28. The Plane

Exercise 28A

1. Question

Find the equation of the plane passing through each group of points:

(i) A(2, 2, -1), B(3, 4, 2) and C(7, 0, 6)

(ii) A(0, -1, -1), B(4, 5, 1) and C(3, 9, 4)

(iii) A(-2, 6, -6), B(-3, 10, 9) and

Answer

(i) A(2, 2, -1), B(3, 4, 2) and C(7, 0, 6)

Given Points :

A = (2, 2, -1)

B = (3, 4, 2)

C = (7, 0, 6)

To Find : Equation of plane passing through points A, B & C

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Vector :

If A and B be two points with position vectors $\bar{a}$ & $\bar{b}$ respectively, where

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\overline{AB} = \bar{b} - \bar{a}$

$= (b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

5) Equation of Plane :

If A = (a_1, a_2, a_3), B = (b_1, b_2, b_3), C = (c_1, c_2, c_3) are three non-collinear points,

Then, the vector equation of the plane passing through these points is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (2, 2, -1)

B = (3, 4, 2)

C = (7, 0, 6)

Position vectors are given by,

$\bar{a} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\bar{b} = 3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$

$\bar{c} = 7\hat{\imath} + 6\hat{k}$

Now, vectors $\overline{AB}$ & $\overline{AC}$ are

$\overline{AB} = \bar{b} - \bar{a}$

$= (3-2)\hat{\imath} + (4-2)\hat{\jmath} + (2+1)\hat{k}$

$\therefore \overline{AB} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\overline{AC} = \bar{c} - \bar{a}$

$= (7-2)\hat{\imath} + (0-2)\hat{\jmath} + (6+1)\hat{k}$

$\therefore \overline{AC} = 5\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$

Therefore,

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix}$$

$= \hat{\imath}(2 \times 7 - (-2) \times 3) - \hat{\jmath}(1 \times 7 - 5 \times 3) + \hat{k}(1 \times (-2) - 5 \times 2)$

$= 20\hat{\imath} + 8\hat{\jmath} - 12\hat{k}$

Now,

$\bar{a}.(\overline{AB} \times \overline{AC}) = (2 \times 20) + (2 \times 8) + ((-1) \times (-12))$

= 40 + 16 + 12

= 68

$\therefore \bar{a}.(\overline{AB} \times \overline{AC}) = 68$ ....eq(1)

And

$\bar{r}.(\overline{AB} \times \overline{AC}) = (x \times 20) + (y \times 8) + (z \times (-12))$

= 20x + 8y – 12z

$\therefore \bar{r}.(\overline{AB} \times \overline{AC}) = 20x + 8y - 12z$ ....eq(2)

Vector equation of the plane passing through points A, B & C is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

From eq(1) and eq(2)

20x + 8y – 12z = 68

This is 5x + 2y – 3z = 17 vector equation of required plane.

(ii) Given Points :

A = (0, -1, -1)

B = (4, 5, 1)

C = (3, 9, 4)

To Find : Equation of plane passing through points A, B & C

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Vector :

If A and B be two points with position vectors $\bar{a}$ & $\bar{b}$ respectively, where

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\overline{AB} = \bar{b} - \bar{a}$

$= (b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

5) Equation of Plane :

If A = (a_1, a_2, a_3), B = (b_1, b_2, b_3), C = (c_1, c_2, c_3) are three non-collinear points,

Then, vector equation of the plane passing through these points is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (0, -1, -1)

B = (4, 5, 1)

C = (3, 9, 4)

Position vectors are given by,

$\bar{a} = -\hat{\jmath} - \hat{k}$

$\bar{b} = 4\hat{\imath} + 5\hat{\jmath} + \hat{k}$

$\bar{c} = 3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}$

Now, vectors $\overline{AB}$ & $\overline{AC}$ are

$\overline{AB} = \bar{b} - \bar{a}$

$= (4 - 0)\hat{\imath} + (5 + 1)\hat{\jmath} + (1 + 1)\hat{k}$

$\therefore \overline{AB} = 4\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$

$\overline{AC} = \bar{c} - \bar{a}$

$= (3 - 0)\hat{\imath} + (9 + 1)\hat{\jmath} + (4 + 1)\hat{k}$

$\therefore \overline{AC} = 3\hat{\imath} + 10\hat{\jmath} + 5\hat{k}$

Therefore,

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 6 & 2 \\ 3 & 10 & 5 \end{vmatrix}$$

$= \hat{\imath}(6 \times 5 - 10 \times 2) - \hat{\jmath}(4 \times 5 - 2 \times 3) + \hat{k}(4 \times 10 - 3 \times 6)$

$= 10\hat{\imath} - 14\hat{\jmath} + 22\hat{k}$

Now,

$\bar{a}.(\overline{AB} \times \overline{AC}) = (0 \times 10) + ((-1) \times (-14)) + ((-1) \times 22)$

= 0 + 14 – 22

= - 8

$\therefore \bar{a}.(\overline{AB} \times \overline{AC}) = -8$ ....eq(1)

And

$\bar{r}.(\overline{AB} \times \overline{AC}) = (x \times 10) + (y \times (-14)) + (z \times 22)$

= 10x - 14y + 22z

$\therefore \bar{r}.(\overline{AB} \times \overline{AC}) = 10x - 14y + 22z$ ....eq(2)

Vector equation of plane passing through points A, B & C is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

From eq(1) and eq(2)

10x - 14y + 22z = - 8

This is 5x - 7y + 11z = - 4 vector equation of required plane

(iii) Given Points :

A = (-2, 6, -6)

B = (-3, 10, 9)

C = (-5, 0, -6)

To Find : Equation of plane passing through points A, B & C

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Vector :

If A and B be two points with position vectors $\bar{a}$ & $\bar{b}$ respectively, where

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\overline{AB} = \bar{b} - \bar{a}$

$= (b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

5) Equation of Plane :

If A = (a_1, a_2, a_3), B = (b_1, b_2, b_3), C = (c_1, c_2, c_3) are three non-collinear points,

Then, vector equation of the plane passing through these points is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (-2, 6, -6)

B = (-3, 10, 9)

C = (-5, 0, -6)

Position vectors are given by,

$\bar{a} = -2\hat{\imath} + 6\hat{\jmath} - 6\hat{k}$

$\bar{b} = -3\hat{\imath} + 10\hat{\jmath} + 9\hat{k}$

$\bar{c} = -5\hat{\imath} - 6\hat{k}$

Now, vectors $\overline{AB}$ & $\overline{AC}$ are

$\overline{AB} = \bar{b} - \bar{a}$

$= (-3+2)\hat{\imath} + (10-6)\hat{\jmath} + (9+6)\hat{k}$

$\therefore \overline{AB} = -\hat{\imath} + 4\hat{\jmath} + 15\hat{k}$

$\overline{AC} = \bar{c} - \bar{a}$

$= (-5+2)\hat{\imath} + (0-6)\hat{\jmath} + (-6+6)\hat{k}$

$\therefore \overline{AC} = -3\hat{\imath} - 6\hat{\jmath} + 0\hat{k}$

Therefore,

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 4 & 15 \\ -3 & -6 & 0 \end{vmatrix}$$

$$= \hat{\imath}(4 \times 0 - (-6) \times 15) - \hat{\jmath}((-1) \times 0 - (-3) \times 15) + \hat{k}((-1) \times (-6) - (-3) \times 4)$$

$= 90\hat{\imath} - 45\hat{\jmath} + 18\hat{k}$

Now,

$\bar{a}.(\overline{AB} \times \overline{AC}) = ((-2) \times 90) + (6 \times (-45)) + ((-6) \times 18)$

= - 180 - 270 – 108

= - 558

$\therefore \bar{a}.(\overline{AB} \times \overline{AC}) = -558$ ....eq(1)

And

$\bar{r}.(\overline{AB} \times \overline{AC}) = (x \times 90) + (y \times (-45)) + (z \times 18)$

= 90x - 45y + 18z

$\therefore \bar{r}.(\overline{AB} \times \overline{AC}) = 90x - 45y + 18z$ ....eq(2)

Vector equation of plane passing through points A, B & C is

$\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$

From eq(1) and eq(2)

90x - 45y + 18z = - 558

This is 10x - 5y + 2z = - 62 vector equation of required plane

2. Question

Show that the four points A(3, 2, -5),B(-1, 4, -3), C(-3, 8, -5) and D(-3, 2, 1) are coplanar. Find the equation of the plane containing them.

Answer

Given Points :

A = (3, 2, -5)

B = (-1, 4, -3)

C = (-3, 8, -5)

D = (-3, 2, 1)

To Prove : Points A, B, C & D are coplanar.

To Find : Equation of plane passing through points A, B, C & D.

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Equation of line

If A and B are two points having position vectors $\bar{a}$ & $\bar{b}$ then equation of line passing through two points is given by,

$\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

5) Coplanarity of two lines :

If two lines $\bar{r_1} = \bar{a} + \lambda\bar{b}$ & $\bar{r_2} = \bar{c} + \mu\bar{d}$ are coplanar then

$\bar{a}.(\bar{b} \times \bar{d}) = \bar{c}.(\bar{b} \times \bar{d})$

6) Equation of plane :

If two lines $\bar{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\bar{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (3, 2, -5)

B = (-1, 4, -3)

C = (-3, 8, -5)

D = (-3, 2, 1)

Position vectors are given by,

$\bar{a} = 3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$

$\bar{b} = -1\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$

$\bar{c} = -3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}$

$\bar{d} = -3\hat{\imath} + 2\hat{\jmath} + \hat{k}$

Equation of line passing through points A & B is

$\bar{r_1} = \bar{a} + \lambda(\bar{b} - \bar{a})$

$\bar{b} - \bar{a} = (-1-3)\hat{\imath} + (4-2)\hat{\jmath} + (-3+5)\hat{k}$

$= -4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

$\therefore \bar{r_1} = (3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}) + \lambda(-4\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

Let, $\overline{r_1} = \overline{a_1} + \lambda b_1$

Where,

$\overline{a_1} = 3\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ & $b_1 = -4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

And the equation of the line passing through points C & D is

$\overline{r_2} = \bar{c} + \mu(\bar{d} - \bar{c})$

$\bar{d} - \bar{c} = (-3+3)\hat{\imath} + (2-8)\hat{\jmath} + (1+5)\hat{k}$

$= -6\hat{\jmath} + 6\hat{k}$

$\therefore \overline{r_1} = (-3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}) + \lambda(-6\hat{\jmath} + 6\hat{k})$

Let, $\overline{r_2} = \overline{a_2} + \lambda b_2$

Where,

$\overline{a_2} = -3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}$ & $b_2 = -6\hat{\jmath} + 6\hat{k}$

Now,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -4 & 2 & 2 \\ 0 & -6 & 6 \end{vmatrix}$$

$= \hat{\imath}(12+12) - \hat{\jmath}(-24-0) + \hat{k}(24+0)$

$\therefore (\overline{b_1} \times \overline{b_2}) = 24\hat{\imath} + 24\hat{\jmath} + 24\hat{k}$

Therefore,

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = (3 \times 24) + (2 \times 24) + ((-5) \times 24)$

= 72 + 48 – 120

= 0

$\therefore \overline{a_1}.(\overline{b_1} \times \overline{b_2}) = 0$ .... eq(1)

And

$\overline{a_2}.(\overline{b_1} \times \overline{b_2}) = ((-3) \times 24) + (8 \times 24) + ((-5) \times 24)$

= - 72 + 192 – 120

= 0

$\therefore \overline{a_2}.(\overline{b_1} \times \overline{b_2}) = 0$ .... eq(2)

From eq(1) and eq(2)

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = \overline{a_2}.(\overline{b_1} \times \overline{b_2})$

Hence lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar

Therefore, points A, B, C & D are also coplanar.

As lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar therefore equation of the plane passing through two lines containing four given points is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Now,

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = (x \times 24) + (y \times 24) + (z \times 24)$

= 24x + 24y + 24z

From eq(1)

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = 0$

Therefore, equation of required plane is

24x + 24y + 24z = 0

x + y + z = 0

3. Question

Show that the four points A(0, -1, 0),B(2, 1, -1), C(1, 1, 1) and D(3, 3, 0) are coplanar. Find the equation of the plane containing them.

Answer

Given Points :

A = (0, -1, 0)

B = (2, 1, -1)

C = (1, 1, 1)

D = (3, 3, 0)

To Prove : Points A, B, C & D are coplanar.

To Find : Equation of plane passing through points A, B, C & D.

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Equation of line

If A and B are two points having position vectors $\bar{a}$ & $\bar{b}$ then equation of line passing through two points is given by,

$\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

4) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

5) Coplanarity of two lines :

If two lines $\overline{r_1} = \bar{a} + \lambda\bar{b}$ & $\overline{r_2} = \bar{c} + \mu\bar{d}$ are coplanar then

$\bar{a}.(\bar{b} \times \bar{d}) = \bar{c}.(\bar{b} \times \bar{d})$

6) Equation of plane :

If two lines $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given points,

A = (0, -1, 0)

B = (2, 1, -1)

C = (1, 1, 1)

D = (3, 3, 0)

Position vectors are given by,

$\bar{a} = -\hat{\jmath}$

$\bar{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\bar{c} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\bar{d} = 3\hat{\imath} + 3\hat{\jmath}$

Equation of line passing through points A & D is

$\overline{r_1} = \bar{a} + \lambda(\bar{d} - \bar{a})$

$\bar{d} - \bar{a} = (3-0)\hat{\imath} + (3+1)\hat{\jmath} + (0-0)\hat{k}$

$= 3\hat{\imath} + 4\hat{\jmath}$

$\therefore \overline{r_1} = (-\hat{\jmath}) + \lambda(3\hat{\imath} + 4\hat{\jmath})$

Let, $\overline{r_1} = \overline{a_1} + \lambda b_1$

Where,

$\overline{a_1} = -\hat{\jmath}$ & $b_1 = 3\hat{\imath} + 4\hat{\jmath}$

And equation of line passing through points B & C is

$\overline{r_2} = \bar{b} + \mu(\bar{c} - \bar{b})$

$\bar{c} - \bar{b} = (1-2)\hat{\imath} + (1-1)\hat{\jmath} + (1+1)\hat{k}$

$= -\hat{\imath} + 0\hat{\jmath} + 2\hat{k}$

$\therefore \overline{r_1} = (2\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(-\hat{\imath} + 2\hat{k})$

Let, $\overline{r_2} = \overline{a_2} + \lambda b_2$

Where,

$\overline{a_2} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ & $b_2 = -\hat{\imath} + 2\hat{k}$

Now,

$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 4 & 0 \\ -1 & 0 & 2 \end{vmatrix}$

$= \hat{\imath}(8-0) - \hat{\jmath}(6-0) + \hat{k}(0+4)$

$\therefore (\overline{b_1} \times \overline{b_2}) = 8\hat{\imath} - 6\hat{\jmath} + 4\hat{k}$

Therefore,

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = (0 \times 8) + ((-1) \times (-6)) + (0 \times 4)$

= 0 + 6 + 0

= 6

$\therefore \overline{a_1}.(\overline{b_1} \times \overline{b_2}) = 6$ .... eq(1)

And

$\overline{a_2}.(\overline{b_1} \times \overline{b_2}) = (2 \times 8) + (1 \times (-6)) + ((-1) \times 4)$

= 16 – 6 – 4

= 6

$\therefore \overline{a_2}.(\bar{b} \times \bar{d}) = 6$ .... eq(2)

From eq(1) and eq(2)

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = \overline{a_2}.(\overline{b_1} \times \overline{b_2})$

Hence lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar

Therefore, points A, B, C & D are also coplanar.

As lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar therefore equation of the plane passing through two lines containing four given points is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Now,

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = (x \times 8) + (y \times (-6)) + (z \times 4)$

= 8x - 6y + 4z

From eq(1)

$\overline{a_1}.(\overline{b_1} \times \overline{b_2}) = 6$

Therefore, equation of required plane is

8x - 6y + 4z = 6

4x - 3y + 2z = 3

4. Question

Write the equation of the plane whose intercepts on the coordinate axes are 2, -4 and 5 respectively.

Answer

Given :

X – intercept, a = 2

Y – intercept, b = - 4

Z – intercept, c = 5

To Find : Equation of plane

Formula :

If a, b & c are the intercepts made by plane on X, Y & Z axes respectively, then equation of the plane is given by,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

$$\therefore \frac{x}{2}+\frac{y}{-4}+\frac{z}{5}=1$$

Multiplying above equation throughout by 40,

$$\therefore \frac{40x}{2}+\frac{40y}{-4}+\frac{40z}{5}=40$$

20x – 10y + 8z = 40

10x – 5y + 4z = 20

This the equation of the required plane.

5. Question

Reduce the equation of the plane 4x – 3y + 2z = 12 to the intercept form, and hence find the intercepts made by the plane with the coordinate axes.

Answer

Given :

Equation of plane : 4x – 3y + 2z = 12

To Find :

1) Equation of plane in intercept form

2) Intercepts made by the plane with the co-ordinate axes.

Formula :

If $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

is the equation of a plane in intercept form then intercept made by it with co-ordinate axes are

X-intercept = a

Y-intercept = b

Z-intercept = c

Given the equation of plane:

4x – 3y + 2z = 12

Dividing the above equation throughout by 12

$$\therefore \frac{4x}{12}+\frac{-3y}{12}+\frac{2z}{12}=1$$

$$\therefore \frac{x}{3}+\frac{y}{-4}+\frac{z}{6}=1$$

This is the equation of a plane in intercept form.

Comparing the above equation with

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

We get,

a = 3

b = -4

c = 6

Therefore, intercepts made by plane with co-ordinate axes are

X-intercept = 3

Y-intercept = -4

Z-intercept = 6

6. Question

Find the equation of the plane which passes through the point (2, -3,7) and makes equal intercepts on the coordinate axes.

Answer

Equation of the plane making a, b & c intercepts with X, Y & Z axes respectively is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

But, the plane makes equal intercepts on the co-ordinate axes

Therefore, a = b = c

Therefore the equation of the plane is

$$\frac{x}{a}+\frac{y}{a}+\frac{z}{a}=1$$

x + y + z = a

As plane passes through the point (2, -3, 7),

Substituting x = 2, y = -3 & z = 7

2 – 3 + 7 = a

Therefore, a = 6

Hence, required equation of plane is

x + y + z = 6

7. Question

A plane meets the coordinate axes at A, B and C respectively such that the centroid of ΔABC is (1, -2, 3). Find the equation of the plane.

Answer

Given :

X-intercept = A

Y-intercept = B

Z-intercept = C

Centroid of ΔABC = (1, -2, 3)

To Find : Equation of a plane

Formulae :

1) Centroid Formula :

For ΔABC if co-ordinates of A, B & C are

A = (x_1, x_2, x_3)

B = (y_1, y_2, y_3)

C = (z_1, z_2, z_3)

Then co-ordinates of the centroid of ΔABC are

$$G=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$$

2) Equation of plane :

Equation of the plane making a, b & c intercepts with X, Y & Z axes respectively is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

As the plane makes intercepts at points A, B & C on X, Y & Z axes respectively, let co-ordinates of A, B, C be

A = (a, 0, 0)

B = (0, b, 0)

C = (0, 0, c)

By centroid formula,

The centroid of ΔABC is given by

$$G=\left(\frac{a+0+0}{3},\frac{0+b+0}{3},\frac{0+0+c}{3}\right)$$

$G = \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

But, Centroid of ΔABC = (1, -2, 3) …… given

$\therefore \frac{a}{3} = 1, \frac{b}{3} = -2, \frac{c}{3} = 3$

Therefore, a = 3, b = - 6, c = 9

Therefore,

X-intercept = a = 3

Y-intercept = b = - 6

Z-intercept = c = 9

Therefore, equation of required plane is

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$\therefore \frac{x}{3} + \frac{y}{-6} + \frac{z}{9} = 1$

8. Question

Find the Cartesian and vector equations of a plane passing through the point (1, 2, 3) and perpendicular to a line with direction ratios 2, 3, -4.

Answer

Given :

A = (1, 2, 3)

Direction ratios of perpendicular vector = (2, 3, -4)

To Find : Equation of a plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

If a plane is passing through point A, then the equation of a plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{a} = position\ vector\ of\ A$

$\bar{n} = vector\ perpendicular\ to\ the\ plane$

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For point A = (1, 2, 3), position vector is

$\bar{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Vector perpendicular to the plane with direction ratios (2, 3, -4) is

$\bar{n} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

Now, $\bar{a}.\bar{n} = (1 \times 2) + (2 \times 3) + (3 \times (-4))$

= 2 + 6 – 12

= - 4

Equation of the plane passing through point A and perpendicular to vector $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \bar{r}.\left(2\hat{\imath}+3\hat{\jmath}-4\hat{k}\right)=-4$

As $\bar{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$

$\therefore \bar{r}.\left(2\hat{\imath}+3\hat{\jmath}-4\hat{k}\right)=\left(x\hat{\imath}+y\hat{\jmath}+z\hat{k}\right).\left(2\hat{\imath}+3\hat{\jmath}-4\hat{k}\right)$

= 2x + 3y – 4z

Therefore, equation of the plane is

2x + 3y – 4z = -4

Or

2x + 3y – 4z + 4 = 0

9. Question

If O is the origin and P(1, 2, -3) be a given point, then find the equation of the plane passing through P and perpendicular to OP.

Answer

Given :

P = (1, 2, -3)

O = (0, 0, 0)

$\bar{n}=\overline{OP}$

To Find : Equation of a plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

2) Vector :

If A and B be two points with position vectors $\bar{a}\ \&\ \bar{b}$ respectively, where

$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

then,

$\overline{AB}=\bar{b}-\bar{a}$

$=(b_1-a_1)\hat{\imath}+(b_2-a_2)\hat{\jmath}+(b_3-a_3)\hat{k}$

3) Dot Product :

If $\bar{a}\ \&\ \bar{b}$ are two vectors

$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

then,

$\bar{a}.\bar{b}=(a_1\times b_1)+(a_2\times b_2)+(a_3\times b_3)$

4) Equation of plane :

If a plane is passing through point A, then the equation of a plane is

$\bar{r}.\bar{n}=\bar{a}.\bar{n}$

Where, $\bar{a}=position\ vector\ of\ A$

$\bar{n}=vector\ perpendicular\ to\ the\ plane$

$\bar{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$

For points,

P = (1, 2, -3)

O = (0, 0, 0)

Position vectors are

$\bar{p}=\hat{\imath}+2\hat{\jmath}+3\hat{k}$

$\bar{o}=0\hat{\imath}+0\hat{\jmath}+0\hat{k}$

Vector

$\overline{OP} = \bar{p} - \bar{o}$

$= (1-0)\hat{\imath} + (2-0)\hat{\jmath} + (3-0)\hat{k}$

$\therefore \overline{OP} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Now,

$\bar{p}.\overline{OP} = (1 \times 1) + (2 \times 2) + (3 \times 3)$

= 1 + 4 + 9

= 14

And

$\bar{r}.\overline{OP} = (x \times 1) + (y \times 2) + (z \times 3)$

= x + 2y + 3z

Equation of the plane passing through point A and perpendicular to the vector $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

But, $\bar{n} = \overline{OP}$

Therefore, the equation of the plane is

$\bar{r}.\overline{OP} = \bar{p}.\overline{OP}$

x + 2y + 3z = 14

x + 2y + 3z – 14 = 0

Exercise 28B

1. Question

Find the vector and Cartesian equations of a plane which is at a distance of 5 units from the origin and which has $\hat{k}$ as the unit vector normal to it.

Answer

Given :

d = 5

$\hat{n} = \hat{k}$

To Find : Equation of a plane

Formulae :

1) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

2) Equation of plane :

Equation of plane which is at a distance of 5 units from the origin and having $\hat{n}$ as a unit vector normal to it is

$\bar{r}.\hat{n} = d$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given d = 5 and $\hat{n} = \hat{k}$,

Equation of plane is

$\bar{r}.\hat{n} = d$

$\therefore \bar{r}.\hat{k} = 5$

This is a vector equation of the plane

Now,

$\bar{r}.\hat{k} = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\hat{k}$

$= (x \times 0) + (y \times 0) + (z \times 1)$

= z

$\therefore \bar{r}.\hat{k} = z$

Therefore, the equation of the plane is

This is - the Cartesian z = 5 equation of the plane.

2. Question

Find the vector and Cartesian equations of a plane which is at a distance of 7 units from the origin and whose normal vector from the origin is $(3\hat{i}+5\hat{j}-6\hat{k})$.

Answer

Given :

d = 7

$\bar{n} = 3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}$

To Find : Equation of plane

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$$

Where, $|\bar{a}| = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

Equation of plane which is at a distance of 5 units from the origin and having $\hat{n}$ as a unit vector normal to it is

$\bar{r}.\hat{n} = d$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given normal vector

$\bar{n} = 3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}$

Unit vector normal to the plane is

$$\hat{n} = \frac{\bar{n}}{|\bar{n}|}$$

$$\therefore \hat{n} = \frac{3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}}{\sqrt{3^2 + 5^2 + (-6)^2}}$$

$$\therefore \hat{n} = \frac{3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}}{\sqrt{9 + 25 + 36}}$$

$$\therefore \hat{n} = \frac{3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}}{\sqrt{70}}$$

Equation of the plane is

$\bar{r}.\hat{n} = d$

$$\therefore \bar{r}.\left(\frac{3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}}{\sqrt{70}}\right) = 7$$

$\therefore \bar{r}.(3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}) = 7\sqrt{70}$

This is a vector equation of the plane.

Now,

$\bar{r}.(3\hat{\imath} + 5\hat{\jmath} - 6\hat{k}) = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(3\hat{\imath} + 5\hat{\jmath} - 6\hat{k})$

= (x × 3) + (y × 5) + (z × (-6))

= 3x + 5y – 6z

Therefore equation of the plane is

$3x + 5y - 6z = 7\sqrt{70}$

This is the Cartesian equation of the plane.

3. Question

Find the vector and Cartesian equations of a plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and whose normal vector from the origin is $(2\hat{i} - 3\hat{j} + 4\hat{k})$.

Answer

Given :

$d = \frac{6}{\sqrt{29}}$

$\bar{n} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

To Find : Equation of a plane

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then the unit vector of $\bar{a}$ is

$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$

Where, $|\bar{a}| = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

Equation of plane which is at a distance of 5 units from the origin and having $\hat{n}$ as a unit vector normal to it is

$\bar{r}.\hat{n} = d$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For given normal vector

$\bar{n} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

Unit vector normal to the plane is

$\hat{n} = \frac{\bar{n}}{|\bar{n}|}$

$\therefore \hat{n} = \frac{2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{2^2 + (-3)^2 + 4^2}}$

$\therefore \hat{n} = \frac{2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{4 + 9 + 16}}$

$\therefore \hat{n} = \frac{2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{29}}$

Equation of the plane is

$\bar{r}.\hat{n} = d$

$\therefore \bar{r}.\left(\frac{2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}}{\sqrt{29}}\right) = \frac{6}{\sqrt{29}}$

$\therefore \bar{r}.\left(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}\right) = 6$

This is a vector equation of the plane.

Now,

$\bar{r}.\left(2\hat{\imath}-3\hat{\jmath}+4\hat{k}\right)=\left(x\hat{\imath}+y\hat{\jmath}+z\hat{k}\right).\left(2\hat{\imath}-3\hat{\jmath}+4\hat{k}\right)$

= (x × 2) + (y × (-3)) + (z × 4)

= 2x - 3y + 4z

Therefore equation of the plane is

2x - 3y + 4z = 6

This is the Cartesian equation of the plane.

4. Question

Find the vector and Cartesian equations of a plane which is at a distance of 6 units from the origin and which has a normal with direction ratios 2, -1, -2.

Answer

Given :

d = 6

direction ratios of $\bar{n}$ are (2, -1, -2)

$\therefore \bar{n}=2\hat{\imath}-\hat{\jmath}-2\hat{k}$

To Find : Equation of plane

Formulae :

1) Unit Vector :

Let $\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$ be any vector

Then the unit vector of $\bar{a}$ is

$$\hat{a}=\frac{\bar{a}}{|\bar{a}|}$$

Where, $|\bar{a}|=\sqrt{{a_1}^2+{a_2}^2+{a_3}^2}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

then,

$\bar{a}.\bar{b}=(a_1\times b_1)+(a_2\times b_2)+(a_3\times b_3)$

3) Equation of plane :

Equation of plane which is at a distance of 5 units from the origin and having $\hat{n}$ as a unit vector normal to it is

$\bar{r}.\hat{n}=d$

Where, $\bar{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$

For given normal vector

$\bar{n}=2\hat{\imath}-\hat{\jmath}-2\hat{k}$

Unit vector normal to the plane is

$$\hat{n}=\frac{\bar{n}}{|\bar{n}|}$$

$$\therefore \hat{n}=\frac{2\hat{\imath}-\hat{\jmath}-2\hat{k}}{\sqrt{2^2+(-1)^2+(-2)^2}}$$

$$\therefore \hat{n}=\frac{2\hat{\imath}-\hat{\jmath}-2\hat{k}}{\sqrt{4+1+4}}$$

$$\therefore \hat{n}=\frac{2\hat{\imath}-\hat{\jmath}-2\hat{k}}{\sqrt{9}}$$

$$\therefore \hat{n}=\frac{2\hat{\imath}-\hat{\jmath}-2\hat{k}}{3}$$

Equation of the plane is

$\bar{r}.\hat{n} = d$

$\therefore \bar{r}.\left(\frac{2\hat{\imath} - \hat{\jmath} - 2\hat{k}}{3}\right) = 6$

$\therefore \bar{r}.\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right) = 18$

This is vector equation of the plane.

Now,

$\bar{r}.\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right) = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right)$

= (x × 2) + (y × (-1)) + (z × (-2))

= 2x - y – 2z

Therefore equation of the plane is

$2x - y - 2z = 18$

This is Cartesian equation of the plane.

5. Question

Find the vector, and Cartesian equations of a plane which passes through the point (1, 4, 6) and the normal vector to the plane is $\left(\hat{\imath} - 2\hat{\jmath} + \hat{k}\right)$.

Answer

Given :

A = (1, 4, 6)

$\bar{n} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

To Find : Equation of plane.

Formulae :

1) Position Vector :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

Equation of plane passing through point A and having $\bar{n}$ as a unit vector normal to it is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Position vector of point A = (1, 4, 6) is

$\bar{a} = \hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

Now,

$\bar{a}.\bar{n} = \left(\hat{\imath} + 4\hat{\jmath} + 6\hat{k}\right).\left(\hat{\imath} - 2\hat{\jmath} + \hat{k}\right)$

= (1×1) + (4×(-2)) + (6×1)

= 1 – 8 + 6

= - 1

Equation of plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \bar{r}.\left(\hat{\imath} - 2\hat{\jmath} + \hat{k}\right) = -1$

This is vector equation of the plane.

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Therefore

$\bar{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

= (x × 1) + (y × (-2)) + (z × 1)

= x - 2y + z

Therefore equation of the plane is

$x - 2y + z = -1$

This is Cartesian equation of the plane.

6. Question

Find the length of the perpendicular from the origin to the plane $\bar{r}\cdot(3\hat{i} - 12\hat{j} - 4\hat{k}) + 39 = 0$. Also write the unit normal vector from the origin to the plane.

Answer

Given :

Equation of plane : $\bar{r}.(3\hat{\imath} - 12\hat{\jmath} - 4\hat{k}) + 39 = 0$

To Find :

i) Length of perpendicular = d

ii) Unit normal vector = $\hat{n}$

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$$

Where, $|\bar{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

2) Length of perpendicular :

The length of the perpendicular from the origin to the plane

$\bar{r}.\bar{n} = p$ is given by,

$$d = \frac{p}{|\bar{n}|}$$

Given the equation of the plane is

$\bar{r}.(3\hat{\imath} - 12\hat{\jmath} - 4\hat{k}) + 39 = 0$

$\therefore \bar{r}.(3\hat{\imath} - 12\hat{\jmath} - 4\hat{k}) = -39$

$\therefore \bar{r}.(-3\hat{\imath} + 12\hat{\jmath} + 4\hat{k}) = 39$

Comparing the above equation with

$\bar{r}.\bar{n} = p$

We get,

$\bar{n} = -3\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$ & p = 39

Therefore,

$|\bar{n}| = \sqrt{(-3)^2 + 12^2 + 4^2}$

$= \sqrt{9 + 144 + 16}$

$= \sqrt{169}$

= 13

The length of the perpendicular from the origin to the given plane is

$$d = \frac{p}{|\bar{n}|}$$

$$\therefore d = \frac{39}{13}$$

$\therefore d = 3$

Vector normal to the plane is

$\bar{n} = -3\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$

Therefore, the unit vector normal to the plane is

$\hat{n} = \frac{\bar{n}}{|\bar{n}|}$

$\therefore \hat{n} = \frac{-3\hat{\imath} + 12\hat{\jmath} + 4\hat{k}}{13}$

$\therefore \hat{n} = \frac{-3\hat{\imath}}{13} + \frac{12\hat{\jmath}}{13} + \frac{4\hat{k}}{13}$

7. Question

Find the Cartesian equation of the plane whose vector equation is $\bar{r} \cdot (3\hat{i} + 5\hat{j} - 9\hat{k}) = 8$.

Answer

Given :

Vector equation of the plane is

$\bar{r}.(3\hat{\imath} + 5\hat{\jmath} - 9\hat{k}) = 8$

To Find : Cartesian equation of the given plane.

Formulae :

1) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

Given the equation of the plane is

$\bar{r}.(3\hat{\imath} + 5\hat{\jmath} - 9\hat{k}) = 8$

Here,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.(3\hat{\imath} + 5\hat{\jmath} - 9\hat{k}) = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(3\hat{\imath} + 5\hat{\jmath} - 9\hat{k})$

= (x × 3) + (y × 5) + (z × (-9))

= 3x + 5y – 9z

Therefore equation of the plane is

3x + 5y – 9z = 8

This is the Cartesian equation of the given plane.

8. Question

Find the vector equation of a plane whose Cartesian equation is 5x - 7y + 2z + 4 = 0.

Answer

Given :

Cartesian equation of the plane is

5x – 7y + 2z + 4 = 0

To Find : Vector equation of the given plane.

Formulae :

1) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

Given the equation of the plane is

5x – 7y + 2z + 4 = 0

⇒ 5x – 7y + 2z = - 4

The term (5x – 7y + 2z) can be written as

$(5x - 7y + 2z) = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(5\hat{\imath} - 7\hat{\jmath} + 2\hat{k})$

But $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore (5x - 7y + 2z) = \bar{r}.(5\hat{\imath} - 7\hat{\jmath} + 2\hat{k})$

Therefore the equation of the plane is

$\bar{r}.(5\hat{\imath} - 7\hat{\jmath} + 2\hat{k}) = -4$

or

$\bar{r}.(-5\hat{\imath} + 7\hat{\jmath} - 2\hat{k}) = 4$

This is Vector equation of the given plane.

9. Question

Find a unit vector normal to the planex – 2y + 2z = 6.

Answer

Given :

Equation of plane : x – 2y + 2z = 6

To Find : unit normal vector $= \hat{n}$

Formula :

Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then the unit vector of $\bar{a}$ is

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$$

Where, $|\bar{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

From the given equation of a plane

x – 2y + 2z = 6

direction ratios of vector normal to the plane are (1, -2, 2).

Therefore, the equation of normal vector is

$\bar{n} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

Therefore unit normal vector is given by

$$\hat{n} = \frac{\bar{n}}{|\bar{n}|}$$

$$\therefore \hat{n} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{\sqrt{1^2 + (-2)^2 + 2^2}}$$

$$\therefore \hat{n} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{\sqrt{1 + 4 + 4}}$$

$$\therefore \hat{n} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{\sqrt{9}}$$

$$\therefore \hat{n} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3}$$

$$\therefore \hat{n} = \frac{\hat{\imath}}{3} - \frac{2\hat{\jmath}}{3} + \frac{2\hat{k}}{3}$$

10. Question

Find the direction cosines of the normal to the plane 3x – 6y + 2z = 7.

Answer

Given :

Equation of plane : 3x – 6y + 2z = 7

To Find : Direction cosines of the normal, i.e. $l, m \& n$

Formula :

1) Direction cosines :

If a, b & c are direction ratios of the vector, then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

For the given equation of a plane

3x – 6y + 2z = 7

Direction ratios of normal vector are (3, -6, 2)

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{3^2 + (-6)^2 + 2^2}$$

$$= \sqrt{9 + 36 + 4}$$

$$= \sqrt{49}$$

$$= \pm 7$$

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \pm\frac{3}{7}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \mp\frac{6}{7}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \pm\frac{2}{7}$$

$$(l, m, n) = \pm\left(\frac{3}{7}, \frac{-6}{7}, \frac{2}{7}\right)$$

11. Question

For each of the following planes, find the direction cosines of the normal to the plane and the distance of the plane from the origin:

(i) 2x + 3y - z = 5

(ii) z = 3

(iii) 3y + 5 = 0

Answer

(i) 2x + 3y – z = 5

Given :

Equation of plane : 2x + 3y – z = 5

To Find :

Direction cosines of the normal i.e. $l, m \& n$

Distance of the plane from the origin = d

Formulae :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

2) The distance of the plane from the origin :

Distance of the plane from the origin is given by,

$$d = \frac{p}{|\bar{n}|}$$

For the given equation of plane

2x + 3y – z = 5

Direction ratios of normal vector are (2, 3, -1)

Therefore, equation of normal vector is

$\bar{n} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$

$\sqrt{a^2 + b^2 + c^2} = \sqrt{2^2 + 3^2 + (-1)^2}$

$= \sqrt{4 + 9 + 1}$

$= \sqrt{14}$

Therefore, direction cosines are

$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{2}{\sqrt{14}}$

$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{3}{\sqrt{14}}$

$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{-1}{\sqrt{14}}$

$(l, m, n) = \left(\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right)$

Now, the distance of the plane from the origin is

$d = \frac{p}{|\bar{n}|}$

$\therefore d = \frac{5}{\sqrt{14}}$

(ii) Given :

Equation of plane : z = 3

To Find :

Direction cosines of the normal, i.e. $l, m \,\&\, n$

The distance of the plane from the origin = d

Formulae :

3) Direction cosines :

If a, b & c are direction ratios of the vector, then its direction cosines are given by

$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$

$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$

$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$

4) The distance of the plane from the origin :

Distance of the plane from the origin is given by,

$d = \frac{p}{|\bar{n}|}$

For the given equation of a plane

z = 3

Direction ratios of normal vector are (0, 0, 1)

Therefore, equation of normal vector is

$\bar{n} = \hat{k}$

$\sqrt{a^2 + b^2 + c^2} = \sqrt{0^2 + 0^2 + 1^2}$

$= \sqrt{1}$

$= 1$

Therefore, direction cosines are

$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{0}{1} = 0$

$m = \frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{0}{1} = 0$

$n = \frac{c}{\sqrt{a^2+b^2+c^2}} = \frac{1}{1} = 1$

$(l, m, n) = (0,0,1)$

Now, the distance of the plane from the origin is

$d = \frac{p}{|\bar{n}|}$

$\therefore d = \frac{3}{1}$

$\therefore d = 3$

(iii) Given :

Equation of plane : 3y + 5 = 0

To Find :

Direction cosines of the normal, i.e. $l, m \,\&\, n$

The distance of the plane from the origin = d

Formulae :

1) Direction cosines :

If a, b & c are direction ratios of the vector, then its direction cosines are given by

$l = \frac{a}{\sqrt{a^2+b^2+c^2}}$

$m = \frac{b}{\sqrt{a^2+b^2+c^2}}$

$n = \frac{c}{\sqrt{a^2+b^2+c^2}}$

2) Distance of the plane from the origin :

Distance of the plane from the origin is given by,

$d = \frac{p}{|\bar{n}|}$

For the given equation of a plane

3y + 5 = 0

⇒-3y = 5

Direction ratios of normal vector are (0, -3, 0)

Therefore, equation of normal vector is

$\bar{n} = -3\hat{j}$

$\sqrt{a^2+b^2+c^2} = \sqrt{0^2+(-3)^2+0^2}$

$= \sqrt{9}$

$= 3$

Therefore, direction cosines are

$l = \frac{a}{\sqrt{a^2+b^2+c^2}} = \frac{0}{3} = 0$

$m = \frac{b}{\sqrt{a^2+b^2+c^2}} = \frac{-3}{3} = -1$

$n = \frac{c}{\sqrt{a^2+b^2+c^2}} = \frac{0}{3} = 0$

$(l, m, n) = (0,-1,0)$

Now, distance of the plane from the origin is

$d = \frac{p}{|\bar{n}|}$

$\therefore d = \frac{5}{3}$

12. Question

Find the vector and Cartesian equations of the plane passing through the point (2, -1, 1) and perpendicular to the line having direction ratios 4, 2, -3.

Answer

Given :

A = (2, -1, 1)

Direction ratios of perpendicular vector = (4, 2, -3)

To Find : Equation of a plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

If a plane is passing through point A, then the equation of a plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{a} = position\ vector\ of\ A$

$\bar{n} = vector\ perpendicular\ to\ the\ plane$

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

For point A = (2, -1, 1), position vector is

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

Vector perpendicular to the plane with direction ratios (4, 2, -3) is

$\bar{n} = 4\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

Now, $\bar{a}.\bar{n} = (2 \times 4) + ((-1) \times 2) + (1 \times (-3))$

= 8 – 2 – 3

= 3

Equation of the plane passing through point A and perpendicular to vector $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \bar{r}.\left(4\hat{\imath} + 2\hat{\jmath} - 3\hat{k}\right) = 3$

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.\left(4\hat{\imath} + 2\hat{\jmath} - 3\hat{k}\right) = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(4\hat{\imath} + 2\hat{\jmath} - 3\hat{k}\right)$

= 4x + 2y – 3z

Therefore, the equation of the plane is

4x + 2y – 3z = 3

Or

4x + 2y – 3z - 3 = 0

13. Question

Find the coordinates of the foot of the perpendicular drawn from the origin to the plane

(i) 2x + 3y + 4z -12 = 0

(ii) 5y + 8 = 0

Answer

(i) 2x + 3y + 4z - 12 = 0

Given :

Equation of plane : 2x + 3y + 4z + 12 = 0

To Find :

coordinates of the foot of the perpendicular

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

From the given equation of the plane

2x + 3y + 4z – 12 = 0

⇒ 2x + 3y + 4z = 12

Direction ratios of the vector normal to the plane are (2, 3, 4)

Let, P = (x, y, z) be the foot of perpendicular perpendicular drawn from origin to the plane.

Therefore perpendicular drawn is $\overline{OP}$.

$$\therefore \overline{OP} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

Let direction ratios of $\overline{OP}$ are (x, y, z)

As normal vector and $\overline{OP}$ are parallel

$$\therefore \frac{x}{2} = \frac{y}{3} = \frac{z}{4} = k(say)$$

⇒x = 2k, y = 3k, z = 4k

As point P lies on the plane, we can write

2(2k) + 3(3k) + 4(4k) = 12

⇒ 4k + 9k + 16k = 12

⇒ 29k = 12

$$\therefore k = \frac{12}{29}$$

$$\therefore x = 2k = \frac{24}{29},$$

$$y = 3k = \frac{36}{29}$$

$$z = 4k = \frac{48}{29}$$

Therefore co-ordinates of the foot of perpendicular are

P(x, y, z) = $\left(\frac{24}{29}, \frac{36}{29}, \frac{48}{29}\right)$

P = $\left(\frac{24}{29}, \frac{36}{29}, \frac{48}{29}\right)$

(ii) Given :

Equation of plane : 5y + 8 = 0

To Find :

coordinates of the foot of the perpendicular

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

From the given equation of the plane

5y + 8 = 0

⇒ 5y = - 8

Direction ratios of the vector normal to the plane are (0, 5, 0)

Let, P = (x, y, z) be the foot of perpendicular perpendicular drawn from origin to the plane.

Therefore perpendicular drawn is $\overline{OP}$.

$\therefore \overline{OP} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Let direction ratios of $\overline{OP}$ are (x, y, z)

As normal vector and $\overline{OP}$ are parallel

$\therefore \frac{0}{x} = \frac{5}{y} = \frac{0}{z} = \frac{1}{k}(say)$

⇒x = 0, y = 5k, z = 0

As point P lies on the plane, we can write

5(5k) = -8

⇒ 25k = -8

$\therefore k = \frac{-8}{25}$

$\therefore x = 0,$

$y = 5k = 5 \times \frac{-8}{25} = \frac{-8}{5}$

$z = 0$

Therefore co-ordinates of the foot of perpendicular are

P(x, y, z) = $\left(0, \frac{-8}{5}, 0\right)$

P = $\left(0, \frac{-8}{5}, 0\right)$

14. Question

Find the length and the foot of perpendicular drawn from the point (2, 3, 7) to the plane 3x – y – z = 7.

Answer

Given :

Equation of plane : 3x – y – z = 7

A = (2, 3, 7)

To Find :

i) Length of perpendicular = d

ii) coordinates of the foot of the perpendicular

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$

Where, $|\bar{a}| = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}$

2) Length of perpendicular :

The length of the perpendicular from point A with position vector $\bar{a}$ to the plane is given by,

$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

Given equation of the plane is

3x – y – z = 7 ....eq(1)

Therefore direction ratios of normal vector of the plane are

(3, -1, -1)

Therefore normal vector of the plane is

$\bar{n} = 3\hat{\imath} - \hat{\jmath} - \hat{k}$

$\therefore |\bar{n}| = \sqrt{3^2 + (-1)^2 + (-1)^2}$

$= \sqrt{9 + 1 + 1}$

$= \sqrt{11}$

From eq(1), p = 7

Given point A = (2, 3, 7)

Position vector of A is

$\bar{a} = 2\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$

Now,

$\bar{a}.\bar{n} = \left(2\hat{\imath} + 3\hat{\jmath} + 7\hat{k}\right).\left(3\hat{\imath} - \hat{\jmath} - \hat{k}\right)$

= (2×3) + (3×(-1)) + (7×(-1))

= 6 – 3 – 7

= -4

Length of the perpendicular from point A to the plane is

$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$

$\therefore d = \frac{|-4 - 7|}{\sqrt{11}}$

$\therefore d = \frac{11}{\sqrt{11}}$

$\therefore d = \sqrt{11}$

Let P be the foot of perpendicular drawn from point A to the given plane,

Let P = (x, y, z)

$\overline{AP} = (x-2)\hat{\imath} + (y-3)\hat{\jmath} + (z-7)\hat{k}$

As normal vector and $\overline{AP}$ are parallel

$\therefore \frac{x-2}{3} = \frac{y-3}{-1} = \frac{z-7}{-1} = k(say)$

⇒x = 3k+2, y = - k+3, z = -k+7

As point P lies on the plane, we can write

3(3k+2) - (- k+3) - (-k+7) = 7

⇒ 9k + 6 + k – 3 + k – 7 = 7

⇒ 11k = 11

$\therefore k = 1$

$\therefore x = 3k + 2 = 5,$

$y = -k + 3 = 2$

$z = -k + 7 = 6$

Therefore co-ordinates of the foot of perpendicular are

P(x, y, z) = $(5, 2, 6)$

P = (5, 2, 6)

15. Question

Find the length and the foot of the perpendicular drawn from the point (1, 1, 2) to the plane $\bar{r} \cdot \left(2\hat{i} - 2\hat{j} + 4\hat{k}\right) + 5 = 0.$

Answer

Given :

Equation of plane : $\bar{r}.\left(2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}\right) + 5 = 0$

A = (1, 1, 2)

To Find :

i) Length of perpendicular = d

ii) coordinates of the foot of the perpendicular

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$$

Where, $|\bar{a}| = \sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}$

2) Length of perpendicular :

The length of the perpendicular from point A with position vector $\bar{a}$ to the plane is given by,

$$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$$

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

Given equation of the plane is

$\bar{r}.\left(2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}\right) + 5 = 0$ ....eq(1)

$\therefore \bar{r}.\left(2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}\right) = -5$

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Therefore equation of plane is

2x – 2y + 4z = -5 .... eq(2)

From eq(1) normal vector of the plane is

$\bar{n} = 2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$

$\therefore |\bar{n}| = \sqrt{2^2 + (-2)^2 + 4^2}$

$= \sqrt{4 + 4 + 16}$

$= \sqrt{24}$

From eq(1), p = -5

Given point A = (1, 1, 2)

Position vector of A is

$\bar{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$

Now,

$\bar{a}.\bar{n} = \left(\hat{\imath} + \hat{\jmath} + 2\hat{k}\right).\left(2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}\right)$

= (1×2) + (1×(-2)) + (2×4)

= 2 – 2 + 8

= 8

Length of the perpendicular from point A to the plane is

$$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$$

$$\therefore d = \frac{|8 + 5|}{\sqrt{24}}$$

$$\therefore d = \frac{13}{\sqrt{24}}$$

$$\therefore d = \frac{13\sqrt{6}}{\sqrt{24}.\sqrt{6}}$$

$$\therefore d = \frac{13\sqrt{6}}{\sqrt{144}}$$

$\therefore d = \frac{13\sqrt{6}}{12}$

Let P be the foot of perpendicular drawn from point A to the given plane,

Let P = (x, y, z)

$\overline{AP} = (x-1)\hat{\imath} + (y-1)\hat{\jmath} + (z-2)\hat{k}$

As normal vector and $\overline{AP}$ are parallel

$\therefore \frac{x-1}{2} = \frac{y-1}{-2} = \frac{z-2}{4} = k(say)$

⇒x = 2k+1, y = -2k+1, z = 4k+2

As point P lies on the plane, we can write

2(2k+1) – 2(-2k+1) + 4(4k+2) = -5

⇒ 4k + 2 + 4k – 2 + 16k + 8 = -5

⇒ 24k = -13

$\therefore k = \frac{-13}{24}$

$\therefore x = 2\left(\frac{-13}{24}\right) + 1 = \frac{-1}{12},$

$y = -2\left(\frac{-13}{24}\right) + 1 = \frac{25}{12}$

$z = 4\left(\frac{-13}{24}\right) + 2 = \frac{-1}{6}$

Therefore co-ordinates of the foot of perpendicular are

P(x, y, z) = $\left(\frac{-1}{12}, \frac{25}{12}, \frac{-1}{6}\right)$

P ≡ $\left(\frac{-1}{12}, \frac{25}{12}, \frac{-1}{6}\right)$

16. Question

From the point P(1, 2, 4), a perpendicular is drawn on the plane 2x + y - 2z + 3 = 0. Find the equation, the length and the coordinates of the foot of the perpendicular.

Answer

Given :

Equation of plane : 2x + y – 2z + 3 = 0

P = (1, 2, 4)

To Find :

i) Equation of perpendicular

ii) Length of perpendicular = d

iii) coordinates of the foot of the perpendicular

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$

Where, $|\bar{a}| = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}$

2) Length of perpendicular :

The length of the perpendicular from point A with position vector $\bar{a}$ to the plane is given by,

$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$

Given equation of the plane is

2x + y – 2z + 3 = 0

⇒2x + y – 2z = -3 ....eq(1)

From eq(1) direction ratios of normal vector of the plane are

(2, 1, -2)

Therefore, equation of normal vector is

$\bar{n} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$

$\therefore |\bar{n}| = \sqrt{2^2 + 1^2 + (-2)^2}$

$= \sqrt{4 + 1 + 4}$

$= \sqrt{9}$

= 3

From eq(1), p = -3

Given point P = (1, 2, 4)

Position vector of A is

$\bar{p} = \hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

Here, $\bar{a} = \bar{p}$

Now,

$\therefore \bar{a}.\bar{n} = \left(\hat{\imath} + 2\hat{\jmath} + 4\hat{k}\right).\left(2\hat{\imath} + \hat{\jmath} - 2\hat{k}\right)$

= (1×2) + (2×1) + (4×(-2))

= 2 + 2 - 8

= -4

Length of the perpendicular from point A to the plane is

$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$

$\therefore d = \frac{|-4 + 3|}{3}$

$\therefore d = \frac{1}{3}$

Let Q be the foot of perpendicular drawn from point P to the given plane,

Let Q = (x, y, z)

$\overline{PQ} = (x - 1)\hat{\imath} + (y - 2)\hat{\jmath} + (z - 4)\hat{k}$

As normal vector and $\overline{PQ}$ are parallel, we can write,

$\therefore \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2}$

This is the equation of perpendicular.

$\therefore \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-4}{-2} = k(say)$

⇒x = 2k+1, y = k+2, z = -2k+4

As point Q lies on the plane, we can write

2(2k+1) + (k+2) - 2(-2k+4) = -3

⇒ 4k + 2 + k + 2 + 4k - 8 = -3

⇒ 9k = 1

$\therefore k = \frac{1}{9}$

$\therefore x = 2\left(\frac{1}{9}\right) + 1 = \frac{11}{9},$

$y = \frac{1}{9} + 2 = \frac{19}{9}$

$z = -2\left(\frac{1}{9}\right) + 4 = \frac{34}{9}$

Therefore co-ordinates of the foot of perpendicular are

Q(x, y, z) = $\left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)$

Q ≡ $\left(\frac{11}{9}, \frac{19}{9}, \frac{34}{9}\right)$

17. Question

Find the coordinates of the foot of the perpendicular and the perpendicular distance from the point P(3, 2, 1) to the plane 2x – y + z + 1 = 0.

Find also the image of the point P in the plane.

Answer

Given :

Equation of plane : 2x - y + z + 1 = 0

P = (3, 2, 1)

To Find :

i) Length of perpendicular = d

ii) coordinates of the foot of the perpendicular

iii) Image of point P in the plane.

Formulae :

1) Unit Vector :

Let $\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$$\hat{a} = \frac{\bar{a}}{|\bar{a}|}$$

Where, $|\bar{a}| = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}$

2) Length of perpendicular :

The length of the perpendicular from point A with position vector $\bar{a}$ to the plane is given by,

$$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$$

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

Given equation of the plane is

2x - y + z + 1 = 0

⇒2x - y + z = -1 ....eq(1)

From eq(1) direction ratios of normal vector of the plane are

(2, -1, 1)

Therefore, equation of normal vector is

$$\bar{n} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$$

$$\therefore |\bar{n}| = \sqrt{2^2 + (-1)^2 + 1^2}$$

$$= \sqrt{4 + 1 + 1}$$

$$= \sqrt{6}$$

From eq(1), p = -1

Given point P = (3, 2, 1)

Position vector of A is

$$\bar{p} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$$

Here, $\bar{a} = \bar{p}$

Now,

$$\therefore \bar{a}.\bar{n} = (3\hat{\imath} + 2\hat{\jmath} + \hat{k}).(2\hat{\imath} - \hat{\jmath} + \hat{k})$$

= (3×2) + (2×(-1)) + (1×1)

= 6 - 2 + 1

= 5

Length of the perpendicular from point A to the plane is

$d = \frac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$

$\therefore d = \frac{|5+1|}{\sqrt{6}}$

$\therefore d = \frac{6}{\sqrt{6}}$

$\therefore d = \sqrt{6}$

Let Q be the foot of perpendicular drawn from point P to the given plane,

Let Q = (x, y, z)

$\overline{PQ} = (x-3)\hat{\imath} + (y-2)\hat{\jmath} + (z-1)\hat{k}$

As normal vector and $\overline{PA}$ are parallel, we can write,

$\therefore \frac{x-3}{2} = \frac{y-2}{-1} = \frac{z-1}{1} = k(say)$

⇒x = 2k+3, y = -k+2, z = k+1

As point A lies on the plane, we can write

2(2k+3) - (-k+2) + (k+1) = -1

⇒ 4k + 6 + k – 2 + k + 1 = -1

⇒ 6k = -6

$\therefore k = -1$

$\therefore x = 2(-1) + 3 = 1,$

$y = -(-1) + 2 = 3$

$z = (-1) + 1 = 0$

Therefore, co-ordinates of the foot of perpendicular are

Q(x, y, z) = $(1,3,0)$

Q ≡ $(1,3,0)$

Let, R(a, b, c) be image of point P in the given plane.

Therefore, the power of points P and R in the given plane will be equal and opposite.

2a – b + c + 1 = - (2(3) – 2 + 1 + 1)

⇒2a – b + c + 1 = - 6

⇒2a – b + c = - 7 ....eq(2)

Now, $\overline{PR} = (a-3)\hat{\imath} + (b-2)\hat{\jmath} + (c-1)\hat{k}$

As $\overline{PR}$ & $\bar{n}$ are parallel

$\therefore \frac{a-3}{2} = \frac{b-2}{-1} = \frac{c-1}{1} = k(say)$

⇒a = 2k+3, b = -k+2, c = k+1

substituting a, b, c in eq(2)

2(2k+3) - (-k+2) + (k+1) = -7

⇒ 4k + 6 + k – 2 + k + 1 = -7

⇒ 6k = -12

$\therefore k = -2$

$\therefore a = 2(-2) + 3 = -1,$

$b = -(-2) + 2 = 4$

$c = (-2) + 1 = -1$

Therefore, co-ordinates of the image of P are

R(a, b, c) = $(-1,4,-1)$

R ≡ $(-1,4,-1)$

18. Question

Find the coordinates of the image of the point P(1, 3, 4) in the plane 2x - y + z + 3 = 0.

Answer

Given :

Equation of plane : 2x - y + z + 3 = 0

P = (1, 3, 4)

To Find : Image of point P in the plane.

Note :

If two vectors with direction ratios (a_1, a_2, a_3) & (b_1, b_2, b_3) are parallel then

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$$

Given equation of the plane is

2x - y + z + 3 = 0

⇒2x - y + z = -3 ....eq(1)

From eq(1) direction ratios of normal vector of the plane are

(2, -1, 1)

Therefore, equation of normal vector is

$\bar{n} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

Given point is P = (1, 3, 4)

Let, R(a, b, c) be image of point P in the given plane.

Therefore, the power of points P and R in the given plane will be equal and opposite.

⇒2a – b + c + 3 = - (2(1) – 3 + 4 + 3)

⇒2a – b + c + 3 = - 6

⇒2a – b + c = - 9 ....eq(2)

Now, $\overline{PR} = (a-1)\hat{\imath} + (b-3)\hat{\jmath} + (c-4)\hat{k}$

As $\overline{PR}$ & $\bar{n}$ are parallel

$$\therefore \frac{a-1}{2} = \frac{b-3}{-1} = \frac{c-4}{1} = k(say)$$

⇒a = 2k+1, b = -k+3, c = k+4

substituting a, b, c in eq(2)

2(2k+1) - (-k+3) + (k+4) = -9

⇒ 4k + 2 + k – 3 + k + 4 = -9

⇒ 6k = -12

$\therefore k = -2$

$\therefore a = 2(-2) + 1 = -3,$

$b = -(-2) + 3 = 5$

$c = (-2) + 4 = 2$

Therefore, co-ordinates of the image of P are

R(a, b, c) = $(-3, 5, 2)$

19. Question

Find the point where the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+3}{4}$ meets the plane 2x + 4y – z = 1.

Answer

Given :

Equation of plane : 2x + 4y - z = 1

Equation of line :

$\frac{x-1}{2}=\frac{y-2}{-3}=\frac{z+3}{4}$

To Find : Point of intersection of line and plane.

Let P(a, b, c) be point of intersection of plane and line.

As point P lies on the line, we can write,

$\frac{a-1}{2}=\frac{b-2}{-3}=\frac{c+3}{4}=k(say)$

⇒a = 2k+1, b = -3k+2, c = 4k-3 ....(1)

Also point P lies on the plane

2a + 4b – c = 1

⇒2(2k+1) + 4(-3k+2) – (4k-3) = 1 .....from (1)

⇒4k + 2 – 12k + 8 – 4k + 3 = 1

⇒-12k = -12

⇒k = 1

$\therefore a=2(1)+1=3,$

$b=-3(1)+2=-1$

$c=4(1)-3=1$

Therefore, co-ordinates of point of intersection of given line and plane are

P ≡ $(3,-1,1)$

20. Question

Find the coordinates of the point where the line through (3, -4, -5) and (2, -3, 1) crosses the plane 2x + y + z = 7.

Answer

Given :

Equation of plane : 2x + y + z = 7

Points :

A = (3, -4, -5)

B = (2, -3, 1)

To Find : Point of intersection of line and plane.

Formula :

Equation of line passing through A = (x_1, y_1, z_1) &

B = (x_2, y_2, z_2) is

$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$

Equation of line passing through A = (3, -4, -5) & B = (2, -3, 1) is

$\frac{x-3}{3-2}=\frac{y+4}{-4+3}=\frac{z+5}{-5-1}$

$\therefore\frac{x-3}{1}=\frac{y+4}{-1}=\frac{z+5}{-6}$

Let P(a, b, c) be point of intersection of plane and line.

As point P lies on the line, we can write,

$\frac{a-3}{1}=\frac{b+4}{-1}=\frac{c+5}{-6}=k(say)$

⇒a = k+3, b = -k - 4, c = -6k-5 ....(1)

Also point P lies on the plane

2a + b + c = 7

⇒2(k+3) + (-k-4) + (-6k-5) = 7 .....from (1)

⇒2k + 6 – k - 4 – 6k - 5 = 7

⇒-5k = 10

⇒k = -2

$\therefore a = (-2) + 3 = 1,$

$b = -(-2) - 4 = -2$

$c = -6(-2) - 5 = 7$

Therefore, co-ordinates of point of intersection of given line and plane are

P ≡ $(1, -2, 7)$

21. Question

Find the distance of the point (2, 3, 4) from the plane 3x + 2y + 2z +5 =0, measured parallel to the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$.

Answer

Given :

Equation of plane : 3x + 2y + 2z + 5 = 0

Equation of line :

$$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$$

Point : P = (2, 3, 4)

To Find : Distance of point P from the given plane parallel to the given line.

Formula :

1) Equation of line :

Equation of line passing through A = (x_1, y_1, z_1) & having direction ratios (a, b, c) is

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

2) Distance formula :

The distance between two points A = (a_1, a_2, a_3) & B = (b_1, b_2, b_3) is

$$d = \sqrt{(a_1-b_1)^2 + (a_2-b_2)^2 + (a_3-b_3)^2}$$

For the given line,

$$\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$$

Direction ratios are (a, b, c) = (3, 6, 2)

Let Q be the point on the plane such that $\overline{PQ}$ is parallel to the given line.

Therefore direction ratios of given line and line PQ will be same.

Therefore equation of line PQ with point P = (2, 3, 4) and with direction ratios (3, 6, 2) is

$$\frac{x-2}{3} = \frac{y-3}{6} = \frac{z-4}{2}$$

Let co-ordinates of Q be (u, v, w)

As point Q lies on the line PQ, we can write,

$$\frac{u-2}{3} = \frac{v-3}{6} = \frac{w-4}{2} = k(say)$$

⇒u = 3k+2, v = 6k+3, w = 2k+4 ....(1)

Also point Q lies on the plane

3u + 2v + 2w = -5

⇒3(3k+2) + 2(6k+3) + 2(2k+4) = -5 .....from (1)

⇒9k + 6 + 12k + 6 + 4k + 8 = -5

⇒25k = -25

⇒k = -1

$\therefore u = 3(-1) + 2 = -1,$

$v = 6(-1) + 3 = -3$

$w = 2(-1) + 4 = 2$

Therefore, co-ordinates of point Q are

Q = $(-1, -3, 2)$

Now distance between points P and Q by distance formula is

$d=\sqrt{(2+1)^2+(3+3)^2+(4-2)^2}$

$=\sqrt{(3)^2+(6)^2+(2)^2}$

$=\sqrt{9+36+4}$

$=\sqrt{49}$

= 7

Therefore distance of point P from the given plane measured parallel to the given line is

d = 7 units

22. Question

Find the distance of the point (0, -3, 2) from the plane x + 2y -z = 1, measured parallel to the line $\frac{x+1}{3}=\frac{y+1}{2}=\frac{z}{3}$.

Answer

Given :

Equation of plane : x + 2y - z = 1

Equation of line :

$$\frac{x+1}{3}=\frac{y+1}{2}=\frac{z}{3}$$

Point : P = (0, -3, 2)

To Find : Distance of point P from the given plane parallel to the given line.

Formula :

1) Equation of line :

Equation of line passing through A = (x_1, y_1, z_1) & having direction ratios (a, b, c) is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

2) Distance formula :

The distance between two points A = (a_1, a_2, a_3) & B = (b_1, b_2, b_3) is

$$d=\sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$$

For the given line,

$$\frac{x+1}{3}=\frac{y+1}{2}=\frac{z}{3}$$

Direction ratios are (a, b, c) = (3, 2, 3)

Let Q be the point on the plane such that $\overline{PQ}$ is parallel to the given line.

Therefore direction ratios of given line and line PQ will be same.

Therefore equation of line PQ with point P = (0, -3, 2) and with direction ratios (3, 2, 3) is

$$\frac{x-0}{3}=\frac{y+3}{2}=\frac{z-2}{3}$$

Let co-ordinates of Q be (u, v, w)

As point Q lies on the line PQ, we can write,

$$\frac{u}{3}=\frac{v+3}{2}=\frac{w-2}{3}=k(say)$$

⇒u = 3k, v = 2k-3, w = 3k+2 ....(1)

Also point Q lies on the plane

u + 2v - w = 1

⇒(3k) + 2(2k-3) - (3k+2) = 1 .....from (1)

⇒3k + 4k - 6 - 3k - 2 = 1

⇒4k = 9

$\Rightarrow k=\frac{9}{4}$

$\therefore u=3\left(\frac{9}{4}\right)=\frac{27}{4},$

$v = 2\left(\frac{9}{4}\right) - 3 = \frac{6}{4}$

$w = 3\left(\frac{9}{4}\right) + 2 = \frac{35}{4}$

Therefore, co-ordinates of point Q are

Q ≡ $\left(\frac{27}{4}, \frac{6}{4}, \frac{35}{4}\right)$

Now distance between points P and Q by distance formula is

$$d = \sqrt{\left(0 - \frac{27}{4}\right)^2 + \left(-3 - \frac{6}{4}\right)^2 + \left(2 - \frac{35}{4}\right)^2}$$

$$= \sqrt{\left(\frac{-27}{4}\right)^2 + \left(\frac{-18}{4}\right)^2 + \left(\frac{-27}{4}\right)^2}$$

$$= \sqrt{45.5625 + 20.25 + 45.5625}$$

$$= \sqrt{111.375}$$

= 10.55

Therefore distance of point P from the given plane measured parallel to the given line is

d = 10.55 units

23. Question

Find the equation of the line passing through the point P(4, 6, 2) and the point of intersection of the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$ and the plane x + y –Z = 8.

Answer

Given :

Equation of plane : x + y - z = 8

Equation of line :

$$\frac{x-1}{3} = \frac{y}{2} = \frac{z+1}{7}$$

Point : P = (4, 6, 2)

To Find : Equation of line.

Formula :

Equation of line passing through A = (x_1, y_1, z_1) &

B = (x_2, y_2, z_2) is

$$\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2} = \frac{z - z_1}{z_1 - z_2}$$

let Q (a, b, c) be point of intersection of plane and line.

As point Q lies on the line, we can write,

$$\frac{a-1}{3} = \frac{b}{2} = \frac{c+1}{7} = k(say)$$

⇒a =3k+1, b= 2k, c= 7k-1

Also point Q lies on the plane,

a + b – c = 8

⇒(3k+1) + (2k) – (7k-1) = 8

⇒3k + 1 + 2k – 7k + 1 = 8

⇒-2k = 6

⇒k = -3

$\therefore a = 3(-3) + 1 = -8,$

$b = -2(-3) = -6$

$c = 7(-3) - 1 = -22$

Therefore, co-ordinates of point of intersection of given line and plane are Q = (-8, -6, -22)

Now, equation of line passing through P(4,6,2) and

Q(-8, -6, -22) is

$$\frac{x-4}{4+8}=\frac{y-6}{6+6}=\frac{z-2}{2+22}$$

$$\therefore \frac{x-4}{12}=\frac{y-6}{12}=\frac{z-2}{24}$$

$$\therefore \frac{x-4}{1}=\frac{y-6}{1}=\frac{z-2}{2}$$

This is the equation of required line

24. Question

Show that the distance of the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane x -y + z = 5 from the point (-1, -5 -10) is 13 units.

Answer

Given :

Equation of plane : x - y + z = 5

Equation of line :

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$$

Point : P = (-1, -5, -10)

To Prove : Distance of point P from the given plane parallel to the given line is 13 units.

Formula :

1) Equation of line :

Equation of line passing through A = (x_1, y_1, z_1) & having direction ratios (a, b, c) is

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

2) Distance formula :

The distance between two points A = (a_1, a_2, a_3) & B = (b_1, b_2, b_3) is

$$d=\sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$$

For the given line,

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$$

Direction ratios are (a, b, c) = (3, 4, 12)

Let Q be the point on the plane such that $\overline{PQ}$ is parallel to the given line.

Therefore direction ratios of given line and line PQ will be same.

Therefore equation of line PQ with point P = (-1, -5, -10) and with direction ratios (3, 4, 12) is

$$\frac{x+1}{3}=\frac{y+5}{4}=\frac{z+10}{12}$$

Let co-ordinates of Q be (u, v, w)

As point Q lies on the line PQ, we can write,

$$\frac{u+1}{3}=\frac{v+5}{4}=\frac{w+10}{12}=k(say)$$

⇒u = 3k-1, v = 4k-5, w = 12k-10 ....(1)

Also point Q lies on the plane

u - v + w = 5

⇒(3k-1) - (4k-5) + (12k-10) = 5 .....from (1)

⇒3k - 1 - 4k + 5 + 12k - 10 = 5

⇒11k = 11

⇒$k=1$

$\therefore u=3(1)-1=2,$

$v=4(1)-5=-1$

$w=12(1)-10=2$

Therefore, co-ordinates of point Q are

Q ≡ $(2, -1, 2)$

Now distance between points P and Q by distance formula is

$d = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2}$

$= \sqrt{(-3)^2 + (-4)^2 + (-12)^2}$

$= \sqrt{9 + 16 + 144}$

$= \sqrt{169}$

= 13

Therefore distance of point P from the given plane measured parallel to the given line is

d = 13 units

Hence proved.

25. Question

Find the distance of the point A(-1, -5, -10) from the point of intersection of the line $\bar{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\bar{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5.$

HINT: Convert the equations of the line and the plane to Cartesian form.

Answer

Given :

Equation of plane : $\bar{r}.(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$

Equation of line :

$\bar{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$

Point : P = (-1, -5, -10)

To Find : Distance of point P from the given plane parallel to the given line.

Formula :

1) Equation of line :

Equation of line passing through A = (x_1, y_1, z_1) & having direction ratios (a, b, c) is

$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

2) Distance formula :

The distance between two points A = (a_1, a_2, a_3) & B = (b_1, b_2, b_3) is

$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$

for the given plane,

$\bar{r}.(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$

Here, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$(x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(\hat{\imath} - \hat{\jmath} + \hat{k}) = 5$

⇒x – y + z = 5 ....eq(1)

For the given line,

$\bar{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$

Here, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore (3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})\lambda = (x\hat{\imath} + y\hat{\jmath} + z\hat{k}) - (2\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$\therefore 3\lambda\hat{\imath} + 4\lambda\hat{\jmath} + 2\lambda\hat{k} = (x-2)\hat{\imath} + (y+1)\hat{\jmath} + (z-2)\hat{k}$

Comparing coefficients of $\hat{\imath}, \hat{\jmath}$ & $\hat{k}$

⇒$3\lambda = (x-2), 4\lambda = (y+1)$ & $2\lambda = (z-2)$

⇒$\lambda = \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$ ....eq(2)

Direction ratios for above line are (a, b, c) = (3, 4, 2)

Let Q be the point on the plane such that $\overline{PQ}$ is parallel to the given line.

Therefore direction ratios of given line and line PQ will be same.

Therefore equation of line PQ with point P = (-1, -5, -10) and with direction ratios (3, 4, 2) is

$$\frac{x+1}{3}=\frac{y+5}{4}=\frac{z+10}{2}$$

Let co-ordinates of Q be (u, v, w)

As point Q lies on the line PQ, we can write,

$$\frac{u+1}{3}=\frac{v+5}{4}=\frac{w+10}{2}=k(say)$$

⇒u = 3k-1, v = 4k-5, w = 2k-10 ....(3)

Also point Q lies on the given plane

Therefore from eq(1), we can write,

u - v + w = 5

⇒(3k-1) - (4k-5) + (2k-10) = 5 .....from (3)

⇒3k - 1 - 4k + 5 + 2k - 10 = 5

⇒k = 11

⇒$k = 11$

$\therefore u = 3(11) - 1 = 32,$

$v = 4(11) - 5 = 39$

$w = 2(11) - 10 = 12$

Therefore, co-ordinates of point Q are

Q ≡ $(32, 39, 12)$

Now the distance between points P and Q by distance formula is

$$d=\sqrt{(-1-32)^2+(-5-39)^2+(-10-12)^2}$$

$$=\sqrt{(-33)^2+(-44)^2+(-22)^2}$$

$$=\sqrt{1089+1936+484}$$

$$=\sqrt{3509}$$

= 59.24

Therefore distance of point P from the given plane measured parallel to the given line is

d = 59.24 units

26. Question

Prove that the normals to the planes 4x + 11y + 2z + 3 = 0 and 3x - 2y + 5z = 8 are perpendicular to each other.

Answer

Given :

Equations of plane are :

4x + 11y + 2z + 3 = 0

3x – 2y + 5z = 8

To Prove : $\overline{n_1}$ & $\overline{n_2}$ are perpendicular.

Formula :

1) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then,

$$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$$

Note :

Direction ratios of the plane given by

ax + by + cz = d

are (a, b, c).

For plane

4x + 11y + 2z + 3 = 0

direction ratios of normal vector are (4, 11, 2)

therefore, equation of normal vector is

$\overline{n_1} = 4\hat{\imath} + 11\hat{\jmath} + 2\hat{k}$

And for plane

3x – 2y + 5z = 8

direction ratios of the normal vector are (3, -2, 5)

therefore, the equation of normal vector is

$\overline{n_2} = 3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$

Now,

$\overline{n_1}.\overline{n_2} = \left(4\hat{\imath} + 11\hat{\jmath} + 2\hat{k}\right).\left(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}\right)$

$= (4 \times 3) + (11 \times (-2)) + (2 \times 5)$

= 12 – 22 + 10

= 0

$\therefore \overline{n_1}.\overline{n_2} = 0$

Therefore, normals to the given planes are perpendicular.

27. Question

Show that the line $\bar{r} = \left(2\hat{i} - 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + 4\hat{k}\right)$ is parallel to the plane $\bar{r} \cdot \left(\hat{i} + 5\hat{j} + \hat{k}\right) = 7$.

Answer

Given :

Equation of plane : : $\bar{r}.\left(\hat{\imath} + 5\hat{\jmath} + \hat{k}\right) = 7$

Equation of a line :

$\bar{r} = \left(2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}\right) + \lambda\left(\hat{\imath} - \hat{\jmath} + 4\hat{k}\right)$

To Prove : Given line is parallel to the given plane.

Comparing given plane i.e.

$\bar{r}.\left(\hat{\imath} + 5\hat{\jmath} + \hat{k}\right) = 7$

with $\bar{r}.\bar{n} = \bar{a}.\bar{n}$, we get,

$\bar{n} = \hat{\imath} + 5\hat{\jmath} + \hat{k}$

This is the vector perpendicular to the given plane.

Now, comparing the given the equation of line i.e.

$\bar{r} = \left(2\hat{\imath} - 2\hat{\jmath} + 3\hat{k}\right) + \lambda\left(\hat{\imath} - \hat{\jmath} + 4\hat{k}\right)$

with $\bar{r} = \bar{a} + \lambda\bar{b}$, we get,

$\bar{b} = \hat{\imath} - \hat{\jmath} + 4\hat{k}$

Now,

$\bar{n}.\bar{b} = \left(\hat{\imath} + 5\hat{\jmath} + \hat{k}\right).\left(\hat{\imath} - \hat{\jmath} + 4\hat{k}\right)$

$= (1 \times 1) + (5 \times (-1)) + (1 \times 4)$

= 1 – 5 + 4

= 0

$\therefore \bar{n}.\bar{b} = 0$

Therefore, a vector normal to the plane is perpendicular to the vector parallel to the line.

Hence, the given line is parallel to the given plane.

28. Question

Find the equation of a plane which is at a distance of 3√3 units from the origin and the normal to which is equally inclined to the coordinate axes.

Answer

Given :

$d = 3\sqrt{3}$

$\alpha = \beta = \gamma$

To Find : Equation of plane

Formulae :

1) Distance of plane from the origin :

If $\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ is the vector normal to the plane, then distance of the plane from the origin is

$$d = \frac{p}{|\bar{n}|}$$

Where, $|\bar{n}| = \sqrt{a^2 + b^2 + c^2}$

2) $l^2 + m^2 + n^2 = 1$

Where $l = \cos\alpha, m = \cos\beta, n = \cos\gamma$

3) Equation of plane :

If $\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ is the vector normal to the plane, then equation of the plane is

$\bar{r}.\bar{n} = p$

As $\alpha = \beta = \gamma$

$\therefore \cos\alpha = \cos\beta = \cos\gamma$

⇒ l = m = n

$l^2 + m^2 + n^2 = 1$

$\therefore 3l^2 = 1$

$$\therefore l = \frac{1}{\sqrt{3}}$$

Therefore equation of normal vector of the plane having direction cosines l, m, n is

$\bar{n} = l\hat{\imath} + m\hat{\jmath} + n\hat{k}$

$$\therefore \bar{n} = \frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$$

$$\therefore |\bar{n}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}}$$

= √1

= 1

Now,

distance of the plane from the origin is

$$d = \frac{p}{|\bar{n}|}$$

$$\therefore 3\sqrt{3} = \frac{p}{1}$$

$\therefore p = 3\sqrt{3}$

Therefore equation of required plane is

$\bar{r}.\bar{n} = p$

$$\therefore (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).\left(\frac{1}{\sqrt{3}}\hat{\imath} + \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}\right) = 3\sqrt{3}$$

$$\therefore \frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 3\sqrt{3}$$

$\therefore x + y + z = 3\sqrt{3}.\sqrt{3}$

$\therefore x + y + z = 9$

This is the required equation of the plane.

29. Question

A vector $\bar{n}$ of magnitude 8 units is inclined to the x-axis at 45°, y-axis at 60° and an acute angle with the z-axis, if a plane passes through a point (√2, -1, 1) and is normal to find its equation in vector form.

Answer

Given :

$|\bar{n}| = 8$

$\alpha = 45°$

$\beta = 60°$

P = (√2, -1, 1)

To Find : Equation of plane

Formulae :

1) $l^2 + m^2 + n^2 = 1$

Where $l = \cos\alpha, m = \cos\beta, n = \cos\gamma$

2) Equation of plane :

If $\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ is the vector normal to the plane, then equation of the plane is

$\bar{r}.\bar{n} = p$

As $\alpha = 45° \;\&\; \beta = 60°$

$\therefore l = \cos\alpha = \cos 45° = \frac{1}{\sqrt{2}}$ and

$m = \cos\beta = \cos 60° = \frac{1}{2}$

But, $l^2 + m^2 + n^2 = 1$

$\therefore \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + n^2 = 1$

$\therefore \frac{1}{2} + \frac{1}{4} + n^2 = 1$

$\therefore n^2 = 1 - \frac{3}{4}$

$\therefore n^2 = \frac{1}{4}$

$\therefore n = \frac{1}{2}$

Therefore direction cosines of the normal vector of the plane are (l, m, n)

Hence direction ratios are (kl, km, kn)

Therefore the equation of normal vector is

$\bar{n} = kl\hat{\imath} + km\hat{\jmath} + kn\hat{k}$

$\therefore |\bar{n}| = \sqrt{(kl)^2 + (km)^2 + (kn)^2}$

$\therefore |\bar{n}| = \sqrt{\left(\frac{k}{\sqrt{2}}\right)^2 + \left(\frac{k}{2}\right)^2 + \left(\frac{k}{2}\right)^2}$

$\therefore 8 = \sqrt{\frac{k^2}{2} + \frac{k^2}{4} + \frac{k^2}{4}}$

$\therefore 8 = \sqrt{k^2}$

$\therefore k = 8$

$\bar{n} = \left(\frac{8}{\sqrt{2}}\right)\hat{\imath} + \left(\frac{8}{2}\right)\hat{\jmath} + \left(\frac{8}{2}\right)\hat{k}$

$\therefore \bar{n} = 4\sqrt{2}\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$

Now, equation of the plane is

$\bar{r}.\bar{n} = p$

$\therefore \bar{r}.\left(4\sqrt{2}\hat{\imath} + 4\hat{\jmath} + 4\hat{k}\right) = p$ ....eq(1)

But $\bar{r} = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right)$

$\therefore \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(4\sqrt{2}\hat{\imath} + 4\hat{\jmath} + 4\hat{k}\right) = p$

⇒4√2x + 4y + 4z = p

As point P (√2, -1, 1) lies on the plane by substituting it in above equation,

4√2(√2) + 4(-1) + 4(1) = p

⇒8 – 4 + 4 = p

⇒P = 8

From eq(1)

$\therefore \bar{r}.\left(4\sqrt{2}\hat{\imath} + 4\hat{\jmath} + 4\hat{k}\right) = 8$

Dividing throughout by 4

$\therefore \bar{r}.\left(\sqrt{2}\hat{\imath} + \hat{\jmath} + \hat{k}\right) = 2$

This is the equation of required plane.

30. Question

Find the vector equation of a line passing through the point $\left(2\hat{i} - 3\hat{j} - 5\hat{k}\right)$ and perpendicular to the plane $\vec{r}\cdot\left(6\hat{i} - 3\hat{j} + 5\hat{k}\right) + 2 = 0.$

Also, find the point of intersection of this line and the plane.

Answer

Given :

$\bar{a} = 2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

Equation of plane : $\bar{r}.\left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right) = -2$

To Find :

Equation of line

Point of intersection

Formula :

Equation of line passing through point A with position vector $\bar{a}$ and parallel to vector $\bar{b}$ is

$\bar{r} = \bar{a} + \lambda\bar{b}$

Where, $\bar{r} = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right)$

From the given equation of the plane

$\bar{r}.\left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right) = -2$ ....eq(1)

The normal vector of the plane is

$\bar{n} = 6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$

As the given line is perpendicular to the plane therefore $\bar{n}$ will be parallel to the line.

$\therefore \bar{n} = \bar{b}$

Now, the equation of the line passing through $\bar{a} = \left(2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}\right)$ and parallel to $\bar{b} = \left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right)$ is

$\bar{r} = \bar{a} + \lambda\bar{b}$

$\therefore \bar{r} = \left(2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}\right) + \lambda\left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right)$

.........eq(2)

This is the required equation line.

Substituting $\bar{r} = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right)$ in eq(1)

$\left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right) = -2$

⇒6x – 3y + 5z = -2 ....eq(3)

Also substituting $\bar{r} = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right)$ in eq(2)

$\left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right) = \left(2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}\right) + \lambda\left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right)$

$\therefore \left(6\hat{\imath} - 3\hat{\jmath} + 5\hat{k}\right)\lambda = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right) - \left(2\hat{\imath} - 3\hat{\jmath} - 5\hat{k}\right)$

$\therefore 6\lambda\hat{\imath} - 3\lambda\hat{\jmath} + 5\lambda\hat{k} = (x-2)\hat{\imath} + (y+3)\hat{\jmath} + (z+5)\hat{k}$

Comparing coefficients of $\hat{\imath}, \hat{\jmath}$ & $\hat{k}$

$\Rightarrow 6\lambda = (x-2), -3\lambda = (y+3)$ & $5\lambda = (z+5)$

$\lambda = \frac{x-2}{6} = \frac{y+3}{-3} = \frac{z+5}{5}$ ....eq(4)

Let Q(a, b, c) be the point of intersection of given line and plane

As point Q lies on the given line.

Therefore from eq(4)

$$\frac{a-2}{6} = \frac{b+3}{-3} = \frac{c+5}{5} = k(say)$$

⇒a = 6k+2, b = -3k-3, c = 5k-5

Also point Q lies on the plane.

Therefore from eq(3)

6a – 3b + 5c = -2

⇒6(6k+2) – 3(-3k-3) + 5(5k-5) = -2

⇒36k + 12 + 9k + 9 + 25k – 25 = -2

⇒70k = 2

$\Rightarrow k = \frac{1}{35}$

$\therefore a = 6\left(\frac{1}{35}\right) + 2 = \frac{76}{35}$

$b = -3\left(\frac{1}{35}\right) - 3 = \frac{-108}{35}$

$c = 5\left(\frac{1}{35}\right) - 5 = \frac{-170}{35} = \frac{-34}{7}$

Therefore co-ordinates of the point of intersection of line and plane are

Q ≡ $\left(\frac{76}{35}, \frac{-108}{35}, \frac{-34}{7}\right)$

Exercise 28C

1. Question

Find the distance of the point $\left(2\hat{i} - \hat{j} - 4\hat{k}\right)$ from the plane $\vec{r} \cdot \left(3\hat{i} - 4\hat{j} + 12\hat{k}\right) = 9.$

Answer

Formula : $Distance\ = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

Plane $r.(3i - 4j\ +\ 12k)\ =\ 9$ can be written in cartesian form as

3x - 4y + 12z = 9

3x - 4y + 12z - 9 = 0

Point = (2i - j - 4k)

Which can be also written as

Point = (2 , - 1 , - 4)

Distance $= \frac{|(2\times3) + (-1\times-4) + (-4\times12) + (-9)|}{\sqrt{(3)^2 + (-4)^2 + 12^2}}$

$$= \frac{|6\ +\ 4 - 48 - 9|}{\sqrt{9\ +\ 16\ +\ 144}}$$

$$= \frac{|-47|}{\sqrt{169}}$$

$$= \frac{47}{13}\ units$$

2. Question

Find the distance of the point $\left(\hat{i} + 2\hat{j} + 5\hat{k}\right)$ from the plane $\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) + 17 = 0.$

Answer

Formula : $Distance\ =\frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

Plane $r.(i\ +\ j\ +\ k)\ +\ 17\ =\ 0$ can be written in cartesian form as

x + y + z + 17 = 0

Point = (i + 2j + 5k)

Which can be also written as

Point = (1 , 2 , 5)

Distance = $\frac{|(1\times1)+(2\times1)+(5\times1)+(17)|}{\sqrt{(1)^2+(1)^2+1^2}}$

$=\frac{|1\ +\ 2\ +\ 5\ +\ 17|}{\sqrt{1\ +\ 1\ +\ 1}}$

$=\frac{|25|}{\sqrt{3}}$

$=\frac{25\sqrt{3}}{3}\ units$

3. Question

Find the distance of the point (3, 4, 5) from the plane $\vec{r}\cdot\left(2\hat{i}-5\hat{j}+3\hat{k}\right)=13.$

Answer

Formula : $Distance\ =\frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

Plane $r.(2i-5j\ +\ 3k)\ =\ 13$ can be written in cartesian form as

2x - 5y + 3z = 13

2x - 5y + 3z - 13 = 0

Point = (3 , 4 , 5)

Distance = $\frac{|(3\times2)+(4\times-5)+(5\times3)-(13)|}{\sqrt{(2)^2+(-5)^2+3^2}}$

$=\frac{|6-20\ +\ 15-13|}{\sqrt{4\ +\ 25\ +\ 9}}$

$=\frac{|-12|}{\sqrt{38}}$

$=\frac{12\sqrt{38}}{38}=\frac{6\sqrt{38}}{19}\ units$

4. Question

Find the distance of the point (1, 1, 2) from the plane $\vec{r}\cdot\left(2\hat{i}-2\hat{j}+4\hat{k}\right)+5=0.$

Answer

Formula : $Distance\ =\frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

Plane $r.(2i-2j\ +\ 4k)\ +\ 5\ =\ 0$ can be written in cartesian form as

2x - 2y + 4z + 5 = 0

Point = (1 , 1 , 2)

Distance = $\frac{|(1\times2)+(1\times-2)+(2\times4)+(5)|}{\sqrt{(2)^2+(-2)^2+(4)^2}}$

$= \frac{|2 - 2 + 8 + 5|}{\sqrt{4 + 4 + 16}}$

$= \frac{|13|}{\sqrt{24}}$

$= \frac{13}{2\sqrt{6}} = \frac{13\sqrt{6}}{12}\ units$

5. Question

Find the distance of the point (2, 1, 0) from the plane 2x + y + 2z + 5 = 0.

Answer

Formula : $Distance\ = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

2x + y + 2z + 5 = 0

Point = (2 , 1 , 0)

Distance = $\frac{|(2\times2) + (1\times1) + (0\times2) + (5)|}{\sqrt{(2)^2 + (1)^2 + (2)^2}}$

$= \frac{|4 + 1 + 0 + 5|}{\sqrt{4 + 1 + 4}}$

$= \frac{|10|}{\sqrt{9}}$

$= \frac{10}{3}\ units$

6. Question

Find the distance of the point (2, 1, - 1) from the plane x – 2y + 4z = 9.

Answer

Formula : $Distance\ = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

x - 2y + 4z = 9

x - 2y + 4z – 9 = 0

Point = (2 , 1 , - 1)

Distance = $\frac{|(2\times1) + (1\times-2) + (-1\times4) - (9)|}{\sqrt{(1)^2 + (-2)^2 + (4)^2}}$

$= \frac{|2 - 2 - 4 - 9|}{\sqrt{1 + 4 + 16}}$

$= \frac{|-13|}{\sqrt{21}}$

$= \frac{13}{\sqrt{21}} = \frac{13\sqrt{21}}{21}\ units$

7. Question

Show that the point (1, 2, 1) is equidistant from the planes $\vec{r}\cdot\left(\hat{i}+2\hat{j}-2\hat{k}\right)=5$ and $\vec{r}\cdot\left(2\hat{i}-2\hat{j}+\hat{k}\right)+3=0.$

Answer

Formula : $Distance\ = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

First Plane $r.(i + 2j - 2k) = 5$ can be written in cartesian form as

x + 2y - 2z = 5

x + 2y - 2z – 5 = 0

Point = (1 , 2 , 1)

Distance for first plane = $\frac{|(1\times1)+(2\times2)+(1\times-2)-(5)|}{\sqrt{(1)^2+(2)^2+(-2)^2}}$

$= \frac{|1 + 4 - 2 - 5|}{\sqrt{1 + 4 + 4}}$

$= \frac{|-2|}{\sqrt{9}}$

$= \frac{2}{3}\ units$

Second Plane $r.(2i - 2j + k) + 3 = 0$ can be written in cartesian form as

2x - 2y + z + 3 = 0

Point = (1 , 2 , 1)

Distance for second plane = $\frac{|(1\times2)+(2\times-2)+(1\times1)+(3)|}{\sqrt{(2)^2+(-2)^2+(1)^2}}$

$= \frac{|2 - 4 + 1 + 3|}{\sqrt{4 + 4 + 1}}$

$= \frac{|2|}{\sqrt{9}}$

$= \frac{2}{3}\ units$

Hence proved.

8. Question

Show that the points (- 3, 0, 1) and (1, 1, 1) are equidistant from the plane 3x + 4y – 12z + 13 = 0.

Answer

Formula : $Distance\ = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2+B^2+C^2}}$

where (x_1, y_1, z_1) is point from which distance is to be calculated

Therefore ,

Plane = 3x + 4y - 12z + 13 = 0

First Point = (- 3 , 0 , 1)

Distance for first point = $\frac{|(-3\times3)+(0\times4)+(1\times-12)+(13)|}{\sqrt{(3)^2+(4)^2+(-12)^2}}$

$= \frac{|-9 + 0 - 12 + 13|}{\sqrt{9 + 16 + 144}}$

$= \frac{|-8|}{\sqrt{169}}$

$= \frac{8}{13}\ units$

Plane = 3x + 4y - 12z + 13 = 0

Second Point = (1 , 1 , 1)

Distance for first point = $\frac{|(1\times3)+(1\times4)+(1\times-12)+(13)|}{\sqrt{(3)^2+(4)^2+(-12)^2}}$

$= \frac{|3 + 4 - 12 + 13|}{\sqrt{9 + 16 + 144}}$

$= \frac{|8|}{\sqrt{169}}$

$= \frac{8}{13}\ units$

Hence proved.

9. Question

Find the distance between the parallel planes 2x + 3y + 4 = 4 and 4x + 6y + 8z = 12.

Answer

Formula : The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Plane 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

where (d_1, d_2) are constants of the planes

Therefore ,

First Plane 2x + 3y + 4 = 4

2x + 3y + 4 – 4 = 0 ...... (1)

Second plane 4x + 6y + 8z = 12

4x + 6y + 8z - 12 = 0

2(2x + 3y + 4z – 6) = 0

2x + 3y + 4z – 6 = 0 ...... (2)

Using equation (1) and (2)

Distance between both planes = $\frac{|-6-(-4)|}{\sqrt{(2)^2+(3)^2+(4)^2}}$

$$= \frac{|-6 + 4|}{\sqrt{4 + 9 + 16}}$$

$$= \frac{|-2|}{\sqrt{29}}$$

$$= \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}\ units$$

10. Question

Find the distance between the parallel planes x + 2y - 2z + 4 = 0 and x + 2y – 2z – 8 = 0.

Answer

Formula : The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Palne 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

where (d_1, d_2) are costants of the planes

Therefore ,

First Plane x + 2y - 2z + 4 = 0 ...... (1)

Second plane x + 2y – 2z – 8 = 0 ...... (2)

Using equation (1) and (2)

Distance between both planes = $\frac{|-8-(4)|}{\sqrt{(1)^2+(2)^2+(2)^2}}$

$$= \frac{|-12|}{\sqrt{1 + 4 + 4}}$$

$$= \frac{12}{\sqrt{9}}$$

$$= \frac{12}{3} = 4\ units$$

11. Question

Find the equation of the planes parallel to the plane x – 2y + 2z – 3 = 0, each one of which is at a unit distance from the point (1, 1, 1).

Answer

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same

Therefore ,

Parallel Plane x – 2y + 2z – 3 = 0

Normal vector = (i - 2j + 2k)

∴ Normal vector of required plane = (i - 2j + 2k)

Equation of required planes r . (i - 2j + 2k) = d

In cartesian form x – 2y + 2y = d

It should be at unit distance from point (1,1,1)

Distance = $\frac{|(1\times1)+(1\times-2)+(1\times2)-(d)|}{\sqrt{(1)^2+(-2)^2+(2)^2}}$

$= \frac{|1-2+2-d|}{\sqrt{1+4+4}}$

$= \frac{|1-d|}{\sqrt{9}}$

$1 = \frac{\pm(1-d)}{3}$

$3 = \pm(1-d)$

For + sign = > 3 = 1 - d = > d = - 2

For - sign = > 3 = - 1 + d = > d = 4

Therefore equations of planes are : -

For d = - 2 For d = 4

x – 2y + 2y = d x – 2y + 2y = d

x – 2y + 2y = - 2 x – 2y + 2y = 4

x – 2y + 2y + 2 = 0 x – 2y + 2y – 4 = 0

Required planes = x – 2y + 2y + 2 = 0

x – 2y + 2y – 4 = 0

12. Question

Find the equation of the plane parallel to the plane 2x – 3y + 5z + 7 = 0 and passing through the point (3, 4, - 1). Also, find the distance between the two planes.

Answer

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Palne 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

If two planes are parallel , then their normal vectors are same

Therefore ,

Parallel Plane 2x – 3y + 5z + 7 = 0

Normal vector = (2i - 3j + 5k)

∴ Normal vector of required plane = (2i - 3j + 5k)

Equation of required plane r . (2i - 3j + 5k) = d

In cartesian form 2x – 3y + 5y = d

Plane passes through point (3,4, - 1) therefore it will satisfy it.

2(3) – 3(4) + 5(- 1) = d

6 – 12 - 5 = d

d = - 11

Equation of required plane 2x – 3y + 5z = - 11

2x – 3y + 5z + 11 = 0

Therefore ,

First Plane 2x – 3y + 5z + 7 = 0 …... (1)

Second plane 2x – 3y + 5z + 11 = 0 …... (2)

Using equation (1) and (2)

Distance between both planes $= \frac{|11-(7)|}{\sqrt{(2)^2+(-3)^2+(5)^2}}$

$= \frac{|4|}{\sqrt{4 + 9 + 25}}$

$= \frac{4}{\sqrt{38}}$

$= \frac{4\sqrt{38}}{38} = \frac{2\sqrt{38}}{19}\ units$

13. Question

Find the equation of the plane mid - parallel to the planes 2x – 3y + 6z + 21 = 0 and 2x – 3y + 6z – 14 = 0

Answer

Formula : The equation of mid parallel plane is , say

Plane 1:ax + by + cz + d1 = 0 &

Plane 2:ax + by + cz + d2 = 0 is given by the formula

$$Mid\ parallel\ plane\ = ax\ +\ by\ +\ cy\ +\ \frac{(d_1\ +\ d_2)}{2}\ =\ 0$$

where (d_1, d_2) are constants of the planes

Therefore ,

First Plane 2x – 3y + 6z + 21 = 0 …... (1)

Second plane 2x – 3y + 6z – 14 = 0 …... (2)

Using equation (1) and (2)

Mid parallel plane $= 2x - 3y\ +\ 6z\ +\ \frac{21-14}{2}\ =\ 0$

4x – 6y + 12z + 7 = 0

Exercise 28D

1. Question

Show that the planes 2x – y + 6z = 5 and 5x – 2.5y + 15z = 12 are parallel.

Answer

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are either same or proportional to each other

Therefore ,

Plane 1 : - 2x – y + 6z = 5

Normal vector (Plane 1) = (2i – j + 6k) …..(1)

Plane 2 : - 5x – 2.5y + 15z = 12

Normal vector (Plane 2) = (5i – 2.5j + 15k) …..(2)

Multiply equation(1) by 5 and equation(2) by 2

Normal vector (Plane 1) = 5(2i – j + 6k)

= 10i – 5j + 30k

Normal vector (Plane 2) = 2(5i – 2.5j + 15k)

= 10i – 5j + 30k

Since, both normal vectors are same .Therefore both planes are parallel

2. Question

Find the vector equation of the plane through the point $\left(3\hat{i}+4\hat{j}-\hat{k}\right)$ and parallel to the plane $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+5\hat{k}\right)+5=0.$

Answer

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane r . (2i – 3j + 5k) + 5 = 0

Normal vector = (2i - 3j + 5k)

∴ Normal vector of required plane = (2i - 3j + 5k)

Equation of required plane r . (2i - 3j + 5k) = d

In cartesian form 2x – 3y + 5z = d

Plane passes through point (3,4, - 1) therefore it will satisfy it.

2(3) – 3(4) + 5(- 1) = d

6 – 12 - 5 = d

d = - 11

Equation of required plane r . (2i - 3j + 5k) = - 11

r . (2i - 3j + 5k) + 11 = 0

3. Question

Find the vector equation of the plane passing through the point (a, b, b) and parallel to the plane $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=2.$

There is a error in question …… the point should be (a,b,c) instead of (a,b,b) to get the required answer.

Answer

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane r . (i + j + k) = 2

Normal vector = (i + j + k)

∴ Normal vector of required plane = (i + j + k)

Equation of required plane r . (i + j + k) = d

In cartesian form x + y + z = d

Plane passes through point (a,b,c) therefore it will satisfy it.

(a) + (b) + (c) = d

d = a + b + c

Equation of required plane r . (i + j + k) = a + b + c

4. Question

Find the vector equation of the plane passing through the point (1, 1, 1) and parallel to the plane $\vec{r}\cdot\left(2\hat{i}-\hat{j}+2\hat{k}\right)=5.$

Answer

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane r . (2i - j + 2k) = 5

Normal vector = (2i - j + 2k)

∴ Normal vector of required plane = (2i - j + 2k)

Equation of required plane r . (2i - j + 2k) = d

In cartesian form 2x - y + 2z = d

Plane passes through point (1,1,1) therefore it will satisfy it.

2(1) - (1) + 2(1) = d

d = 2 – 1 + 2 = 3

Equation of required plane r . (2i - j + 2k) = 3

5. Question

Find the equation of the plane passing through the point (1, 4, - 2) and parallel to the plane 2x – y + 3z + 7 = 0.

Answer

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane 2x – y + 3z + 7 = 0

Normal vector = (2i - j + 3k)

∴ Normal vector of required plane = (2i - j + 3k)

Equation of required plane r . (2i - j + 3k) = d

In cartesian form 2x - y + 3z = d

Plane passes through point (1,4, - 2) therefore it will satisfy it.

2(1) - (4) + 3(- 2) = d

d = 2 – 4 – 6 = - 8

Equation of required plane 2x - y + 3z = - 8

2x - y + 3z + 8 = 0

6. Question

Find the equations of the plane passing through the origin and parallel to the plane 2x – 3y + 7z + 13 = 0.

Answer

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane 2x – 3y + 7z + 13 = 0

Normal vector = (2i - 3j + 7k)

∴ Normal vector of required plane = (2i - 3j + 7k)

Equation of required plane r . (2i - 3j + 7k) = d

In cartesian form 2x - 3y + 7z = d

Plane passes through point (0,0,0) therefore it will satisfy it.

2(0) - (0) + 3(0) = d

d = 0

Equation of required plane 2x - 3y + 7z = 0

7. Question

Find the equations of the plane passing through the point (- 1, 0, 7) and parallel to the plane 3x – 5y + 4z = 11.

Answer

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same.

Therefore ,

Parallel Plane 3x – 5y + 4z = 11

Normal vector = (3i - 5j + 4k)

∴ Normal vector of required plane = (3i - 5j + 4k)

Equation of required plane r . (3i - 5j + 4k) = d

In cartesian form 3x - 5y + 4z = d

Plane passes through point (- 1,0,7) therefore it will satisfy it.

3(- 1) - 5(0) + 4(7) = d

d = - 3 + 28 = 25

Equation of required plane 3x - 5y + 4z = 25

8. Question

Find the equations of planes parallel to the plane x – 2y + 2z = 3 which are at a unit distance from the point (1, 2, 3).

Answer

Formula : Plane = r . (n) = d

Where r = any random point

n = normal vector of plane

d = distance of plane from origin

If two planes are parallel , then their normal vectors are same

Therefore ,

Parallel Plane x – 2y + 2z – 3 = 0

Normal vector = (i - 2j + 2k)

∴ Normal vector of required plane = (i - 2j + 2k)

Equation of required planes r . (i - 2j + 2k) = d

In cartesian form x – 2y + 2y = d

It should be at unit distance from point (1,2,3)

Distance = $\frac{|(1\times 1)+(2\times -2)+(3\times 2)-(d)|}{\sqrt{(1)^2+(-2)^2+(2)^2}}$

$= \frac{|1-4+6-d|}{\sqrt{1+4+4}}$

$= \frac{|3-d|}{\sqrt{9}}$

$1 = \frac{\pm(3-d)}{3}$

$3 = \pm(3-d)$

For + sign = > 3 = 3 - d = > d = 0

For - sign = > 3 = - 3 + d = > d = 6

Therefore equations of planes are : -

For d = 0 For d = 6

x – 2y + 2y = d x – 2y + 2y = d

x – 2y + 2y = 0 x – 2y + 2y = 6

Required planes = x – 2y + 2y = 0

x – 2y + 2y – 6 = 0

9. Question

Find the distance between the planes x + 2y + 3z + 7 = 0 and 2x + 4y + 6z + 7 = 0.

Answer

Formula : The distance between two parallel planes, say

Plane 1:ax + by + cz + d1 = 0 &

Plane 2:ax + by + cz + d2 = 0 is given by the formula

$$Distance = \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$$

where (d_1, d_2) are constants of the planes

Therefore ,

First Plane x + 2y + 3z + 7 = 0

2(x + 2y + 3z + 7) = 0

2x + 4y + 6z + 14 = 0 …… (1)

Second plane 2x + 4y + 6z + 7 = 0 …… (2)

Using equation (1) and (2)

Distance between both planes $= \frac{|7-(14)|}{\sqrt{(2)^2+(4)^2+(6)^2}}$

$$= \frac{|-7|}{\sqrt{4 + 16 + 36}}$$

$$= \frac{|-7|}{\sqrt{56}}$$

$$= \frac{7}{\sqrt{56}} \text{ units}$$

Exercise 28E

1. Question

Find the equation of the plane through the line of intersection of the planes x + y + z = 6 and 2x + 2y + 4z + 5 = 0, and passing through the point (1, 1, 1).

Answer

Equation of plane through the line of intersection of planes in Cartesian form is

$A_1x + B_1y + C_1z + D_1 + \lambda(A_2x + B_2y + C_2z + D_2) = 0$ (1)

For the standard equation of planes,

$A_1x + B_1y + C_1z + D_1$ and $A_2x + B_2y + C_2z + D_2$

So, putting in equation (1), we have

x + y + z-6 + λ(2x + 2y + 4z + 5)=0

(1 + 2λ)x + (1 + 2λ)y + (1 + 4λ)z-6 + 5λ=0 (2)

Now plane passes through (1,1,1) then it must satisfy the plane equation,

(1 + 2λ).1 + (1 + 2λ).1 + (1 + 4λ).1-6 + 5λ=0

1 + 2λ + 1 + 2λ + 1 + 4λ-6 + 5λ=0

3 + 8λ-6 + 5λ=0

13λ=3

$$\lambda = \frac{3}{13}$$

Putting in equation (2)

$$\left(1+2.\frac{3}{13}\right)x+\left(1+2.\frac{3}{13}\right)y+\left(1+4.\frac{3}{13}\right)z-6+5.\frac{3}{13}=0$$

$$\left(\frac{13+6}{13}\right)x+\left(\frac{13+6}{13}\right)y+\left(\frac{13+12}{13}\right)z+\frac{-78+15}{13}=0$$

19x + 19y + 25z-63=0

So, the required equation of plane is 19x + 19y + 25z=63.

2. Question

Find the equation of the plane through the line of intersection of the planes x - 3y + z + 6 = 0 and x + 2y + 3z + 5 = 0, and passing through the origin.

Answer

Equation of plane through the line of intersection of planes in Cartesian form is

$A_1x+B_1y+C_1z+D_1+\lambda\left(A_2x+B_2y+C_2z+D_2\right)=0$ (1)

For the standard equation of planes,

$A_1x+B_1y+C_1z+D_1$ and $A_2x+B_2y+C_2z+D_2$

So, putting in equation (1), we have

x-3y + z + 6 + λ(x + 2y + 3z + 5)=0

(1 + λ)x + (-3 + 2λ)y + (1 + 3λ)z + 6 + 5λ=0 (2)

Now plane passes through (0,0,0) then it must satisfy the plane equation,

(1 + λ).0 + (-3 + 2λ).0 + (1 + 3λ).0 + 6 + 5λ=0

5λ=-6

$$\lambda=\frac{-6}{5}$$

Putting in equation (2)

$$\left(1+\frac{-6}{5}\right)x+\left(-3+2.\frac{-6}{5}\right)y+\left(1+3.\frac{-6}{5}\right)z+6+5.\frac{-6}{5}=0$$

$$\left(\frac{5+(-6)}{5}\right)x+\left(\frac{-15-12}{5}\right)y+\left(\frac{5+(-18)}{5}\right)z+\frac{30+(-30)}{5}=0$$

-x-27y-13z=0

x + 27y + 13z=0

So, required equation of plane is x + 27y + 13z=0.

3. Question

Find the equation of the plane passing through the intersection of the planes 2x + 3y – z + 1 = 0 and x + y – 2z + 3 = 0, and perpendicular to the plane 3x - y -2z -4 = 0.

Answer

Equation of plane through the line of intersection of planes in Cartesian form is

$A_1x+B_1y+C_1z+D_1+\lambda\left(A_2x+B_2y+C_2z+D_2\right)=0$ (1)

For the standard equation of planes,

$A_1x+B_1y+C_1z+D_1$ and $A_2x+B_2y+C_2z+D_2$

So, putting in equation (1), we have

2x + 3y–z + 1 + λ(x + y–2z + 3)=0

(2 + λ)x + (3 + λ)y + (-1-2λ)z + 1 + 3λ=0 (2)

Now as the plane 3x-y-2z-4=0 is perpendicular the given plane,

For θ=90°, cos90°=0

$A_1A_2+B_1B_2+C_1C_2=0$ (3)

On comparing with standard equations in Cartesian form,

$A_1=2+\lambda, B_1=3+\lambda, C_1=-1-2\lambda$ and $A_2=3, B_2=-1, C_2=-2$

Putting values in equation (3), we have

(2 + λ).3 + (3 + λ).(-1) + (-1-2λ).(-2)=0

6 + 3λ-3-λ + 2 + 4λ=0

5 + 6λ=0

$$\lambda = \frac{-5}{6}$$

Putting in equation(2)

$$\left(2+\frac{-5}{6}\right)x+\left(3+\frac{-5}{6}\right)y+\left(-1-2.\frac{-5}{6}\right)z+1+3.\frac{-5}{6}=0$$

$$\left(\frac{12-5}{6}\right)x+\left(\frac{18-5}{6}\right)y+\left(\frac{-6+10}{6}\right)z+\frac{6-15}{6}=0$$

7x + 13y + 4z-9=0

7x + 13y + 4z=9

So, required equation of plane is 7x + 13y + 4z=9.

4. Question

Find the equation of the plane passing through the line of intersection of the planes 2x - y = 0 and 3z - y = 0, and perpendicular to the plane 4x + 5y - 3z = 9.

Answer

Equation of plane through the line of intersection of planes in Cartesian form is

$A_1x+B_1y+C_1z+D_1+\lambda\left(A_2x+B_2y+C_2z+D_2\right)=0$ (1)

For the standard equation of planes,

$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$

So, putting in equation (1), we have

2x-y + λ(3z-y)=0

2x + (-1-λ)y + 3λz=0 (2)

Now as the plane is perpendicular the given plane,

For θ=90°, cos90°=0

$A_1A_2+B_1B_2+C_1C_2=0$ (3)

On comparing with standard equations in Cartesian form,

$A_1=2, B_1=-1-\lambda, C_1=3\lambda \text{ and } A_2=4, B_2=5, C_2=-3$

Putting values in equation(3),

2.4 + (-1-λ).5 + 3λ.-3=0

8-5-5λ-9λ=0

-14λ=-3

$$\lambda = \frac{3}{14}$$

Putting in equation(2)

$$2x+\left(-1-\frac{3}{14}\right)y+3\left(\frac{3}{14}\right)z=0$$

$$2x+\left(\frac{-14-3}{14}\right)y+\frac{9}{14}.z=0$$

28x-17y + 9z=0

So, required equation of plane is 28x-17y + 9z=0.

5. Question

Find the equation of the plane passing through the intersection of the planes x - 2y + z = 1 and 2x + y + z = 8, and parallel to the line with direction ratios 1, 2, 1. Also, find the perpendicular distance of (1, 1, 1) from the plane.

Answer

Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda\left(A_2x+B_2y+C_2z+D_2\right)=0 \text{ (1)}$$

For the standard equation of planes,

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

x-2y + z-1 + λ(2x + y + z-8)=0

(1 + 2λ)x + (-2 + λ)y + (1 + λ)z-1-8λ=0 (2)

For plane the normal is perpendicular to line given parallel to this i.e.

$$A_1A_2+B_1B_2+C_1C_2=0$$

Where A_1, B_1, C_1 are direction ratios of plane and A_2, B_2, C_2 are of line.

(1 + 2λ).1 + (-2 + λ).2 + (1 + λ).1=0

1 + 2λ-4 + 2λ + 1 + λ=0

-2 + 5λ=0

$$\lambda=\frac{2}{5}$$

Putting the value of λ in equation (2)

$$\left(1+2.\left(\frac{2}{5}\right)\right).x+\left(-2+\frac{2}{5}\right).y+\left(1+\frac{2}{5}\right).z-1-8.\left(\frac{2}{5}\right)=0$$

$$\left(\frac{5+4}{5}\right)x+\left(\frac{-10+2}{5}\right)y+\left(\frac{5+2}{5}\right)z+\frac{-5-16}{5}=0$$

9x-8y + 7z-21=0

9x-8y + 7z=21

For the equation of plane Ax + By + Cz=D and point (x1,y1,z1), a distance of a point from a plane can be calculated as

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

$$\left|\frac{9.1-8.1+7.1-21}{\sqrt{(9)^2+(-8)^2+(7)^2}}\right| \Rightarrow \left|\frac{9-8+7-21}{\sqrt{81+64+49}}\right|=\left|\frac{13}{\sqrt{194}}\right|$$

So, the required equation of the plane is 9x-8y + 7z=21, and distance of the plane from (1,1,1) is

$$d=\frac{13}{\sqrt{194}}$$

6. Question

Find the equation of the plane passing through the line of intersection of the planes x + 2y + 3z – 5 = 0 and 3x - 2y –z + 1 = 0 and cutting off equal intercepts on the x-axis and z-axis.

Answer

Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda\left(A_2x+B_2y+C_2z+D_2\right)=0 \text{ (1)}$$

For the standard equation of planes in Cartesian form

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation 1 we have

x + 2y + 3 z-5 + λ(3x -2y -z + 1)=0

(1 + 3λ)x + (2-2λ)y + (3-λ)z-5 + λ=0

Now equation of plane in intercept form

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

As given equal intercept means a=c

First, we transform equation of a plane in intercept form

$$\frac{x}{\frac{1}{(1+3\lambda)}}+\frac{y}{\frac{1}{(2-2\lambda)}}+\frac{z}{\frac{1}{(3-\lambda)}}=5-\lambda$$

$$\frac{x}{\frac{5-\lambda}{(1+3\lambda)}}+\frac{y}{\frac{5-\lambda}{(2-2\lambda)}}+\frac{z}{\frac{5-\lambda}{(3-\lambda)}}=1$$

On comparing with the standard equation of a plane in intercept form

$$a=\frac{5-\lambda}{(1+3\lambda)}, c=\frac{5-\lambda}{(3-\lambda)}$$

Now as a=b=c

$$\frac{5-\lambda}{(1+3\lambda)}=\frac{5-\lambda}{(3-\lambda)} \Rightarrow 3-\lambda=1+3\lambda$$

$$4\lambda=2 \Rightarrow \lambda=\frac{1}{2}$$

Putting in equation (2), we have

$$\left(1+3.\frac{1}{2}\right)x+\left(2-2.\frac{1}{2}\right)y+\left(3-\frac{1}{2}\right)z-5+\frac{1}{2}=0$$

$$\left(\frac{2+3}{2}\right)x+\left(\frac{4-2}{2}\right)y+\left(\frac{6-1}{2}\right)z+\frac{-10+1}{2}=0$$

5x + 2y + 5z-9=0

5x + 2y + 5z=9

So, required equation of plane is 5x + 2y + 5z=9.

7. Question

Find the equation of the plane through the intersection of the planes 3x – 4y + 5z =10 and 2x + 2y - 3z = 4 and parallel to the line x = 2y = 3z.

Answer

Equation of plane through the line of intersection of planes in Cartesian form is

$$A_1x+B_1y+C_1z+D_1+\lambda(A_2x+B_2y+C_2z+D_2)=0$$ (1)

For the standard equation of planes in Cartesian form

$$A_1x+B_1y+C_1z+D_1 \text{ and } A_2x+B_2y+C_2z+D_2$$

So, putting in equation (1), we have

3x – 4y + 5z -10 + λ(2x + 2y - 3z - 4)=0

(3 + 2λ)x + (-4 + 2λ)y + (5-3λ)z-10-4λ=0

Given line is parallel to plane then the normal of plane is perpendicular to line,

$$A_1A_2+B_1B_2+C_1C_2=0$$

Where A_1, B_1, C_1 are direction ratios of plane and A_2, B_2, C_2 are of line.

(3 + 2λ).6 + (-4 + 2λ).3 + (5-3λ).2=0

18 + 12λ-12 + 6λ + 10-6λ=0

16 + 12λ=0

$$\lambda=\frac{-16}{12} \Rightarrow \frac{-4}{3}$$

Putting the value of λ in equation (2)

$$\left(3+2.\left(\frac{-4}{3}\right)\right)x+\left(-4+2.\left(\frac{-4}{3}\right)\right)y+\left(5-3\left(\frac{-4}{3}\right)\right)z-10-4.\left(\frac{-4}{3}\right)=0$$

$$\left(\frac{9-8}{3}\right)x+\left(\frac{-12-8}{3}\right)y+\left(\frac{15+12}{3}\right)z+\frac{-30+16}{3}=0$$

x-20y + 27z-14=0

So, required equation of plane is x-20y + 27z-14=0.

8. Question

Find the vector equation of the plane through the intersection of the planes $\vec{r}.\left(\hat{i}+3\hat{j}-\hat{k}\right)=0$ and $\vec{r}.\left(\hat{j}+2\hat{k}\right)=0,$ and passing through the point (2, 1, -1).

Answer

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation(1)

$$\vec{r}(\hat{i}+3\hat{j}-\hat{k}+\lambda\left(\hat{j}+2\hat{k}\right)=0+\lambda.0$$

$\vec{r}\left(\hat{i}+(3+\lambda)\hat{j}+(-1+2\lambda)\hat{k}\right)=0$ (2)

Now as the plane passes through (2,1,-1)

$$\vec{r}=2\hat{i}+\hat{j}-\hat{k}$$

Putting in equation (2)

$$\left(2\hat{i}+\hat{j}-\hat{k}\right)(\left(\hat{i}+(3+\lambda)\hat{j}+(-1+2\lambda)\hat{k}\right)=0$$

2.1 + 1.(3 + λ) + (-1)(-1 + 2λ)=0

2 + 3 + λ + 1-2λ=0

λ=6

Putting the value of λ in equation (2)

$$\vec{r}\left(\hat{i}+(3+6)\hat{j}+(-1+2(6))\hat{k}\right)=0$$

$$\vec{r}\left(\hat{i}+9\hat{j}+11\hat{k}\right)=0$$

So, required equation of plane is $\vec{r}\left(\hat{i}+9\hat{j}+11\hat{k}\right)=0$.

9. Question

Find the vector equation of the plane through the point (1, 1, 1), and passing through the intersection of the planes $\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)+1=0$ and $\vec{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)-5=0.$

Answer

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation(1)

$$\vec{r}(\hat{i}-\hat{j}+3\hat{k}+\lambda\left(2\hat{i}+\hat{j}-\hat{k}\right)=-1+\lambda.5$$

$\vec{r}\left((1+2\lambda)\hat{i}+(-1+\lambda)\hat{j}+(3-\lambda)\hat{k}\right)=-1+5\lambda$ (2)

Now as the plane passes through (1,1,1)

$$\vec{r}=\hat{i}+\hat{j}+\hat{k}$$

Putting in equation (2)

$$\left(\hat{i}+\hat{j}+\hat{k}\right)(\left((1+2\lambda)\hat{i}+(-1+\lambda)\hat{j}+(3-\lambda)\hat{k}\right)=-1+5\lambda$$

1.(1 + 2λ) + 1.(-1 + λ) + 1.(3-λ)=-1 + 5λ

1 + 2λ-1 + λ + 3- λ + 1-5λ=0

-3λ + 4=0

$$\lambda=\frac{4}{3}$$

Putting the value of λ in equation (2)

$$\vec{r}\left(\left(1+2.\frac{4}{3}\right)\hat{i}+\left(-1+\frac{4}{3}\right)\hat{j}+\left(3-\frac{4}{3}\right)\hat{k}\right)=-1+5.\frac{4}{3}$$

$$\vec{r}\left(\left(\frac{3+8}{3}\right)\hat{i}+\left(\frac{-3+4}{3}\right)\hat{j}+\left(\frac{9-4}{3}\right)\hat{k}\right)=\frac{-3+20}{3}$$

$$\vec{r}\left(11\hat{i}+\hat{j}+5\hat{k}\right)=17$$

So, required equation of plane is $\vec{r}\left(11\hat{i}+\hat{j}+5\hat{k}\right)=17.$

10. Question

Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.\left(2\hat{i}-7\hat{j}+4\hat{k}\right)=3$ and $\vec{r}.\left(3\hat{i}-5\hat{j}+4\hat{k}\right)+11=0,$ and passing through the point (-2, 1, 3).

Answer

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

Putting values in equation(1)

$$\vec{r}(2\hat{i}-7\hat{j}+4\hat{k}+\lambda\left(3\hat{i}-5\hat{j}+4\hat{k}\right)=3-\lambda.11$$

$\vec{r}\left((2+3\lambda)\hat{i}+(-7-5\lambda)\hat{j}+(4+4\lambda)\hat{k}\right)=3-11\lambda$ (2)

Now as the plane passes through (-2,1,3)

$$\vec{r}=-2\hat{i}+\hat{j}+3\hat{k}$$

Putting in equation (2)

$$\left(-2\hat{i}+\hat{j}+3\hat{k}\right)(\left((2+3\lambda)\hat{i}+(-7-5\lambda)\hat{j}+(4+4\lambda)\hat{k}\right)=3-11\lambda$$

-2.(2 + 3λ) + 1.(-7-5λ) + 3.(4 + 4λ)=3-11λ

-4-6λ-7-5λ + 12 + 12λ-3 + 11λ=0

-14 + 12 + 12λ=0

$$\lambda=\frac{1}{6}$$

Putting the value of λ in equation (2)

$$\vec{r}\left(\left(2+3.\frac{1}{6}\right)\hat{i}+\left(-7-5.\frac{1}{6}\right)\hat{j}+\left(4+4\frac{1}{6}\right)\hat{k}\right)=3-11.\frac{1}{6}$$

$$\vec{r}\left(\left(\frac{12+3}{6}\right)\hat{i}+\left(\frac{-42-5}{6}\right)\hat{j}+\left(\frac{24+4}{6}\right)\hat{k}\right)=\frac{18-11}{6}$$

$$\vec{r}\left(15\hat{i}-47\hat{j}+28\hat{k}\right)=7$$

$\text{So, required equation of plane is } \vec{r}\left(15\hat{i}-47\hat{j}+28\hat{k}\right)=7.$

11. Question

Find the equation of the plane through the line of intersection of the planes $\vec{r}.\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=1$ and $\vec{r}.\left(\hat{i}-\hat{j}\right)+4=0$ and perpendicular to the plane $\vec{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)+8=0.$

Answer

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2 \text{ where } \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1 \text{ and } \vec{r}.\vec{n}_2=d_2$

Putting values in equation (1), we have

$\vec{r}(2\hat{i}-3\hat{j}+4\hat{k}+\lambda\left(\hat{i}-\hat{j}\right)=1-\lambda.4$

$\vec{r}\left((2+\lambda)\hat{i}+(-3-\lambda)\hat{j}+4\hat{k}\right)=1-4\lambda$ (2)

Given a plane perpendicular to this plane, So if n1 and n2 are normal

vectors of planes

$\vec{n}_1.\vec{n}_2=0$

$(2\hat{i}-\hat{j}+\hat{k}).\left((2+\lambda)\hat{i}+(-3-\lambda)\hat{j}+4\hat{k}\right)=0$

2.(2 + λ) + (-1).(-3-λ) + 1.4=0

4 + 2λ + 3 + λ + 4=0

11 + 3λ=0

$\lambda=\frac{-11}{3}$

Putting the value of λ in equation (2)

$\vec{r}\left(\left(2+\frac{-11}{3}\right)\hat{i}+\left(-3-\frac{-11}{3}\right)\hat{j}+4\hat{k}\right)=1-4.\frac{-11}{3}$

$\vec{r}\left(\left(\frac{6-11}{3}\right)\hat{i}+\left(\frac{-9+11}{3}\right)\hat{j}+4\hat{k}\right)=\frac{3+44}{3}$

$\vec{r}\left(-5\hat{i}-2\hat{j}+12\hat{k}\right)=47$

$\text{So required equation of plane is } \vec{r}\left(-5\hat{i}-2\hat{j}+12\hat{k}\right)=47.$

12. Question

Find the Cartesian and vector equations of the planes through the line of intersection of the planes $\vec{r}.\left(\hat{i}-\hat{j}\right)+6=0$ and $\vec{r}.\left(3\hat{i}+3\hat{j}-4\hat{k}\right)=0,$ which are at a unit distance from the origin.

Answer

Equation of plane through the line of intersection of two planes in vector form is

$\vec{r}.(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2 \text{ where } \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ (1)

Where the standard equation of planes are

$\vec{r}.\vec{n}_1=d_1 \text{ and } \vec{r}.\vec{n}_2=d_2$

Putting values in equation (1)

$\vec{r}(\hat{i}-\hat{j}+\lambda\left(3\hat{i}+3\hat{j}-4\hat{k}\right)=6+\lambda.0$

$\bar{r}\left((1+3\lambda)\hat{i}+(-1+3\lambda)\hat{j}+(-4\lambda)\hat{k}\right)=6$ (2)

For the equation of plane Ax + By + Cz=D and point (x1,y1,z1), a distance of a point from a plane can be calculated as

$$\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$$

$$\left|\frac{(1+3\lambda)0+(-1+3\lambda).0+(-4\lambda).0-6}{\sqrt{(1+3\lambda)^2+(-1+3\lambda)^2+(-4\lambda)^2)}}\right|=1$$

$$\left|\frac{-6}{\sqrt{1+9\lambda^2+6\lambda+1+9\lambda^2-6\lambda+16\lambda^2}}\right|=1$$

$$\sqrt{2+34\lambda^2}=-6$$

$$2+34\lambda^2=(-6)^2$$

$$34\lambda^2=36-2$$

$$34\lambda^2=34$$

$$\lambda^2=1\Rightarrow\lambda=1,-1$$

Putting value of λ in equation (2)

λ=1

$$\bar{r}\left((1+3.1)\hat{i}+(-1+3.1)\hat{j}+(-4.1)\hat{k}\right)=6$$

$$\bar{r}\left(4\hat{i}+2\hat{j}-4\hat{k}\right)=6\Rightarrow\bar{r}.\left(2\hat{i}+\hat{j}-2\hat{k}\right)=3$$

λ=-1

$$\bar{r}\left((1+3.(-1))\hat{i}+(-1+3(-1))\hat{j}+(-4(-1))\hat{k}\right)=6$$

$$\bar{r}\left(-2\hat{i}-4\hat{j}+4\hat{k}\right)=6\Rightarrow\bar{r}.\left(\hat{i}+2\hat{j}-2\hat{k}\right)=-3$$

For equations in Cartesian form put

$$\bar{r}=x\hat{i}+y\hat{j}+z\hat{k}$$

For λ=1

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(2\hat{i}+\hat{j}-2\hat{k}-3\right)=0$$

x.2 + y.1 + z.(-2)-3=0

2x + y-2z-3=0

For λ=-1

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(\hat{i}+2\hat{j}-2\hat{k}+3\right)=0$$

x.1 + y.2 + z.(-2) + 3=0

x + 2y-2z + 3=0

So, required equation of plane

in vector form are $\bar{r}.\left(2\hat{i}+\hat{j}-2\hat{k}\right)=3$ for $\lambda=1$

$\bar{r}.\left(\hat{i}+2\hat{j}-2\hat{k}\right)=-3$ for $\lambda=-1$

In Cartesian form are 2x + y-2z-3=0 & x + 2y-2z + 3=0

Exercise 28F

1. Question

Find the acute angle between the following planes :

(i) $\vec{r}\cdot(\hat{i}+\hat{j}-2\hat{k})=5$ and $\vec{r}\cdot(2\hat{i}+2\hat{j}-\hat{k})=9$

(ii) $\vec{r}\cdot(\hat{i}+2\hat{j}-\hat{k})=6$ and $\vec{r}\cdot(2\hat{i}-\hat{j}-\hat{k})+3=0$

(iii) $\vec{r}\cdot(2\hat{i}-3\hat{j}+4\hat{k})=1$ and $\vec{r}\cdot(-\hat{i}+\hat{j})=4$

(iv) $\vec{r}\cdot(2\hat{i}-3\hat{j}+6\hat{k})=8$ and $\vec{r}\cdot(3\hat{i}+4\hat{j}-12\hat{k})+7=0$

Answer

To find the angle between two planes, we simply find the angle between the normal vectors of planes. So if n1 and n2 are normal vectors and θ is the angle between both then,

$$\cos\theta=\left|\frac{\vec{n}_1\overline{n_2}}{|\overline{n_1}||\vec{n}_2|}\right|$$

(i)On comparing with the standard equation of planes in vector form

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

$\vec{n}_1=\hat{i}+\hat{j}-2\hat{k}$ and $\vec{n}_2=2\hat{i}+2\hat{j}-\hat{k}$

Then

$$\cos\theta=\left|\frac{(\hat{i}+\hat{j}-2\hat{k}).(2\hat{i}+2\hat{j}-\hat{k})}{|\hat{i}+\hat{j}-2\hat{k}||2\hat{i}+2\hat{j}-\hat{k}|}\right|\Rightarrow\left|\frac{1.2+1.2+(-2).(-1)}{\left(\sqrt{1^2+1^2+(-2)^2}\right).\left(\sqrt{2^2+2^2+(-1)^2}\right)}\right|=\left|\frac{2+2+2}{\sqrt{1+1+4}\sqrt{4+4+1}}\right|$$

$$\Rightarrow\left|\frac{6}{\sqrt{6}.\sqrt{9}}\right|=\left|\frac{\sqrt{6}}{3}\right|$$

$$\theta=\cos^{-1}\left(\frac{\sqrt{6}}{3}\right)$$

(ii) On comparing with the standard equation of planes in vector form

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

$\vec{n}_1=\hat{i}+\widehat{2j}-\hat{k}$ and $\vec{n}_2=2\hat{i}-\hat{j}-\hat{k}$

Then

$$\cos\theta=\left|\frac{(\hat{i}+\widehat{2j}-\hat{k}).(2\hat{i}-\hat{j}-\hat{k})}{|\hat{i}+2\hat{j}-\hat{k}||2\hat{i}-\hat{j}-\hat{k}|}\right|\Rightarrow\left|\frac{1.2+2.(-1)+(-1).(-1)}{\left(\sqrt{1^2+2^2+(-1)^2}\right).\left(\sqrt{2^2+(-1)^2+(-1)^2}\right)}\right|=\left|\frac{2-2+1}{\sqrt{1+4+1}\sqrt{4+1+1}}\right|$$

$$\Rightarrow\left|\frac{1}{\sqrt{6}.\sqrt{6}}\right|=\left|\frac{1}{6}\right|$$

$$\theta=\cos^{-1}\left(\frac{1}{6}\right)$$

(iii) On comparing with the standard equation of planes in vector form

$\vec{r}.\vec{n}_1=d_1$ and $\vec{r}.\vec{n}_2=d_2$

$\vec{n}_1=2\hat{i}-3\hat{j}+4\hat{k}$ and $\vec{n}_2=-\hat{i}+\hat{j}$

Then

$$\cos\theta=\left|\frac{(2\hat{i}-3\hat{j}+4\hat{k}).(-\hat{i}+\hat{j})}{|2\hat{i}-3\hat{j}+4\hat{k}||-\hat{i}+\hat{j}|}\right|\Rightarrow\left|\frac{2.(-1)+(-3).1+4.0}{\left(\sqrt{2^2+(-3)^2+4^2}\right).\left(\sqrt{(-1)^2+1^2}\right)}\right|=\left|\frac{-2+(-3)}{\left(\sqrt{4+9+16}\right)\left(\sqrt{1+1}\right)}\right|$$

$$\Rightarrow\left|\frac{-5}{\sqrt{29}\sqrt{2}}\right|=\left|\frac{-5}{\sqrt{58}}\right|$$

$$\theta = \cos^{-1}\left(\frac{5}{\sqrt{58}}\right)$$

(iv)On comparing with the standard equation of planes in vector for

$$\vec{r}.\vec{n}_1 = d_1 \text{ and } \vec{r}.\vec{n}_2 = d_2$$

$$\vec{n}_1 = 2\hat{i} - 3\hat{j} + 6\hat{k} \text{ and } \vec{n}_2 = 3\hat{i} + 4\hat{j} - 12\hat{k}$$

Then

$$\cos\theta = \left|\frac{\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right).\left(3\hat{i} + 4\hat{j} - 12\hat{k}\right)}{\left|2\hat{i} - 3\hat{j} + 6\hat{k}\right|\left|3\hat{i} + 4\hat{j} - 12\hat{k}\right|}\right| \Rightarrow \left|\frac{2.3 + (-3).4 + 6.(-12)}{\left(\sqrt{2^2} + (-3)^2 + 6^2\right).(\sqrt{3^2} + 4^2 + (-12)^2}\right|$$

$$= \left|\frac{6 + (-12) + (-72)}{\left(\sqrt{4} + 9 + 36\right)\left(\sqrt{9} + 16 + 144\right)}\right|$$

$$\Rightarrow \left|\frac{-78}{\sqrt{49}\sqrt{169}}\right| = \left|\frac{-78}{7.13}\right|$$

$$\theta = \cos^{-1}\left(\frac{6}{7}\right)$$

2. Question

Show that the following planes are at right angles:

(i) $\vec{r} \cdot \left(4\hat{i} - 7\hat{j} - 8\hat{k}\right) = 5$ and $\vec{r} \cdot \left(3\hat{i} - 4\hat{j} + 5\hat{k}\right) + 10 = 0$

(ii) $\vec{r} \cdot \left(2\hat{i} + 6\hat{j} + 6\hat{k}\right) = 13$ and $\vec{r} \cdot \left(3\hat{i} + 4\hat{j} - 5\hat{k}\right) + 7 = 0$

Answer

To show the right angle between two planes, we simply find the angle between the normal vectors of planes. So if n1 and n2 are normal vectors and θ is the angle between both then

$\cos\theta = \left|\frac{\vec{n}_1\overline{n_2}}{\left|\overline{n_1}\right|\left|\vec{n}_2\right|}\right|$.for right angle θ=90°

Cos90°=0

$\vec{n}_1.\vec{n}_2 = 0$ (1)

(i)On comparing with standard equation

$$\vec{n}_1 = 4\hat{i} - 7\hat{j} - 8\hat{k} \text{ and } \vec{n}_2 = 3\hat{i} - 4\hat{j} + 5\hat{k}$$

$$\text{LHS} = \vec{n}_1.\vec{n}_2 \Rightarrow \left(4\hat{i} - 7\hat{j} - 8\hat{k}\right).\left(3\hat{i} - 4\hat{j} + 5\hat{k}\right) = 4.3 + (-7).(-4) + (-8).5$$

$$\Rightarrow 12 + 28 - 40 = 40 - 40 \Rightarrow 0 = \text{RHS}$$

Hence proved planes at right angles.

(ii) On comparing with the standard equation of a plane

$$\vec{n}_1 = 2\hat{i} + 6\hat{j} + 6\hat{k} \text{ and } \vec{n}_2 = 3\hat{i} + 4\hat{j} - 5\hat{k}$$

$$\text{LHS} = \vec{n}_1.\vec{n}_2 \Rightarrow \left(2\hat{i} + 6\hat{j} + 6\hat{k}\right).\left(3\hat{i} + 4\hat{j} - 5\hat{k}\right) = 2.3 + 6.4 + 6.(-5)$$

$$\Rightarrow 6 + 24 - 30 = 30 - 30 \Rightarrow 0 = \text{RHS}$$

Hence proved planes at right angles.

3. Question

Find the value of λ for which the given planes are perpendicular to each other:

(i) $\vec{r} \cdot \left(2\hat{i} - \hat{j} - \lambda\hat{k}\right) = 7$ and $\vec{r} \cdot \left(3\hat{i} + 2\hat{j} + 2\hat{k}\right) = 9$

(ii) $\vec{r} \cdot \left(\lambda\hat{i} + 2\hat{j} + 3\hat{k}\right) = 5$ and $\vec{r} \cdot \left(\hat{i} + 2\hat{j} - 7\hat{k}\right) + 11 = 0$

Answer

For planes perpendicular Cos90°=0

$\vec{n}_1.\vec{n}_2 = 0$ (1)

(i)On comparing with the standard equation of a plane

$\vec{n}_1 = 2\hat{i} - \hat{j} - \lambda\hat{k}$ and $\vec{n}_2 = 3\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{n}_1.\vec{n}_2 = \left(2\hat{i} - \hat{j} - \lambda\hat{k}\right).\left(3\hat{i} + 2\hat{j} + 2\hat{k}\right) = 0$

2.3 + (-1).2 + (-λ).2=0

6-2-2λ=0

2λ=4

λ=2

(ii) On comparing with the standard equation of a plane

$\vec{n}_1 = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{n}_2 = \hat{i} + 2\hat{j} - 7\hat{k}$

$\vec{n}_1.\vec{n}_2 = \left(\lambda\hat{i} + 2\hat{j} + 3\hat{k}\right).\left(\hat{i} + 2\hat{j} - 7\hat{k}\right) = 0$

λ.1 + 2.2 + 3.(-7)=0 λ + 4-21=0 λ=17

4. Question

Find the acute angle between the following planes:

(i) 2X – y + z = 5 and x + y + 2z = 7

(ii) x + 2y + 2z = 3 and 2x - 3y + 6z = 8

(iii) x + y - z = 4 and x + 2y + z = 9

(iv) x + y - 2z = 6 and 2x - 2y + z = 11

Answer

To find angle in Cartesian form, for standard equation of planes

$A_1x + B_1y + C_1z + D_1 = 0$ and $A_1x + B_2y + C_2z + D_2 = 0$

$$\cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{\left(A_1^2 + B_1^2 + C_1^2\right)}\sqrt{\left(A_2^2 + B_2^2 + C_2^2\right)}}\right|$$

(i)On comparing with the standard equation of planes

$A_1 = 2, B_1 = -1, C_1 = 1$ and $A_2 = 1, B_2 = 1, C_2 = 2$

$$\cos\theta = \left|\frac{2.1 + (-1).1 + 1.2}{\sqrt{2^2 + (-1)^2 + 1^2}\sqrt{1^2 + 1^2 + 2^2}}\right| \Rightarrow \left|\frac{2 + (-1) + 2}{\sqrt{4 + 1 + 1}\sqrt{1 + 1 + 4}}\right| = \left|\frac{3}{\sqrt{6}\sqrt{6}}\right|$$

$$= \frac{3}{6} \Rightarrow \frac{1}{2}$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \frac{\pi}{3}$$

(ii)On comparing with the standard equation of planes

$A_1 = 1, B_1 = 2, C_1 = 2$ and $A_2 = 2, B_2 = -3, C_2 = 6$

$$\cos\theta = \left|\frac{1.2 + 2.(-3) + 2.6}{\sqrt{1^2 + 2^2 + 2^2}\sqrt{2^2 + (-3)^2 + 6^2}}\right| \Rightarrow \left|\frac{2 + (-6) + 12}{\sqrt{1 + 4 + 4}\sqrt{4 + 9 + 36}}\right| = \left|\frac{8}{\sqrt{9}\sqrt{49}}\right|$$

$$= \frac{8}{3.7} \Rightarrow \frac{8}{21}$$

$$\theta = \cos^{-1}\left(\frac{8}{21}\right)$$

(iii) On comparing with standard equation of planes

$A_1 = 1, B_1 = 1, C_1 = -1 \text{ and } A_2 = 1, B_2 = 2, C_2 = 1$

$$\cos\theta = \left|\frac{1.1 + 1.2 + (-1).1}{\sqrt{1^2 + 1^2 + (-1)^2}\sqrt{1^2 + 2^2 + 1^2}}\right| \Rightarrow \left|\frac{1 + 2 + (-1)}{\sqrt{1+1+1}\sqrt{1+4+1}}\right| = \left|\frac{2}{\sqrt{3}\sqrt{6}}\right|$$

$$= \frac{\sqrt{2}}{3}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

(iv)On comparing with the standard equation of planes

$A_1 = 1, B_1 = 1, C_1 = -2 \text{ and } A_2 = 2, B_2 = -2, C_2 = 1$

$$\cos\theta = \left|\frac{1.2 + 1.(-2) + (-2).1}{\sqrt{1^2 + 1^2 + (-2)^2}\sqrt{2^2 + (-2)^2 + 1^2}}\right| \Rightarrow \left|\frac{2 + (-2) + (-2)}{\sqrt{1+1+4}\sqrt{4+4+1}}\right| = \left|\frac{-2}{\sqrt{6}\sqrt{9}}\right|$$

$$= \frac{2}{\sqrt{6}.3}$$

$$\theta = \cos^{-1}\left(\frac{2}{3\sqrt{6}}\right)$$

5. Question

Show that each of the following pairs of planes are at right angles:

(i) 3x + 4y - 5z = 7 and 2x + 6y + 6z + 7 = 0

(ii) x - 2y + 4z = 10 and 18x + 17y + 4z = 49

Answer

To find angle in Cartesian form, for standard equation of planes

$$A_1x + B_1y + C_1z + D_1 = 0 \text{ and } A_1x + B_2y + C_2z + D_2 = 0 \ \cos\theta = \left|\frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{\left(A_1^2 + B_1^2 + C_1^2\right)}\sqrt{\left(A_2^2 + B_2^2 + C_2^2\right)}}\right|$$

For θ=90°, cos90°=0

$A_1A_2 + B_1B_2 + C_1C_2 = 0$

(i)On comparing with the standard equation of a plane

$A_1 = 3, B_1 = 4, C_1 = -5 \text{ and } A_2 = 2, B_2 = 6, C_2 = 6$

$\text{LHS} = A_1A_2 + B_1B_2 + C_1C_2 \Rightarrow 3.2 + 4.6 + (-5).6 = 6 + 24 - 30$

=0=RHS

Hence proved that the angle between planes is 90°.

(ii) On comparing with the standard equation of a plane

$A_1 = 1, B_1 = -2, C_1 = 4 \text{ and } A_2 = 18, B_2 = 17, C_2 = 4$

$\text{LHS} = A_1A_2 + B_1B_2 + C_1C_2 \Rightarrow 1.18 + (-2).17 + 4.4 = 18 + (-34) + 16$

=0=RHS

Hence proved that angle between planes is 90°.

6. Question

Prove that the plane 2x + 2y + 4z = 9 is perpendicular to each of the planes x + 2y + 2z – 7 = 0 and 5x + 6y + 7z = 23.

Answer

To show that planes are perpendicular

$A_1A_2 + B_1B_2 + C_1C_2 = 0$

Where A_1, B_1, C_1 are direction ratios of plane and A_2, B_2, C_2 are of other

plane.

$2.1 + 2.2 + 4.2 = 2 + 4 + 8 = 14 \neq 0$

Hence, planes are not perpendicular.

Similarly for the other plane

$2.5 + 2.6 + 2.7 = 10 + 12 + 14 = 36 \neq 0$

Hence, planes are not perpendicular.

7. Question

Show that the planes 2x – 2y + 4z + 5 = 0 and 3x – 3y + 6z - 1 = 0 are parallel.

Answer

To show that planes are parallel

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

On comparing with the standard equation of a plane

$$A_1 = 2, B_1 = -2, C_1 = 4 \text{ and } A_2 = 3, B_2 = -3, C_2 = 6$$

$$\frac{A_1}{A_2} = \frac{2}{3}, \frac{B_1}{B_2} = \frac{-2}{-3} \Rightarrow \frac{2}{3}, \frac{C_1}{C_2} = \frac{4}{6} \Rightarrow \frac{2}{3}$$

So,

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = \frac{2}{3}$$

Hence proved that planes are parallel.

8. Question

Find the value of λ for which the planes x – 4y + λz + 3 = 0 and 2x + 2y + 3z = 5 are perpendicular to each other.

Answer

To find an angle in Cartesian form, for the standard equation of planes

$$A_1x + B_1y + C_1z + D_1 = 0 \text{ and } A_1x + B_2y + C_2z + D_2 = 0$$

$$\cos\theta = \left| \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{\left(A_1^2 + B_1^2 + C_1^2\right)}\sqrt{\left(A_2^2 + B_2^2 + C_2^2\right)}} \right|$$

For θ=90°, cos90°=0

$$A_1A_2 + B_1B_2 + C_1C_2 = 0$$

On comparing with the standard equation of the plane,

$$A_1 = 1, B_1 = -4, C_1 = \lambda \text{ and } A_2 = 2, B_2 = 2, C_2 = 3$$

$$A_1A_2 + B_1B_2 + C_1C_2 \Rightarrow 1.2 + (-4).2 + \lambda.3 = 0$$

$2 + (-8) + 3\lambda = 0$

$-6 + 3\lambda = 0$

$\lambda = 2$

9. Question

Write the equation of the plane passing through the origin and parallel to the plane 5x - 3y + 7z + 11 = 0.

Answer

Let the equation of plane be

$$A_1x + B_1y + C_1z + D_1 = 0$$

Direction ratios of parallel planes are related to each other as

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = k\left(\text{constant}\right)$$

Putting the values from the equation of a given parallel plane,

$$\frac{A_1}{5}=\frac{B_1}{-3}=\frac{C_1}{7}=k$$

$$A_1=5k, B_1=-3k, C_1=7k$$

Putting in equation plane

$$5kx-3ky+7kz+D_1=0$$

As the plane is passing through (0,0,0), it must satisfy the plane,

$$5k.0-3k.0+7k.0+D_1=0$$

$$D_1=0$$

5kx-3ky + 7kz=0

5x-3y + 7z=0

So, required equation of plane is 5x-3y + 7z=0.

10. Question

Find the equation of the plane passing through the point (a, b, c) and parallel to the plane $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=2$.

Answer

Let the equation of a plane

$$\bar{r}.\left(x_1\hat{i}+y_1\hat{j}+z_1\hat{k}\right)=d \quad (1)$$

Direction ratios of parallel planes are related to each other as

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=\lambda(\text{constant})$$

Putting the values from the equation of a given parallel plane,

$$\frac{x_1}{1}=\frac{x_1}{1}=\frac{z_1}{1}=\lambda$$

$$x_1=y_1=z_1=\lambda$$

Putting values in equation (1), we have

$$\bar{r}.\left(\lambda\hat{i}+\lambda\hat{j}+\lambda\hat{k}\right)=d \quad (2)$$

A plane passes through (a,b,c) then it must satisfy the equation of a plane

$$\left(a\hat{i}+b\hat{j}+c\hat{k}\right).\left(\lambda\hat{i}+\lambda\hat{j}+\lambda\hat{k}\right)=d$$

$$\lambda\left(a\hat{i}+b\hat{j}+c\hat{k}\right)\left(\hat{i}+\hat{j}+\hat{k}\right)=d$$

λ(a.1 + b.1 + c.1)=d

λ(a + b + c)=d

Putting value in equation (2)

$$\bar{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)\lambda=\lambda(a+\ b+\ c)$$

$$\bar{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=a+\ b+\ c$$

So, required equtaion of plane is $\bar{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=a+\ b+\ c.$

11. Question

Find the equation of the plane passing through the point (1, -2, 7) and parallel to the plane 5x + 4y - 11z = 6.

Answer

Let the equation of plane be

$$A_1x+B_1y+C_1z+D_1=0$$

Direction ratios of parallel planes are related to each other as

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}=k\left(\text{constant}\right)$$

Putting the values from the equation of a given parallel plane,

$$\frac{A_1}{5}=\frac{B_1}{4}=\frac{C_1}{-11}=k$$

$$A_1=5k,B_1=4k,C_1=-11k$$

Putting in the equation of a plane

$$5kx+4ky-11kz+D_1=0$$

As the plane is passing through (1,-2,7), it must satisfy the plane,

$$5k.1+4k.\left(-2\right)-11k.7+D_1=0 \quad (1)$$

$$5k-8k-77k+D_1=0$$

$$D_1=80k$$

Putting value in equation (1), we have

$$5kx+4ky-11kz+80k=0$$

5x + 4y-11z + 80=0

So, the required equation of the plane is 5x + 4y-11z + 80=0.

12. Question

Find the equation of the plane passing through the point A(-1, -1, 2) and perpendicular to each of the planes 3x + 2y - 3z = 1 and 5x – 4y + z = 5.

Answer

Applying the condition of perpendicularity between planes

$$AA_1+BB_1+CC_1=0$$

Where A, B, C are direction ratios of plane and A_1, B_1, C_1 are of another plane.

$$3.A_1+2B_1-3C_1=0 \quad (1)$$

$$5.A_1-4B_1+C_1=0 \quad (2)$$

And plane passes through (-1,-1,2),

A(x + 1) + B(y + 1) + C(z-2)=0 (3)

On solving equation (1) and (2)

$$A=\frac{5B}{9}\text{and }C=\frac{11B}{9}$$

Putting values in equation (3)

$$\frac{5B}{9}.\left(x+1\right)+B\left(y+1\right)+\frac{11B}{9}.\left(z-2\right)=0$$

B(5x + 5 + 9y + 9 + 11z-22)=0

5x + 9y + 11z-8=0

So, required equation of plane is 5x + 9y + 11z=8.

13. Question

Find the equation of the plane passing through the origin and perpendicular to each of the planes x + 2y - z = 1 and 3x - 4y + z = 5.

Answer

Applying condition of perpendicularity between planes,

$$AA_1+BB_1+CC_1=0$$

Where A, B, C are direction ratios of plane and A_1, B_1, C_1 are of other

plane.

$$1.A+2.B-1.C=0$$

$$A+2B-C=0 \quad (1)$$

$3.A - 4.B + C = 0$

$3A - 4B + C = 0$ (2)

And plane passes through (0, 0, 0),

A(x-0) + B(y-0) + C(z-0)=0

Ax + By + Cz=0 (3)

On solving equation (1) and (2)

$$A = \frac{B}{2} \text{and} \, C = \frac{5B}{2}$$

Putting values in equation(3)

$$\frac{B}{2}.x + By + \frac{5B}{2}.z = 0$$

B(x + 2y + 5z)=0

x + 2y + 5z=0

So, required equation of plane is x + 2y + 5z=0.

14. Question

Find the equation of the plane that contains the point A(1, -1, 2) and is perpendicular to both the planes 3x + 3y – 2z = 5 and x + 2y - 3z = 8. Hence, find the distance of the point P(-2, 5, 5) from the plane obtained above.

Answer

Applying condition of perpendicularity between planes,

$$AA_1 + BB_1 + CC_1 = 0$$

Where A, B, C are direction ratios of plane and A_1, B_1, C_1 are of other

plane.

$3.A + 3.B - 2.C = 0$

$3A + 3B - 2C = 0$ (1)

$1.A + 2.B - 3C = 0$

$A + 2B - 3C = 0$ (2)

And plane contains the point (1,-1,2),

A(x-1) + B(y + 1) + C(z-2)=0 (3)

On solving equation (1) and (2)

$$A = \frac{-5B}{7} \text{and} \, C = \frac{3B}{7}$$

Putting values in equation (3)

$$\frac{-5B}{7}.(x-1) + B(y+1) + \frac{3B}{7}.(z-2) = 0$$

$$B(-5(x-1) + 7(y+1) + 3(z-2)) = 0$$

-5x + 5 + 7y + 7 + 3z-6=0

-5x + 7y + 3z + 6=0

5x-7y-3z-6=0

For equation of plane Ax + By + Cz=D and point (x1,y1,z1), distance of a

point from a plane can be calculated as

$$\left|\frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}}\right|$$

$$\left|\frac{5.(-2) - 7.5 - 3.5 - 6}{\sqrt{(5)^2 + (-7)^2 + (-3)^2)}}\right| \Rightarrow \left|\frac{-10 - 35 - 15 - 6}{\sqrt{25} + 49 + 9}\right| = \left|\frac{-66}{\sqrt{83}}\right| \Rightarrow \frac{66}{\sqrt{83}}$$

15. Question

Find the equation of the plane passing through the points A(1, 1, 2) and B(2, -2, 2) and perpendicular to the plane 6x – 2y + 2z = 9.

Answer

Plane passes through (1,1,2) and (2,-2,2),

A(x-1) + B(y-1) + C(z-2)=0 (1)

A(x-2) + B(y + 2) + C(z-2)=0 (2)

Subtracting (1) from (2),

A(x-2-x + 1) + B(y + 2-y-1)=0

A-3B=0 (3)

Now plane is perpendicular to 6x-2y + 2z=9

6A-2B + 2C=0 (4)

Using (3) in (4)

18A-2B + 2C=0

16B + 2C=0

C=-8B

Putting values in equation (1)

3B(x-1) + B(y + 2)-8B(z-2)=0

B(3x-3 + y + 2-8z + 16)=0

3x + y-8z + 15=0

16. Question

Find the equation of the plane passing through the points A(-1, 1, 1) and B(1, -1, 1) and perpendicular to the plane x + 2y + 2z = 5.

Answer

Plane passes through (-1,1,1) and (1,-1,1),

A(x + 1) + B(y-1) + C(z-1)=0 (1)

A(x-1) + B(y + 1) + C(z-1)=0 (2)

Subtracting (1) from (2),

A(x-1-x-1) + B(y + 1-y + 1)=0

-2A + 2B=0

A=B (3)

Now plane is perpendicular to x + 2y + 2z=5

A + 2B + 2C=0 (4)

Using (3) in (4)

B + 2B + 2C=0

3B + 2C=0

$$C = \frac{-3}{2}B$$

Putting values in equation (1)

$$B(x+1)+B(y-1)+\frac{-3}{2}B(z-1)=0$$

B(2(x + 1) + 2(y-1)-3(z-1)=0

2x + 2y-3z + 2-2-3=0

2x + 2y-3z-3=0

17. Question

Find the equation of the plane through the points A(3, 4, 2) and B(7, 0, 6) and perpendicular to the plane 2x – 5y = 15.

HINT: The given plane is 2x – 5y + 0z = 15

Answer

Plane passes through (3,4,2) and (7,0,6),

A(x-3) + B(y-4) + C(z-2)=0 (1)

A(x-7) + B(y-0) + C(z-6)=0 (2)

Subtracting (1) from (2),

A(x-7-x + 3) + B(y-y + 4) + C(z-6-z + 2)=0

-4A + 4B-4C=0

A-B + C=0

B=A + C (3)

Now plane is perpendicular to 2x-5y=15

2A-5B=0 (4)

Using (3) in (4)

2A-5(A + C)=0

2A-5A-5C=0

-3A-5C=0

$$C = \frac{-3}{5}A$$

$$B = A + \frac{-3}{5}A \Rightarrow \frac{2}{5}A$$

Putting values in equation (1)

$$A\left(x-3\right)+\frac{2}{5}A\left(y-4\right)+\frac{-3}{5}A\left(z-2\right)=0$$

A(5(x-3) + 2(y-4)-3(z-2)=0

5x + 2y-3z-15-8 + 6=0

5x + 2y-3z-17=0

So, required equation of plane is 5x + 2y-3z-17=0.

18. Question

Find the equation of the plane through the points A(2, 1, -1) and B(-1, 3, 4) and perpendicular to the plane x – 2y + 4z = 10. Also, show that the plane thus obtained contains the line $\vec{r} = \left(-\hat{i}+3\hat{j}+4\hat{k}\right)+\lambda\left(3\hat{i}-2\hat{j}-5\hat{k}\right)$

Answer

Plane passes through (2,1,-1) and (-1,3,4),

A(x-2) + B(y-1) + C(z + 1)=0 (1)

A(x + 1) + B(y-3) + C(z-4)=0 (2)

Subtracting (1) from (2),

A(x + 1-x + 2) + B(y-3-y + 1) + C(z-4-z-1)=0

3A-2B-5C=0 (3)

Now plane is perpendicular to x-2y + 4z=10

A-2B + 4C=0 (4)

Using (3) in (4)

2A-9C=0

$$C = \frac{2}{9}A$$

$$2B = A + 4.\frac{2}{9}A \Rightarrow \left(\frac{9+8}{9}\right)A = \frac{17}{9}A$$

$$B = \frac{17}{18}A$$

Putting values in equation (1)

$$A\left(x-2\right)+\frac{17}{18}A\left(y-1\right)+\frac{2}{9}A\left(z+1\right)=0$$

A(18(x-2) + 17(y-1) + 4(z + 1)=0

18x + 17y + 4z-36-17 + 4=0

18x + 17y + 4z-49=0

So, the required equation of plane is 18x + 17y + 4z-49=0

If plane contains $\vec{r} = -\hat{i} + 3\hat{j} + 4\hat{k} + \left(3\hat{i} - 2\hat{j} - 5\hat{k}\right)$ then $(-1, 3, 4)$ satisfies plane and normal vector of plane is perpendicular to vector of line

LHS=18(-1) + 17.3 + 4.4-49

=-18 + 51 + 16-49

=-2 + 2=0=RHS

In vector form normal of plane

$\vec{n} = 18\hat{i} + 17\hat{j} + 4\hat{k}$

LHS=18.3 + 17(-2) + 4.(-5)=54-34-20=0=RHS

Hence line is contained in plane.

Exercise 28G

1. Question

Find the angle between the line $\vec{r} = \left(\hat{i} + 2\hat{j} - \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and the plane $\vec{r} \cdot \left(2\hat{i} - \hat{j} + \hat{k}\right) = 4$.

Answer

Given - $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r}.(2\hat{\imath} - \hat{\jmath} + \hat{k}) = 4$

To find – The angle between the line and the plane

Direction ratios of the line = (1, - 1, 1)

Direction ratios of the normal of the plane = (2, - 1, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the line and the plane

$= \sin^{-1}\left(\frac{1 \times 2 + (-1) \times (-1) + 1 \times 1}{\sqrt{1^2 + 1^2 + 1^2}\sqrt{2^2 + 1^2 + 1^2}}\right)$

$= \sin^{-1}\left(\frac{2 + 1 + 1}{\sqrt{3}\sqrt{6}}\right)$

$= \sin^{-1}\left(\frac{4}{3\sqrt{2}}\right)$

$= \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

2. Question

Find the angle between the line $\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ and the plane $\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 3$.

Answer

Given - $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(3\hat{\imath} - \hat{\jmath} + 2\hat{k})$ and $\vec{r}.(\hat{\imath} + \hat{\jmath} + \hat{k}) = 3$

To find – The angle between the line and the plane

Direction ratios of the line = (3, - 1, 2)

Direction ratios of the normal of the plane = (1, 1, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the line and the plane

$= \sin^{-1}\left(\frac{3 \times 1 + (-1) \times 1 + 2 \times 1}{\sqrt{3^2 + 1^2 + 2^2}\sqrt{1^2 + 1^2 + 1^2}}\right)$

$= \sin^{-1}\left(\frac{3 - 1 + 2}{\sqrt{14}\sqrt{3}}\right)$

$= \sin^{-1}\left(\frac{4}{\sqrt{42}}\right)$

3. Question

Find the angle between the line $\vec{r}=\left(3\hat{i}+\hat{k}\right)+\lambda\left(\hat{j}+\hat{k}\right)$ and the plane $\vec{r}\cdot\left(2\hat{i}-\hat{j}+2\hat{k}\right)=1.$

Answer

Given - $\vec{r} = (3\hat{i} + \hat{k}) + \lambda(\hat{j} + \hat{k})$ and $\vec{r}.(2\hat{i} - \hat{j} + 2\hat{k}) = 1$

To find – The angle between the line and the plane

Direction ratios of the line = (0, 1, 1)

Direction ratios of the normal of the plane = (2, - 1, 2)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{0\times 2 + 1\times(-1) + 1\times 2}{\sqrt{0^2 + 1^2 + 1^2}\sqrt{2^2 + 1^2 + 2^2}}\right)$$

$$= \sin^{-1}\left(\frac{-1 + 2}{3\sqrt{2}}\right)$$

$$= \mathbf{\sin^{-1}\left(\frac{1}{3\sqrt{2}}\right)}$$

4. Question

Find the angle between the line $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$ and the plane 3x + 4y + z + 5 = 0.

Answer

Given - $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-3}{2}$ and $3x + 4y + z + 5 = 0$

To find – The angle between the line and the plane

Direction ratios of the line = (3, - 1, 2)

Direction ratios of the normal of the plane = (3, 4, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the line and the plane

$$= \sin^{-1}\left(\frac{3\times 3 + (-1)\times 4 + 2\times 1}{\sqrt{3^2 + 1^2 + 2^2}\sqrt{3^2 + 4^2 + 1^2}}\right)$$

$$= \sin^{-1}\left(\frac{9 - 4 + 2}{\sqrt{14}\sqrt{26}}\right)$$

$$= \sin^{-1}\left(\frac{7}{\sqrt{2}\sqrt{7}\times\sqrt{2}\times\sqrt{13}}\right)$$

$$= \mathbf{\sin^{-1}\left(\frac{7}{2\sqrt{91}}\right)}$$

5. Question

Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane 10x + 2y – 11z = 3.

Answer

Given - $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and $10x + 2y - 11z = 3$

To find – The angle between the line and the plane

Direction ratios of the line = (2, 3, 6)

Direction ratios of the normal of the plane = (10, 2, - 11)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a', b', c') be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the line and the plane

$= \sin^{-1}\left(\frac{2 \times 10 + 3 \times 2 + 6 \times (-11)}{\sqrt{2^2 + 3^2 + 6^2}\sqrt{10^2 + 2^2 + 11^2}}\right)$

$= \sin^{-1}\left(\frac{20 + 6 - 66}{7 \times 15}\right)$

$= \sin^{-1}\left(\frac{-40}{7 \times 15}\right)$

$= \mathbf{\sin^{-1}\left(-\frac{8}{21}\right)}$

6. Question

Find the angle between the line joining the points A(3, - 4, - 2) and B(12, 2, 0) and the plane 3x – y + z = 1.

Answer

Given - A = (3, - 4, - 2) , B = (12, 2, 0) and 3x - y + z = 1

To find – The angle between the line joining the points A and B and the plane

Tip – If P = (a, b, c) and Q = (a’, b’, c’), then the direction ratios of the line PQ is given by ((a’ - a), (b’ - b), (c’ - c))

The direction ratios of the line AB can be given by

((12 - 3), (2 + 4), (0 + 2))

= (9, 6, 2)

Direction ratios of the normal of the plane = (3, - 1, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a’, b’, c’) be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the line and the plane

$= \sin^{-1}\left(\frac{9 \times 3 + 6 \times (-1) + 2 \times 1}{\sqrt{9^2 + 6^2 + 2^2}\sqrt{3^2 + 1^2 + 1^2}}\right)$

$= \sin^{-1}\left(\frac{27 - 6 + 2}{11 \times \sqrt{11}}\right)$

$= \mathbf{\sin^{-1}\left(\frac{23}{11\sqrt{11}}\right)}$

7. Question

If the plane 2x – 3y – 6z = 13 makes an angle $\sin^{-1}$ (λ) with the x - axis, then find the value of λ.

Answer

Given - $y = z = 0$ and $2x - 3y - 6z = 13$

To find – The angle between the line and the plane

Direction ratios of the line = (1, 0, 0)

Direction ratios of the normal of the plane = (2, - 3, - 6)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a’, b’, c’) be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a \times a' + b \times b' + c \times c'}{\sqrt{a^2 + b^2 + c^2}\sqrt{a'^2 + b'^2 + c'^2}}\right)$

The angle between the line and the plane

$= \sin^{-1}\left(\frac{1 \times 2 + 0 \times (-3) + 0 \times (-6)}{\sqrt{1^2 + 0^2 + 0^2}\sqrt{2^2 + 3^2 + 9^2}}\right)$

$= \mathbf{\sin^{-1}\left(\frac{2}{7}\right)}$

8. Question

Show that the line $\vec{r} = \left(2\hat{i} + 5\hat{j} + 7\hat{k}\right) + \lambda\left(\hat{i} + 3\hat{j} + 4\hat{k}\right)$ is parallel to the plane $\vec{r} \cdot \left(\hat{i} + \hat{j} - \hat{k}\right) = 7$. Also, find the distance between them.

Answer

Given - $\vec{r} = (2\hat{\imath} + 5\hat{\jmath} + 7\hat{k}) + \lambda(\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ and $\vec{r}.(\hat{\imath} + \hat{\jmath} - \hat{k}) = 7$

To prove – The line and the plane are parallel &

To find – The distance between them

Direction ratios of the line = (1, 3, 4)

Direction ratios of the normal of the plane = (1, 1, - 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a’, b’, c’) be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$

The angle between the line and the plane

$= \sin^{-1}\left(\frac{1\times 1 + 3\times 1 + 4\times(-1)}{\sqrt{1^2 + 3^2 + 4^2}\sqrt{1^2 + 1^2 + 1^2}}\right)$

$= \sin^{-1}\left(\frac{1 + 3 - 4}{\sqrt{26}\sqrt{3}}\right)$

$= \sin^{-1}(0)$

$= 0$

Hence, **the line and the plane are parallel.**

Now, the equation of the plane may be written as x + y - z = 7.

Tip – If ax + by + c + d = 0 be a plane and $\vec{r} = (a'\hat{\imath} + b'\hat{\jmath} + c'\hat{k}) + \lambda(a''\hat{\imath} + b''\hat{\jmath} + c''\hat{k})$ be a line vector, then the distance between them is given by $\left|\frac{a\times a' + b\times b' + c\times c' + d}{\sqrt{a^2+b^2+c^2}}\right|$

The distance between the plane and the line

$= \left|\frac{1\times 2 + 1\times 5 - 1\times 7 - 7}{\sqrt{1^2 + 1^2 + 1^2}}\right|$

$= \left|\frac{2 + 5 - 7 - 7}{\sqrt{3}}\right|$

$= \frac{7}{\sqrt{3}}$ **units**

9. Question

Find the value of m for which the line $\vec{r} = (\hat{i} + 2\hat{k}) + \lambda(2\hat{i} - m\hat{j} - 3\hat{k})$ is parallel to the plane $\vec{r}\cdot(m\hat{i} + 3\hat{j} + \hat{k}) = 4$.

Answer

Given - $\vec{r} = (\hat{\imath} + 2\hat{k}) + \lambda(2\hat{\imath} - m\hat{\jmath} - 3\hat{k})$ and $\vec{r}.(m\hat{\imath} + 3\hat{\jmath} + \hat{k}) = 4$ and they are parallel

To find – The value of m

Direction ratios of the line = (2, - m, - 3)

Direction ratios of the normal of the plane = (m, 3, 1)

Formula to be used – If (a, b, c) be the direction ratios of a line and (a’, b’, c’) be the direction ratios of the normal to the plane, then, the angle between the two is given by $\sin^{-1}\left(\frac{a\times a' + b\times b' + c\times c'}{\sqrt{a^2+b^2+c^2}\sqrt{a'^2+b'^2+c'^2}}\right)$

$\therefore \sin^{-1}\left(\frac{2\times m + (-m)\times 3 + (-3)\times 1}{\sqrt{2^2 + m^2 + 3^2}\sqrt{m^2 + 3^2 + 1^2}}\right) = 0$

$\Rightarrow \sin^{-1}\left(\frac{2m - 3m - 3}{\sqrt{13 + m^2}\sqrt{10 + m^2}}\right) = 0$

$\Rightarrow \frac{-m - 3}{\sqrt{13 + m^2}\sqrt{10 + m^2}} = 0$

$\Rightarrow \mathbf{m = -3}$

10. Question

Find the vector equation of a line passing through the origin and perpendicular to the plane $\vec{r}\cdot(\hat{i} + 2\hat{j} + 3\hat{k}) = 3$.

Answer

Given - $\vec{r}.(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) = 3$

To find – The vector equation of the line passing through the origin and perpendicular to the given plane

Tip – The equation of a plane can be given by $\vec{r}.\hat{n} = d$ where $\hat{n}$ is the normal of the plane

A line parallel to the given plane will be in the direction of the normal and will have the direction ratios same as that of the normal.

Formula to be used – If a line passes through the point (a, b, c) and has the direction ratios as (a’, b’, c’), then its vector equation is given by $\vec{r} = (a\hat{i} + b\hat{j} + c\hat{k}) + \lambda(a'\hat{i} + b'\hat{j} + c'\hat{k})$ where λ is any scalar constant

The required equation will be $\vec{r} = (0.\hat{i} + 0.\hat{j} + 0.\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$= \boldsymbol{\lambda(\hat{i} + 2\hat{j} + 3\hat{k})}$ **for some scalar λ**

11. Question

Find the vector equation of the line passing through the point with position vector $\left(\hat{i} - 2\hat{j} + 5\hat{k}\right)$ and perpendicular to the plane $\vec{r} \cdot \left(2\hat{i} - 3\hat{j} - \hat{k}\right) = 0.$

Answer

Given - $\vec{r}.(2\hat{i} - 3\hat{j} - \hat{k}) = 0$ and the vector has position vector $(\hat{i} - 2\hat{j} + 5\hat{k})$

To find – The vector equation of the line passing through (1, - 2, 5) and perpendicular to the given plane

Tip – The equation of a plane can be given by $\vec{r}.\hat{n} = d$ where $\hat{n}$ is the normal of the plane

A line parallel to the given plane will be in the direction of the normal and will have the direction ratios same as that of the normal.

Formula to be used – If a line passes through the point (a, b, c) and has the direction ratios as (a', b', c'), then its vector equation is given by $\vec{r} = (a\hat{i} + b\hat{j} + c\hat{k}) + \lambda(a'\hat{i} + b'\hat{j} + c'\hat{k})$ where λ is any scalar constant

The required equation will be $\boldsymbol{\vec{r} = (\hat{i} - 2\hat{j} + 5\hat{k}) + \lambda(2\hat{i} - 3\hat{j} - \hat{k})}$ **for some scalar λ**

12. Question

Show that the equation ax + by + d = 0 represents a plane parallel to the z - axis. Hence, find the equation of a plane which is parallel to the z - axis and passes through the points A(2, - 3, 1) and B(- 4, 7, 6).

Answer

Given – The equation of the plane is given by ax + by + d = 0

To prove – The plane is parallel to z - axis

Tip – If ax + by + cz + d is the equation of the plane then its angle with the z - axis will be given by $\sin^{-1}\left(\frac{c}{\sqrt{a^2 + b^2 + c^2}}\right)$

Considering the equation, the direction ratios of its normal is given by (a, b, 0)

The angle the plane makes with the z - axis = $\sin^{-1}[0/\sqrt{(a^2 + b^2)}] = 0$

Hence, **the plane is parallel to the z - axis**

To find – Equation of the plane parallel to z - axis and passing through points A = (2, - 3, 1) and B = (- 4, 7, 6)

The given equation ax + by + d = 0 passes through (2, - 3, 1) & (- 4, 7, 6)

$\therefore 2a - 3b + d = 0$.........(i)

$\therefore -4a + 7b + d = 0$......(ii)

Solving (i) and (ii),

$\therefore \frac{a}{\begin{vmatrix} -3 & 1 \\ 7 & 1 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 1 & 2 \\ 1 & -4 \end{vmatrix}} = \alpha$ [α → arbitrary constant]

$\therefore a = -10\alpha$

$\therefore b = -6\alpha$

Substituting the values of a and b in eqn (i), we get,

- 2X10α + 3X6α + d = 0 i.e. d = - 2α

Putting the value of a, b and d in the equation ax + by + d = 0,

(- 10α)x + (- 6α)y + (- 2α) = 0

i.e. **5x + 3y + 1 = 0**

13. Question

Find the equation of the plane passing through the points (1, 2, 3) and (0, - 1, 0) and parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$.

Answer

Given – A plane passes through points (1, 2, 3) and (0, - 1, 0) and is parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$

To find – Equation of the plane

Tip – If a plane passes through points (a', b', c'), then its equation may be given as a(x - a') + b(y - b') + c(z - c') = 0

Taking points (1, 2, 3):

a(x - 1) + b(y - 2) + c(z - 3) = 0............(i)

The plane passes through (0, - 1, 0):

a(0 - 1) + b(- 1 - 2) + c(0 - 3) = 0

i.e. a + 3b + 3c = 0.................(ii)

The plane is parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$

Tip – The normal of the plane will be normal to the given line since both the line and plane are parallel.

Direction ratios of the line is (2, 3, - 3)

Direction ratios of the normal of the plane is (a, b, c)

So, 2a + 3b - 3c = 0...........................(iii)

Solving equations (ii) and (iii),

$\therefore \frac{a}{\begin{vmatrix} 3 & 3 \\ 3 & -3 \end{vmatrix}} = -\frac{b}{\begin{vmatrix} 1 & 3 \\ 2 & -3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix}} = \alpha$ [α → arbitrary constant]

$\therefore a = -18\alpha$

$\therefore b = 9\alpha$

$\therefore c = -3\alpha$

Putting these values in equation (i) we get,

$-18\alpha(x-1) + 9\alpha(y-2) - 3\alpha(z-3) = 0$

$\Rightarrow 18(x-1) - 9(y-2) + 3(z-3) = 0$

$\Rightarrow 6(x-1) - 3(y-2) + (z-3) = 0$

$\Rightarrow 6x - 3y + z - 3 = 0$

$\Rightarrow \mathbf{6x - 3y + z = 3}$

14. Question

Find the equation of a plane passing through the point (2, - 1, 5), perpendicular to the plane x + 2y - 3z = 7 and parallel to the line $\frac{x+5}{3} = \frac{y+1}{-1} = \frac{z-2}{1}$.

Answer

Given – A plane passes through (2, - 1, 5), perpendicular to the plane x + 2y - 3z = 7 and parallel to the line $\frac{x+5}{3} = \frac{y+1}{-1} = \frac{z-2}{1}$

To find – The equation of the plane

Let the equation of the required plane be ax + by + cz + d = 0......(a)

The plane passes through (2, - 1, 5)

So, 2a - b + 5c + d = 0.....................(i)

The direction ratios of the normal of the plane is given by (a, b, c)

Now, this plane is perpendicular to the plane x + 2y - 3z = 7 having direction ratios (1, 2, - 3)

So, a + 2b - 3c = 0..............(ii)

This plane is also parallel to the line having direction ratios (3, - 1, 1)

So, the direction of the normal of the required plane is also at right angles to the given line.

So, 3a - b + c = 0..................(iii)

Solving equations (ii) and (iii),

$\therefore \frac{a}{\begin{vmatrix} 2 & -3 \\ -1 & 1 \end{vmatrix}} = -\frac{b}{\begin{vmatrix} 1 & -3 \\ 3 & 1 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}} = \alpha$ [α → arbitrary constant]

$\therefore a = -\alpha$

$\therefore b = -10\alpha$

$\therefore c = -7\alpha$

Putting these values in equation (i) we get,

2X(- α) - (- 10α) + 5(- 7α) + d = 0 i.e. d = 27α

Substituting all the values of a, b, c and d in equation (a) we get,

$-\alpha x - 10\alpha y - 7\alpha z + 27\alpha = 0$

$\Rightarrow$ **$x + 10y + 7z + 27 = 0$**

15. Question

Find the equation of the plane passing through the intersection of the planes5x - y + z = 10 and x + y - z = 4 and parallel to the line with direction ratios2, 1, 1. Find also the perpendicular distance of (1, 1, 1) from this plane.

Answer

Given – A plane passes through the intersection of 5x - y + z = 10 and x + y - z = 4 and parallel to the line with direction ratios (2, 1, 1)

To find – Equation of the plane

Tip – If ax + by + cz + d = 0 and a'x + b'y + c'z + d' = 0 be two planes, then the equation of the plane passing through their intersection will be given by

(ax + by + cz + d) + λ(a'x + b'y + c'z + d') = 0, where λ is any scalar constant

So, the equation of the plane maybe written as

(5x - y + z - 10) + λ(x + y - z - 4) = 0

⇒ (5 + λ)x + (- 1 + λ)y + (1 - λ)z + (- 10 - 4λ) = 0

This is plane parallel to a line with direction ratios (2, 1, 1)

So, the normal of this line with direction ratios ((5 + λ), (- 1 + λ), (1 - λ)) will be perpendicular to the given line.

Hence,

2(5 + λ) + (- 1 + λ) + (1 - λ) = 0

⇒ λ = - 5

The equation of the plane will be

(5 + (- 5))x + (- 1 + (- 5))y + (1 - X(- 5))z + (- 10 - 4X(- 5)) = 0

⇒ - 6y + 6z + 10 = 0

⇒ **3y - 3z = 5**

To find – Perpendicular distance of point (1, 1, 1) from the plane

Formula to be used - If ax + by + c + d = 0 be a plane and (a', b', c') be the point, then the distance between them is given by $\left|\frac{a \times a' + b \times b' + c \times c' + d}{\sqrt{a^2 + b^2 + c^2}}\right|$

The distance between the plane and the line

$= \left|\frac{0 \times 2 + 3 \times 1 - 3 \times 1 - 5}{\sqrt{0^2 + 3^2 + 3^2}}\right|$

$= \left|\frac{3 - 3 - 5}{2\sqrt{3}}\right|$

$= \frac{5}{2\sqrt{3}}$ **units**

Exercise 28H

1. Question

Find the vector and Cartesian equations of the plane passing through the origin and parallel to the vectors $\left(\hat{i} + \hat{j} - \hat{k}\right)$ and $\left(3\hat{i} - \hat{k}\right)$.

Answer

Given - $\vec{r} = \hat{i} + \hat{j} - \hat{k}$ & $\vec{r'} = 3\hat{i} - \hat{k}$ are two lines to which a plane is parallel and it passes through the origin.

To find – The equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

$\therefore \vec{r} \times \vec{r'}$

$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$

$= \hat{i}(-1 - 0) + \hat{j}(-3 + 1) + \hat{k}(0 - 3)$

$= -\hat{i} - 2\hat{j} - 3\hat{k}$

The plane passes through origin (0, 0, 0).

Formula to be used – If a line passes through the point (a, b, c) and has the direction ratios as (a', b', c'), then its vector equation is given by $\vec{r} = \left(a\hat{i} + b\hat{j} + c\hat{k}\right) + \lambda\left(a'\hat{i} + b'\hat{j} + c'\hat{k}\right)$ where λ is any scalar constant

The required plane will be

$\vec{r} = (0\times\hat{i} + 0\times\hat{j} + 0\times\hat{k}) + \lambda'(-\hat{i} - 2\hat{j} - 3\hat{k})$

$\Rightarrow \vec{r} = \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

The vector equation : $\mathbf{\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) = 0}$

The Cartesian equation : **x + 2y + 3z = 0**

3. Question

Find the vector and Cartesian equations of the plane passing through the point(3, - 1, 2) and parallel to the lines $\vec{r} = (-\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 5\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} - 3\hat{j} + \hat{k}) + \mu(-5\hat{i} + 4\hat{j})$.

Answer

Given - $\vec{r} = (-\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 5\hat{j} - \hat{k})$ & $\vec{r} = (\hat{i} - 3\hat{j} + \hat{k}) + \mu(-5\hat{i} + 4\hat{j})$. A plane is parallel to both these lines and passes through (3, - 1, 2).

To find – The equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

$\vec{R} = 2\hat{i} - 5\hat{j} - \hat{k}$ & $\vec{R'} = -5\hat{i} + 4\hat{j}$, where the two vectors represent the directions

$\therefore \vec{R} \times \vec{R'}$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & -1 \\ -5 & 4 & 0 \end{bmatrix}$$

$= \hat{i}(0 + 4) + \hat{j}(5 - 0) + \hat{k}(8 - 25)$

$= 4\hat{i} + 5\hat{j} - 17\hat{k}$

The equation of the plane maybe represented as 4x + 5y - 17z + d = 0

Now, this plane passes through the point (3, - 1, 2)

Hence,

4 × 3 + 5 × (- 1) – 17 × 2 + d = 0

⇒ d = 27

The Cartesian equation of the plane : **4x + 5y - 17z + 27 = 0**

The vector equation : $\mathbf{\vec{r}.(4\hat{i} + 5\hat{j} - 17\hat{k}) + 27 = 0}$

3. Question

Find the vector equation of a plane passing through the point (1, 2, 3) and parallel to the lines whose direction ratios are 1, - 1, - 2, and - 1, 0, 2.

Answer

Given – The lines have direction ratios of (1, - 1, - 2) and (- 1, 0, 2). The plane parallel to these lines passes through (1, 2, 3)

To find – The vector equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

$\vec{R} = \hat{i} - \hat{j} - 2\hat{k}$ & $\vec{R'} = -\hat{i} + 2\hat{k}$, where the two vectors represent the directions

$\therefore \vec{R} \times \vec{R'}$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ -1 & 0 & 2 \end{bmatrix}$$

$= \hat{i}(-2 - 0) + \hat{j}(2 - 2) + \hat{k}(0 - 1)$

$= -2\hat{i} - \hat{k}$

The equation of the plane maybe represented as - 2x - z + d = 0

Now, this plane passes through the point (1, 2, 3)

Hence,

(- 2) × 1 - 3 + d = 0

⇒ d = 5

The Cartesian equation of the plane : - 2x - z + 5 = 0 i.e. 2x + z = 5

The vector equation : $\mathbf{\vec{r}.(2\hat{i} + \hat{k}) = 5}$

4. Question

Find the Cartesian and vector equations of a plane passing through the point (1, 2, - 4) and parallel to the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}$ and $\frac{x-1}{1}=\frac{y+3}{1}=\frac{z}{-1}$.

Answer

Given - $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}$ & $\frac{x-1}{1}=\frac{y+3}{1}=\frac{z}{-1}$. A plane is parallel to both these lines and passes through (1, 2, - 4).

To find – The equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

The direction ratios of the given lines are (2, 3, 6) and (1, 1, - 1)

$\therefore \vec{R} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ & $\vec{R'} = \hat{i} + \hat{j} - \hat{k}$

$\therefore \vec{R} \times \vec{R'}$

$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{bmatrix}$

$= \hat{i}(-3-6) + \hat{j}(6+2) + \hat{k}(2-3)$

$= -9\hat{i} + 8\hat{j} - \hat{k}$

The equation of the plane maybe represented as - 9x + 8y - z + d = 0

Now, this plane passes through the point (1, 2, - 4)

Hence,

(- 9) × 1 + 8 × 2 - (- 4) + d = 0

⇒ d = - 11

The Cartesian equation of the plane : - 9x + 8y - z - 11 = 0 i.e. **9x - 8y + z + 11 = 0**

The vector equation : $\mathbf{\vec{r}.(9\hat{i} - 8\hat{j} + \hat{k}) + 11 = 0}$

5. Question

Find the vector equation of the plane passing through the point $(3\hat{i}+4\hat{j}+2\hat{k})$ and parallel to the vectors $(\hat{i}+2\hat{j}+3\hat{k})$ and $(\hat{i}-\hat{j}+\hat{k})$.

Answer

Given - $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k}$ & $\vec{r'} = \hat{i} - \hat{j} + \hat{k}$ are two lines to which a plane is parallel and it passes through the point $3\hat{i} + 4\hat{j} + 2\hat{k}$

To find – The equation of the plane

Tip – A plane parallel to two vectors will have its normal in a direction perpendicular to both the vectors, which can be evaluated by taking their cross product

$\therefore \vec{r} \times \vec{r'}$

$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$

$= \hat{i}(2+3) + \hat{j}(3-1) + \hat{k}(-1-2)$

$= 5\hat{i} + 2\hat{j} - 3\hat{k}$

The equation of the plane maybe represented as 5x + 2y - 3z + d = 0

Now, this plane passes through the point (3, 4, 2)

Hence,

5 × 3 + 2 × 4 – 3 × 2 + d = 0

⇒ d = - 17

The Cartesian equation of the plane : 5x + 2y - 3z - 17 = 0 i.e. 5x + 2y - 3z = 17

The vector equation : $\mathbf{\vec{r}.(5\hat{i} + 2\hat{j} - 3\hat{k}) = 17}$

Exercise 28I

1. Question

Show that the lines $\bar{r} = \left(2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $\bar{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ are coplanar.

Also find the equation of the plane containing these lines.

Answer

Given : Equations of lines -

$\overline{r_1} = \left(2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{\imath} + 2\hat{j} + 3\hat{k}\right)$

$\overline{r_2} = \left(2\hat{\imath} + 6\hat{j} + 3\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{j} + 4\hat{k}\right)$

To Prove : $\overline{r_1}$ & $\overline{r_2}$ are coplanar.

To Find : Equation of plane.

Formulae :

1) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{j} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{j} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Coplanarity of two lines :

If two lines $\overline{r_1} = \bar{a} + \lambda\bar{b}$ & $\overline{r_2} = \bar{c} + \mu\bar{d}$ are coplanar then

$\bar{a}.\left(\bar{b} \times \bar{d}\right) = \bar{c}.\left(\bar{b} \times \bar{d}\right)$

4) Equation of plane :

If two lines $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$\bar{r}.\left(\overline{b_1} \times \overline{b_2}\right) = \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right)$

Where,

$\bar{r} = x\hat{\imath} + y\hat{j} + z\hat{k}$

Answer :

Given equations of lines are

$\overline{r_1} = \left(2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{\imath} + 2\hat{j} + 3\hat{k}\right)$

$\overline{r_2} = \left(2\hat{\imath} + 6\hat{j} + 3\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{j} + 4\hat{k}\right)$

Let, $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$

Where,

$\overline{a_1} = 2\hat{j} - 3\hat{k}$

$\overline{b_1} = \hat{\imath} + 2\hat{j} + 3\hat{k}$

$\overline{a_2} = 2\hat{\imath} + 6\hat{j} + 3\hat{k}$

$\overline{b_2} = 2\hat{\imath} + 3\hat{j} + 4\hat{k}$

Now,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$= \hat{\imath}(8 - 9) - \hat{j}(4 - 6) + \hat{k}(3 - 4)$

$\therefore (\bar{b_1} \times \bar{b_2}) = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$

Therefore,

$\bar{a_1}.(\bar{b_1} \times \bar{b_2}) = (0 \times (-1)) + (2 \times 2) + ((-3) \times (-1))$

= 0 + 4 + 3

= 7

$\therefore \bar{a_1}.(\bar{b_1} \times \bar{b_2}) = 7$ ……… eq(1)

And

$\bar{a_2}.(\bar{b_1} \times \bar{b_2}) = (2 \times (-1)) + (6 \times 2) + (3 \times (-1))$

= - 2 + 12 – 3

= 7

$\therefore \bar{a_2}.(\bar{b_1} \times \bar{b_2}) = 7$ ……… eq(2)

From eq(1) and eq(2)

$\bar{a_1}.(\bar{b_1} \times \bar{b_2}) = \bar{a_2}.(\bar{b_1} \times \bar{b_2})$

Hence lines $\bar{r_1}$ & $\bar{r_2}$ are coplanar.

Equation of plane containing lines $\bar{r_1}$ & $\bar{r_2}$ is

$\bar{r}.(\bar{b_1} \times \bar{b_2}) = \bar{a_1}.(\bar{b_1} \times \bar{b_2})$

Now,

$\bar{b_1} \times \bar{b_2} = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$

From eq(1)

$\bar{a_1}.(\bar{b_1} \times \bar{b_2}) = 7$

Therefore, equation of required plane is

$\bar{r}.(-\hat{\imath} + 2\hat{\jmath} - \hat{k}) = 7$

$\therefore \bar{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) = -7$

$\therefore \bar{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) + 7 = 0$

$\bar{r}.(\hat{\imath} - 2\hat{\jmath} + \hat{k}) + 7 = 0$

2. Question

Find the vector and Cartesian forms of the equations of the plane containing the two lines $\bar{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and .

$\bar{r} = (9\hat{i} + 5\hat{j} - \hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$..

Answer

Given : Equations of lines -

$\bar{r_1} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

$\bar{r_2} = (9\hat{\imath} + 5\hat{\jmath} - \hat{k}) + \mu(-2\hat{\imath} + 3\hat{\jmath} + 8\hat{k})$

To Find : Equation of plane.

Formulae :

1) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

If two lines $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$\bar{r}.\left(\overline{b_1} \times \overline{b_2}\right) = \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right)$

Where,

Given equations of lines are

$\overline{r_1} = \left(\hat{\imath} + 2\hat{\jmath} - 4\hat{k}\right) + \lambda\left(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}\right)$

$\overline{r_2} = \left(9\hat{\imath} + 5\hat{\jmath} - \hat{k}\right) + \mu\left(-2\hat{\imath} + 3\hat{\jmath} + 8\hat{k}\right)$

Let, $\overline{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\overline{r_2} = \overline{a_2} + \lambda\overline{b_2}$

Where,

$\overline{a_1} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

$\overline{b_1} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$

$\overline{a_2} = 9\hat{\imath} + 5\hat{\jmath} - \hat{k}$

$\overline{b_2} = -2\hat{\imath} + 3\hat{\jmath} + 8\hat{k}$

Now,

$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix}$

$= \hat{\imath}(24 - 18) - \hat{\jmath}(16 + 12) + \hat{k}(6 + 6)$

$\therefore \left(\overline{b_1} \times \overline{b_2}\right) = 6\hat{\imath} - 28\hat{\jmath} + 12\hat{k}$

Therefore,

$\overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = (1 \times 6) + (2 \times (-28)) + ((-4) \times 12)$

= 6 – 56 – 48

= - 98

$\therefore \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = -98$ ....eq(1)

Equation of plane containing lines $\overline{r_1}$ & $\overline{r_2}$ is

$\bar{r}.\left(\overline{b_1} \times \overline{b_2}\right) = \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right)$

Now,

$\overline{b_1} \times \overline{b_2} = 6\hat{\imath} - 28\hat{\jmath} + 12\hat{k}$

From eq(1)

$\overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = -98$

Therefore, equation of required plane is

$\bar{r}.\left(6\hat{\imath} - 28\hat{\jmath} + 12\hat{k}\right) = -98$

$\therefore \bar{r}.\left(6\hat{\imath} - 28\hat{\jmath} + 12\hat{k}\right) + 98 = 0$

This vector equation of plane.

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.\left(\overline{b_1} \times \overline{b_2}\right) = (x \times 6) + (y \times (-28)) + (z \times 12)$

= 6x – 28y + 12z

Therefore, equation of plane is

6x – 28y + 12z = -98

6x – 28y + 12z + 98 = 0

This Cartesian equation of plane.

3. Question

Find the vector and Cartesian equations of a plane containing the two lines $\bar{r} = (2\hat{i} + \hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$ and $\bar{r} = (3\hat{i} + 3\hat{j} + 2\hat{k}) + \mu(3\hat{i} - 2\hat{j} + 5\hat{k})$. Also show that the lines $\bar{r} = (2\hat{i} + 5\hat{j} + 2\hat{k}) + p\ (3\hat{i} - 2\hat{j} + 5\hat{k})$ lies in the plane.

Answer

Given : Equations of lines -

$\bar{r_1} = (2\hat{\imath} + \hat{\jmath} - 3\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 5\hat{k})$

$\bar{r_2} = (3\hat{\imath} + 3\hat{\jmath} + 2\hat{k}) + \mu(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k})$

To Prove : $\bar{r_1}$ & $\bar{r_2}$ are coplanar.

To Find : Equation of plane.

Formulae :

1) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Coplanarity of two lines :

If two lines $\bar{r_1} = \bar{a} + \lambda\bar{b}$ & $\bar{r_2} = \bar{c} + \mu\bar{d}$ are coplanar then

$\bar{a}.(\bar{b} \times \bar{d}) = \bar{c}.(\bar{b} \times \bar{d})$

4) Equation of plane :

If two lines $\bar{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\bar{r_2} = \overline{a_2} + \lambda\overline{b_2}$ are coplanar then equation of the plane containing them is

$\bar{r}.(\overline{b_1} \times \overline{b_2}) = \overline{a_1}.(\overline{b_1} \times \overline{b_2})$

Where,

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Answer :

Given equations of lines are

$\bar{r_1} = (2\hat{\imath} + \hat{\jmath} - 3\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} + 5\hat{k})$

$\bar{r_2} = (3\hat{\imath} + 3\hat{\jmath} + 2\hat{k}) + \mu(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k})$

Let, $\bar{r_1} = \overline{a_1} + \lambda\overline{b_1}$ & $\bar{r_2} = \overline{a_2} + \lambda\overline{b_2}$

Where,

$\overline{a_1} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$

$\overline{b_1} = \hat{\imath} + 2\hat{\jmath} + 5\hat{k}$

$\overline{a_2} = 3\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

$\overline{b_2} = 3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$

Now,

$$\overline{b_1} \times \overline{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix}$$

$= \hat{\imath}(10 + 10) - \hat{\jmath}(5 - 15) + \hat{k}(-2 - 6)$

$\therefore \left(\overline{b_1} \times \overline{b_2}\right) = 20\hat{\imath} + 10\hat{\jmath} - 8\hat{k}$

Therefore,

$\overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = (2 \times 20) + (1 \times 10) + ((-3) \times (-8))$

= 40 + 10 + 24

= 74

$\therefore \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = 74$ .... eq(1)

And

$\overline{a_2}.\left(\overline{b_1} \times \overline{b_2}\right) = (3 \times 20) + (3 \times 10) + (2 \times (-8))$

= 60 + 30 – 16

= 74

$\therefore \overline{a_2}.\left(\overline{b_1} \times \overline{b_2}\right) = 74$ .... eq(2)

From eq(1) and eq(2)

$\overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = \overline{a_2}.\left(\overline{b_1} \times \overline{b_2}\right)$

Hence lines $\overline{r_1}$ & $\overline{r_2}$ are coplanar.

Equation of plane containing lines $\overline{r_1}$ & $\overline{r_2}$ is

$\bar{r}.\left(\overline{b_1} \times \overline{b_2}\right) = \overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right)$

Now,

$\overline{b_1} \times \overline{b_2} = 20\hat{\imath} + 10\hat{\jmath} - 8\hat{k}$

From eq(1)

$\overline{a_1}.\left(\overline{b_1} \times \overline{b_2}\right) = 74$

Therefore, equation of required plane is

$\bar{r}.\left(20\hat{\imath} + 10\hat{\jmath} - 8\hat{k}\right) = 74$

$\therefore \bar{r}.\left(10\hat{\imath} + 5\hat{\jmath} - 4\hat{k}\right) = 37$

$\therefore \bar{r}.\left(10\hat{\imath} + 5\hat{\jmath} - 4\hat{k}\right) - 37 = 0$

This vector equation of plane.

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.\left(\overline{b_1} \times \overline{b_2}\right) = (x \times 20) + (y \times 10) + (z \times (-8))$

= 20x + 10y – 8z

Therefore, equation of plane is

20x + 10y – 8z = 74

20x + 10y – 8z – 74 = 0

10x + 5y – 4z – 37 = 0

This Cartesian equation of plane.

4. Question

Prove that the lines $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ are coplanar. Also find the equation of the plane containing these lines.

Answer

Given : Equations of lines –

Line 1 : $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$

Line 2 : $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

Answer :

Given lines –

Line 1 : $\frac{x}{1}=\frac{y-2}{2}=\frac{z+3}{3}$

Line 2 : $\frac{x-2}{2}=\frac{y-6}{3}=\frac{z-3}{4}$

Here, $x_1 = 0$, $y_1 = 2$, $z_1 = -3$, $a_1 = 1$, $b_1 = 2$, $c_1 = 3$

$x_2 = 2$, $y_2 = 6$, $z_2 = 3$, $a_2 = 2$, $b_2 = 3$, $c_2 = 4$

Now,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=\begin{vmatrix} 2-0 & 6-2 & 3+3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$$=\begin{vmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}$$

$= 2(8-9)-4(4-6)+6(3-4)$

$= 2(-1)-4(-2)+6(-1)$

= - 2 + 8 – 6

= 0

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

Hence, given two lines are coplanar.

Equation of plane passing through line1 and line 2 is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\therefore \begin{vmatrix} x-0 & y-2 & z+3 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}=0$$

$\therefore (x-0)\times(8-9)-(y-2)\times(4-6)+(z+3)\times(3-4)=0$

$\therefore -1(x)-(y-2)(-2)+(z+3)(-1)=0$

-x + 2y – 4 – z – 3 = 0

- x + 2y – z – 7 = 0

x – 2y + z + 7 = 0

Therefore, equation of plane is

x – 2y + z + 7 = 0

5. Question

Prove that the lines $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$ and $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ are coplanar. Also find the equation of the plane containing these lines.

Answer

Given : Equations of lines –

Line 1 : $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$

Line 2 : $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Answer :

Given lines –

Line 1 : $\frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$

Line 2 : $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$

Here, $x_1 = 2$, $y_1 = 4$, $z_1 = 6$, $a_1 = 1$, $b_1 = 4$, $c_1 = 7$

$x_2 = -1$, $y_2 = -3$, $z_2 = -5$, $a_2 = 3$, $b_2 = 5$, $c_2 = 7$

Now,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} -1-2 & -3-4 & -5-6 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & -7 & -11 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix}$$

$= -3(28-35) - (-7)(7-21) - 11(5-12)$

$= -3(-7) + 7(-14) - 11(-7)$

= 21 - 98 + 77

= 0

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Hence, given two lines are coplanar.

Equation of plane passing through line 1 and line 2 is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} x-2 & y-4 & z-6 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix} = 0$$

$\therefore (x-2) \times (28-35) - (y-4) \times (7-21) + (z-6) \times (5-12) = 0$

$\therefore -7(x-2) - (y-4)(-14) + (z-6)(-7) = 0$

-7x + 14 + 14y - 56 – 7z + 42 = 0

- 7x + 14y – 7z = 0

x – 2y + z = 0

Therefore, equation of plane is

x – 2y + z = 0

6. Question

Show that the lines $\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{2y-8}{2}=\frac{z-5}{3}$ are coplanar. Find the equation of the plane containing these lines.

Answer

Given : Equations of lines –

Line 1 : $\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{-5}$ or $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$

Line 2 : $\frac{x-8}{7}=\frac{2y-8}{2}=\frac{z-5}{3}$ or $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

Answer :

Given lines –

Line 1 : $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$

Line 2 : $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$

Here, $x_1 = 5$, $y_1 = 7$, $z_1 = -3$, $a_1 = 4$, $b_1 = 4$, $c_1 = -5$

$x_2 = 8$, $y_2 = 4$, $z_2 = 5$, $a_2 = 7$, $b_2 = 1$, $c_2 = 3$

Now,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=\begin{vmatrix} 8-5 & 4-7 & 5+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$$=\begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$=3(12+5)-(-3)(12+35)+8(4-28)$

$=3(17)+3(47)+8(-24)$

= 51 + 141 – 192

= 0

$$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

Hence, given two lines are coplanar.

Equation of plane passing through line1 and line 2 is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

$$\therefore \begin{vmatrix} x-5 & y-7 & z+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix} = 0$$

$\therefore (x-5) \times (12+5) - (y-7) \times (12+35) + (z+3) \times (4-28) = 0$

$\therefore 17(x-5) - 47(y-7) + (z+3)(-24) = 0$

17x – 85 – 47y + 329 – 24z – 72 = 0

17x - 47y – 24z + 172 = 0

Therefore, equation of plane is

17x - 47y – 24z + 172 = 0

7. Question

Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar. Find the equation of the plane containing these lines.

Answer

Given : Equations of lines –

Line 1 : $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$

Line 2 : $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, then these lines are coplanar, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by,

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Answer :

Given lines –

Line 1 : $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$

Line 2 : $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$

Here, x_1 = -1 , y_1 = 3 , z_1 = -2 , a_1 = -3 , b_1 = 2 , c_1 = 1

x_2 = 0 , y_2 = 7 , z_2 = -7 , a_2 = 1 , b_2 = -3 , c_2 = 2

Now,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 0+1 & 7-3 & -7+2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 4 & -5 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$= 1(4+3) - 4(-6-1) - 5(9-2)$

$= 1(7) - 4(-7) - 5(7)$

= 7 + 28 – 35

= 0

$\therefore \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

Hence, given two lines are coplanar.

Equation of plane passing through line1 and line 2 is given by,

$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

$\therefore \begin{vmatrix} x+1 & y-3 & z+2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} = 0$

$\therefore (x+1) \times (4+3) - (y-3) \times (-6-1) + (z+2) \times (9-2) = 0$

$\therefore 7(x+1) - (y-3)(-7) + (z+2)(7) = 0$

7x + 7 + 7y – 21 + 7z + 14 = 0

7x + 7y + 7z = 0

x + y + z = 0

Therefore, equation of plane is

8. Question

Show that the lines $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z}{-1}$ and $\frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{-1}$ are coplanar. Also find the equation of the plane containing these lines.

Answer

Given : Equations of lines –

Line 1 : $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z}{-1}$

Line 2 : $\frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{-1}$

To Prove : Line 1 & line 2 are coplanar.

To Find : Equation of plane.

Formulae :

1) Coplanarity of two lines :

If two lines are given by,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, then these lines are coplanar, if

$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

2) Equation of plane :

The equation of plane containing two coplanar lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$

& $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ is given by,

$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

Answer :

Given lines –

Line 1 : $\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z}{-1}$

Line 2 : $\frac{x-4}{3} = \frac{y-1}{-2} = \frac{z-1}{-1}$

Here, $x_1 = 1$, $y_1 = 3$, $z_1 = 0$, $a_1 = 2$, $b_1 = -1$, $c_1 = -1$

$x_2 = 4$, $y_2 = 1$, $z_2 = 1$, $a_2 = 3$, $b_2 = -2$, $c_2 = -1$

Now,

$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 4-1 & 1-3 & 1-0 \\ 2 & -1 & -1 \\ 3 & -2 & -1 \end{vmatrix}$

$$=\begin{vmatrix}3 & -2 & 1\\ 2 & -1 & -1\\ 3 & -2 & -1\end{vmatrix}$$

$= 3(1-2)-(-2)(-2+3)+1(-4+3)$

$= 3(-1)+2(1)+1(-1)$

= - 2

$$\therefore \begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix} \neq 0$$

Hence, given two lines are not coplanar.

9. Question

Find the equation of the plane which contains two parallel lines given by $\frac{x-3}{1}=\frac{y+2}{-4}=\frac{z}{5}$ and $\frac{x-4}{1}=\frac{y-3}{-4}=\frac{z-2}{5}$.

Answer

Given : Equations of lines –

Line 1 : $\frac{x-3}{1}=\frac{y+2}{-4}=\frac{z}{5}$

Line 2 : $\frac{x-4}{1}=\frac{y-3}{-4}=\frac{z-2}{5}$

To Find : Equation of plane.

Formulae :

Equation of plane :

The equation of plane containing two parallel lines $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

& $\frac{x-x_2}{a}=\frac{y-y_2}{b}=\frac{z-z_2}{c}$ is given by,

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ a & b & c\end{vmatrix}=0$$

Answer :

Given lines –

Line 1 : $\frac{x-3}{1}=\frac{y+2}{-4}=\frac{z}{5}$

Line 2 : $\frac{x-4}{1}=\frac{y-3}{-4}=\frac{z-2}{5}$

Here, $x_1 = 3$, $y_1 = -2$, $z_1 = 0$, a= 1 , b = -4 , c = 5

$x_2 = 4$, $y_2 = 3$, $z_2 = 2$

Therefore, equation of plane containing line 1 & line 2 is given by,

$$\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ a & b & c\end{vmatrix}=0$$

$$\therefore \begin{vmatrix}x-3 & y+2 & z-0\\ 4-3 & 3+2 & 2-0\\ 1 & -4 & 5\end{vmatrix}=0$$

$$\therefore \begin{vmatrix}x-3 & y+2 & z\\ 1 & 5 & 2\\ 1 & -4 & 5\end{vmatrix}=0$$

$\therefore (x-3)\times(25+8)-(y+2)\times(5-2)+(z)\times(-4-5)=0$

$\therefore 33(x-3)-(y+2)(3)+(z)(-9)=0$

33x – 99 – 3y – 6 – 9z = 0

33x – 3y – 9z – 105 = 0

11x – y – 3z = 35

Therefore, equation of plane is

Exercise 28J

1. Question

Find the direction ratios of the normal to the plane x + 2y - 3z = 5.

Answer

Given :

Equation of plane : x + 2y – 3z = 5

To Find : direction ratios of normal

Answer :

Given equation of plane : x + 2y – 3z = 5

It can be written as

$$(x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(\hat{\imath} + 2\hat{\jmath} - 3\hat{k}) = 5$$

Comparing with $\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Therefore, normal vector is $\bar{n} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

Hence, direction ratios of normal are (1, 2, -3).

2. Question

Find the direction cosines of the normal to the plane 2x + 3y - z = 4.

Answer

Given :

Equation of plane : 2x + 3y – z = 4

To Find : Direction cosines of the normal i.e. $l, m \,\&\, n$

Formula :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Answer :

For the given equation of plane

2x + 3y – z = 4

Direction ratios of normal vector are (2, 3, -1)

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{2^2 + 3^2 + (-1)^2}$$

$$= \sqrt{4 + 9 + 1}$$

$$= \sqrt{14}$$

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{2}{\sqrt{14}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{3}{\sqrt{14}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{-1}{\sqrt{14}}$$

$$(l, m, n) = \left(\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{-1}{\sqrt{14}}\right)$$

3. Question

Find the direction cosines of the normal to the plane y = 3.

Answer

Given :

Equation of plane : y = 3

To Find : Direction cosines of the normal i.e. $l, m \,\&\, n$

Formula :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Answer :

For the given equation of plane

y = 3

Direction ratios of normal vector are (0, 1, 0)

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{0^2 + 1^2 + 0^2}$$

$$= \sqrt{0 + 1 + 0}$$

$$= \sqrt{1}$$

= 1

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{0}{1} = 0$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{1}{1} = 1$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{0}{1} = 0$$

$$(l, m, n) = (0,1,0)$$

4. Question

Find the direction cosines of the normal to the plane 3x + 4 = 0.

Answer

Given :

Equation of plane : 3x + 4 = 0

To Find : Direction cosines of the normal i.e. l, m & n

Formula :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$$

$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Answer :

For the given equation of plane

-3x = 4

Direction ratios of normal vector are (-3, 0, 0)

$$\sqrt{a^2 + b^2 + c^2} = \sqrt{(-3)^2 + 0^2 + 0^2}$$

$$= \sqrt{9 + 0 + 0}$$

$$= \sqrt{9}$$

= 3

Therefore, direction cosines are

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{-3}{3} = -1$$

$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{0}{3} = 0$

$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{0}{3} = 0$

$(l, m, n) = (-1, 0, 0)$

5. Question

Write the equation of the plane parallel to XY-plane and passing through the point (4, -2, 3).

Answer

Given :

Point : (4, -2, 3)

To Find : equation of plane

Formula :

1) Equation of plane :

Equation of plane passing through point A with position vector $\bar{a}$ and perpendicular to vector $\bar{n}$ is given by,

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Answer :

Position vector for given point A ≡ (4, -2, 3) is

$\bar{a} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

As required plane is parallel to XY plane, therefore Z-axis is perpendicular to the plane.

$\therefore \bar{n} = \hat{k}$

Therefore, equation of plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(\hat{k}) = (4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}).(\hat{k})$

$\therefore (x \times 0) + (y \times 0) + (z \times 1) = (4 \times 0) + (-2 \times 0) + (3 \times 1)$

$\therefore z = 3$

This is required equation of plane.

6. Question

Write the equation of the plane parallel to YZ-plane and passing through the point(-3, 2, 0).

Answer

Given :

Point : (-3, 2, 0)

To Find : equation of plane

Formula :

1) Equation of plane :

Equation of plane passing through point A with position vector $\bar{a}$ and perpendicular to vector $\bar{n}$ is given by,

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Answer :

Position vector for given point A ≡ (-3, 2, 0) is

$\bar{a} = -3\hat{\imath} + 2\hat{\jmath} + 0\hat{k}$

As required plane is parallel to YZ plane, therefore X-axis is perpendicular to the plane.

$\therefore \bar{n} = \hat{\imath}$

Therefore, equation of plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(\hat{\imath}) = (-3\hat{\imath} + 2\hat{\jmath} + 0\hat{k}).(\hat{\imath})$

$\therefore (x \times 1) + (y \times 0) + (z \times 0) = (-3 \times 1) + (2 \times 0) + (0 \times 0)$

$\therefore x = -3$

This is required equation of plane.

7. Question

Write the general equation of a plane parallel to the x-axis.

Answer

Let, normal vector of plane be

$\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

Equation of plane is given by,

$\bar{r}.\bar{n} = d$

$\therefore (x\hat{\imath} + y\hat{\jmath} + z\hat{k}).(a\hat{\imath} + b\hat{\jmath} + c\hat{k}) = d$

$\therefore ax + by + cz = d$

As the required plane is parallel to the given plane, hence normal vector of plane is perpendicular to x-axis.

$\therefore \bar{n}.\hat{\imath} = 0$

$\therefore (a\hat{\imath} + b\hat{\jmath} + c\hat{k}).\hat{\imath} = 0$

a = 0

Therefore, equation of plane is

by + cz = d

8. Question

Write the intercept cut off by the plane 2x + y - z = 5 on the x-axis.

Answer

Given :

Equation of plane : 2x + y – z = 5

To Find : Intercept made by the plane with the X-axis.

Formula :

If $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

is the equation of plane in intercept form then intercept made by it with co-ordinate axes are

X-intercept = a

Y-intercept = b

Z-intercept = c

Answer :

Given equation of plane:

2x + y – z = 5

Dividing above equation throughout by 5

$\therefore \frac{2x}{5} + \frac{y}{5} + \frac{-z}{5} = 1$

$\therefore \frac{x}{5/2} + \frac{y}{5} + \frac{z}{-5} = 1$

Comparing above equation with

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

We get,

a = 5/2

Therefore, intercepts made by plane with X-axis are

X-intercept = 5/2

9. Question

Write the intercepts made by the plane4x - 3y + 2z = 12 on the coordinate axes.

Answer

Given :

Equation of plane : 4x – 3y + 2z = 12

To Find :

1) Equation of plane in intercept form

2) Intercepts made by the plane with the co-ordinate axes.

Formula :

If $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

is the equation of plane in intercept form then intercept made by it with co-ordinate axes are

X-intercept = a

Y-intercept = b

Z-intercept = c

Answer :

Given equation of plane:

4x – 3y + 2z = 12

Dividing above equation throughout by 12

$\therefore \frac{4x}{12}+\frac{-3y}{12}+\frac{2z}{12}=1$

$\therefore \frac{x}{3}+\frac{y}{-4}+\frac{z}{6}=1$

This is the equation of plane in intercept form.

Comparing above equation with

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

We get,

a = 3

b = -4

c = 6

Therefore, intercepts made by plane with co-ordinate axes are

X-intercept = 3

Y-intercept = -4

Z-intercept = 6

10. Question

Reduce the equation 2x – 3y + 5z + 4 = 0 to intercept form and find the intercepts made by it on the coordinate axes.

Answer

Given :

Equation of plane : 2x – 3y + 5z + 4 = 0

To Find :

1) Equation of plane in intercept form

2) Intercepts made by the plane with the co-ordinate axes.

Formula :

If $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

is the equation of plane in intercept form then intercept made by it with co-ordinate axes are

X-intercept = a

Y-intercept = b

Z-intercept = c

Answer :

Given equation of plane:

2x – 3y + 5z = -4

Dividing above equation throughout by -4

$\therefore \frac{2x}{-4} + \frac{-3y}{-4} + \frac{5z}{-4} = 1$

$\therefore \frac{x}{-2} + \frac{y}{4/3} + \frac{z}{-4/5} = 1$

This is the equation of plane in intercept form.

Comparing above equation with

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

We get,

a = -2

$b = \frac{4}{3}$

$c = \frac{-4}{5}$

Therefore, intercepts made by plane with co-ordinate axes are

X-intercept = -2

$Y - intercept = \frac{4}{3}$

$Z - intercept = -\frac{4}{5}$

11. Question

Find the equation of a plane passing through the points A(a, 0, 0), B(0, b, 0) and C(0, 0, c).

Answer

Given : Plane is passing through points

A ≡ (a, 0, 0)

B ≡ (0, b, 0)

C ≡ (0, 0, c)

To Find : Equation of plane

Formulae :

Equation of plane making intercepts (a, b, c) on X, Y & Z axes respectively is given by,

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Answer : As plane is passing through points A ≡ (a, 0, 0),

B ≡ (0, b, 0) & C ≡ (0, 0, c)

Therefore, intercepts made by it on X, Y & Z axes respectively are

a, b & c.

hence, equation of plane is

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

12. Question

Write the value of k for which the planes 2x – 5y + kz = 4 and x + 2y – z = 6 are perpendicular to each other.

Answer

Given : equations of perpendicular planes-

2x – 5y + kz = 4

x + 2y – z = 6

To Find : k

Formulae :

Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

Answer :

For given planes –

2x – 5y + kz = 4

x + 2y – z = 6

normal vectors are

$\overline{n_1} = 2\hat{\imath} - 5\hat{\jmath} + k\hat{k}$

$\overline{n_2} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

As given vectors are perpendicular, hence their normal vectors are also perpendicular to each other.

$\therefore \overline{n_1}.\overline{n_2} = 0$

$\therefore \left(2\hat{\imath} - 5\hat{\jmath} + k\hat{k}\right).\left(\hat{\imath} + 2\hat{\jmath} - \hat{k}\right) = 0$

(2×1) + (-5×2) + (k×(-1)) = 0

2 – 10 – k = 0

- 8 – k = 0

k = -8

13. Question

Find the angle between the planes 2x + y – 2z = 5 and 3x – 6y – 2z = 7.

Answer

Given : equations of planes-

2x + y – 2z = 5

3x – 6y – 2z = 7

To Find : angle between two planes

Formulae :

1) Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

2) Angle between two planes :

The angle Θ between the planes $\bar{r}.\overline{n_1} = p_1$ and $\bar{r}.\overline{n_2} = p_2$ is given by

$$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$$

Answer :

For given planes

2x + y – 2z = 5

3x – 6y – 2z = 7

Normal vectors are

$\overline{n_1} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and

$\overline{n_2} = 3\hat{\imath} - 6\hat{\jmath} - 2\hat{k}$

$\therefore |\overline{n_1}| = \sqrt{2^2 + 1^2 + (-2)^2} = \sqrt{4+1+4} = \sqrt{9} = 3$

$\therefore |\overline{n_2}| = \sqrt{3^2 + (-6)^2 + (-2)^2} = \sqrt{9+36+4} = \sqrt{49} = 7$

Therefore, angle between two planes is

$$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$$

$$\therefore \cos\theta = \frac{\left(2\hat{\imath} + \hat{\jmath} - 2\hat{k}\right).\left(3\hat{\imath} - 6\hat{\jmath} - 2\hat{k}\right)}{3 \times 7}$$

$$\therefore \cos\theta = \frac{(2\times 3) + (1\times(-6)) + ((-2)\times(-2))}{21}$$

$$\therefore \cos\theta = \frac{6-6+4}{21}$$

$\therefore \cos\theta = \frac{4}{21}$

$\therefore \theta = cos^{-1}\left(\frac{4}{21}\right)$

14. Question

Find the angle between the planes $\bar{r}\cdot\left(\hat{i}+\hat{j}\right)=1$ and $\bar{r}\cdot\left(\hat{i}+\hat{k}\right)=3$.

Answer

Given : equations of planes-

$\bar{r}.(\hat{\imath}+\hat{\jmath}) = 1$

$\bar{r}.(\hat{\jmath}+\hat{k}) = 3$

To Find : angle between two planes

Formulae :

Angle between two planes :

The angle Θ between the planes $\bar{r}.\overline{n_1} = p_1$ and $\bar{r}.\overline{n_2} = p_2$ is given by

$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$

Answer :

For given planes

$\bar{r}.(\hat{\imath}+\hat{\jmath}) = 1$

$\bar{r}.(\hat{\jmath}+\hat{k}) = 3$

Normal vectors are

$\overline{n_1} = \hat{\imath}+\hat{\jmath}$ and

$\overline{n_2} = \hat{\jmath}+\hat{k}$

$\therefore |\overline{n_1}| = \sqrt{1^2+1^2+0^2} = \sqrt{1+1+0} = \sqrt{2}$

$\therefore |\overline{n_2}| = \sqrt{0^2+1^2+1^2} = \sqrt{0+1+1} = \sqrt{2}$

Therefore, angle between two planes is

$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$

$\therefore \cos\theta = \frac{(\hat{\imath}+\hat{\jmath}).(\hat{\jmath}+\hat{k})}{\sqrt{2}\times\sqrt{2}}$

$\therefore \cos\theta = \frac{(1\times 0)+(1\times 1)+(0\times 1)}{2}$

$\therefore \cos\theta = \frac{0+1+0}{2}$

$\therefore \cos\theta = \frac{1}{2}$

$\therefore \theta = cos^{-1}\left(\frac{1}{2}\right)$

$\therefore \theta = \frac{\pi}{3}$

15. Question

Find the angle between the planes $\bar{r}\cdot\left(3\hat{i}-4\hat{j}+5\hat{k}\right)=0$ and $\bar{r}\cdot\left(2\hat{i}-\hat{j}-2\hat{k}\right)=7$.

Answer

Given : equations of planes-

$\bar{r}.(3\hat{\imath}-4\hat{\jmath}+5\hat{k}) = 0$

$\bar{r}.(2\hat{\imath}-\hat{\jmath}-2\hat{k}) = 7$

To Find : angle between two planes

Formulae :

Angle between two planes :

The angle Θ between the planes $\bar{r}.\overline{n_1} = p_1$ and $\bar{r}.\overline{n_2} = p_2$ is given by

$$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$$

Answer :

For given planes

$$\bar{r}.\left(3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}\right) = 0$$

$$\bar{r}.\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right) = 7$$

Normal vectors are

$\overline{n_1} = 3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and

$$\overline{n_2} = 2\hat{\imath} - \hat{\jmath} - 2\hat{k}$$

$$\therefore |\overline{n_1}| = \sqrt{3^2 + (-4)^2 + 5^2} = \sqrt{9 + 16 + 25} = \sqrt{50} = 5\sqrt{2}$$

$$\therefore |\overline{n_2}| = \sqrt{2^2 + (-1)^2 + (-2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

Therefore, angle between two planes is

$$\cos\theta = \frac{\overline{n_1}.\overline{n_2}}{|\overline{n_1}|.|\overline{n_2}|}$$

$$\therefore \cos\theta = \frac{\left(3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}\right).\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right)}{5\sqrt{2} \times 3}$$

$$\therefore \cos\theta = \frac{(3 \times 2) + ((-4) \times (-1)) + (5 \times (-2))}{15\sqrt{2}}$$

$$\therefore \cos\theta = \frac{6 + 4 - 10}{2}$$

$$\therefore \cos\theta = 0$$

$$\therefore \theta = cos^{-1}(0)$$

$$\therefore \theta = \frac{\pi}{2}$$

16. Question

Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the planes 10x + 2y – 11z = 3.

Answer

Given :

Equation of line : $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$

Equation of plane : 10x + 2y – 11z = 3

To Find : angle between line and plane

Formulae :

1) Parallel vector to the line :

If equation of the line is $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ then,

Vector parallel to the line is given by,

$$\bar{b} = a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k}$$

2) Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$$\bar{n} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$$

3) Angle between a line and a plane :

If Θ is a angle between the line $\bar{r} = \bar{a} + \lambda\bar{b}$ and the plane $\bar{r}.\bar{n} = p$, then

$$\sin\theta = \frac{\bar{b}.\bar{n}}{|\bar{b}|.|\bar{n}|}$$

Where, $\bar{b}$ is vector parallel to the line and

$\bar{n}$ is the vector normal to the plane.

Answer :

For given equation of line,

$$\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$$

Parallel vector to the line is

$$\bar{b}=2\hat{\imath}+3\hat{\jmath}+6\hat{k}$$

$$\therefore |\bar{b}|=\sqrt{2^2+3^2+6^2}=\sqrt{4+9+36}=\sqrt{49}=7$$

For given equation of plane,

10x + 2y – 11z = 3

normal vector to the plane is

$$\bar{n}=10\hat{\imath}+2\hat{\jmath}-11\hat{k}$$

$$\therefore |\bar{n}|=\sqrt{10^2+2^2+(-11)^2}=\sqrt{100+4+121}=\sqrt{225}=15$$

Therefore, angle between given line and plane is

$$\sin\theta=\frac{\bar{b}\,.\,\bar{n}}{|\bar{b}|\,.\,|\bar{n}|}$$

$$\therefore \sin\theta=\frac{\left(2\hat{\imath}+3\hat{\jmath}+6\hat{k}\right).\left(10\hat{\imath}+2\hat{\jmath}-11\hat{k}\right)}{7\times 15}$$

$$\therefore \sin\theta=\frac{(2\times 10)+(3\times 2)+(6\times(-11))}{105}$$

$$\therefore \sin\theta=\frac{20+6-66}{105}$$

$$\therefore \sin\theta=\frac{-40}{105}$$

$$\therefore \sin\theta=\frac{-8}{21}$$

$$\therefore \theta=sin^{-1}\left(\frac{-8}{21}\right)$$

17. Question

Find the angle between the line $\bar{r}=\left(\hat{i}+\hat{j}-2\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$ and the plane $\bar{r}\cdot\left(2\hat{i}-\hat{j}+\hat{k}\right)=4$.

Answer

Given :

Equation of line : $\bar{r}=\left(\hat{\imath}+\hat{\jmath}-2\hat{k}\right)+\lambda\left(\hat{\imath}-\hat{\jmath}+\hat{k}\right)$

Equation of plane : $\bar{r}.\left(2\hat{\imath}-\hat{\jmath}+\hat{k}\right)=4$

To Find : angle between line and plane

Formulae :

1) Angle between a line and a plane :

If Θ is a angle between the line $\bar{r}=\bar{a}+\lambda\bar{b}$ and the plane $\bar{r}.\bar{n}=p$, then

$$\sin\theta=\frac{\bar{b}\,.\,\bar{n}}{|\bar{b}|\,.\,|\bar{n}|}$$

Where, $\bar{b}$ is vector parallel to the line and

$\bar{n}$ is the vector normal to the plane.

Answer :

For given equation of line,

$$\bar{r}=\left(\hat{\imath}+\hat{\jmath}-2\hat{k}\right)+\lambda\left(\hat{\imath}-\hat{\jmath}+\hat{k}\right)$$

Parallel vector to the line is

$$\bar{b}=\hat{\imath}-\hat{\jmath}+\hat{k}$$

$$\therefore |\bar{b}|=\sqrt{1^2+(-1)^2+1^2}=\sqrt{1+1+1}=\sqrt{3}$$

For given equation of plane,

$\bar{r}.(2\hat{\imath}-\hat{\jmath}+\hat{k})=4$

normal vector to the plane is

$\bar{n}=2\hat{\imath}-\hat{\jmath}+\hat{k}$

$\therefore |\bar{n}|=\sqrt{2^2+(-1)^2+1^2}=\sqrt{4+1+1}=\sqrt{6}$

Therefore, angle between given line and plane is

$$\sin\theta=\frac{\bar{b}\,.\,\bar{n}}{|\bar{b}|\,.\,|\bar{n}|}$$

$$\therefore \sin\theta=\frac{(\hat{\imath}-\hat{\jmath}+\hat{k}).(2\hat{\imath}-\hat{\jmath}+\hat{k})}{\sqrt{3}\times\sqrt{6}}$$

$$\therefore \sin\theta=\frac{(1\times 2)+((-1)\times(-1))+(1\times 1)}{\sqrt{18}}$$

$$\therefore \sin\theta=\frac{2+1+1}{3\sqrt{2}}$$

$$\therefore \sin\theta=\frac{4}{3\sqrt{2}}$$

$$\therefore \sin\theta=\frac{2\times 2}{3\sqrt{2}}$$

$$\therefore \sin\theta=\frac{2\sqrt{2}}{3}$$

$$\therefore \theta=sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

18. Question

Find the value of λ such that the line $\frac{x-2}{6}=\frac{y-1}{\lambda}=\frac{z+5}{4}$ is perpendicular to the plane 3x - y – 2z = 7.

Answer

Given :

Equation of line : $\frac{x-2}{6}=\frac{y-1}{\lambda}=\frac{z+5}{4}$

Equation of plane : 3x – y – 2z = 7

To Find : λ

Formulae :

1) Parallel vector to the line :

If equation of the line is $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ then,

Vector parallel to the line is given by,

$\bar{b}=a_1\hat{\imath}+b_1\hat{\jmath}+c_1\hat{k}$

2) Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$\bar{n}=a\hat{\imath}+b\hat{\jmath}+c\hat{k}$

3) Cross Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$

$\bar{b}=b_1\hat{\imath}+b_2\hat{\jmath}+b_3\hat{k}$

then,

$$\bar{a}\times\bar{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$$

Answer :

For given equation of line,

$\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{4}$

Parallel vector to the line is

$\bar{b} = 6\hat{\imath} + \lambda\hat{\jmath} + 4\hat{k}$

For given equation of plane,

3x – y – 2z = 7

normal vector to the plane is

$\bar{n} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$

As given line and plane are perpendicular to each other.

$\therefore \bar{b} \times \bar{n} = 0$

$\therefore \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 6 & \lambda & 4 \\ 3 & -1 & -2 \end{vmatrix} = 0$

$\therefore \hat{\imath}(-2\lambda + 4) - \hat{\jmath}(-12 - 12) + \hat{k}(-6 - 3\lambda) = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$

Comparing coefficients of $\hat{k}$ on both sides

$\therefore -6 - 3\lambda = 0$

$3\lambda = -6$

$\lambda = -2$

19. Question

Write the equation of the plane passing through the point (a, b, c) and parallel to the plane $\bar{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$.

Answer

Given :

A ≡ (a, b, c)

Equation of plane parallel to required plane

$\therefore \bar{r}.(\hat{\imath} + \hat{\jmath} + \hat{k}) = 2$

To Find : Equation of plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

If a plane is passing through point A, then equation of plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{a} = position\ vector\ of\ A$

$\bar{n} = vector\ perpendicular\ to\ the\ plane$

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Answer :

For point A ≡ (a, b, c), position vector is

$\bar{a} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

As plane $\bar{r}.(\hat{\imath} + \hat{\jmath} + \hat{k}) = 2$ is parallel to the required plane, the vector normal to required plane is

$\bar{n} = \hat{\imath} + \hat{\jmath} + \hat{k}$

Now, $\bar{a}.\bar{n} = (a \times 1) + (b \times 1) + (c \times 1)$

= a + b + c

Equation of the plane passing through point A and perpendicular to vector $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \bar{r}.\left(\hat{\imath} + \hat{\jmath} + \hat{k}\right) = a + b + c$

20. Question

Find the length of perpendicular drawn from the origin to the plane 2x – 3y + 6z + 21 = 0.

Answer

Given :

Equation of plane : 2x – 3y + 6z + 21 = 0

To Find :

Length of perpendicular drawn from origin to the plane = d

Formulae :

1) Distance of the plane from the origin :

Distance of the plane from the origin is given by,

$d = \frac{p}{|\bar{n}|}$

Answer :

For the given equation of plane

2x – 3y + 6z = -21

Direction ratios of normal vector are (2, -3, 6)

Therefore, equation of normal vector is

$\bar{n} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$

$\therefore |\bar{n}| = \sqrt{2^2 + (-3)^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

= 7

From given equation of plane,

p = -21

Now, distance of the plane from the origin is

$d = \frac{p}{|\bar{n}|}$

$\therefore d = \frac{-21}{7}$

d = 3 units

21. Question

Find the direction cosines of the perpendicular from the origin to the plane $\bar{r} \cdot \left(6\hat{i} - 3\hat{j} - 2\hat{k}\right) + 1 = 0$.

Answer

Given :

Equation of plane : $\bar{r}.\left(6\hat{\imath} - 3\hat{\jmath} - 2\hat{k}\right) + 1 = 0$

To Find :

Direction cosines of the normal i.e. $l, m \,\&\, n$

Formulae :

1) Direction cosines :

If a, b & c are direction ratios of the vector then its direction cosines are given by

$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}$

$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}$

$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$

Answer :

For the given equation of plane

$\bar{r}.(6\hat{\imath} - 3\hat{\jmath} - 2\hat{k}) + 1 = 0$

Equation of normal vector is

$\bar{n} = 6\hat{\imath} - 3\hat{\jmath} - 2\hat{k}$

$\sqrt{a^2 + b^2 + c^2} = \sqrt{6^2 + (-3)^2 + (-2)^2}$

$= \sqrt{36 + 9 + 4}$

$= \sqrt{49}$

= 7

Therefore, direction cosines are

$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{6}{7}$

$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{-3}{7}$

$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{-2}{7}$

$(l, m, n) = \left(\frac{6}{7}, \frac{-3}{7}, \frac{-2}{7}\right)$

22. Question

Show that the line $\bar{r} = (4\hat{i} - 7\hat{k}) + \lambda(4\hat{i} - 2\hat{j} + 3\hat{k})$ is parallel to the plane $\bar{r} \cdot (5\hat{i} + 4\hat{j} - 4\hat{k}) = 7$.

Answer

Given :

Equation of plane : : $\bar{r}.(5\hat{\imath} + 4\hat{\jmath} - 4\hat{k}) = 7$

Equation of line :

$\bar{r} = (4\hat{\imath} - 7\hat{k}) + \lambda(4\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$

To Prove : Given line is parallel to the given plane.

Answer :

Comparing given plane i.e.

$\bar{r}.(5\hat{\imath} + 4\hat{\jmath} - 4\hat{k}) = 7$

with $\bar{r}.\bar{n} = \bar{a}.\bar{n}$, we get,

$\bar{n} = 5\hat{\imath} + 4\hat{\jmath} - 4\hat{k}$

This is the vector perpendicular to the given plane.

Now, comparing given equation of line i.e.

$\bar{r} = (4\hat{\imath} - 7\hat{k}) + \lambda(4\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$

with $\bar{r} = \bar{a} + \lambda\bar{b}$, we get,

$\bar{b} = 4\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

Now,

$\bar{n}.\bar{b} = (5\hat{\imath} + 4\hat{\jmath} - 4\hat{k}).(4\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$

$= (5 \times 4) + (4 \times (-2)) + ((-4) \times 3)$

= 20 – 8 – 12

= 0

$\therefore \bar{n}.\bar{b} = 0$

Therefore, vector normal to the plane is perpendicular to the vector parallel to the line.

Hence, the given line is parallel to the given plane.

23. Question

Find the length of perpendicular from the origin to the plane $\bar{r}\cdot(2\hat{i}-3\hat{j}+6\hat{k})+14=0$.

Answer

Given :

Equation of plane : $\bar{r}.\left(2\hat{\imath}-3\hat{\jmath}+6\hat{k}\right)+14=0$

To Find : Length of perpendicular = d

Formulae :

1) Unit Vector :

Let $\bar{a}=a_1\hat{\imath}+a_2\hat{\jmath}+a_3\hat{k}$ be any vector

Then unit vector of $\bar{a}$ is

$\hat{a}=\frac{\bar{a}}{|\bar{a}|}$

Where, $|\bar{a}|=\sqrt{{a_1}^2+{a_2}^2+{a_3}^2}$

2) Length of perpendicular :

The length of the perpendicular from the origin to the plane

$\bar{r}.\bar{n}=p$ is given by,

$d=\frac{p}{|\bar{n}|}$

Answer :

Given equation of the plane is

$\bar{r}.\left(2\hat{\imath}-3\hat{\jmath}+6\hat{k}\right)+14=0$

$\therefore \bar{r}.\left(2\hat{\imath}-3\hat{\jmath}+6\hat{k}\right)=-14$

$\therefore \bar{r}.\left(-2\hat{\imath}+3\hat{\jmath}-6\hat{k}\right)=14$

Comparing above equation with

$\bar{r}.\bar{n}=p$

We get,

$\bar{n}=-2\hat{\imath}+3\hat{\jmath}-6\hat{k}$ & p = 14

Therefore,

$|\bar{n}|=\sqrt{(-2)^2+3^2+(-6)^2}$

$=\sqrt{4+9+36}$

$=\sqrt{49}$

= 7

The length of the perpendicular from the origin to the given plane is

$d=\frac{p}{|\bar{n}|}$

$\therefore d=\frac{14}{7}$

$\therefore d=2\ units$

24. Question

Find the value of λ for which the line

$\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{\lambda}$ is parallel to the plane $\bar{r}.\left(2\hat{\imath}+3\hat{\jmath}+4\hat{k}\right)=4$

Answer

Given :

Equation of line : $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{\lambda}$

Equation of plane : $\bar{r}.\left(2\hat{\imath}+3\hat{\jmath}+4\hat{k}\right)=4$

To Find : λ

Formulae :

1) Parallel vector to the line :

If equation of the line is $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ then,

Vector parallel to the line is given by,

$\bar{b}=a_1\hat{\imath}+b_1\hat{\jmath}+c_1\hat{k}$

2) Angle between a line and a plane :

If Θ is a angle between the line $\bar{r}=\bar{a}+\lambda\bar{b}$ and the plane $\bar{r}.\bar{n}=p$, then

$$\sin\theta=\frac{\bar{b}\,.\bar{n}}{|\bar{b}|\,.|\bar{n}|}$$

Where, $\bar{b}$ is vector parallel to the line and

$\bar{n}$ is the vector normal to the plane.

Answer :

For given equation of line,

$$\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{\lambda}$$

Parallel vector to the line is

$\bar{b}=2\hat{\imath}+3\hat{\jmath}+\lambda\hat{k}$

For given equation of plane,

$\bar{r}.\left(2\hat{\imath}+3\hat{\jmath}+4\hat{k}\right)=4$

normal vector to the plane is

$\bar{n}=2\hat{\imath}+3\hat{\jmath}+4\hat{k}$

Therefore, angle between given line and plane is

$$\sin\theta=\frac{\bar{b}\,.\bar{n}}{|\bar{b}|\,.|\bar{n}|}$$

As given line is parallel too the given plane, angle between them is 0.

$\therefore\theta=0$

$\therefore\sin\theta=0$

$\therefore\bar{b}\,.\bar{n}=0$

$\therefore\left(2\hat{\imath}+3\hat{\jmath}+\lambda\hat{k}\right).\left(2\hat{\imath}+3\hat{\jmath}+4\hat{k}\right)=0$

$\therefore(2\times2)+(3\times3)+(\lambda\times4)=0$

4 + 9 + 4 λ = 0

13 + 4λ = 0

4λ = -13

$$\therefore\lambda=-\frac{13}{4}$$

$$\lambda=-\frac{13}{4}$$

25. Question

Write the angle between the line

$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-2}$ and the plane x + y + 4 = 0.

Answer

Given :

Equation of line : $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-2}$

Equation of plane : x + y + 4 = 0

To Find : angle between line and plane

Formulae :

1) Parallel vector to the line :

If equation of the line is $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ then,

Vector parallel to the line is given by,

$\bar{b}=a_1\hat{\imath}+b_1\hat{\jmath}+c_1\hat{k}$

2) Normal vector to the plane :

If equation of the plane is ax + by + cz = d then,

Vector normal to the plane is given by,

$\bar{n}=a\hat{\imath}+b\hat{\jmath}+c\hat{k}$

3) Angle between a line and a plane :

If Θ is a angle between the line $\bar{r}=\bar{a}+\lambda\bar{b}$ and the plane $\bar{r}.\bar{n}=p$, then

$$\sin\theta=\frac{\bar{b}\,.\bar{n}}{|\bar{b}|\,.|\bar{n}|}$$

Where, $\bar{b}$ is vector parallel to the line and

$\bar{n}$ is the vector normal to the plane.

Answer :

For given equation of line,

$$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-2}$$

Parallel vector to the line is

$\bar{b}=2\hat{\imath}+\hat{\jmath}-2\hat{k}$

$\therefore |\bar{b}|=\sqrt{2^2+1^2+(-2)^2}=\sqrt{4+1+4}=\sqrt{9}=3$

For given equation of plane,

x + y + 4 = 0

normal vector to the plane is

$\bar{n}=\hat{\imath}+\hat{\jmath}+0\hat{k}$

$\therefore |\bar{n}|=\sqrt{1^2+1^2+0^2}=\sqrt{1+1+0}=\sqrt{2}$

Therefore, angle between given line and plane is

$$\sin\theta=\frac{\bar{b}\,.\bar{n}}{|\bar{b}|\,.|\bar{n}|}$$

$$\therefore \sin\theta=\frac{\left(2\hat{\imath}+\hat{\jmath}-2\hat{k}\right).\left(\hat{\imath}+\hat{\jmath}+0\hat{k}\right)}{3\times\sqrt{2}}$$

$$\therefore \sin\theta=\frac{(2\times1)+(1\times1)+((-2)\times0)}{3\sqrt{2}}$$

$$\therefore \sin\theta=\frac{2+1-0}{3\sqrt{2}}$$

$$\therefore \sin\theta=\frac{3}{3\sqrt{2}}$$

$$\therefore \sin\theta=\frac{1}{\sqrt{2}}$$

$$\therefore \theta=sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\theta=\frac{\pi}{4}$$

26. Question

Write the equation of a plane passing through the point (2, -1, 1) and parallel to the plane 3x + 2y - z = 7.

Answer

Given :

A ≡ (2, -1, 1)

Plane parallel to the required plane : 3x + 2y – z = 7

To Find : Equation of plane

Formulae :

1) Position vectors :

If A is a point having co-ordinates (a_1, a_2, a_3), then its position vector is given by,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

2) Dot Product :

If $\bar{a}$ & $\bar{b}$ are two vectors

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then,

$\bar{a}.\bar{b} = (a_1 \times b_1) + (a_2 \times b_2) + (a_3 \times b_3)$

3) Equation of plane :

If a plane is passing through point A, then equation of plane is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Where, $\bar{a} = position\ vector\ of\ A$

$\bar{n} = vector\ perpendicular\ to\ the\ plane$

$\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Answer :

For point A ≡ (2, -1, 1), position vector is

$\bar{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

As required plane is parallel to 3x + 2y – z = 7.

Therefore, normal vector of given plane is also perpendicular to required plane

$\bar{n} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$

Now, $\bar{a}.\bar{n} = (2 \times 3) + ((-1) \times 2) + (1 \times (-1))$

= 6 – 2 – 1

= 3

Equation of the plane passing through point A and perpendicular to vector $\bar{n}$ is

$\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\therefore \bar{r}.\left(3\hat{\imath} + 2\hat{\jmath} - \hat{k}\right) = 3$

As $\bar{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\therefore \bar{r}.\left(3\hat{\imath} + 2\hat{\jmath} - \hat{k}\right) = \left(x\hat{\imath} + y\hat{\jmath} + z\hat{k}\right).\left(3\hat{\imath} + 2\hat{\jmath} - \hat{k}\right)$

= 3x + 2y – z

Therefore, equation of the plane is

3x + 2y – z = 3

3x + 2y – z - 3 = 0

Objective Questions

1. Question

Mark against the correct answer in each of the following:

The direction cosines of the perpendicular from the origin to the plane $\vec{r} \cdot \left(6\hat{i} - 3\hat{j} + 2\hat{k}\right) + 1 = 0$ are

A. $\frac{6}{7}, \frac{3}{7}, \frac{-2}{7}$

B. $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$

C. $\frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$

D. None of these

Answer

Given: Equation of plane is $\vec{r}.(6\hat{\imath}-3\hat{\jmath}+2\hat{k})+1=0$

Formula Used: Equation of a plane is $\hat{u}.\vec{r}=p$ where $\hat{u}$ is the unit vector normal to the plane, $\vec{r}$ represents a point on the plane and p is the distance of the plane from the origin.

Explanation:

The equation of the given plane is $\vec{r}.(6\hat{\imath}-3\hat{\jmath}+2\hat{k})=-1$ ... (1)

Now, $|6\hat{\imath}-3\hat{\jmath}+2\hat{k}|=\sqrt{36+9+4}$

= 7

$\therefore \frac{6}{7}\hat{\imath}-\frac{3}{7}\hat{\jmath}+\frac{2}{7}\hat{k}$ is a unit vector.

(1) can be rewritten as

$$\vec{r}.\left(\frac{6}{7}\hat{\imath}-\frac{3}{7}\hat{\jmath}+\frac{2}{7}\hat{k}\right)=-\frac{1}{7}$$

$$\Rightarrow \vec{r}.\left(\frac{-6}{7}\hat{\imath}+\frac{3}{7}\hat{\jmath}-\frac{2}{7}\hat{k}\right)=\frac{1}{7}$$

which is of the form $\hat{u}.\vec{r}=p$

Perpendicular vector from the origin to the plane is

$$\hat{u}=\frac{-6}{7}\hat{\imath}+\frac{3}{7}\hat{\jmath}-\frac{2}{7}\hat{k}$$

So, direction cosines of the vector perpendicular from the origin to the plane is $\left(\frac{-6}{7},\frac{3}{7},\frac{-2}{7}\right)$

2. Question

Mark against the correct answer in each of the following:

The direction cosines of the normal to the plane 5y + 4 = 0 are

A. $0,\frac{-4}{5},0$

B. 0, 1, 0

C. 0, -1, 0

D. None of these

Answer

Given: Equation of plane is 5y + 4 = 0

Formula Used: Equation of a plane is lx + my + nz = p where (l, m, n) are the direction cosines of the normal to the plane and (x, y, z) is a point on the plane and p is the distance of plane from origin.

Explanation:

Given equation is 5y = -4

Dividing by -5,

$$-y=\frac{4}{5}$$

which is of the form lx + my + nz = p where l = 0, m = -1, n = 0

Therefore, direction cosines of the normal to the plane is (0, -1, 0)

3. Question

Mark against the correct answer in each of the following:

The length of perpendicular from the origin to the plane $\vec{r}\cdot(3\hat{i}-4\hat{j}-12\hat{k})+39=0$ is

A. 3 units

B. $\frac{13}{5}$ units

C. $\frac{5}{3}$ units

D. None of these

Answer

Given: Equation of plane is $\vec{r}.\left(3\hat{\imath}-4\hat{\jmath}-12\hat{k}\right)+39=0$

Formula Used: Equation of a plane is $\hat{u}.\vec{r}=p$ where $\hat{u}$ is the unit vector normal to the plane, $\vec{r}$ represents a point on the plane and p is the distance of the plane from the origin.

Explanation:

Given equation is $\vec{r}.\left(3\hat{\imath}-4\hat{\jmath}-12\hat{k}\right)=-39$ ... (1)

Now, $\left|3\hat{\imath}-4\hat{\jmath}-12\hat{k}\right|=\sqrt{9+16+144}=\sqrt{169}$

= 13

Dividing (1) by 13 and multiplying by -1,

$\vec{r}.\left(\frac{-3}{13}\hat{\imath}+\frac{4}{13}\hat{\jmath}+\frac{12}{13}\hat{k}\right)=3$

which is of the form $\hat{u}.\vec{r}=p$

Therefore, length of perpendicular from origin to plane is 3 units.

4. Question

Mark against the correct answer in each of the following:

The equation of a plane passing through the point A(2, -3, 7) and making equal intercepts on the axes, is

A. x + y + z = 3

B. x + y + z = 6

C. x + y + z = 9

D. x + y + z = 4

Answer

Given: A(2, -3, 7) is a point on the plane making equal intercepts on the axes.

Formula Used: Equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Explanation:

Let the equation of the plane be

$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ ... (1)

Here a = b = c = p (let's say)

Since (2, -3, 7) is a point on the plane,

(1) becomes

$\frac{2-3+7}{p}=1$

p = 6

Therefore equation of the plane is

x + y + z = 6

5. Question

Mark against the correct answer in each of the following:

A plane cuts off intercepts 3, -4, 6 on the coordinate axes. The length of perpendicular from the origin to this plane is

A. $\frac{5}{\sqrt{29}}$ units

B. $\frac{8}{\sqrt{29}}$ units

C. $\frac{6}{\sqrt{29}}$ units

D. $\frac{12}{\sqrt{29}}$ units

Answer

Given: Plane makes intercepts 3, -4 and 6 with the coordinate axes.

Formula Used: Equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Normal Form of a plane ⇒ lx + my + nz = p where (l, m, n) is the direction cosines and p is the distance of perpendicular to the plane from the origin.

Explanation:

Equation of the given plane is

$$\frac{x}{3}+\frac{y}{-4}+\frac{z}{6}=1$$

i.e., 4x – 3y + 2z = 12 … (1)

which is of the form ax + by + cz = d

Direction ratios are (4, -3, 12)

So, $\sqrt{4^2+(-3)^2+2^2} = \sqrt{16+9+4}$

= √29

Dividing (1) by 13,

$$\frac{4}{\sqrt{29}}x-\frac{3}{\sqrt{29}}+\frac{2}{\sqrt{29}}=\frac{12}{\sqrt{29}}$$

which is in the normal form

Therefore length of perpendicular from the origin is $\frac{12}{\sqrt{29}}$ units

6. Question

Mark against the correct answer in each of the following:

If the line $\frac{x+1}{3}=\frac{y-2}{4}=\frac{z+6}{5}$ is parallel to the plane 2x – 3y + kz = 0, then the value of k is

A. $\frac{5}{6}$

B. $\frac{6}{5}$

C. $\frac{3}{4}$

D. $\frac{4}{5}$

Answer

Given:

1. Equation of line is $\frac{x+1}{3}=\frac{y-2}{4}=\frac{z+6}{5}$

2. Equation of plane is 2x – 3y + kz = 0

Formula Used: If two direction ratios are perpendicular, then

a �$_1a_2 + b_1b_2 + c_1c_2 = 0$

Explanation:

Direction ratios of given line is (3, 4, 5)

Direction ratios of given plane is (2, -3, k)

Since the given line is parallel to the plane, the normal to the plane is perpendicular to the line.

So direction ratio of line is perpendicular to direction ratios of plane.

⇒ 3× 2 + 4× -3+ 5× k = 0

⇒ 6 – 12 + 5k = 0

$$\Rightarrow k=\frac{6}{5}$$

Therefore, $k=\frac{6}{5}$

7. Question

Mark against the correct answer in each of the following:

If O is the origin and P(1, 2, -3) is a given point, then the equation of the plane through P and perpendicular to OP is

A. x + 2y – 3z = 14

B. x – 2y + 3z = 12

C. x – 2y – 3z = 14

D. None of these

Answer

Given: P(1, 2, -3) is a point on the plane. OP is perpendicular to the plane.

Explanation:

Let equation of plane be ax + by + cz = d … (1)

Substituting point P,

⇒ a + 2b -3c = d … (2)

$\overrightarrow{OP} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

Since OP is perpendicular to the plane, direction ratio of the normal is (1, 2, -3)

Substituting in (2)

1 + 4 + 9 = d

d = 14

Substituting the direction ratios and value of 'd' in (1), we get

x + 2y – 3z = 14

Therefore equation of plane is x + 2y – 3z = 14

8. Question

Mark against the correct answer in each of the following:

If the line $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$ lies in the plane 2x – 4y + z = 7, then the value of k is

A. -7

B. 7

C. 4

D. -4

Answer

Given: Equation of plane is 2x – 4y + z = 7

Line $\frac{(x-4)}{1} = \frac{(y-2)}{1} = \frac{(z-k)}{2}$ lies on the given plane.

Formula Used: Equation of a line is

$$\frac{(x-x_1)}{b_1} = \frac{(y-y_1)}{b_2} = \frac{(z-z_1)}{b_3} = \lambda$$

Where (x_1, y_1, z_1) is a point on the line and b_1, b_2, b_3 : direction ratios of line.

Explanation:

Let $\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2} = \lambda$

So the given line passes through the point (4, 2, k)

Since the line lies on the given plane, (4, 2, k) is a point on the plane.

Therefore, substituting the point on the equation for the plane,

⇒ 8 – 8 + k = 7

⇒ k = 7

9. Question

Mark against the correct answer in each of the following:

The plane 2x + 3y + 4z =12 meets the coordinate axes in A, B and C. The centroid of ΔABC is

A. (2, 3, 4)

B. (6, 4, 3)

C. $\left(2, \frac{4}{3}, 1\right)$

D. None of these

Answer

Given: The plane 2x + 3y + 4z = 12 meets coordinate axes at A, B and C.

To find: Centroid of ΔABC

Formula Used: Equation of plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Centroid of a triangle $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

Explanation:

Equation of given plane is 2x + 3y + 4z = 12

Dividing by 12,

$\frac{x}{6} + \frac{y}{4} + \frac{z}{3} = 1$

Therefore the intercepts on x, y and z-axis are 6, 6 and 3 respectively.

So, the vertices of ΔABC are (6, 0, 0), (0, 4, 0) and (0, 0, 3)

Centroid $= \left(\frac{6+0+0}{3}, \frac{0+4+0}{3}, \frac{0+0+3}{3}\right)$

= (2, 4/3, 1)

Therefore, the centroid of ΔABC is (2, 4/3, 1)

10. Question

Mark against the correct answer in each of the following:

If a plane meets the coordinate axes in A, B and C such that the centroid of ΔABC is (1, 2, 4), then the equation of the plane is

A. x + 2y + 4z = 6

B. 4x + 2y + z = 12

C. x + 2y + 4z = 7

D. 4x + 2y + z = 7

Answer

Given: Centroid of ΔABC is (1, 2, 4)

To find: Equation of plane.

Formula Used: Equation of plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Centroid of a triangle $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$

Explanation:

Let the equation of plane be

$\frac{x}{A} + \frac{y}{B} + \frac{z}{C} = 1$ ... (1)

Therefore, A = 3a, B = 3b, C = 3c where (a, b, c) is the centroid of the triangle with vertices (A, 0, 0), (0, B, 0) and (0, 0, C)

Substituting in (1),

$\Rightarrow \frac{x}{3a} + \frac{y}{3b} + \frac{z}{3c} = 1$

Here a = 1, b = 2 and c = 4

$\Rightarrow \frac{x}{3} + \frac{y}{6} + \frac{z}{12} = 1$

Multiplying by 12,

4x + 2y + z = 12

Therefore equation of required plane is 4x + 2y + z = 12

11. Question

Mark against the correct answer in each of the following:

The equation of a plane through the point A(1, 0, -1) and perpendicular to the line $\frac{x+1}{2} = \frac{y+3}{4} = \frac{z+7}{-3}$ is

A. 2x + 4y – 3z = 3

B. 2x – 4y + 3z = 5

C. 2x + 4y – 3z = 5

D. x + 3y + 7z = -6

Answer

Given: Plane passes through the point A(1, 0, -1).

Plane is perpendicular to the line

$$\frac{x+1}{2} = \frac{y+3}{4} = \frac{z+7}{-3}$$

To find: Equation of the plane.

Formula Used: Equation of a plane is ax + by + cz = d where (a, b, c) are the direction ratios of the normal to the plane.

Explanation:

Let the equation of the plane be

ax + by + cz = d … (1)

Substituting point A,

a – z = d

Since the given line is perpendicular to the plane, it is the normal.

Direction ratios of line is 2, 4, -3

Therefore, 2 + 3 = d

d = 5

So the direction ratios of perpendicular to plane is 2, 4, -3 and d = 5

Substituting in (1),

2x + 4y – 3z = 5

Therefore, equation of plane is 2x + 4y – 3z = 5

12. Question

Mark against the correct answer in each of the following:

The line $\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{-3}$ meets the plane 2x + 3y – z = 14 in the point

A. (2, 5, 7)

B. (3, 5, 7)

C. (5, 7, 3)

D. (6, 5, 3)

Answer

Given: Line $\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{-3}$ meets plane 2x + 3y – z = 14

To find: Point of intersection of line and plane.

Explanation:

Let the equation of the line be

$$\frac{x-1}{2} = \frac{y-2}{4} = \frac{z-3}{-3} = \lambda$$

Therefore, any point on the line is (2λ + 1, 4λ +2, -3λ +3)

Since this point also lies on the plane,

2(2λ + 1) + 3(4λ +2) – (-3λ +3) =14

4λ + 2 + 12λ + 6 + 3λ – 3 = 14

19λ + 5 = 14

$$\lambda = \frac{19}{19} = 1$$

Therefore the required point is (3, 5, 7).

13. Question

Mark against the correct answer in each of the following:

The equation of the plane passing through the points A(2, 2, 1) and B(9, 3, 6) and perpendicular to the plane 2x + 6y + 6z = 1, is

A. x + 2y – 3z + 5 = 0

B. 2x – 3y + 4z – 6 = 0

C. 4x + 5y – 6z + 3 = 0

D. 3x + 4y – 5z – 9 = 0

Answer

Given: Plane passes through A(2, 2, 1) and B(9, 3, 6). Plane is perpendicular to 2x + 6y + 6z = 1

To find: Equation of the plane

Formula Used: Equation of a plane is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

where a:b:c is the direction ratios of the normal to the plane.

(x_1, y_1, z_1) is a point on the plane.

Explanation:

Let the equation of plane be $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Since (2, 2, 1) is a point in the plane,

a(x – 2) + b(y – 2) + c(z – 1) = 0 … (1)

Since B(9, 3, 6) is another point on the plane,

a(9 – 2) + b(3 – 2) + c(6 – 1) = 0

7a + b + 5c = 0 … (1)

Since this plane is perpendicular to the plane 2x + 6y + 6z = 1, the direction ratios of the normal to the plane will also be perpendicular.

So, 2a + 6b + 6c = 0 ⇒ a + 3b + 3c = 0 … (2)

Solving (1) and (2),

$$\frac{a}{\begin{vmatrix}1 & 5\\3 & 3\end{vmatrix}} = \frac{-b}{\begin{vmatrix}7 & 5\\1 & 3\end{vmatrix}} = \frac{c}{\begin{vmatrix}7 & 1\\1 & 3\end{vmatrix}}$$

$$\frac{a}{-12} = \frac{b}{-16} = \frac{c}{20}$$

$$\frac{a}{3} = \frac{b}{4} = \frac{c}{-5}$$

a : b : c = 3 : 4 : -5

Substituting in (1),

3x – 6 + 4y – 8 – 5z + 5 = 0

3x + 4y – 5z – 9 = 0

Therefore the equation of the plane is 3x + 4y – 5z – 9 = 0

14. Question

Mark against the correct answer in each of the following:

The equation of the plane passing through the intersection of the planes 3x - y + 2z – 4 = 0 and x + y + z - 2 = 0 and passing through the point A(2, 2, 1) is given by

A. 7x + 5y – 4z – 8 = 0

B. 7x – 5y + 4z – 8 = 0

C. 5x – 7y + 4z – 8 = 0

D. 5x + 7y – 4z + 8 = 0

Answer

Given: Plane passes through the intersection of planes 3x – y + 2z – 4 = 0 and x + y + z – 2 = 0. Point A(2, 2, 1) lies on the plane.

To find: Equation of the plane.

Formula Used: Equation of plane passing through the intersection of 2 planes P �$_1$ and P_2 is given by $P_1 + \lambda P_2 = 0$

Explanation:

Equation of plane is

$3x - y + 2z - 4 + \lambda (x + y + z - 2) = 0$ … (1)

Since A(2, 2, 1) lies on the plane,

$6 - 2 + 2 - 4 + \lambda (2 + 2 + 1 - 2) = 0$

$2 + 3\lambda = 0$

$$\lambda = \frac{-2}{3}$$

Substituting in (1) and multiplying by 3,

$9x - 3y + 6z - 12 - 2 (x + y + z - 2) = 0$

$9x - 3y + 6z - 12 - 2x - 2y - 2z + 4 = 0$

$7x - 5y + 4z - 8 = 0$

Therefore the equation of the plane is $7x - 5y + 4z - 8 = 0$

15. Question

Mark against the correct answer in each of the following:

The equation of the plane passing through the points A(0, -1, 0), B(2, 1, -1) and C(1, 1, 1) is given by

A. $4x + 3y - 2z - 3 = 0$

B. $4x - 3y + 2z + 3 = 0$

C. $4x - 3y + 2z - 3 = 0$

D. None of these

Answer

Given: Plane passes through A(0, -1, 0), B(2, 1, -1) and C(1, 1, 1)

To find: Equation of the plane

Formula Used: Equation of a plane is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

where a:b:c is the direction ratios of the normal to the plane.

(x_1, y_1, z_1) is a point on the plane.

Explanation:

Let the equation of plane be $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Substituting point A,

$ax + b(y + 1) + cz = 0$ … (1)

Substituting points B and C,

$2a + 2b - c = 0$ and $a + 2b + c = 0$

Solving,

$$\frac{a}{\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 2 & 2 \\ 1 & 2 \end{vmatrix}}$$

$$\frac{a}{4} = \frac{b}{-3} = \frac{c}{2}$$

Therefore, a : b : c = 4 : -3 : 2

Substituting in (1),

$4x - 3 (y + 1) + 2z = 0$

$4x - 3y + 2z - 3 = 0$

Therefore equation of plane is $4x - 3y + 2z - 3 = 0$

16. Question

Mark against the correct answer in each of the following:

If the plane $2x - y + z = 0$ is parallel to the line $\frac{2x-1}{2} = \frac{2-y}{2} = \frac{z+1}{a}$, then the value of a is

A. -4

B. -2

C. 4

D. 2

Answer

Given: Plane 2x – y + z = 0 is parallel to the line

$$\frac{2x-1}{2}=\frac{2-y}{2}=\frac{z+1}{a},$$

To find: value of a

Formula Used: If two lines with direction ratios $a_1:a_2:a_3$ and $b_1:b_2:b_3$ are perpendicular, then

$a_1b_1 + a_2b_2 + a_3b_3 = 0$

Explanation:

Since the plane is parallel to the line, the normal to the plane will be perpendicular to the line.

Equation of the line can be rewritten as

$$\frac{x-\frac{1}{2}}{1}=\frac{y-2}{-2}=\frac{z-(-1)}{a}$$

Direction ratio of the normal to the plane is 2 : -1 : 1

Direction ratio of line is 1 : -2 : a

Therefore,

2 + 2 + a = 0

a = -4

Therefore, a = -4

17. Question

Mark against the correct answer in each of the following:

The angle between the line $\frac{x+1}{1}=\frac{y}{2}=\frac{z-1}{1}$ and a normal to the plane x – y + z = 0 is

A. 0°

B. 30°

C. 45°

D. 90°

Answer

Given: Equation of line is $\frac{x+1}{1}=\frac{y}{2}=\frac{z-1}{1}$

Equation of plane is x – y + z = 0

To find: Angle between a line and the normal to a plane.

Formula Used: If θ is the angle between two lines with direction ratios $b_1:b_2:b_3$ and $c_1:c_2:c_3$, then

$$\cos\theta=\frac{b_1c_1+b_2c_2+b_3c_3}{\sqrt{b_1^2+b_2^2+b_3^2}\times\sqrt{c_1^2+c_2^2+c_3^2}}$$

Explanation:

Direction ratios of given line is 1 : 2 : 1

Direction ratios of the normal to the plane is 1 : -1 : 1

Therefore,

$$\cos\theta=\frac{1-2+1}{\sqrt{1+4+1}\times\sqrt{1+1+1}}$$

cos θ = 0

θ = 90°

Therefore angle between them is 90°

18. Question

Mark against the correct answer in each of the following:

The point of intersection of the line $\frac{x-1}{3}=\frac{y+2}{4}=\frac{z-3}{-2}$ and the plane 2x – y + 3z – 1 = 0, is

A. (-10, 10, 3)

B. (10, 10, -3)

C. (10, -10, 3)

D. (10, -10, -3)

Answer

Given: Line $\frac{x-1}{3}=\frac{y+2}{4}=\frac{z-3}{-2}$ meets plane 2x – y + 3z – 1 = 0

To find: Point of intersection of line and plane.

Explanation:

Let the equation of the line be

$$\frac{x-1}{3}=\frac{y+2}{4}=\frac{z-3}{-2}=\lambda$$

Therefore, any point on the line is (3λ + 1, 4λ - 2, -2λ + 3)

Since this point also lies on the plane,

2(3λ + 1) - (4λ -2) + 3(-2λ +3) = 1

6λ + 2 – 4λ + 2 – 6λ + 9 = 1

-4λ = -12

λ = 3

Therefore required point is (10, 10, -3)

19. Question

Mark against the correct answer in each of the following:

The equation of a plane passing through the points A(a, 0, 0), B(0, b, 0) and C(0, 0, c) is given by

A. ax + by + cz = 0

B. ax + by + cz = 1

C. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$

D. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Answer

Given: Plane passes through the points A(a, 0, 0), B(0, b, 0) and C(0, 0, c)

To find: Equation of plane.

Explanation:

The given points lie on the co-ordinate axes.

Therefore, the plane makes intercepts of a, b and c on the x, y and z-axis respectively.

Equation of the plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

20. Question

Mark against the correct answer in each of the following:

If θ is the angle between the planes 2x – y + 2z = 3 and 6x – 2y + 3z = 5, then cos θ = ?

A. $\frac{11}{20}$

B. $\frac{12}{23}$

C. $\frac{17}{25}$

D. $\frac{20}{21}$

Answer

Given: Equation of two planes are 2x – y + 2z = 3 and 6x – 2y + 3z = 5

To find: cos θ where θ: angle between the planes

Formula Used: Angle between two planes $a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$ is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \times \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

where θ : angle between the planes,

Explanation:

Here $a_1 = 2$, $b_1 = -1$, $c_1 = 2$

$a_2 = 6$, $b_2 = -2$, $c_2 = 3$

$$\Rightarrow \cos\theta = \frac{12 + 2 + 6}{\sqrt{4 + 1 + 4} \times \sqrt{36 + 4 + 9}}$$

$$\Rightarrow \cos\theta = \frac{20}{3 \times 7}$$

$$\Rightarrow \cos\theta = \frac{20}{21}$$

Therefore, $\cos\theta = \frac{20}{21}$

21. Question

Mark against the correct answer in each of the following:

The angle between the planes 2x – y + z = 6 and x + y + 2z = 7, is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer

Given: Equation of two planes are 2x – y + z = 6 and x + y + 2z = 7

To find: Angle between the two planes

Formula Used: Angle between two planes $a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$ is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \times \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

where θ : angle between the planes,

Explanation:

Here $a_1 = 2$, $b_1 = -1$, $c_1 = 1$

$a_2 = 1$, $b_2 = 1$, $c_2 = 2$

$$\Rightarrow \cos\theta = \frac{2 - 1 + 2}{\sqrt{4 + 1 + 1} \times \sqrt{1 + 1 + 4}}$$

$$\Rightarrow \cos\theta = \frac{3}{\sqrt{6} \times \sqrt{6}}$$

$$\Rightarrow \cos\theta = \frac{3}{6}$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$\Rightarrow \theta = \frac{\pi}{3}$

Therefore angle between the planes is $\frac{\pi}{3}$

22. Question

Mark against the correct answer in each of the following:

The angle between the planes $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 4$ and $\vec{r} \cdot (2\hat{i} - \hat{j} + 2\hat{k}) = 3$, is

A. $\cos^{-1}\left(\frac{16}{21}\right)$

B. $\cos^{-1}\left(\frac{4}{21}\right)$

C. $\cos^{-1}\left(\frac{3}{4}\right)$

D. $\cos^{-1}\left(\frac{1}{4}\right)$

Answer

Given: Equation of two planes are $\vec{r}.(3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}) = 4$ and $\vec{r}.(2\hat{\imath} - \hat{\jmath} + 2\hat{k}) = 3$

To find: Angle between the two planes

Formula Used: Angle between two planes $a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$ is

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \times \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

where θ : angle between the planes,

Explanation:

Since $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, the given equation of planes can rewritten as:

$3x - 6y + 2z = 4$ and $2x - y + 2z = 3$

Here $a_1 = 3$, $b_1 = -6$, $c_1 = 2$

$a_2 = 2$, $b_2 = -1$, $c_2 = 2$

$$\Rightarrow \cos\theta = \frac{6 + 6 + 4}{\sqrt{9 + 36 + 4} \times \sqrt{4 + 1 + 4}}$$

$$\Rightarrow \cos\theta = \frac{16}{7 \times 3}$$

$$\Rightarrow \cos\theta = \frac{16}{21}$$

$$\Rightarrow \theta = \cos^{-1}\frac{16}{21}$$

Therefore angle between the planes is $\cos^{-1}\frac{16}{21}$

23. Question

Mark against the correct answer in each of the following:

The equation of the plane through the points A(2, 3, 1) and B(4, -5, 3), parallel to the x-axis, is

A. $x + y - 3z = 2$

B. $y + 4z = 7$

C. $y + 3z = 6$

D. $x + 5y - 3z = 4$

Answer

Given: Plane passes through the points A(2, 3, 1) and B(4, -5, 3) and is parallel to x-axis

To find: Equation of plane

Formula Used: Equation of a plane parallel o x-axis is

$b(y - y_1) + c(z - z_1) = 0$

Explanation:

Let the equation of the plane be

$b(y - y_1) + c(z - z_1) = 0$

Since A(2, 3, 1) lies on the plane,

b(y – 3) + c(z – 1) = 0 … (1)

Since B(4, -5, 3) lies on the plane,

b(-5 – 3) + c(3 – 1) = 0

-8b + 2c = 0 or –4b + c = 0

b : c = 1 : 4

Substituting in (1),

y – 3 + 4z – 4 = 0

y + 4z = 7

The equation of the plane is y + 4z = 7

24. Question

Mark against the correct answer in each of the following:

A variable plane moves so that the sum of the reciprocals of its intercepts on the coordinate axes is (1/2). Then, the plane passes through the point

A. (0, 0, 0)

B. (1, 1, 1)

C. $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

D. (2, 2, 2)

Answer

Given: Variable plane moves so that the sum of the reciprocals of its intercepts on the coordinate axes is (1/2)

Formula Used: Equation of a plane is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Explanation:

Let the intercepts made by the plane on the co-ordinate axes be a, b and c.

$$\Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{2}$$

Let the equation of the plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

On solving for each of the given options,

(0, 0, 0) ⇒ LHS ≠ RHS

(1, 1, 1) ⇒ LHS ≠ RHS

$$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \Rightarrow LHS = \frac{1}{2a} + \frac{1}{2b} + \frac{1}{2c} = \frac{1}{2} \times \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = \frac{1}{4} \neq RHS$$

$$(2,2,2) \Rightarrow LHS = \frac{2}{a} + \frac{2}{b} + \frac{2}{c} = 2 \times \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = 1 = RHS$$

Therefore, plane passes through the point (2, 2, 2)

25. Question

Mark against the correct answer in each of the following:

The equation of a plane which is perpendicular to $\left(2\hat{i} - 3\hat{j} + \hat{k}\right)$ and at a distance of 5 units from the origin is

A. 2x – 3y + z = 5

B. 2x – 3y + z = 5√14

C. $\frac{x}{2}-\frac{y}{3}+\frac{z}{1}=5$

D. $\frac{x}{2}-\frac{y}{3}+\frac{z}{1}=\frac{5}{\sqrt{14}}$

Answer

Given: Plane is perpendicular to $(2\hat{\imath}-3\hat{\jmath}+\hat{k})$ and is at a distance of 5 units from origin.

To find: Equation of plane

Formula Used: Equation of a plane is lx + my + nz = p where p is the distance from the origin and l, m and n are the direction cosines of the normal to the plane

Explanation:

Direction ratio of normal to the plane is 2:-3:1

$|2\hat{\imath}-3\hat{\jmath}+\hat{k}| = \sqrt{4+9+1}=\sqrt{14}$

Therefore, direction cosines of the normal to the plane is

l = $\frac{2}{\sqrt{14}}$, m = $\frac{-3}{\sqrt{14}}$, n = $\frac{1}{\sqrt{14}}$

Since the equation of a plane is lx + my + nz = p where p is the distance from the origin,

2x – 3y + z = 5√14

Therefore, equation of the plane is 2x – 3y + z = 5√14

26. Question

Mark against the correct answer in each of the following:

The equation of the plane passing through the point A(2, 3,4) and parallel to the plane 5x - 6y + 7z = 3, is

A. 5x – 6y + 7z = 20

B. 7x – 6y + 5z = 72

C. 20x – 18y + 14z = 11

D. 10x – 18y + 28z = 13

Answer

Given: Point A(2, 3, 4) lies on a plane which is parallel to 5x – 6y + 7z = 3

To find: Equation of the plane

Formula Used: Equation of a plane is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

where a:b:c is the direction ratios of the normal to the plane

(x_1, y_1, z_1) is a point on the plane.

Explanation:

Since the plane (say P_1) is parallel to the plane 5x – 6y + 7z = 3 (say P_2), the direction ratios of the normal to P_1 is same as the direction ratios of the normal to P_2.

i.e., direction ratios of P_1 is 5 : -6 : 7

Let the equation of the required plane be

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

Here a = 5, b = -6 and c = 7

Since (2, 3, 4) lies on the plane,

5(x - 2) – 6 (y - 3) + 7 (z - 4) = 0

5x – 6y + 7z – 10 + 18 – 28 = 0

5x – 6y + 7z = 20

The equation of the plane is 5x – 6y + 7z = 20

27. Question

Mark against the correct answer in each of the following:

The foot of the perpendicular from the point A(7, 14, 5) to the plane 2x + 4y – z = 2 is

A. (3, 1, 8)

B. (1, 2, 8)

C. (3, -3, 5)

D. (5, -3, -4)

Answer

Given: Perpendicular dropped from A(7, 14, 5) on to the plane 2x + 4y – z = 2

To find: co-ordinates of the foot of perpendicular

Formula Used: Equation of a line is

$$\frac{x-x_1}{b_1}=\frac{y-y_2}{b_2}=\frac{z-z_3}{b_3}=\lambda$$

Where $b_1:b_2:b_3$ is the direction ratio and (x_1, x_2, x_3) is a point on the line.

Explanation:

Let the foot of the perpendicular be (a, b, c)

Since this point lies on the plane,

2a + 4b – c = 2 ... (1)

Direction ratio of the normal to the plane is 2 : 4 : -1

Direction ratio perpendicular = direction ratio of normal to the plane

So, equation of the perpendicular is

$$\frac{x-x_1}{2}=\frac{y-y_1}{4}=\frac{z-z_1}{-1}=\lambda$$

Since (a, b, c) is a point on the perpendicular,

$$\frac{x-a}{2}=\frac{y-b}{4}=\frac{z-c}{-1}=\lambda$$

(7, 14, 5) is a point on the perpendicular.

$$\frac{7-a}{2}=\frac{14-b}{4}=\frac{5-c}{-1}=\lambda$$

So, a = 7 - 2λ, b = 14 – 4λ, c = 5 + λ

Substituting in (1),

14 – 4λ + 56 – 16λ – 5 - λ = 2

21λ = 70 – 7 = 63

λ = 3

Therefore, foot of the perpendicular is (1, 2, 8)

28. Question

Mark against the correct answer in each of the following:

The equation of the plane which makes with the coordinate axes, a triangle with centroid (α, β, γ) is given by

A. αx + βy + γz = 1

B. αx + βy + γz = 3

C. $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$

D. $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$

Answer

Given: Centroid of triangle is (α, β, γ)

To find: Equation of plane.

Formula Used: Equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Centroid of a triangle $=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$

Explanation:

Let the equation of plane be

$\frac{x}{A}+\frac{y}{B}+\frac{z}{C}=1$ … (1)

Therefore, A = 3α, B = 3β, C = 3γ where (a, b, c) is the centroid of the triangle with vertices (A, 0, 0), (0, B, 0) and (0, 0, C)

Substituting in (1),

$\Rightarrow \frac{x}{3a}+\frac{y}{3b}+\frac{z}{3c}=1$

Here a = α, b = β and c = γ

$\Rightarrow \frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$

Therefore equation of required plane is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3$

29. Question

Mark against the correct answer in each of the following:

The intercepts made by the plane $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+4\hat{k}\right)=12$ are

A. 2, -3, 4

B. 2, -3, -6

C. 6, -4, 3

D. -6, 4, 3

Answer

Given: Equation of plane is $\vec{r}.\left(2\hat{\imath}-3\hat{\jmath}+4\hat{k}\right)=12$

To find: Intercepts made by the plane.

Formula Used: Equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ where (x, y, z) is a point on the plane and a, b, c are intercepts on x-axis, y-axis and z-axis respectively.

Explanation:

The equation of the plane can be written as

2x – 3y + 4z = 12

Dividing by 12,

$\frac{x}{6}+\frac{y}{-4}+\frac{z}{3}=1$ which is of the form $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Therefore the intercepts made by the plane are 6, -4, 3

30. Question

Mark against the correct answer in each of the following:

The angle between the line $\frac{x-2}{1}=\frac{y+3}{-2}=\frac{z+4}{-3}$ and the plane 2x – 3y + z = 5 is

A. $\cos^{-1}\left(\frac{5}{14}\right)$

B. $\sin^{-1}\left(\frac{5}{14}\right)$

C. $\cos^{-1}\left(\frac{3}{7}\right)$

D. $\sin^{-1}\left(\frac{3}{7}\right)$

Answer

Given: Equation of line is $\frac{x-2}{1}=\frac{y+3}{-2}=\frac{z+4}{-3}$

Equation of the plane is 2x – 3y + z = 5

To find: angle between line and plane

Formula Used: If θ is the angle between a line with direction ratio $b_1:b_2:b_3$ and a plane with direction ratio of normal $n_1:n_2:n_3$, then

$$\sin\theta = \frac{n_1b_1 + n_2b_2 + n_3b_3}{\sqrt{n_1^2 + n_2^2 + n_3^2} \times \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Explanation:

Here direction ratio of the line is 1 : -2 : -3

Direction ratio of normal to the plane is 2 : -3 : 1

Therefore,

$$\sin\theta = \frac{2 + 6 - 3}{\sqrt{1 + 4 + 9} \times \sqrt{4 + 9 + 1}}$$

$$\sin\theta = \frac{5}{\sqrt{14} \times \sqrt{14}}$$

$$\theta = \sin^{-1}\frac{5}{14}$$

Therefore, angle between the line and plane is $\sin^{-1}\frac{5}{14}$

31. Question

Mark against the correct answer in each of the following:

The angle between the line $\vec{r}\cdot(\hat{i} + \hat{j} - 3\hat{k}) + \lambda(2\hat{i} + 2\hat{j} + \hat{k})$ and the plane $\vec{r}\cdot(6\hat{i} - 3\hat{j} + 2\hat{k}) = 5,$ is

A. $\cos^{-1}\left(\frac{8}{21}\right)$

B. $\cos^{-1}\left(\frac{5}{21}\right)$

C. $\sin^{-1}\left(\frac{5}{21}\right)$

D. $\sin^{-1}\left(\frac{8}{21}\right)$

Answer

Given: Equation of line is $\vec{r}.(\hat{\imath} + \hat{\jmath} - 3\hat{k}) + \lambda(2\hat{\imath} + 2\hat{\jmath} + \hat{k})$

Equation of plane is $\vec{r}.(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}) = 5$

To find: angle between line and plane

Formula Used: If θ is the angle between a line with direction ratio b_1:b_2:b_3 and a plane with direction ratio of normal n_1:n_2:n_3, then

$$\sin\theta = \frac{n_1b_1 + n_2b_2 + n_3b_3}{\sqrt{n_1^2 + n_2^2 + n_3^2} \times \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Explanation:

Here direction ratio of the line is 2 : 2 : 1

Direction ratio of normal to the plane is 6 : -3 : 2

Therefore,

$$\sin\theta = \frac{12 - 6 + 2}{\sqrt{4 + 4 + 1} \times \sqrt{36 + 9 + 4}}$$

$$\sin\theta = \frac{8}{3 \times 7}$$

$$\theta = \sin^{-1}\frac{8}{21}$$

Therefore, angle between the line and plane is $\sin^{-1}\frac{8}{21}$

32. Question

Mark against the correct answer in each of the following:

The distance of the point $(\hat{i} + 2\hat{j} + 5\hat{k})$ from the plane $\vec{r}\cdot(\hat{i} + \hat{j} + \hat{k}) + 17 = 0,$ is

A. $\frac{25}{\sqrt{2}}$ units

B. $\frac{25}{\sqrt{3}}$ units

C. 25√2 units

D. 25√3 units

Answer

Given: Point is at $(\hat{\imath}+2\hat{\jmath}+5\hat{k})$ and equation of plane is $\vec{r}.(\hat{\imath}+\hat{\jmath}+\hat{k})+17=0$

To find: distance of point from plane

Formula Used: Perpendicular distance from (x_1, y_1, z_1) to the plane ax + by + cz + d = 0 is

$$\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

Explanation:

The point is at (1, 2, 5) and equation of plane is x + y + z + 17 = 0

Distance $=\frac{1+2+5+17}{\sqrt{1+1+1}}$

$=\frac{25}{\sqrt{3}}$

Therefore, distance $=\frac{25}{\sqrt{3}}$ units

33. Question

Mark against the correct answer in each of the following:

The distance between the parallel planes 2x – 3y + 6z = 5 and 6x – 9y + 18z + 20 = 0, is

A. $\frac{5}{3}$ units

B. 5√3 units

C. $\frac{8}{5}$ units

D. 8√5 units

Answer

Given: The equations of the parallel planes are 2x – 3y + 6z = 5 and 6x – 9y + 18z + 20 = 0

To find: distance between the planes

Formula Used: Distance between two parallel planes ax + by + cz + d_1 = 0 and ax_1 + by_1 + cz_1 + d_1 = 0 is

$$\left|\frac{d_2-d_1}{\sqrt{a^2+b^2+c^2}}\right|$$

Explanation:

The equations of the parallel planes are:

2x – 3y + 6z – 5 = 0

$2x-3y+6z+\frac{20}{3}=0$

Therefore distance between them is

$=\left|\frac{\frac{20}{3}+5}{\sqrt{4+9+36}}\right|$

$=\left|\frac{35}{3\times\sqrt{49}}\right|$

$=\frac{5}{3}$

Therefore distance between the planes is $\frac{5}{3}$ units

34. Question

Mark against the correct answer in each of the following:

The distance between the planes x + 2y – 2z + 1 = 0 and 2x + 4y – 4z – 4z + 5 = 0, is

A. 4 units

B. 2 units

C. $\frac{1}{2}$ units

D. $\frac{1}{4}$ units

Answer

Given: The equations of the planes are x + 2y – 2z + 1 = 0 and 2x + 4y – 4z – 4z + 5 = 0

To find: distance between the planes

Formula Used: Distance between two parallel planes ax + by + cz + d_1 = 0 and $ax_1 + by_1 + cz_1 + d_1 = 0$ is

$$\left|\frac{d_2 - d_1}{\sqrt{a^2 + b^2 + c^2}}\right|$$

Explanation:

The equations of the planes are:

x + 2y – 2z + 1 = 0 and 2x + 4y – 4z – 4z + 5 = 0

Multiplying the equation of first plane by 2,

2x + 4y – 4z + 2 = 0

Therefore distance between them is

$$= \left|\frac{\frac{20}{3} + 5}{\sqrt{4 + 9 + 36}}\right|$$

$$= \left|\frac{35}{3 \times \sqrt{49}}\right|$$

$$= \frac{5}{3}$$

Therefore distance between the planes is $\frac{5}{3}$ units

35. Question

Mark against the correct answer in each of the following:

The image of the point P(1, 3, 4) in the plane 2x – y + z + 3 = 0, is

A. (3, -5, 2)

B. (3, 5, -2)

C. (3, 5, 2)

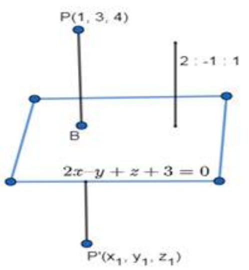
D. (-3, 5, 2)

Answer

Given: Equation of plane is 2x – y + z + 3 = 0. P is at (1, 3, 4)

To find: image of P

Explanation:



From the figure, P’ is the image of P and B is the midpoint of PP’

If B is (a, b, c), then

$a = \frac{1 + x_1}{2}$

$b = \frac{3 + y_1}{2}$

$c = \frac{4 + z_1}{2}$

B lies on the plane.

So, 2a – b + c + 3 = 0 … (1)

Also, equation of line PB is

$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda$

So any point on the line PB will be of the form

x = 2λ +1, y = -λ + 3, z = λ + 4

Since (a, b, c) will also be of this form we can substitute these values in (1)

⇒ 2(2λ + 1) – (-λ + 3) + (λ + 4) + 3 = 0

⇒ 4λ + 2 + λ – 3 + λ + 4 + 3 = 0

⇒ 6λ = -6

⇒ λ = -1

So (a, b, c) = (-1, 4, 3)

Substituting these values in the equations of a, b and c,

$-1 = \frac{1 + x_1}{2}$

$4 = \frac{3 + y_1}{2}$

$3 = \frac{4 + z_1}{2}$

Therefore,

$x_1 = -3$, $y_1 = 5$, $z_1 = 2$

Therefore, the image is (-3, 5, 2)