \hat{k}$

$\overline{a_2} = \hat{i} - 7\hat{j} - 2\hat{k}$

$\overline{b_2} = \hat{i} + 3\hat{j} + 2\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$= \hat{i}(4 - 3) - \hat{j}(-2 - 1) + \hat{k}(-3 - 2)$

$\therefore \overline{b_1} \times \overline{b_2} = \hat{i} + 3\hat{j} - 5\hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{1^2 + 3^2 + (-5)^2}$

$= \sqrt{1 + 9 + 25}$

$= \sqrt{35}$

$\overline{a_2} - \overline{a_1} = (1 - 3)\hat{i} + (-7 - 4)\hat{j} + (-2 + 2)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = -2\hat{i} - 11\hat{j} + 0\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(\hat{i} + 3\hat{j} - 5\hat{k}\right).\left(-2\hat{i} - 11\hat{j} + 0\hat{k}\right)$

$= (1 \times (-2)) + (3 \times (-11)) + ((-5) \times 0)$

= - 2 – 33 + 0

= - 35

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$\therefore d = \left|\frac{-35}{\sqrt{35}}\right|$

$\therefore d = \sqrt{35}$

$d = \sqrt{35}\ units$

**8. Question**

Find the shortest distance between the given lines.

$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k},$

$\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}.$

**Answer**

**<u>Given equations</u> :**

$\bar{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k}$

$\bar{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$

**<u>To Find</u> :** d

**<u>Formula</u> :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\bar{b_1}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b_2}$ is given by,

$$d = \left|\frac{(\bar{b_1} \times \bar{b_2}).(\bar{a_2} - \bar{a_1})}{|\bar{b_1} \times \bar{b_2}|}\right|$$

**<u>Answer</u> :**

Given lines,

$$\bar{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (\lambda + 1)\hat{k}$$

$$\bar{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$$

Above equations can be written as

$$\bar{r} = (-\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$$

$$\bar{r} = (\hat{i} - \hat{j} + 2\hat{k}) + s(-\hat{i} + 2\hat{j} + \hat{k})$$

Here,

$$\bar{a_1} = -\hat{i} + \hat{j} - \hat{k}$$

$$\bar{b_1} = \hat{i} + \hat{j} - \hat{k}$$

$$\bar{a_2} = \hat{i} - \hat{j} + 2\hat{k}$$

$$\bar{b_2} = -\hat{i} + 2\hat{j} + \hat{k}$$

Therefore,

$$\bar{b_1} \times \bar{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(1 + 2) - \hat{j}(1 - 1) + \hat{k}(2 + 1)$$

$$\therefore \bar{b_1} \times \bar{b_2} = 3\hat{i} - 0\hat{j} + 3\hat{k}$$

$$\therefore |\bar{b_1} \times \bar{b_2}| = \sqrt{3^2 + 0^2 + 3^2}$$

$$= \sqrt{9 + 0 + 9}$$

$$= \sqrt{18}$$

$$= 3\sqrt{2}$$

$$\bar{a_2} - \bar{a_1} = (1 + 1)\hat{i} + (-1 - 1)\hat{j} + (2 + 1)\hat{k}$$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} - 2\hat{j} + 3\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(3\hat{i} - 0\hat{j} + 3\hat{k}\right).\left(2\hat{i} - 2\hat{j} + 3\hat{k}\right)$

$= (3 \times 2) + (0 \times (-2)) + (3 \times 3)$

= 6 + 0 + 9

= 15

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{15}{3\sqrt{2}}\right|$$

$$\therefore d = \frac{5}{\sqrt{2}}$$

$$\therefore d = \frac{5\sqrt{2}}{2}$$

$$d = \frac{5\sqrt{2}}{2}\ units$$

**9. Question**

Compute the shortest distance between the lines $\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} - \hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} - \hat{j} - \hat{k}\right)$. Determine whether these lines intersect or not.

**Answer**

**Given equations :**

$\bar{r} = (\hat{i} - \hat{j}) + \lambda\left(2\hat{i} - \hat{k}\right)$

$\bar{r} = (2\hat{i} - \hat{j}) + \mu\left(\hat{i} - \hat{j} - \hat{k}\right)$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \bar{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**<u>Answer</u> :**

For given lines,

$$\bar{r} = (\hat{i} - \hat{j}) + \lambda\left(2\hat{i} - \hat{k}\right)$$

$$\bar{r} = (2\hat{i} - \hat{j}) + \mu\left(\hat{i} - \hat{j} - \hat{k}\right)$$

Here,

$$\bar{a_1} = \hat{i} - \hat{j}$$

$$\overline{b_1} = 2\hat{i} - \hat{k}$$

$$\overline{a_2} = 2\hat{i} - \hat{j}$$

$$\overline{b_2} = \hat{i} - \hat{j} - \hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -1 \\ 1 & -1 & -1 \end{vmatrix}$$

$$= \hat{i}(0 - 1) - \hat{j}(-2 + 1) + \hat{k}(-2 - 0)$$

$\therefore \overline{b_1} \times \overline{b_2} = -\hat{i} + \hat{j} - 2\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-1)^2 + 1^2 + (-2)^2}$

$= \sqrt{1 + 1 + 4}$

$= \sqrt{6}$

$\overline{a_2} - \overline{a_1} = (2 - 1)\hat{i} + (-1 + 1)\hat{j} + (0 - 0)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} + 0\hat{j} + 0\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (-\hat{i} + \hat{j} - 2\hat{k}).(\hat{i} + 0\hat{j} + 0\hat{k})$

$= ((-1) \times 1) + (1 \times 0) + ((-2) \times 0)$

= - 1 + 0 + 0

= - 1

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{-1}{\sqrt{6}}\right|$$

$$\therefore d = \frac{1}{\sqrt{6}}$$

$$\therefore d = \frac{\sqrt{6}}{6}$$

$$d = \frac{\sqrt{6}}{6}\ units$$

As d ≠ 0

Hence, the given lines do not intersect.

**10. Question**

Show that the lines $\vec{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda\ (2\hat{i} - 7\hat{j} + 5\hat{k})$, and $\vec{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu$ $(2\hat{i} + \hat{j} - 3\hat{k})$ do not intersect.

**Answer**

**Given equations :**

$\bar{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k})$

$\bar{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu(2\hat{i} + \hat{j} - 3\hat{k})$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

**Answer :**

For given lines,

$\bar{r} = (3\hat{i} - 15\hat{j} + 9\hat{k}) + \lambda(2\hat{i} - 7\hat{j} + 5\hat{k})$

$\bar{r} = (-\hat{i} + \hat{j} + 9\hat{k}) + \mu(2\hat{i} + \hat{j} - 3\hat{k})$

Here,

$\overline{a_1} = 3\hat{i} - 15\hat{j} + 9\hat{k}$

$\overline{b_1} = 2\hat{i} - 7\hat{j} + 5\hat{k}$

$\overline{a_2} = -\hat{i} + \hat{j} + 9\hat{k}$

$\overline{b_2} = 2\hat{i} + \hat{j} - 3\hat{k}$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$= \hat{i}(21 - 5) - \hat{j}(-6 - 10) + \hat{k}(2 + 14)$

$\therefore \overline{b_1} \times \overline{b_2} = 17\hat{i} + 16\hat{j} + 16\hat{k}$

$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{17^2 + 16^2 + 17^2}$

$= \sqrt{289 + 256 + 289}$

$= \sqrt{834}$

$\overline{a_2} - \overline{a_1} = (-1 - 3)\hat{i} + (1 + 15)\hat{j} + (9 - 9)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = -4\hat{i} + 16\hat{j} + 0\hat{k}$

Now,

$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(17\hat{i} + 16\hat{j} + 16\hat{k}\right).\left(-4\hat{i} + 16\hat{j} + 0\hat{k}\right)$

$= (17 \times (-4)) + (16 \times 16) + (16 \times 0)$

= - 68 + 256 + 0

= 188

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{188}{\sqrt{834}}\right|$$

$$\therefore d = \frac{188}{\sqrt{834}}\text{units}$$

As d ≠ 0

Hence, the given lines do not intersect.

**11. Question**

Show that the lines $\vec{r} = \left(2\hat{i} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu$ $\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ intersect.

Also, find their point of intersection.

**Answer**

**Given equations :**

$$\bar{r} = (2\hat{\imath} - 3\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$$

$$\bar{r} = (2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}) + \mu(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and $\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**Answer :**

For given lines,

$$\bar{r} = \left(2\hat{i} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

$$\bar{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

Here,

$$\overline{a_1} = 2\hat{i} - 3\hat{k}$$

$$\overline{b_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\overline{a_2} = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

$$\overline{b_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(12 - 9) - \hat{j}(4 - 6) + \hat{k}(3 - 4)$$

$$\therefore \overline{b_1} \times \overline{b_2} = 3\hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{3^2 + 2^2 + (-1)^2}$$

$$= \sqrt{9 + 4 + 1}$$

$$= \sqrt{14}$$

$$\overline{a_2} - \overline{a_1} = (2 - 2)\hat{i} + (6 - 0)\hat{j} + (3 + 3)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 0\hat{i} + 6\hat{j} + 6\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(3\hat{i} + 2\hat{j} - \hat{k}\right).\left(0\hat{i} + 6\hat{j} + 6\hat{k}\right)$$

$$= (3 \times 0) + (2 \times 6) + ((-1) \times 6)$$

= 0 + 12 – 6

= 6

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$\therefore d = \left|\frac{6}{\sqrt{14}}\right|$

$\therefore d = \frac{6}{\sqrt{14}}$ units

As d ≠ 0

Hence, the given lines do not intersect.

**12. Question**

Show that the lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\ (2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = (4\hat{i} + \hat{j}) + \mu\ (5\hat{i} + 2\hat{j} + \hat{k})$ intersect.

Also, find their point of intersection.

**Answer**

**Given equations :**

$\bar{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$

$\bar{r} = (4\hat{\imath} + \hat{\jmath}) + \mu(5\hat{\imath} + 2\hat{\jmath} + \hat{k})$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\bar{b_1}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b_2}$ is given by,

$$d = \left|\frac{\left(\bar{b_1} \times \bar{b_2}\right).\left(\bar{a_2} - \bar{a_1}\right)}{\left|\bar{b_1} \times \bar{b_2}\right|}\right|$$

**<u>Answer</u> :**

For given lines,

$$\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

$$\bar{r} = \left(4\hat{i} + \hat{j}\right) + \mu\left(5\hat{i} + 2\hat{j} + \hat{k}\right)$$

Here,

$$\bar{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\bar{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\bar{a_2} = 4\hat{i} + \hat{j}$$

$$\bar{b_2} = 5\hat{i} + 2\hat{j} + \hat{k}$$

Therefore,

$$\bar{b_1} \times \bar{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(3 - 8) - \hat{j}(2 - 20) + \hat{k}(4 - 15)$$

$$\therefore \bar{b_1} \times \bar{b_2} = -5\hat{i} + 18\hat{j} - 11\hat{k}$$

$$\therefore \left|\bar{b_1} \times \bar{b_2}\right| = \sqrt{(-5)^2 + 18^2 + (-11)^2}$$

$$= \sqrt{25 + 324 + 121}$$

$$= \sqrt{470}$$

$$\bar{a_2} - \bar{a_1} = (4 - 1)\hat{i} + (1 - 2)\hat{j} + (0 - 3)\hat{k}$$

$$\therefore \bar{a_2} - \bar{a_1} = 3\hat{i} - \hat{j} - 3\hat{k}$$

Now,

$$\left(\bar{b_1} \times \bar{b_2}\right).\left(\bar{a_2} - \bar{a_1}\right) = \left(-5\hat{i} + 18\hat{j} - 11\hat{k}\right).\left(3\hat{i} - \hat{j} - 3\hat{k}\right)$$

$$= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3))$$

= - 15 – 18 + 33

= 0

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{0}{\sqrt{470}}\right|$$

$$\therefore d = 0 \text{ units}$$

As d = 0

Hence, the given lines not intersect each other.

Now, to find point of intersection, let us convert given vector equations into Cartesian equations.

For that substituting $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in given equations,

$$\therefore L1: \; x\hat{i} + y\hat{j} + z\hat{k} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

$$\therefore L2: \; x\hat{i} + y\hat{j} + z\hat{k} = \left(4\hat{i} + \hat{j}\right) + \mu\left(5\hat{i} + 2\hat{j} + \hat{k}\right)$$

$$\therefore L1: (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k} = 2\lambda\hat{i} + 3\lambda\hat{j} + 4\lambda\hat{k}$$

$$\therefore L2: (x-4)\hat{i} + (y-1)\hat{j} + (z-0)\hat{k} = 5\mu\hat{i} + 2\mu\hat{j} + \mu\hat{k}$$

$$\therefore L1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

$$\therefore L2: \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$$

General point on L1 is

$x_1 = 2\lambda+1$ , $y_1 = 3\lambda+2$ , $z_1 = 4\lambda+3$

let, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Therefore, point P satisfies equation of line L2.

$$\therefore \frac{2\lambda + 1 - 4}{5} = \frac{3\lambda + 2 - 1}{2} = \frac{4\lambda + 3 - 0}{1}$$

$$\therefore \frac{2\lambda - 3}{5} = \frac{3\lambda + 1}{2}$$

⇒ $4\lambda - 6 = 15\lambda + 5$

⇒ $11\lambda = -11$

⇒ $\lambda = -1$

Therefore, $x_1 = 2(-1)+1$ , $y_1 = 3(-1)+2$ , $z_1 = 4(-1)+3$

$\Rightarrow x_1 = -1$ , $y_1 = -1$ , $z_1 = -1$

Hence point of intersection of given lines is (-1, -1, -1).

**13. Question**

Find the shortest distance between the lines $L_1$ and $L_2$ whose vector equations are

$\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$ and $\vec{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$.

**HINT:** The given lines are parallel.

**Answer**

**Given equations :**

$$\bar{r} = \left(\hat{\imath} + 2\hat{\jmath} - 4\hat{k}\right) + \lambda\left(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}\right)$$

$$\bar{r} = \left(3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}\right)$$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**3. Shortest distance between two parallel lines :**

The shortest distance between the parallel lines $\bar{r} = \bar{a_1} + \lambda\bar{b}$ and

$\bar{r} = \overline{a_2} + \lambda\bar{b}$ is given by,

$$d = \left|\frac{\left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right|}{\left|\bar{b}\right|}\right|$$

**Answer :**

For given lines,

$$\bar{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$\bar{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

Here,

$$\overline{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\overline{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\overline{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\overline{b_2} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

As $\overline{b_1} = \overline{b_2} = \bar{b}\ (\text{say})$ , given lines are parallel to each other.

Therefore,

$$\bar{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\therefore \left|\bar{b}\right| = \sqrt{2^2 + 3^2 + 6^2}$$

$$= \sqrt{4 + 9 + 36}$$

$$= \sqrt{49}$$

= 7

$$\overline{a_2} - \overline{a_1} = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (-5 + 4)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + \hat{j} - \hat{k}$$

$$(\overline{a_2} - \overline{a_1}) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= \hat{i}(6 + 3) - \hat{j}(12 + 2) + \hat{k}(6 - 2)$$

$$\therefore (\overline{a_2} - \overline{a_1}) \times \bar{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$$

$$\therefore \left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right| = \sqrt{9^2 + (-14)^2 + 4^2}$$

$$= \sqrt{81 + 196 + 16}$$

$= \sqrt{293}$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{|(\overline{a_2} - \overline{a_1}) \times \overline{b}|}{|\overline{b}|}\right|$$

$$\therefore d = \left|\frac{\sqrt{293}}{7}\right|$$

$$d = \frac{\sqrt{293}}{7}\ units$$

**14. Question**

Find the distance between the parallel lines $L_1$ and $L_2$ whose vector equations are $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$, and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$.

**Answer**

**Given equations :**

$$\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$$

**To Find :** d

**Formula :**

**1. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**2. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**3. Shortest distance between two parallel lines :**

The shortest distance between the parallel lines $\bar{r} = \overline{a_1} + \lambda\bar{b}$ and $\bar{r} = \overline{a_2} + \lambda\bar{b}$ is given by,

$$d = \left|\frac{\left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right|}{\left|\bar{b}\right|}\right|$$

**<u>Answer</u> :**

For given lines,

$\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$

$\bar{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$

Here,

$\overline{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overline{a_2} = 2\hat{i} - \hat{j} - \hat{k}$

$\bar{b} = \hat{i} - \hat{j} + \hat{k}$

$\therefore \left|\bar{b}\right| = \sqrt{1^2 + (-1)^2 + 1^2}$

$= \sqrt{1 + 1 + 1}$

$= \sqrt{3}$

$\overline{a_2} - \overline{a_1} = (2 - 1)\hat{i} + (-1 - 2)\hat{j} + (-1 - 3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} - 3\hat{j} - 4\hat{k}$

$$(\overline{a_2} - \overline{a_1}) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -4 \\ 1 & -1 & 1 \end{vmatrix}$$

$= \hat{i}(-3 - 4) - \hat{j}(1 + 4) + \hat{k}(-1 + 3)$

$\therefore (\overline{a_2} - \overline{a_1}) \times \bar{b} = -7\hat{i} - 5\hat{j} + 2\hat{k}$

$\therefore \left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right| = \sqrt{(-7)^2 + (-5)^2 + 2^2}$

$= \sqrt{49 + 25 + 4}$

$= \sqrt{78}$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{|(\bar{a_2} - \bar{a_1}) \times \bar{b}|}{|\bar{b}|}\right|$$

$$\therefore d = \left|\frac{\sqrt{78}}{\sqrt{3}}\right|$$

$$\therefore d = \sqrt{26}$$

$$d = \sqrt{26}\, units$$

**15. Question**

Find the vector equation of a line passing through the point (2, 3, 2) and parallel to the line $\vec{r} = \left(-2\hat{i} + 3\hat{j}\right) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$. Also, find the distance between these lines.

HINT: The given line is

$$L_1 : \vec{r} = \left(-2\hat{i} + 3\hat{j}\right) + \lambda\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right).$$

The required line is

$$L_2 : \vec{r} = \left(2\hat{i} + 3\hat{j} + 2\hat{k}\right) + \mu\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right).$$

Now, find the distance between the parallel lines $L_1$ and $L_2$.

**Answer**

**<u>Given</u> :** point A ≡ (2, 3, 2)

Equation of line : $\bar{r} = (-2\hat{\imath} + 3\hat{\jmath}) + \lambda\left(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}\right)$

**<u>To Find</u> :** i) equation of line

ii) distance d

**<u>Formulae</u> :**

**1. Equation of line :**

Equation of line passing through point A ($a_1$, $a_2$, $a_3$) and parallel to vector $\bar{b} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is given by

$$\bar{r} = \bar{a} + \lambda\bar{b}$$

Where, $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

**2. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**3. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**4. Shortest distance between two parallel lines :**

The shortest distance between the parallel lines $\bar{r} = \overline{a_1} + \lambda\bar{b}$ and

$\bar{r} = \overline{a_2} + \lambda\bar{b}$ is given by,

$$d = \left| \frac{\left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right|}{\left|\bar{b}\right|} \right|$$

**Answer :**

As the required line is parallel to the line

$\bar{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$

Therefore, the vector parallel to the required line is

$\bar{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Given point A ≡ (2, 3, 2)

$\therefore \bar{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

Therefore, equation of line passing through A and parallel to $\bar{b}$ is

$\bar{r} = \bar{a} + \mu\bar{b}$

$\therefore \bar{r} = (2\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$

Now, to calculate distance between above line and given line,

$\bar{r} = (2\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(2\hat{i} - 3\hat{j} + 6\hat{k})$

$\bar{r} = (-2\hat{i} + 3\hat{j}) + \lambda(2\hat{i} - 3\hat{j} + 6\hat{k})$

Here,

$\bar{a_1} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

$\bar{a_2} = -2\hat{i} + 3\hat{j}$

$\bar{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$\therefore |\bar{b}| = \sqrt{2^2 + (-3)^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

= 7

$\bar{a_2} - \bar{a_1} = (-2 - 2)\hat{i} + (3 - 3)\hat{j} + (0 - 2)\hat{k}$

$\therefore \bar{a_2} - \bar{a_1} = -4\hat{i} + 0\hat{j} - 2\hat{k}$

$$(\bar{a_2} - \bar{a_1}) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 0 & -2 \\ 2 & -3 & 6 \end{vmatrix}$$

$= \hat{i}(0 - 6) - \hat{j}(-24 + 4) + \hat{k}(12 - 0)$

$\therefore (\bar{a_2} - \bar{a_1}) \times \bar{b} = -6\hat{i} + 20\hat{j} + 12\hat{k}$

$\therefore |(\bar{a_2} - \bar{a_1}) \times \bar{b}| = \sqrt{(-6)^2 + 20^2 + 12^2}$

$= \sqrt{36 + 400 + 144}$

$= \sqrt{580}$

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{|(\bar{a_2} - \bar{a_1}) \times \bar{b}|}{|\bar{b}|}\right|$$

$$\therefore d = \left|\frac{\sqrt{580}}{7}\right|$$

$$\therefore d = \frac{\sqrt{580}}{7}$$

$$d = \frac{\sqrt{580}}{7}\ units$$

**16. Question**

Write the vector equation of each of the following lines and hence determine the distance between them :

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6} \text{ and } \frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}.$$

HINT: The given lines are

$$L_1 : \vec{r}=\left(-2\hat{i}+3\hat{j}\right)+\lambda\left(2\hat{i}-3\hat{j}+6\hat{k}\right)$$

$$L_2 : \vec{r}=\left(3\hat{i}+3\hat{j}-5\hat{k}\right)+2\mu\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$$

Now, find the distance between the parallel lines $L_1$ and $L_2$.

**Answer**

**<u>Given</u> :** Cartesian equations of lines

$$L1 : \frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$$

$$L2 : \frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}$$

**<u>To Find</u> :** i) vector equations of given lines

ii) distance d

**<u>Formulae</u> :**

**1. Equation of line :**

Equation of line passing through point A ($a_1$, $a_2$, $a_3$) and having direction ratios ($b_1$, $b_2$, $b_3$) is

$$\bar{r}=\bar{a}+\lambda\bar{b}$$

Where, $\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

And $\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

**2. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$$

then,

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

**3. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**4. Shortest distance between two parallel lines :**

The shortest distance between the parallel lines $\bar{r} = \overline{a_1} + \lambda\bar{b}$ and

$\bar{r} = \overline{a_2} + \lambda\bar{b}$ is given by,

$$d = \left|\frac{\left|(\overline{a_2} - \overline{a_1}) \times \bar{b}\right|}{|\bar{b}|}\right|$$

**Answer :**

Given Cartesian equations of lines

$$L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

Line L1 is passing through point (1, 2, -4) and has direction ratios (2, 3, 6)

Therefore, vector equation of line L1 is

$$\bar{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

And

$$L2 : \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

Line L2 is passing through point (3, 3, -5) and has direction ratios (4, 6, 12)

Therefore, vector equation of line L2 is

$$\bar{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(4\hat{i} + 6\hat{j} + 12\hat{k}\right)$$

$$\therefore \bar{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + 2\mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

Now, to calculate distance between the lines,

$$\bar{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$\bar{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + 2\mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

Here,

$\overline{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\overline{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\overline{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

As $\overline{b_1} = \overline{b_2} = \bar{b}\ (\text{say})$ , given lines are parallel to each other.

Therefore,

$\bar{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\therefore |\bar{b}| = \sqrt{2^2 + 3^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

= 7

$\overline{a_2} - \overline{a_1} = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (-5 + 4)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + \hat{j} - \hat{k}$

$$(\overline{a_2} - \overline{a_1}) \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$= \hat{i}(6 + 3) - \hat{j}(12 + 2) + \hat{k}(6 - 2)$

$\therefore (\overline{a_2} - \overline{a_1}) \times \bar{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$

$\therefore |(\overline{a_2} - \overline{a_1}) \times \bar{b}| = \sqrt{9^2 + (-14)^2 + 4^2}$

$= \sqrt{81 + 196 + 16}$

$= \sqrt{293}$

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{|(\overline{a_2} - \overline{a_1}) \times \bar{b}|}{|\bar{b}|} \right|$$

$$\therefore d = \left| \frac{\sqrt{293}}{7} \right|$$

$$d = \frac{\sqrt{293}}{7}\ units$$

**17. Question**

Write the vector equation of the following lines and hence find the shortest distance between them :

$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-5}{5}$.

**Answer**

**Given :** Cartesian equations of lines

$$L1 : \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$$

$$L2 : \frac{x-2}{3}=\frac{y-3}{4}=\frac{z-5}{5}$$

**To Find :** i) vector equations of given lines

ii) distance d

**Formulae :**

**1. Equation of line :**

Equation of line passing through point A ($a_1$, $a_2$, $a_3$) and having direction ratios ($b_1$, $b_2$, $b_3$) is

$$\bar{r}=\bar{a}+\lambda\bar{b}$$

Where, $\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

And $\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

**2. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$$

then,

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

**3. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**4. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

<u>**Answer**</u> **:**

Given Cartesian equations of lines

$$L1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

Line L1 is passing through point (1, 2, 3) and has direction ratios (2, 3, 4)

Therefore, vector equation of line L1 is

$$\bar{r} = \left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right) + \lambda\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right)$$

And

$$L2 : \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$$

Line L2 is passing through point (2, 3, 5) and has direction ratios (3, 4, 5)

Therefore, vector equation of line L2 is

$$\bar{r} = \left(3\hat{\imath} + 3\hat{\jmath} + 5\hat{k}\right) + \mu\left(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}\right)$$

Now, to calculate distance between the lines,

$$\bar{r} = \left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right) + \lambda\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right)$$

$$\bar{r} = \left(3\hat{\imath} + 3\hat{\jmath} + 5\hat{k}\right) + \mu\left(3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}\right)$$

Here,

$$\overline{a_1} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$$

$$\overline{b_1} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$$

$$\overline{a_2} = 3\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$$

$$\overline{b_2} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-1)^2 + 2^2 + (-1)^2}$$

$$= \sqrt{1 + 4 + 1}$$

$$= \sqrt{6}$$

$$\overline{a_2} - \overline{a_1} = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (5 - 3)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + \hat{j} + 2\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right) = \left(-\hat{i} + 2\hat{j} - \hat{k}\right).\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$$

$$= ((-1) \times 2) + (2 \times 1) + ((-1) \times 2)$$

= - 2 + 2 - 2

= -2

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{-2}{\sqrt{6}}\right|$$

$$\therefore d = \frac{2}{\sqrt{3}.\sqrt{2}}$$

$$\therefore d = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\therefore d = \sqrt{\frac{2}{3}}$$

$$d = \sqrt{\frac{2}{3}}\ units$$

**18. Question**

Find the shortest distance between the lines given below:

$$\frac{x-1}{-1}=\frac{y+2}{1}=\frac{z-3}{-2} \text{ and } \frac{x-1}{2}=\frac{y+1}{2}\ \frac{z+1}{-2}.$$

**Answer**

**Given :** Cartesian equations of lines

$$L1 : \frac{x-1}{-1}=\frac{y+2}{1}=\frac{z-3}{-2}$$

$$L2 : \frac{x-1}{2}=\frac{y+1}{2}=\frac{z+1}{-2}$$

**To Find :** distance d

**Formulae :**

**1. Equation of line :**

Equation of line passing through point A ($a_1$, $a_2$, $a_3$) and having direction ratios ($b_1$, $b_2$, $b_3$) is

$$\bar{r}=\bar{a}+\lambda\bar{b}$$

Where, $\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

And $\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

**2. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$$

then,

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

**3. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$$

then,

$$\bar{a}.\bar{b}=(a_1\times b_1)+(a_2\times b_2)+(a_3\times b_3)$$

**4. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \bar{a_1} + \lambda\bar{b_1}$ and

$\bar{r} = \bar{a_2} + \lambda\bar{b_2}$ is given by,

$$d = \left|\frac{(\bar{b_1} \times \bar{b_2}).(\bar{a_2} - \bar{a_1})}{|\bar{b_1} \times \bar{b_2}|}\right|$$

__**Answer**__ **:**

Given Cartesian equations of lines

$$L1 : \frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}$$

Line L1 is passing through point (1, -2, 3) and has direction ratios (-1, 1, -2)

Therefore, vector equation of line L1 is

$$\bar{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$$

And

$$L2 : \frac{x-1}{2} = \frac{y+1}{2} = \frac{z+1}{-2}$$

Line L2 is passing through point (1, -1, -1) and has direction ratios (2, 2, -2)

Therefore, vector equation of line L2 is

$$\bar{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + 2\hat{j} - 2\hat{k})$$

Now, to calculate distance between the lines,

$$\bar{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$$

$$\bar{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + 2\hat{j} - 2\hat{k})$$

Here,

$$\bar{a_1} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\bar{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$$

$$\bar{a_2} = \hat{i} - \hat{j} - \hat{k}$$

$$\bar{b_2} = 2\hat{i} + 2\hat{j} - 2\hat{k}$$

Therefore,

$$\bar{b_1} \times \bar{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 2 & 2 & -2 \end{vmatrix}$$

$= \hat{i}(-2+4) - \hat{j}(2+4) + \hat{k}(-2-2)$

$\therefore \overline{b_1} \times \overline{b_2} = 2\hat{i} - 6\hat{j} - 4\hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{2^2 + (-6)^2 + (-4)^2}$

$= \sqrt{4 + 36 + 16}$

$= \sqrt{56}$

$\overline{a_2} - \overline{a_1} = (1-1)\hat{i} + (-1+2)\hat{j} + (-1-3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 0\hat{i} + \hat{j} - 4\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1}) = (2\hat{i} - 6\hat{j} - 4\hat{k}) . (0\hat{i} + \hat{j} - 4\hat{k})$

$= (2 \times 0) + ((-6) \times 1) + ((-4) \times (-4))$

= 0 - 6 + 16

= 10

Therefore, the shortest distance between the given lines is

$$d = \left| \frac{(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|} \right|$$

$$\therefore d = \left| \frac{10}{\sqrt{56}} \right|$$

$$\therefore d = \frac{10}{\sqrt{56}}$$

$$d = \frac{10}{\sqrt{56}} \; units$$

**19. Question**

Find the shortest distance between the lines given below:

$$\frac{x-12}{-9} = \frac{y-1}{4} = \frac{z-5}{2} \text{ and } \frac{x-23}{-6} = \frac{y-10}{-4} = \frac{z-25}{3}.$$

**HINT:** Change the given equations in vector form.

**Answer**

**Given :** Cartesian equations of lines

$$L1 : \frac{x-12}{-9} = \frac{y-1}{4} = \frac{z-5}{2}$$

$$L2: \frac{x-23}{-6} = \frac{y-10}{-4} = \frac{z-23}{3}$$

**To Find :** distance d

**Formulae :**

**1. Equation of line :**

Equation of line passing through point A ($a_1$, $a_2$, $a_3$) and having direction ratios ($b_1$, $b_2$, $b_3$) is

$$\bar{r} = \bar{a} + \lambda\bar{b}$$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

**2. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**3. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

**4. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**Answer :**

Given Cartesian equations of lines

$$L1: \frac{x-12}{-9} = \frac{y-1}{4} = \frac{z-5}{2}$$

Line L1 is passing through point (12, 1, 5) and has direction ratios (-9, 4, 2)

Therefore, vector equation of line L1 is

$$\bar{r} = \left(12\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(-9\hat{i} + 4\hat{j} + 2\hat{k}\right)$$

And

$$L2: \frac{x-23}{-6} = \frac{y-10}{-4} = \frac{z-23}{3}$$

Line L2 is passing through point (23, 10, 23) and has direction ratios (-6, -4, 3)

Therefore, vector equation of line L2 is

$$\bar{r} = \left(23\hat{i} + 10\hat{j} + 23\hat{k}\right) + \mu\left(-6\hat{i} - 4\hat{j} + 3\hat{k}\right)$$

Now, to calculate distance between the lines,

$$\bar{r} = \left(12\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(-9\hat{i} + 4\hat{j} + 2\hat{k}\right)$$

$$\bar{r} = \left(23\hat{i} + 10\hat{j} + 23\hat{k}\right) + \mu\left(-6\hat{i} - 4\hat{j} + 3\hat{k}\right)$$

Here,

$$\overline{a_1} = 12\hat{i} + \hat{j} + 5\hat{k}$$

$$\overline{b_1} = -9\hat{i} + 4\hat{j} + 2\hat{k}$$

$$\overline{a_2} = 23\hat{i} + 10\hat{j} + 23\hat{k}$$

$$\overline{b_2} = -6\hat{i} - 4\hat{j} + 3\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -9 & 4 & 2 \\ -6 & -4 & 3 \end{vmatrix}$$

$$= \hat{i}(12+8) - \hat{j}(-27+12) + \hat{k}(36+24)$$

$$\therefore \overline{b_1} \times \overline{b_2} = 20\hat{i} + 15\hat{j} + 60\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{20^2 + 15^2 + 60^2}$$

$$= \sqrt{400 + 225 + 3600}$$

$$= \sqrt{4225}$$

= 65

$$\overline{a_2} - \overline{a_1} = (23 - 12)\hat{i} + (10 - 1)\hat{j} + (23 - 5)\hat{k}$$

$$\therefore \overline{a_2} - \overline{a_1} = 11\hat{i} + 9\hat{j} + 18\hat{k}$$

Now,

$$\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1}) = \left(20\hat{i} + 15\hat{j} + 60\hat{k}\right).\left(11\hat{i} + 9\hat{j} + 18\hat{k}\right)$$

$$= (20 \times 11) + (15 \times 9) + (60 \times 18)$$

= 220 + 135 + 1080

= 1435

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).(\overline{a_2} - \overline{a_1})}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

$$\therefore d = \left|\frac{1435}{65}\right|$$

$$\therefore d = \frac{287}{13}$$

$$d = \frac{287}{13}\ units$$

## Exercise 27E

**1. Question**

Find the length and the equations of the line of shortest distance between the lines given by:

$\frac{x-3}{3} = \frac{y-8}{-1} = z-3$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$.

**Answer**

**Given :** Cartesian equations of lines

$$L1 : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$

$$L2 : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

**Formulae :**

**1. Condition for perpendicularity :**

If line L1 has direction ratios ($a_1$, $a_2$, $a_3$) and that of line L2 are ($b_1$, $b_2$, $b_3$) then lines L1 and L2 will be perpendicular to each other if

$$(a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3) = 0$$

**2. Distance formula :**

Distance between two points A≡($a_1$, $a_2$, $a_3$) and B≡($b_1$, $b_2$, $b_3$) is given by,

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

**3. Equation of line :**

Equation of line passing through points A≡($x_1$, $y_1$, $z_1$) and B≡($x_2$, $y_2$, $z_2$) is given by,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

**Answer :**

Given equations of lines

$$L1 : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$

$$L2 : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

Direction ratios of L1 and L2 are (3, -1, 1) and (-3, 2, 4) respectively.

Let, general point on line L1 is P≡($x_1$, $y_1$, $z_1$)

$x_1$ = 3s+3 , $y_1$ = -s+8 , $z_1$ = s+3

and let, general point on line L2 is Q≡($x_2$, $y_2$, $z_2$)

$x_2$ = -3t – 3 , $y_2$ = 2t – 7 , $z_2$ = 4t + 6

$$\therefore \overline{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$= (-3t - 3 - 3s - 3)\hat{i} + (2t - 7 + s - 8)\hat{j} + (4t + 6 - s - 3)\hat{k}$$

$$\therefore \overline{PQ} = (-3t - 3s - 6)\hat{i} + (2t + s - 15)\hat{j} + (4t - s + 3)\hat{k}$$

Direction ratios of $\overline{PQ}$ are ((-3t – 3s - 6), (2t + s - 15), (4t – s + 3))

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

3(-3t – 3s - 6) – 1(2t + s - 15) + 1(4t – s + 3) = 0 and

-3(-3t – 3s - 6) + 2(2t + s - 15) + 4(4t – s + 3) = 0

⇒ -9t – 9s – 18 – 2t – s + 15 + 4t – s + 3 = 0 and

9t + 9s + 18 + 4t + 2s – 30 + 16t – 4s + 12 = 0

⇒ -7t – 11s = 0 and

29t + 7s = 0

Solving above two equations, we get,

t = 0 and s = 0

therefore,

P ≡ (3, 8, 3) and Q ≡ (-3, -7, 6)

Now, distance between points P and Q is

$$d = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2}$$

$$= \sqrt{(6)^2 + (15)^2 + (-3)^2}$$

$$= \sqrt{36 + 225 + 9}$$

$$= \sqrt{270}$$

$$= 3\sqrt{30}$$

Therefore, the shortest distance between two given lines is

$$d = 3\sqrt{30}\ units$$

Now, equation of line passing through points P and Q is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

$$\therefore \frac{x-3}{3+3} = \frac{y-8}{8+7} = \frac{z-3}{3-6}$$

$$\therefore \frac{x-3}{6} = \frac{y-8}{15} = \frac{z-3}{-3}$$

$$\therefore \frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

Therefore, equation of line of shortest distance between two given lines is

$$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

**2. Question**

Find the length and the equations of the line of shortest distance between the lines given by:

$$\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1} \text{ and } \frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}.$$

**Answer**

**Given :** Cartesian equations of lines

$$L1: \frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$$

$$L2: \frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$$

**Formulae :**

**1. Condition for perpendicularity :**

If line L1 has direction ratios ($a_1$, $a_2$, $a_3$) and that of line L2 are ($b_1$, $b_2$, $b_3$) then lines L1 and L2 will be perpendicular to each other if

$$(a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3) = 0$$

**2. Distance formula :**

Distance between two points A≡($a_1$, $a_2$, $a_3$) and B≡($b_1$, $b_2$, $b_3$) is given by,

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

**3. Equation of line :**

Equation of line passing through points A≡($x_1$, $y_1$, $z_1$) and B≡($x_2$, $y_2$, $z_2$) is given by,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

**Answer :**

Given equations of lines

$$L1: \frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$$

$$L2: \frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$$

Direction ratios of L1 and L2 are (-1, 2, 1) and (1, 3, 2) respectively.

Let, general point on line L1 is P≡($x_1$, $y_1$, $z_1$)

$x_1$ = -s+3 , $y_1$ = 2s+4 , $z_1$ = s-2

and let, general point on line L2 is Q≡($x_2$, $y_2$, $z_2$)

$x_2$ = t+1 , $y_2$ = 3t – 7 , $z_2$ = 2t - 2

$$\therefore \overline{PQ} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$$

$$= (t + 1 + s - 3)\hat{\imath} + (3t - 7 - 2s - 4)\hat{\jmath} + (2t - 2 - s + 2)\hat{k}$$

$\therefore \overline{PQ} = (t + s - 2)\hat{i} + (3t - 2s - 11)\hat{j} + (2t - s)\hat{k}$

Direction ratios of $\overline{PQ}$ are ((t + s − 2), (3t − 2s − 11), (2t − s))

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

-1(t + s − 2) + 2(3t − 2s − 11) + 1(2t − s) = 0 and

1(t + s − 2) + 3(3t − 2s − 11) + 2(2t − s) = 0

⇒ - t − s + 2 + 6t − 4s − 22 + 2t − s = 0 and

t + s − 2 + 9t − 6s − 33 + 4t − 2s = 0

⇒ 7t − 6s = 20 and

14t - 7s = 35

Solving above two equations, we get,

t = 2 and s = -1

therefore,

P ≡ (4, 2, -3) and Q ≡ (3, -1, 2)

Now, distance between points P and Q is

$$d = \sqrt{(4-3)^2 + (2+1)^2 + (-3-2)^2}$$

$$= \sqrt{(1)^2 + (3)^2 + (-5)^2}$$

$$= \sqrt{1 + 9 + 25}$$

$$= \sqrt{35}$$

Therefore, the shortest distance between two given lines is

$$d = \sqrt{35}\ units$$

Now, equation of line passing through points P and Q is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

$$\therefore \frac{x-4}{4-3} = \frac{y-2}{2+1} = \frac{z+3}{-3-2}$$

$$\therefore \frac{x-4}{1} = \frac{y-2}{3} = \frac{z+3}{-5}$$

$$\therefore \frac{x-4}{-1} = \frac{y-2}{-3} = \frac{z+3}{5}$$

Therefore, equation of line of shortest distance between two given lines is

$$\frac{x-4}{-1}=\frac{y-2}{-3}=\frac{z+3}{5}$$

**3. Question**

Find the length and the equations of the line of shortest distance between the lines given by:

$$\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3} \text{ and } \frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}.$$

**Answer**

**Given :** Cartesian equations of lines

$$L1:\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$$

$$L2:\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$$

**Formulae :**

**1. Condition for perpendicularity :**

If line L1 has direction ratios ($a_1$, $a_2$, $a_3$) and that of line L2 are ($b_1$, $b_2$, $b_3$) then lines L1 and L2 will be perpendicular to each other if

$$(a_1\times b_1)+(a_2\times b_2)+(a_3\times b_3)=0$$

**2. Distance formula :**

Distance between two points A≡($a_1$, $a_2$, $a_3$) and B≡($b_1$, $b_2$, $b_3$) is given by,

$$d=\sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$$

**3. Equation of line :**

Equation of line passing through points A≡($x_1$, $y_1$, $z_1$) and B≡($x_2$, $y_2$, $z_2$) is given by,

$$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$$

**Answer :**

Given equations of lines

$$L1:\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$$

$$L2:\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$$

Direction ratios of L1 and L2 are (2, 1, -3) and (2, -7, 5) respectively.

Let, general point on line L1 is P≡($x_1$, $y_1$, $z_1$)

$x_1$ = 2s-1 , $y_1$ = s+1 , $z_1$ = -3s+9

and let, general point on line L2 is Q≡($x_2$, $y_2$, $z_2$)

$x_2$ = 2t+3 , $y_2$ = -7t – 15 , $z_2$ = 5t + 9

$$\therefore \overline{PQ} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$$

$$= (5t + 9 - 2s + 1)\hat{\imath} + (-7t - 15 - s - 1)\hat{\jmath} + (5t + 9 + 3s - 9)\hat{k}$$

$$\therefore \overline{PQ} = (5t - 2s + 10)\hat{\imath} + (-7t - s - 16)\hat{\jmath} + (5t + 3s)\hat{k}$$

Direction ratios of $\overline{PQ}$ are ((5t - 2s + 10), (-7t – s – 16), (5t + 3s))

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

2(5t - 2s + 10) + 1(-7t – s – 16) - 3(5t + 3s) = 0 and

2(5t - 2s + 10) – 7(-7t – s – 16) + 5(5t + 3s) = 0

⇒ 10t – 4s + 20 - 7t – s - 16 - 15t – 9s = 0 and

10t - 4s + 20 + 49t + 7s + 112 + 25t + 15s = 0

⇒ -12t – 14s = -4 and

84t + 18s = -132

Solving above two equations, we get,

t = -2 and s = 2

therefore,

P ≡ (3, 3, 3) and Q ≡ (-1, -1, -1)

Now, distance between points P and Q is

$$d = \sqrt{(3+1)^2 + (3+1)^2 + (3+1)^2}$$

$$= \sqrt{(4)^2 + (4)^2 + (4)^2}$$

$$= \sqrt{16 + 16 + 16}$$

$$= \sqrt{48}$$

$$= 4\sqrt{3}$$

Therefore, the shortest distance between two given lines is

$$d = 4\sqrt{3}\ units$$

Now, equation of line passing through points P and Q is,

$$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$$

$$\therefore \frac{x-3}{3+1}=\frac{y-3}{3+1}=\frac{z-3}{3+1}$$

$$\therefore \frac{x-3}{4}=\frac{y-3}{4}=\frac{z-3}{4}$$

$$\therefore x-3=y-3=z-3$$

$$\Rightarrow x=y=z$$

Therefore, equation of line of shortest distance between two given lines is

x = y = z

**4. Question**

Find the length and the equations of the line of shortest distance between the lines given by:

$$\frac{x-6}{3}=\frac{y-7}{-1}=\frac{z-4}{1} \text{ and } \frac{x}{-3}=\frac{y+9}{2}=\frac{z-2}{4}.$$

**Answer**

**Given :** Cartesian equations of lines

$$L1: \frac{x-6}{3}=\frac{y-7}{-1}=\frac{z-4}{1}$$

$$L2: \frac{x}{-3}=\frac{y+9}{2}=\frac{z-2}{4}$$

**Formulae :**

**1. Condition for perpendicularity :**

If line L1 has direction ratios ($a_1$, $a_2$, $a_3$) and that of line L2 are ($b_1$, $b_2$, $b_3$) then lines L1 and L2 will be perpendicular to each other if

$$(a_1\times b_1)+(a_2\times b_2)+(a_3\times b_3)=0$$

**2. Distance formula :**

Distance between two points A≡($a_1$, $a_2$, $a_3$) and B≡($b_1$, $b_2$, $b_3$) is given by,

$$d=\sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$$

**3. Equation of line :**

Equation of line passing through points A≡($x_1$, $y_1$, $z_1$) and B≡($x_2$, $y_2$, $z_2$) is given by,

$$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$$

**Answer :**

Given equations of lines

$$L1:\ \frac{x-6}{3}=\frac{y-7}{-1}=\frac{z-4}{1}$$

$$L2:\frac{x}{-3}=\frac{y+9}{2}=\frac{z-2}{4}$$

Direction ratios of L1 and L2 are (3, -1, 1) and (-3, 2, 4) respectively.

Let, general point on line L1 is P≡($x_1$, $y_1$, $z_1$)

$x_1$ = 3s+6 , $y_1$ = -s+7 , $z_1$ = s+4

and let, general point on line L2 is Q≡($x_2$, $y_2$, $z_2$)

$x_2$ = -3t , $y_2$ = 2t – 9 , $z_2$ = 4t + 2

$$\therefore \overline{PQ}=(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$$

$$=(-3t-3s-6)\hat{i}+(2t-9+s-7)\hat{j}+(4t+2-s-4)\hat{k}$$

$$\therefore \overline{PQ}=(-3t-3s-6)\hat{i}+(2t+s-16)\hat{j}+(4t-s-2)\hat{k}$$

Direction ratios of $\overline{PQ}$ are ((-3t - 3s - 6), (2t + s – 16), (4t – s – 2))

PQ will be the shortest distance if it perpendicular to both the given lines

Therefore, by the condition of perpendicularity,

3(-3t - 3s - 6) - 1(2t + s – 16) + 1(4t – s – 2) = 0 and

-3(-3t - 3s - 6) + 2(2t + s – 16) + 4(4t – s – 2) = 0

⇒ -9t - 9s - 18 - 2t – s + 16 + 4t – s – 2 = 0 and

9t + 9s + 18 + 4t + 2s – 32 + 16t – 4s – 8 = 0

⇒ -7t – 11s = 4 and

29t + 7s = -22

Solving above two equations, we get,

t = 1 and s = -1

therefore,

P ≡ (3, 8, 3) and Q ≡ (-3, -7, 6)

Now, distance between points P and Q is

$d = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2}$

$= \sqrt{(6)^2 + (15)^2 + (-3)^2}$

$= \sqrt{36 + 225 + 9}$

$= \sqrt{270}$

$= 3\sqrt{30}$

Therefore, the shortest distance between two given lines is

$d = 3\sqrt{30}\ units$

Now, equation of line passing through points P and Q is,

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

$$\therefore \frac{x-3}{3+3} = \frac{y-8}{8+7} = \frac{z-3}{3-6}$$

$$\therefore \frac{x-3}{6} = \frac{y-8}{15} = \frac{z-3}{-3}$$

$$\therefore \frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

Therefore, equation of line of shortest distance between two given lines is

$$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

**5. Question**

Show that the lines $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ intersect and find their point of intersection.

**Answer**

**Given :** Cartesian equations of lines

$$L1 : \frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$$

$$L2 : \frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$$

**To Find :** distance d

**Formulae :**

**1. Equation of line :**

Equation of line passing through point A ($a_1$, $a_2$, $a_3$) and having direction ratios ($b_1$, $b_2$, $b_3$) is

$\bar{r} = \bar{a} + \lambda\bar{b}$

Where, $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

And $\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

**2. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

**3. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

**4. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1}$ and

$\bar{r} = \overline{a_2} + \lambda\overline{b_2}$ is given by,

$$d = \left|\frac{\left(\overline{b_1} \times \overline{b_2}\right).\left(\overline{a_2} - \overline{a_1}\right)}{\left|\overline{b_1} \times \overline{b_2}\right|}\right|$$

**Answer :**

Given Cartesian equations of lines

$$L1 : \frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$$

Line L1 is passing through point (0, 2, -3) and has direction ratios (1, 2, 3)

Therefore, vector equation of line L1 is

$\bar{r} = (0\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

And

$L2 : \frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$

Line L2 is passing through point (2, 6, 3) and has direction ratios (2, 3, 4)

Therefore, vector equation of line L2 is

$\bar{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$

Now, to calculate distance between the lines,

$\bar{r} = (0\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$\bar{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$

Here,

$\overline{a_1} = 0\hat{i} + 2\hat{j} - 3\hat{k}$

$\overline{b_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overline{a_2} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

$\overline{b_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Therefore,

$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$

$= \hat{i}(8-9) - \hat{j}(4-6) + \hat{k}(3-4)$

$\therefore \overline{b_1} \times \overline{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$

$\therefore |\overline{b_1} \times \overline{b_2}| = \sqrt{(-1)^2 + 2^2 + (-1)^2}$

$= \sqrt{1+4+1}$

$= \sqrt{6}$

$\overline{a_2} - \overline{a_1} = (2-0)\hat{i} + (6-2)\hat{j} + (3+3)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = 2\hat{i} + 4\hat{j} + 6\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1}) = (-\hat{i} + 2\hat{j} - \hat{k}).(2\hat{i} + 4\hat{j} + 6\hat{k})$

$= ((-1) \times 2) + (2 \times 4) + ((-1) \times 6)$

= - 2 + 8 – 6

= 0

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}).(\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{0}{\sqrt{14}}\right|$$

$$\therefore d = 0 \text{ units}$$

As d = 0

Hence, given lines intersect each other.

Now, general point on L1 is

$x_1 = \lambda$ , $y_1 = 2\lambda+2$ , $z_1 = 3\lambda-3$

let, P($x_1$, $y_1$, $z_1$) be point of intersection of two given lines.

Therefore, point P satisfies equation of line L2.

$$\therefore \frac{\lambda - 2}{2} = \frac{2\lambda + 2 - 6}{3} = \frac{3\lambda - 3 - 3}{4}$$

$$\therefore \frac{\lambda - 2}{2} = \frac{2\lambda - 4}{3}$$

⇒ 3λ – 6 = 4λ – 8

⇒ λ = 2

Therefore, $x_1 = 2$ , $y_1 = 2(2)+2$ , $z_1 = 3(2)-3$

⇒ $x_1 = 2$ , $y_1 = 6$ , $z_1 = 3$

Hence point of intersection of given lines is (2, 6, 3).

**6. Question**

Show that the lines $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$ do not intersect each other.

**Answer**

**Given :** Cartesian equations of lines

$$L1 : \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$$

$$L2:\frac{x-2}{2}=\frac{y-1}{3}=\frac{z+1}{-2}$$

**To Find :** distance d

**Formulae :**

**1. Equation of line :**

Equation of line passing through point A ($a_1$, $a_2$, $a_3$) and having direction ratios ($b_1$, $b_2$, $b_3$) is

$$\bar{r}=\bar{a}+\lambda\bar{b}$$

Where, $\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

And $\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$

**2. Cross Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$$

then,

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

**3. Dot Product :**

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$$

$$\bar{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$$

then,

$$\bar{a}.\bar{b}=(a_1\times b_1)+(a_2\times b_2)+(a_3\times b_3)$$

**4. Shortest distance between two lines :**

The shortest distance between the skew lines $\bar{r}=\overline{a_1}+\lambda\overline{b_1}$ and

$\bar{r}=\overline{a_2}+\lambda\overline{b_2}$ is given by,

$$d=\left|\frac{\left(\overline{b_1}\times\overline{b_2}\right).\left(\overline{a_2}-\overline{a_1}\right)}{\left|\overline{b_1}\times\overline{b_2}\right|}\right|$$

**Answer :**

Given Cartesian equations of lines

$$L1 : \frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$$

Line L1 is passing through point (1, -1, 1) and has direction ratios (3, 2, 5)

Therefore, vector equation of line L1 is

$$\bar{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$$

And

$$L2 : \frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$$

Line L2 is passing through point (2, 1, -1) and has direction ratios (2, 3, -2)

Therefore, vector equation of line L2 is

$$\bar{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} + 3\hat{j} - 2\hat{k})$$

Now, to calculate distance between the lines,

$$\bar{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 5\hat{k})$$

$$\bar{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} + 3\hat{j} - 2\hat{k})$$

Here,

$$\overline{a_1} = \hat{i} - \hat{j} + \hat{k}$$

$$\overline{b_1} = 3\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\overline{a_2} = 2\hat{i} + \hat{j} - \hat{k}$$

$$\overline{b_2} = 2\hat{i} + 3\hat{j} - 2\hat{k}$$

Therefore,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 5 \\ 2 & 3 & -2 \end{vmatrix}$$

$$= \hat{i}(-4 - 15) - \hat{j}(-6 - 10) + \hat{k}(9 - 4)$$

$$\therefore \overline{b_1} \times \overline{b_2} = -19\hat{i} + 16\hat{j} + 5\hat{k}$$

$$\therefore \left|\overline{b_1} \times \overline{b_2}\right| = \sqrt{(-19)^2 + 16^2 + 5^2}$$

$$= \sqrt{361 + 256 + 25}$$

$$= \sqrt{642}$$

$\overline{a_2} - \overline{a_1} = (2-1)\hat{i} + (1+1)\hat{j} + (-1-1)\hat{k}$

$\therefore \overline{a_2} - \overline{a_1} = \hat{i} + 2\hat{j} - 2\hat{k}$

Now,

$(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1}) = (-19\hat{i} + 16\hat{j} + 5\hat{k}) . (\hat{i} + 2\hat{j} - 2\hat{k})$

$= ((-19) \times 1) + (16 \times 2) + (5 \times (-2))$

= - 19 + 32 – 10

= 3

Therefore, the shortest distance between the given lines is

$$d = \left|\frac{(\overline{b_1} \times \overline{b_2}) . (\overline{a_2} - \overline{a_1})}{|\overline{b_1} \times \overline{b_2}|}\right|$$

$$\therefore d = \left|\frac{3}{\sqrt{642}}\right|$$

$$\therefore d = \frac{3}{\sqrt{642}} \text{ units}$$

As d ≠ 0

Hence, given lines do not intersect each other.

## Exercise 27F

### 1. Question

If a line has direction ratios 2, -1, -2 then what are its direction cosines?

**Answer**

Given : A line has direction ratios 2, -1, -2

To find : Direction cosines of the line

Formula used : If (l,m,n) are the direction ratios of a given line then direction cosines are given by $\frac{l}{\sqrt{l^2 + m^2 + n^2}}, \frac{m}{\sqrt{l^2 + m^2 + n^2}}, \frac{n}{\sqrt{l^2 + m^2 + n^2}}$

Here l = 2 , m = -1 , n = -2

Direction cosines of the line with direction ratios 2, -1, -2 is

$$\frac{2}{\sqrt{2^2 + (-1)^2 + (-2)^2}}, \frac{-1}{\sqrt{2^2 + (-1)^2 + (-2)^2}}, \frac{-2}{\sqrt{2^2 + (-1)^2 + (-2)^2}}$$

$$= \frac{2}{\sqrt{4+1+4}}, \frac{-1}{\sqrt{4+1+4}}, \frac{-2}{\sqrt{4+1+4}} = \frac{2}{\sqrt{9}}, \frac{-1}{\sqrt{9}}, \frac{-2}{\sqrt{9}}$$

$$= \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$$

Direction cosines of the line with direction ratios 2, -1, -2 is $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$

**2. Question**

Find the direction cosines of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

**Answer**

Given : A line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

To find : Direction cosines of the line

Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}$, $\frac{m}{\sqrt{l^2+m^2+n^2}}$, $\frac{n}{\sqrt{l^2+m^2+n^2}}$

The line is $\frac{x-4}{-2} = \frac{y-0}{6} = \frac{z-1}{-3}$

Here l = -2 , m = 6 , n = -3

Direction cosines of the line $\frac{x-4}{-2} = \frac{y-0}{6} = \frac{z-1}{-3}$ is

$\frac{-2}{\sqrt{(-2)^2+(6)^2+(-3)^2}}, \frac{6}{\sqrt{(-2)^2+(6)^2+(-3)^2}}, \frac{-3}{\sqrt{(-2)^2+(6)^2+(-3)^2}}$

$= \frac{-2}{\sqrt{4+36+9}}, \frac{6}{\sqrt{4+36+9}}, \frac{-3}{\sqrt{4+36+9}} = \frac{-2}{\sqrt{49}}, \frac{6}{\sqrt{49}}, \frac{-3}{\sqrt{49}}$

$= \frac{-2}{7}, \frac{6}{7}, \frac{-3}{7}$

Direction cosines of the line $\frac{x-4}{-2} = \frac{y-0}{6} = \frac{z-1}{-3}$ is $\frac{-2}{7}, \frac{6}{7}, \frac{-3}{7}$

**3. Question**

If the equations of a line are $\frac{3-x}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$, find the direction cosines of a line parallel to the given line.

**Answer**

Given : A line $\frac{3-x}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$,

To find : Direction cosines of the line parallel to $\frac{3-x}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$, Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}$ , $\frac{m}{\sqrt{l^2+m^2+n^2}}$ , $\frac{n}{\sqrt{l^2+m^2+n^2}}$

The line is $\frac{x-3}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$

Parallel lines have same direction ratios and direction cosines

Here l = 3 , m = -2 , n = 6

Direction cosines of the line $\frac{x-3}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$ is

$\frac{3}{\sqrt{(3)^2+(-2)^2+(6)^2}}$ , $\frac{-2}{\sqrt{(3)^2+(-2)^2+(6)^2}}$ , $\frac{6}{\sqrt{(3)^2+(-2)^2+(6)^2}}$

$= \frac{3}{\sqrt{9+4+36}}$ , $\frac{-2}{\sqrt{9+4+36}}$ , $\frac{6}{\sqrt{9+4+36}} = \frac{3}{\sqrt{49}}$ , $\frac{-2}{\sqrt{49}}$ , $\frac{6}{\sqrt{49}}$

$= \frac{3}{7}$ , $\frac{-2}{7}$ , $\frac{6}{7}$

Direction cosines of the line parallel to the line $\frac{x-3}{-3} = \frac{y+2}{-2} = \frac{z+2}{6}$ is

$\frac{3}{7}$ , $\frac{-2}{7}$ , $\frac{6}{7}$

**4. Question**

Write the equations of a line parallel to the line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through the point (1, -2, 3).

**Answer**

Given : A line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$

To find : equations of a line parallel to the line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through the point (1, -2, 3).

Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ then equation of parallel

line passing through the point (p,q,r) is given by $\frac{x-p}{l} = \frac{y-q}{m} = \frac{z-r}{n}$

Here l = -3 , m = 2 , n = 6 and p = 1 , q = -2 , r = 3

The line parallel to the line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through the point (1,-2,3)

is given by

$\frac{x-1}{-3}=\frac{y-(-2)}{2}=\frac{z-3}{6}$

$\frac{x-1}{-3}=\frac{y+2}{2}=\frac{z-3}{6}$

The line parallel to the line $\frac{x-2}{-3}=\frac{y+3}{2}=\frac{z+5}{6}$ and passing through the point

(1,-2,3) is given by $\frac{x-1}{-3}=\frac{y+2}{2}=\frac{z-3}{6}$

**5. Question**

Find the Cartesian equations of the line which passes through the point (-2, 4, -5) and which is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

**Answer**

Given : A line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

To find : equations of a line parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

and passing through the point (-2, 4, -5).

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ then equation of parallel

line passing through the point (p,q,r) is given by $\frac{x-p}{l}=\frac{y-q}{m}=\frac{z-r}{n}$

The given line is $\frac{x+3}{3}=\frac{y-4}{-5}=\frac{z+8}{6}$

Here l = 3 , m = -5 , n = 6 and p = -2 , q = 4 , r = -5

The line parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by

$\frac{x-(-2)}{3}=\frac{y-4}{-5}=\frac{z+5}{6}$

$\frac{x+2}{3}=\frac{y-4}{-5}=\frac{z+5}{6}$

The line parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by $\frac{x+2}{3}=\frac{y-4}{-5}=\frac{z+5}{6}$

**6. Question**

Write the vector equation of a line whose Cartesian equations are $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$.

**Answer**

Given : A line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$.

To find : vector equation of a line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$.

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}=\lambda$ then vector equation of the line is given by $\vec{r}=a\vec{\imath}+b\vec{\jmath}+c\vec{k}+\lambda(l\vec{\imath}+m\vec{\jmath}+n\vec{k})$

Here a = 5 , b = -4 , c = 6 and l = 3 , m = 7 , n = -2

Substituting the above values,we get

$\vec{r}=5\vec{\imath}-4\vec{\jmath}+6\vec{k}+\lambda(3\vec{\imath}+7\vec{\jmath}-2\vec{k})$

The vector equation of a line $\frac{x-5}{3}=\frac{y+4}{7}=\frac{6-z}{2}$. is given by

$\vec{r}=5\vec{\imath}-4\vec{\jmath}+6\vec{k}+\lambda(3\vec{\imath}+7\vec{\jmath}-2\vec{k})$

**7. Question**

The Cartesian equations of a line are $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$. Write the vector equation of the line.

**Answer**

Given : A line $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$.

To find : vector equation of a line $\frac{3-x}{5}=\frac{y+4}{7}=\frac{2z-6}{4}$.

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}=\lambda$ then vector equation of the line is given by $\vec{r}=a\vec{\imath}+b\vec{\jmath}+c\vec{k}+\lambda(l\vec{\imath}+m\vec{\jmath}+n\vec{k})$

The given line is $\frac{x-3}{-5}=\frac{y+4}{7}=\frac{z-3}{2}$

Here a = 3 , b = -4 , c = 3 and l = -5 , m = 7 , n = 2

Substituting the above values,we get

$\vec{r} = 3\vec{i} - 4\vec{j} + 3\vec{k} + \lambda(-5\vec{i} + 7\vec{j} + 2\vec{k})$

The vector equation of a line is given by $\frac{x-3}{-5} = \frac{y+4}{7} = \frac{z-3}{2}$

$\vec{r} = 3\vec{i} - 4\vec{j} + 3\vec{k} + \lambda(-5\vec{i} + 7\vec{j} + 2\vec{k})$

**8. Question**

Write the vector equation of a line passing through the point (1, -1, 2) and parallel to the line whose equations are $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.

**Answer**

Given : A line $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.

To find : vector equation of a line passing through the point (1, -1, 2) and parallel

to the line whose equations are $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$.

Formula used : If a line is parallel to $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ and passing through the point (p,q,r) then vector equation of the line is given by $\vec{r} = p\vec{i} + q\vec{j} + r\vec{k} + \lambda(l\vec{i} + m\vec{j} + n\vec{k})$

The given line is $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$

Here p = 1 , q = -1 , c = 2 and l = 1 , m = 2 , n = 2

Substituting the above values,we get

$\vec{r} = 1\vec{i} - 1\vec{j} + 2\vec{k} + \lambda(1\vec{i} + 2\vec{j} + 2\vec{k})$

The vector equation of a line passing through the point (1, -1, 2) and

parallel to the line whose equations are $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$. is given by

$\vec{r} = \vec{i} - \vec{j} + 2\vec{k} + \lambda(\vec{i} + 2\vec{j} + 2\vec{k})$

**9. Question**

If P(1, 5, 4) and Q(4, 1, -2) be two given points, find the direction ratios of PQ.

**Answer**

Given : P(1, 5, 4) and Q(4, 1, -2) be two given points

To find : direction ratios of PQ

Formula used : if $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two given points then direction

ratios of PQ is given by $x_2 - x_1$ , $y_2 - y_1$ , $z_2 - z_1$

$x_1 = 1$, $y_1 = 5$ , $z_1 = 4$ and $x_2 = 4$, $y_2 = 1$ , $z_2 = -2$

Direction ratios of PQ is given by $x_2 - x_1$ , $y_2 - y_1$ , $z_2 - z_1$

Direction ratios of PQ is given by $4 - 1, 1 - 5, -2 - 4$

Direction ratios of PQ is given by $3, -4, -6$

Direction ratios of PQ is given by $3, -4, -6$

**10. Question**

The equations of a line are $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$. Find the direction cosines of a line parallel to this line.

**Answer**

Given : A line $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$.

To find : Direction cosines of the line parallel to $\frac{4-x}{2} = \frac{y+3}{2} = \frac{z+2}{1}$.

Formula used : If a line is given by $\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$ then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}$ , $\frac{m}{\sqrt{l^2+m^2+n^2}}$ , $\frac{n}{\sqrt{l^2+m^2+n^2}}$

The line is $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$

Parallel lines have same direction ratios and direction cosines

Here l = -2 , m = 2 , n = 1

Direction cosines of the line $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$ is

$$\frac{-2}{\sqrt{(-2)^2+(2)^2+(1)^2}}, \frac{2}{\sqrt{(-2)^2+(2)^2+(1)^2}}, \frac{1}{\sqrt{(-2)^2+(2)^2+(1)^2}}$$

$$= \frac{-2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{1}{\sqrt{4+4+1}} = \frac{-2}{\sqrt{9}}, \frac{2}{\sqrt{9}}, \frac{1}{\sqrt{9}}$$

$$= \frac{-2}{3}, \frac{2}{3}, \frac{1}{3}$$

Direction cosines of the line parallel to the line $\frac{x-4}{-2} = \frac{y+3}{2} = \frac{z+2}{1}$ is

$\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}$

**11. Question**

The Cartesian equations of a line are $\frac{x-1}{2}=\frac{y+2}{3}=\frac{5-z}{1}$. Find its vector equation.

**Answer**

Given : A line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{5-z}{1}$.

To find : vector equation of a line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{5-z}{1}$.

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}=\lambda$ then vector equation of the line is given by $\vec{r} = a\vec{i} + b\vec{j} + c\vec{k} + \lambda\,(l\vec{i} + m\vec{j} + n\vec{k})$

The given line is $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-5}{-1}$

Here a = 1 , b = -2 , c = 5 and l = 2 , m = 3 , n = -1

Substituting the above values,we get

$\vec{r} = 1\vec{i} - 2\vec{j} + 5\vec{k} + \lambda\,(2\vec{i} + 3\vec{j} - 1\vec{k})$

The vector equation of a line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{5-z}{1}$. is given by

$\vec{r} = 1\vec{i} - 2\vec{j} + 5\vec{k} + \lambda\,(2\vec{i} + 3\vec{j} - 1\vec{k})$

**12. Question**

Find the vector equation of a line passing through the point (1, 2, 3) and parallel to the vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.

**Answer**

Given : A vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.

To find : vector equation of a line passing through the point (1, 2, 3) and parallel

to the vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.

Formula used : If a line is parallel to the vector $(l\vec{i} + m\vec{j} + n\vec{k})$

and passing through the point (p,q,r) then vector equation of the line is given by

$\vec{r} = p\vec{i} + q\vec{j} + r\vec{k} + \lambda\,(l\vec{i} + m\vec{j} + n\vec{k})$

Here p = 1 , q = 2 , c = 3 and l = 3 , m = 2 , n = -2

Substituting the above values,we get

$\vec{r} = 1\vec{i} + 2\vec{j} + 3\vec{k} + \lambda\,(3\vec{i} + 2\vec{j} - 2\vec{k})$

The vector equation of a line passing through the point (1, 2, 3) and

parallel to the vector $\left(3\hat{i}+2\hat{j}-2\hat{k}\right)$.is $\vec{r} = \vec{i} + 2\vec{j} + 3\vec{k} + \lambda\,(3\vec{i} + 2\vec{j} - 2\vec{k})$

**13. Question**

The vector equation of a line is $\vec{r}=\left(2\hat{i}+\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$. Find its Cartesian equation.

**Answer**

Given : The vector equation of a line is $\vec{r}=\left(2\hat{i}+\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$.

To find : Cartesian equation of the line

Formula used : If the vector equation of the line is given by

$\vec{r} = p\vec{i} + q\vec{j} + r\vec{k} + \lambda\,(l\vec{i} + m\vec{j} + n\vec{k})$ then its Cartesian equation is given by

$\frac{x-p}{l}=\frac{y-q}{m}=\frac{z-r}{n}$

The vector equation of a line is $\vec{r}=\left(2\hat{i}+\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}-\hat{k}\right)$.

Here p = 2, q = 1, r = -4 and l = 1,m = -1,n = -1

Cartesian equation is given by

$\frac{x-2}{1}=\frac{y-1}{-1}=\frac{z-(-4)}{-1}$

$\frac{x-2}{1}=\frac{y-1}{-1}=\frac{z+4}{-1}$

Cartesian equation of the line is given by $\frac{x-2}{1}=\frac{y-1}{-1}=\frac{z+4}{-1}$

**14. Question**

Find the Cartesian equation of a line which passes through the point (-2, 4, -5) and which is parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

**Answer**

Given : A line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

To find : cartesian equations of a line parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

and passing through the point (-2, 4, -5).

Formula used : If a line is given by $\frac{x-a}{l}=\frac{y-b}{m}=\frac{z-c}{n}$ then equation of parallel

line passing through the point (p,q,r) is given by $\frac{x-p}{l}=\frac{y-q}{m}=\frac{z-r}{n}$

The given line is $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$

Here l = 3 , m = 5 , n = 6 and p = -2 , q = 4 , r = -5

The line parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by

$$\frac{x-(-2)}{3}=\frac{y-4}{5}=\frac{z-(-5)}{6}$$

$$\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$$

The line parallel to the line $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$. and passing through the point

(-2,4,-5) is given by $\frac{x+2}{3}=\frac{y-4}{5}=\frac{z+5}{6}$

**15. Question**

Find the Cartesian equation of a line which passes through the point having position vector $\left(2\hat{i}-\hat{j}+4\hat{k}\right)$ and is in the direction of the vector $\left(\hat{i}+2\hat{j}-\hat{k}\right)$.

**Answer**

Given : A line which passes through the point having position vector $\left(2\hat{i}-\hat{j}+4\hat{k}\right)$

and is in the direction of the vector $\left(\hat{i}+2\hat{j}-\hat{k}\right)$.

To find : cartesian equations of a line

Formula used : If a line which passes through the point having position vector

$p\vec{i}$ + $q\vec{j}$ + $r\vec{k}$ and is in the direction of the vector $l\vec{i}$ + $m\vec{j}$ + $n\vec{k}$ then its Cartesian

equation is given by

$\frac{x-p}{l}=\frac{y-q}{m}=\frac{z-r}{n}$

A line which passes through the point having position vector $\left(2\hat{i}-\hat{j}+4\hat{k}\right)$

and is in the direction of the vector $\left(\hat{i}+2\hat{j}-\hat{k}\right)$.

Here l = 1 , m = 2 , n = -1 and p = 2 , q = -1 , r = 4

$\frac{x-2}{1}=\frac{y-(-1)}{2}=\frac{z-4}{-1}$

$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-4}{-1}$

The Cartesian equation of a line which passes through the point having

position vector $\left(2\hat{i}-\hat{j}+4\hat{k}\right)$ and is in the direction of the vector $\left(\hat{i}+2\hat{j}-\hat{k}\right)$. is

$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z-4}{-1}$

**16. Question**

Find the angle between the lines $\vec{r}=\left(2\hat{i}-5\hat{j}+\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$ and

$\vec{r}=\left(7\hat{i}-6\hat{k}\right)+\mu\left(\hat{i}+2\hat{j}+2\hat{k}\right)$.

**Answer**

Given : the lines $\vec{r}=\left(2\hat{i}-5\hat{j}+\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$ and $\vec{r}=\left(7\hat{i}-6\hat{k}\right)+\mu\left(\hat{i}+2\hat{j}+2\hat{k}\right)$.

To find : angle between the lines

Formula used : If the lines are $a\vec{i}+b\vec{j}+c\vec{k}+\lambda(p\vec{i}+q\vec{j}+r\vec{k})$ and $d\vec{i}+e\vec{j}+f\vec{k}+$

$\lambda(l\vec{i}+m\vec{j}+n\vec{k})$ then the angle between the lines 'θ' is given by

$\theta=\cos^{-1}\frac{pl+qm+rn}{\sqrt{p^2+q^2+r^2}\sqrt{l^2+m^2+n^2}}$

the lines $\vec{r}=\left(2\hat{i}-5\hat{j}+\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$ and $\vec{r}=\left(7\hat{i}-6\hat{k}\right)+\mu\left(\hat{i}+2\hat{j}+2\hat{k}\right)$.

Here p = 3 , q = 2 , r = 6 and l = 1 , m = 2 , n = 2

$\theta=\cos^{-1}\frac{3(1)+2(2)+6(2)}{\sqrt{3^2+2^2+6^2}\sqrt{1^2+2^2+2^2}}=\cos^{-1}\frac{3+4+12}{\sqrt{9+4+36}\sqrt{1+4+4}}$

$\theta=\cos^{-1}\frac{3+4+12}{\sqrt{49}\sqrt{9}}=\cos^{-1}\frac{19}{7\times 3}=\cos^{-1}\frac{19}{21}$

$\theta = \cos^{-1}\frac{19}{21}$

The angle between the lines $\vec{r} = (2\hat{i} - 5\hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = (7\hat{i} - 6\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$. is $\cos^{-1}\frac{19}{21}$

**17. Question**

Find the angle between the lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

**Answer**

Given : the lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

To find : angle between the lines

Formula used : If the lines are $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$ and $\frac{x-c}{l} = \frac{y-d}{m} = \frac{z-e}{n}$

then the angle between the lines 'θ' is given by

$\theta = \cos^{-1}\frac{pl + qm + rn}{\sqrt{p^2 + q^2 + r^2}\sqrt{l^2 + m^2 + n^2}}$

The lines are $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

Here p = 3 , q = 5 , r = 4 and l = 1 , m = 1 , n = 2

$\theta = \cos^{-1}\frac{3(1) + 5(1) + 4(2)}{\sqrt{3^2 + 5^2 + 4^2}\sqrt{1^2 + 1^2 + 2^2}} = \cos^{-1}\frac{3 + 5 + 8}{\sqrt{9 + 25 + 16}\sqrt{1 + 1 + 4}}$

$\theta = \cos^{-1}\frac{3 + 5 + 8}{\sqrt{50}\sqrt{6}} = \cos^{-1}\frac{16}{10\sqrt{3}} = \cos^{-1}\frac{8}{5\sqrt{3}}$

$\theta = \cos^{-1}\frac{8\sqrt{3}}{15}$

The angle between the lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

is $\cos^{-1}\frac{8\sqrt{3}}{15}$

**18. Question**

Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are at right angles.

**Answer**

Given : the lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$.

To prove : the lines are at right angles.

Formula used : If the lines are $\frac{x-a}{p}=\frac{y-b}{q}=\frac{z-c}{r}$ and $\frac{x-c}{l}=\frac{y-d}{m}=\frac{z-e}{n}$

then the angle between the lines 'θ' is given by

$\theta = \cos^{-1}\frac{pl+qm+rn}{\sqrt{p^2+q^2+r^2}\sqrt{l^2+m^2+n^2}}$

The lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$.

Here p = 7 , q = -5 , r = 1 and l = 1 , m = 2 , n = 3

$\theta = \cos^{-1}\frac{7(1)+(-5)(2)+1(3)}{\sqrt{7^2+(-5)^2+1}\sqrt{1^2+2^2+3^2}} = \cos^{-1}\frac{7-10+3}{\sqrt{49+25+1}\sqrt{1+4+9}}$

$\theta = \cos^{-1}\frac{0}{\sqrt{75}\sqrt{14}} = \cos^{-1}0 = 90°$

θ = 90°

The Lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ are at right angles.

**19. Question**

The direction ratios of a line are 2, 6, -9. What are its direction cosines?

**Answer**

Given : A line has direction ratios 2, 6 , -9

To find : Direction cosines of the line

Formula used : If (l,m,n) are the direction ratios of a given line then direction cosines are given by
$\frac{l}{\sqrt{l^2+m^2+n^2}}, \frac{m}{\sqrt{l^2+m^2+n^2}}, \frac{n}{\sqrt{l^2+m^2+n^2}}$

Here l = 2 , m = 6 , n = -9

Direction cosines of the line with direction ratios 2, 6 , -9 is

$\frac{2}{\sqrt{2^2+6^2+(-9)^2}}, \frac{6}{\sqrt{2^2+6^2+(-9)^2}}, \frac{-9}{\sqrt{2^2+6^2+(-9)^2}}$

$= \frac{2}{\sqrt{4+36+81}}, \frac{6}{\sqrt{4+36+81}}, \frac{-9}{\sqrt{4+36+81}} = \frac{2}{\sqrt{121}}, \frac{6}{\sqrt{121}}, \frac{-9}{\sqrt{121}}$

$= \frac{2}{11}, \frac{6}{11}, \frac{-9}{11}$

Direction cosines of the line with direction ratios 2, 6, -9 is $\frac{2}{11}, \frac{6}{11}, \frac{-9}{11}$

**20. Question**

A line makes angles $90^o$, $135^o$ and $45^o$ with the positive directions of x-axis, y-axis and z-axis respectively. what are the direction cosines of the line?

**Answer**

Given : A line makes angles $90^o$, $135^o$ and $45^o$ with the positive directions of x-axis, y-axis and z-axis respectively.

To find : Direction cosines of the line

Formula used : If a line makes angles $\alpha^o$, $\beta^o$ and $\gamma^o$ with the positive directions of x-axis, y-axis and z-axis respectively. then direction cosines are given by $\cos\alpha$ , $\cos(180° - \beta)$ , $\cos(180° - \gamma)$

$\alpha = 90^o$, $\beta = 135^o$ and $\gamma = 45^o$

Direction cosines of the line is

$\cos 90°$ , $\cos(180° - 135°)$ , $\cos(180° - 45°)$

$\cos 90°$ , $\cos 45°$ , $\cos(135°)$

$0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Direction cosines of the line is $0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

**21. Question**

What are the direction cosines of the y-axis?

**Answer**

To find : Direction cosines of the y- axis

Formula used : If a line makes angles $\alpha^o$, $\beta^o$ and $\gamma^o$ with the positive directions of x-axis, y-axis and z-axis respectively. then direction cosines are given by $\cos\alpha$ , $\cos\beta$ , $\cos\gamma$

y-axis makes $90^o$ with the x and z axes

$\alpha = 90^o$, $\beta = 0^o$ and $\gamma = 90^o$

Direction cosines of the line is

$\cos 90°$ , $\cos 0°$ , $\cos 90°$

0, 1, 0

Direction cosines of the line is 0, 1 , 0

**22. Question**

What are the direction cosines of the vector $\left(2\hat{i}+\hat{j}-2\hat{k}\right)$?

**Answer**

Given : A vector $\left(2\hat{i}+\hat{j}-2\hat{k}\right)$?

To find : Direction cosines of the vector

Formula used : If a vector is $l\vec{i}$ + $m\vec{j}$ + $n\vec{k}$ then direction cosines are given by $\frac{l}{\sqrt{l^2+m^2+n^2}}$, $\frac{m}{\sqrt{l^2+m^2+n^2}}$, $\frac{n}{\sqrt{l^2+m^2+n^2}}$

Here l = 2 , m = 1 , n = -2

Direction cosines of the line with direction ratios 2, 1, -2 is

$$\frac{2}{\sqrt{2^2+(1)^2+(-2)^2}}, \frac{1}{\sqrt{2^2+(1)^2+(-2)^2}}, \frac{-2}{\sqrt{2^2+(1)^2+(-2)^2}}$$

$$= \frac{2}{\sqrt{4+1+4}}, \frac{1}{\sqrt{4+1+4}}, \frac{-2}{\sqrt{4+1+4}} = \frac{2}{\sqrt{9}}, \frac{1}{\sqrt{9}}, \frac{-2}{\sqrt{9}}$$

$$= \frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$$

Direction cosines of the vector is $\frac{2}{3}, \frac{1}{3}, \frac{-2}{3}$

**23. Question**

What is the angle between the vector $\vec{r}=\left(4\hat{i}+8\hat{j}+\hat{k}\right)$ and the x-axis?

**Answer**

Given : the vector $\vec{r}=\left(4\hat{i}+8\hat{j}+\hat{k}\right)$

To find : angle between the vector and the x-axis

Formula used : If the vector $l\vec{i}$ + $m\vec{j}$ + $n\vec{k}$ and x-axis then the angle between the

lines 'θ' is given by

$$\theta = \cos^{-1}\frac{l}{\sqrt{l^2+m^2+n^2}}$$

Here l = 4 , m = 8 , n = 1

$$\theta = \cos^{-1}\frac{4}{\sqrt{4^2+8^2+1^2}} = \cos^{-1}\frac{4}{\sqrt{16+64+1}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{81}} = \cos^{-1}\frac{4}{9}$$

θ = $\cos^{-1}\frac{4}{9}$

The angle between the vector and the x-axis is $\cos^{-1}\frac{4}{9}$

## Objective Questions

**1. Question**

The direction ratios of two lines are 3, 2, -6 and 1, 2, 2, respectively. The acute angle between these lines is

A. $\cos^{-1}\left(\frac{5}{18}\right)$

B. $\cos^{-1}\left(\frac{3}{20}\right)$

C. $\cos^{-1}\left(\frac{5}{21}\right)$

D. $\cos^{-1}\left(\frac{8}{21}\right)$

**Answer**

Direction ratio are given implies that we can write the parallel vector towards that line, lets consider the first parallel vector to be $|\vec{a}| = 3\hat{i} + 2\hat{j} - 6\hat{k}$ and second parallel vector be $|\vec{b}| = \hat{i} + 2\,\hat{j} + 2\hat{k}$.

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}| = \sqrt{3^2 + 2^2 + (-6)^2}$

$= 7$

$|\vec{b}| = \sqrt{1 + 2^2 + 2^2}$

$= 3$

$$\cos\alpha = \frac{(3\hat{i} + 2\hat{j} - 6\hat{k}).(\hat{i} + 2\,\hat{j} + 2\hat{k})}{7 \times 3}$$

$$\cos\alpha = \frac{3 + 4 - 12}{21}$$

$\cos\alpha = \frac{-5}{21}$

$\alpha = \cos^{-1}\left(-\frac{5}{21}\right)$

The negative sign does not affect anything in cosine as cosine is positive in the fourth quadrant.

$\alpha = \cos^{-1}\left(\frac{5}{21}\right)$

**2. Question**

The direction ratios of two lines are a, b, c and (b − c), (c − a), (a − b) respectively. The angle between these lines is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. $\frac{3\pi}{4}$

**Answer**

Direction ratio are given implies that we can write the parallel vector towards that line, lets consider the first parallel vector to be $|\vec{a}| = a\hat{i} + b\hat{j} + c\hat{k}$ and second parallel vector be $|\vec{b}| = (b-c)\hat{i} + (c-a)\,\hat{j} + (a-b)\hat{k}.$

For the angle, we can use the formula $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}| = \sqrt{a^2 + b^2 + (c)^2}$

$= \sqrt{a^2 + b^2 + c^2}$

$|\vec{b}| = \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}$

$= \sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)}$

$\cos\alpha = \frac{(a\hat{i} + b\hat{j} + c\hat{k}).((b-c)\hat{i} + (c-a)\,\hat{j} + (a-b)\hat{k})}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$

$$\cos\alpha = \frac{ab - ac + bc - ba + ca - cb}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$$

$$\cos\alpha = \frac{0}{\sqrt{2(a^2 + b^2 + c^2 - ab - bc - ca)} \times \sqrt{a^2 + b^2 + c^2}}$$

$$\alpha = \cos^{-1}(0)$$

$$\alpha = \frac{\pi}{2}$$

**3. Question**

The angle between the lines $\frac{x-2}{2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{2} = \frac{z-5}{4}$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\cos^{-1}\left(\frac{3}{8}\right)$

**Answer**

Direction ratio are given implies that we can write the parallel vector towards those line, lets consider first parallel vector to be $|\vec{a}| = 2\hat{\imath} + 7\hat{\jmath} - 3\hat{k}$ and second parallel vector be $|\vec{b}| = -\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$.

For the angle we can use the formula $\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}| \times |\vec{b}|}$

For that we need to find magnitude of these vectors

$$|\vec{a}| = \sqrt{3^2 + 2^2 + (7)^2}$$

$$= \sqrt{62}$$

$$|\vec{b}| = \sqrt{1 + 2^2 + 4^2}$$

$$= \sqrt{21}$$

$$\cos\alpha = \frac{(2\hat{i} + 7\hat{j} - 3\hat{k}).(-\hat{i} + 2\hat{j} + 4\hat{k})}{\sqrt{21} \times \sqrt{62}}$$

$$\cos\alpha = \frac{-2 + 14 - 12}{\sqrt{21} \times \sqrt{62}}$$

$$\cos\alpha = \frac{0}{\sqrt{21} \times \sqrt{62}}$$

$$\alpha = \cos^{-1} 0$$

Negative sign does not affect anything in cosine as cosine is positive in fourth quadrant

$$\alpha = \frac{\pi}{2}$$

**4. Question**

If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular to each other then k = ?

A. $\frac{-5}{7}$

B. $\frac{5}{7}$

C. $\frac{10}{7}$

D. $\frac{-10}{7}$

**Answer**

If the lines are perpendicular to each other then the angle between these lines will be $\frac{\pi}{2}$, me the cosine will be 0

$$\vec{a} = -3\hat{i} + 2k\hat{j} + 2\hat{k}$$

$$|\vec{a}| = \sqrt{3^2 + (2k)^2 + 2^2}$$

$$= \sqrt{13 + 4k^2}$$

$$\vec{b} = 3k\hat{i} + \hat{j} - 5\hat{k}$$

$|\vec{b}| = \sqrt{(3k)^2 + 1 + 5^2}$

$= \sqrt{9k^2 + 26}$

$$\cos\left(\frac{\pi}{2}\right) = \frac{(3k\hat{i} + \hat{j} - 5\hat{k}).(-3\hat{i} + 2k\hat{j} + 2\hat{k})}{\sqrt{13 + 4k^2} \times \sqrt{9k^2 + 26}}$$

$$0 = \frac{-9k + 2k - 10}{\sqrt{13 + 4k^2} \times \sqrt{9k^2 + 26}}$$

$$k = -\frac{10}{7}$$

**5. Question**

A line passes through the points A(2, -1, 4) and B(1, 2, -2). The equations of the line AB are

A. $\frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-4}{-6}$

B. $\frac{x+2}{-1} = \frac{y+1}{2} = \frac{z-4}{6}$

C. $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-4}{6}$

D. none of these

**Answer**

To write the equation of a line we need a parallel vector and a fixed point through which the line is passing

Parallel vector=$((2-1)\hat{i} + (-1-2)\hat{j} + (4+2)\hat{k})$

$= \hat{i} - 3\hat{j} + 6\hat{k}$

Or = $-(\hat{i} - 3\hat{j} + 6\hat{k})$

Fixed point is $2\hat{i} - \hat{j} + 4\hat{k}$

Equation

$$\frac{x-2}{1} = \frac{y-(-1)}{-3} = \frac{z-4}{6}$$

$$\frac{x-2}{1} = \frac{y+1}{-3} = \frac{z-4}{6}$$

Or

$$\frac{x-2}{-1}=\frac{y+1}{3}=\frac{z-4}{-6}$$

**6. Question**

The angle between the lines $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$ is

A. $\cos^{-1}\left(\frac{3}{4}\right)$

B. $\cos^{-1}\left(\frac{5}{6}\right)$

C. $\cos^{-1}\left(\frac{2}{3}\right)$

D. $\frac{\pi}{3}$

**Answer**

Direction cosine of the lines are given 2î +2ĵ +k^ and 4î +ĵ +8k^

$$\vec{a}=2\hat{i}+2\hat{j}+\hat{k}$$

$$|\vec{a}|=\sqrt{2^2+2^2+1}$$

$$|\vec{a}|=3$$

$$\vec{b}=4\hat{i}+\hat{j}+8\hat{k}$$

$$|\vec{b}|=\sqrt{4^2+1+8^2}$$

$$=9$$

$$\cos\alpha=\frac{\vec{a}.\vec{b}}{|\vec{a}|\times|\vec{b}|}$$

$$\cos\alpha=\frac{(2\hat{i}+2\hat{j}+\hat{k}).(4\hat{i}+\hat{j}+8\hat{k})}{3\times9}$$

$$\cos\alpha=\frac{8+8+2}{27}$$

$$\cos\alpha=\frac{2}{3}$$

**7. Question**

The angle between the lines $\vec{r} = \left(3\hat{i} + \hat{j} - 2\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j} - 5\hat{k}\right) + \mu\left(3\hat{i} - 5\hat{j} - 4\hat{k}\right)$ is

A. $\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$

B. $\cos^{-1}\left(\frac{6\sqrt{2}}{5}\right)$

C. $\cos^{-1}\left(\frac{5\sqrt{3}}{8}\right)$

D. $\cos^{-1}\left(\frac{5\sqrt{2}}{6}\right)$

**Answer**

Let $\vec{a} = \hat{\imath} - \hat{\jmath} - 2\hat{k}$ and $\vec{b} = 3\hat{\imath} - 5\hat{\jmath} - 4\hat{k}$ and $|\vec{a}| = \sqrt{1 + 1 + 2^2} = \sqrt{6}$

$|\vec{b}| = \sqrt{3^2 + 5^2 + 4^2} = 5\sqrt{2}$

$$\cos\alpha = \frac{\vec{a}.\vec{b}}{|\vec{a}| \times |\vec{b}|}$$

$$\cos\alpha = \frac{(3\hat{\imath} - 5\hat{\jmath} - 4\hat{k}).(\hat{\imath} - \hat{\jmath} - 2\hat{k})}{5\sqrt{2} \times \sqrt{6}}$$

$$\cos\alpha = \frac{3 + 5 + 8}{5\sqrt{12}}$$

$$\cos\alpha = \frac{8\sqrt{3}}{15}$$

**8. Question**

A line is perpendicular to two lines having direction ratios 1, -2, -2 and 0, 2, 1. The direction cosines of the line are

A. $\frac{-2}{3}, \frac{1}{3}, \frac{2}{3}$

B. $\frac{2}{3}, \frac{1}{3}, \frac{-1}{3}$

C. $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$

D. none of these

**Answer**

If a line is perpendicular to two given lines we can find out the parallel vector by cross product of the given two vectors.

$\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{b} = 2\hat{j} + \hat{k}$

$\vec{a} \times \vec{b} = (\hat{i} - 2\hat{j} - 2\hat{k}) \times (2\hat{j} + \hat{k})$

$= 2\hat{i} - \hat{j} + 2\hat{k}$

So the direction cosines are

$\hat{n} = \frac{1}{\sqrt{2^2 + 1 + 2^2}}$

$\hat{n} = \frac{1}{3}$

Direction cosine

$\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

**9. Question**

A line passes through the point A(5, -2, 4) and it is parallel to the vector $(2\hat{i} - \hat{j} + 3\hat{k})$. The vector equation of the line is

A. $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(5\hat{i} - 2\hat{j} + 4\hat{k})$

B. $\vec{r} = (5\hat{i} - 2\hat{j} + 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$

C. $\vec{r} \cdot (5\hat{i} - 2\hat{j} + 4\hat{k}) = \sqrt{14}$

D. none of these

**Answer**

Fixed point is $5\hat{i} - 2\hat{j} + 4\hat{k}$ and parallel vector is $2\hat{i} - \hat{j} + 3\hat{k}$

Equation $5\hat{i} - 2\hat{j} + 4\hat{k} + \alpha(2\hat{i} - \hat{j} + 3\hat{k})$

**10. Question**

The Cartesian equations of a line are $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-5}{-1}$. Its vector equation is

A. $\vec{r} = \left(-\hat{i} + 2\hat{j} - 5\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - \hat{k}\right)$

B. $\vec{r} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} + 5\hat{k}\right)$

C. $\vec{r} = \left(\hat{i} - 2\hat{j} + 5\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 4\hat{k}\right)$

D. none of these

**Answer**

Dixed point (1,-2,5) and the parallel vector is $2\hat{i} + 3\hat{j} - \hat{k}$

Equation ($\hat{i}$ -2$\hat{j}$ +5k$\hat{}$ )+α $(2\hat{i} + 3\hat{j} - \hat{k})$

**11. Question**

A line passes through the pointA(-2, 4, -5) and is parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$. The vector equation of the line is

A. $\vec{r} = \left(-3\hat{i} + 4\hat{j} - 8\hat{k}\right) + \lambda\left(-2\hat{i} + 4\hat{j} - 5\hat{k}\right)$

B. $r = \left(-2\hat{i} + 4\hat{j} - 5\hat{k}\right) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k})$

C. $\vec{r} = \left(3\hat{i} + 5\hat{j} + 6\hat{k}\right) + \lambda\left(-2\hat{i} + 4\hat{j} - 5\hat{k}\right)$

D. none of these

**Answer**

Fixed point is $-2\hat{i} + 4\hat{j} - 5\hat{k}$ and the parallel vector is $3\hat{i} + 5\hat{j} + 6\hat{k}$

Equation is $r = \left(-2\hat{i} + 4\hat{j} - 5\hat{k}\right) + \lambda(3\hat{i} + 5\hat{j} + 6\hat{k})$

**12. Question**

The coordinates of the point where the line through the points A(5, 1, 6) and B(3, 4, 1) crosses the yz-plane is

A. (0, 17, -13)

B. $\left(0, \frac{-17}{2}, \frac{13}{2}\right)$

C. $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$

D. none of these

**Answer**

We first need to find the equation of a line passing through the two given points

taking fixed point as $5\hat{i} + \hat{j} + 6\hat{k}$

and the parallel vector will be $(5-3)\hat{i} + (1-4)\hat{j} + (6-1)\hat{k} = 2\hat{i} - 3\hat{j} + 5\hat{k}$

equation of the line in cartesian form

$$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5}$$

Assume above equation to be equal to k, a constant

$$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = k$$

And y-z plane have x-coordinate as zero we may get

$$\frac{0-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = k$$

$$k = -\frac{5}{2}$$

Now we can find y and z

$$\frac{y-1}{-3} = -\frac{5}{2}$$

$$y - 1 = \frac{15}{2}$$

$$y = \frac{17}{2}$$

$$\frac{z-6}{5} = -\frac{5}{2}$$

$$z - 6 = -\frac{25}{2}$$

$z = -\frac{13}{2}$

The coordinate where the line meets y-z plane is $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$

**13. Question**

The vector equation of the x-axis is given by

A. $\vec{r} = \hat{i}$

B. $\vec{r} = \hat{j} + \hat{k}$

C. $\vec{r} = \lambda \hat{i}$

D. none of these

**Answer**

Vector equation need a fixed point and a parallel vector

For x-axis fixed point can be anything ranging from negative to positive including origin

And parallel vector is $\hat{i}$

Equation would be $\lambda\, \hat{i}$

**14. Question**

The Cartesian equations of a lines are $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z-3}{-2}$. What is its vector equation?

A. $\vec{r} = \left(2\hat{i} + 3\hat{j} - 2\hat{k}\right) + \lambda\left(2\hat{i} - \hat{j} + 3\hat{k}\right)$

B. $\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$

C. $\vec{r} = \left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$

D. none of these

**Answer**

Fixed point is $2\hat{i} - \hat{j} + 3\hat{k}$ and the vector is $2\hat{i} + 3\hat{j} - 2\hat{k}$

Equation $\left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right)$

**15. Question**

The angle between two lines having direction ratios 1, 1, 2 and $\left(\sqrt{3} - 1\right)$, $\left(-\sqrt{3} - 1\right)$, 4 is

A. $\frac{\pi}{6}$

B. □$\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

**Answer**

Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = (\sqrt{3} - 1)\hat{i} + (-\sqrt{3} - 1)\hat{j} + 4\hat{k}$

$|\vec{a}| = \sqrt{6}\ |\vec{b}| = \sqrt{(4 - 2\sqrt{3}) + (4 + 2\sqrt{3}) + 16} = 2\sqrt{6}$

$$\cos\alpha = \frac{(\hat{i} + \hat{j} + 2\hat{k}).\left((\sqrt{3} - 1)\hat{i} + (-\sqrt{3} - 1)\hat{j} + 4\hat{k}\right)}{\sqrt{6} \times 2\sqrt{6}}$$

$$\cos\alpha = \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{12}$$

$$\cos\alpha = \frac{1}{2}$$

$\alpha = 60°$

**16. Question**

The straight line $\frac{x-2}{3} = \frac{y-3}{1} = \frac{z+1}{0}$ is

A. parallel to the x-axis

B. parallel to the y-axis

C. parallel to the z-axis

D. perpendicular to the z-axis

**Answer**

It is perpendicular to z-axis because cos90° is 0 which implies that it makes 90° with z-axis

**17. Question**

If a line makes angles α, β and γ with the x-axis, y-axis and z-axis respectively then ($\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$) = ?

A. 1

B. 3

C. 2

D. $\frac{3}{2}$

**Answer**

$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 1 - \cos^2 \alpha + 1 - \cos \beta + 1 - \cos^2 \gamma$

$= 3 - (\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma)$

$(\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma)$ is the square of the direction ratios of all three axes which is always equal to 1

$= 3 - 1$

$= 2$

**18. Question**

If ($a_1$, $b_1$, $c_1$) and ($a_2$, $b_2$, $c_2$) be the direction ratios of two parallel lines then

A. $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$

B. $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

C. $a_1^2 + b_1^2 + c_1^2 = a_2^2 + b_2^2 + c_2^2$

D. $a_1a_2 + b_1b_2 + c_1c_2 = 0$

**Answer**

We know that if there is two parallel lines then their direction ratios must have a relation

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

**19. Question**

If the points A(-1, 3, 2), B(-4, 2, -2) and C(5, 5, λ) are collinear then the value of λ is

A. 5

B. 7

C. 8

D. 10

**Answer**

Determinant of these point should be zero

$$\begin{vmatrix} -1 & 3 & 2 \\ -4 & 2 & -2 \\ 5 & 5 & \lambda \end{vmatrix} = 0$$

$$-1(2\lambda + 10) - 3(-4\lambda + 10) + 2(-20 - 10) = 0$$

$$10\lambda - 10 - 30 - 60 = 0$$

$$\lambda = 10$$