

Trigonometric Equations

Exercise 17

Q. 1. Find the principal solutions of each of the following equations:

(i) $\sin x = \frac{\sqrt{3}}{2}$

(ii) $\cos x = \frac{1}{2}$

(iii) $\tan x = \sqrt{3}$

(iv) $\cot x = \sqrt{3}$

(v) $\operatorname{cosec} x = 2$

(vi) $\sec x = \frac{2}{\sqrt{3}}$

Answer : To Find: Principal solution.

[NOTE: The solutions of a trigonometry equation for which $0 \leq x < 2\pi$ is called principal solution]

(i) Given: $\sin x = \frac{\sqrt{3}}{2}$

Formula used: $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in I$

By using above formula, we have

$$\sin x = \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3} \Rightarrow x = n\pi + \frac{\pi}{3}(-1)^n$$

$$\text{Put } n=0 \Rightarrow x = 0 \times \pi + \frac{\pi}{3}(-1)^0 \Rightarrow x = \frac{\pi}{3}$$

$$\text{Put } n=1 \Rightarrow x = 1 \times \pi + \frac{\pi}{3}(-1)^1 \Rightarrow x = 1 \times \pi + \frac{\pi}{3}(-1)^1 \Rightarrow x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

So principal solution is $x = \frac{\pi}{3}$ and $\frac{2\pi}{3}$

(ii) Given: $\cos x = \frac{1}{2}$

Formula used: $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in I$

By using above formula, we have

$$\cos x = \frac{1}{2} = \cos \frac{\pi}{3} \Rightarrow \theta = 2n\pi \pm \alpha, n \in I$$

$$\text{Put } n=0 \Rightarrow x = 2n\pi \pm \frac{\pi}{3} \Rightarrow x = \frac{\pi}{3}$$

$$\text{Put } n=1 \Rightarrow x = 2\pi \pm \frac{\pi}{3} \Rightarrow x = \frac{5\pi}{3}, \frac{7\pi}{3} \Rightarrow x = \frac{5\pi}{3}, \frac{7\pi}{3}$$

$[\frac{7\pi}{3} > 2\pi$ So it is not include in principal solution]

So principal solution is $x = \frac{\pi}{3}$ and $\frac{5\pi}{3}$

(iii) Given: $\tan x = \sqrt{3}$

Formula used: $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$

By using above formula, we have

$$\tan x = \sqrt{3} = \tan \frac{\pi}{3} \Rightarrow x = n\pi + \alpha, n \in I$$

$$\text{Put } n = 0 \Rightarrow x = n\pi + \frac{\pi}{3} \Rightarrow x = \frac{\pi}{3}$$

$$\text{Put } n = 1 \Rightarrow x = \pi + \frac{\pi}{3} \Rightarrow x = \frac{4\pi}{3} \Rightarrow x = \frac{4\pi}{3}$$

So principal solution is $x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$

We know that $\tan\theta \times \cot\theta = 1$

$$\text{So } \cot x = \sqrt{3} \Rightarrow \tan x = \frac{1}{\sqrt{3}}$$

The formula used: $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$

By using the above formula, we have

$$\tan x = \frac{1}{\sqrt{3}} = \tan\frac{\pi}{6} \Rightarrow \theta = n\pi + \alpha, n \in I$$

$$\text{Put } n = 0 \Rightarrow x = n\pi + \frac{\pi}{6} \Rightarrow x = \frac{\pi}{6}$$

$$\text{Put } n = 1 \Rightarrow x = \pi + \frac{\pi}{6} \Rightarrow x = \frac{7\pi}{6}$$

So principal solution is $x = \frac{\pi}{6}$ and $\frac{7\pi}{6}$

(v) Given: $\operatorname{cosec} x = 2$

We know that $\operatorname{cosec} \theta \times \sin \theta = 1$

So $\sin x = \frac{1}{2}$

Formula used: $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in \mathbb{Z}$

By using above formula, we have

$$\sin x = \frac{1}{2} = \sin \frac{\pi}{6} \Rightarrow \theta = n\pi + \frac{\pi}{6}(-1)^n$$

$$\text{Put } n=0 \Rightarrow \theta = 0 \times \pi + \frac{\pi}{6}(-1)^0 \Rightarrow \theta = \frac{\pi}{6}$$

$$\text{Put } n=1 \Rightarrow \theta = 1 \times \pi + \frac{\pi}{6}(-1)^1 \Rightarrow \theta = 1 \times \pi + \frac{\pi}{6}(-1)^1 \Rightarrow \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

So principal solution is $x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

(vi) Given: $\sec x = \frac{2}{\sqrt{3}}$

We know that $\sec \theta \times \cos \theta = 1$

$$\text{So } \cos x = \frac{\sqrt{3}}{2}$$

Formula used: $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in \mathbb{I}$

By using the above formula, we have

$$\cos x = \frac{\sqrt{3}}{2} = \cos \frac{\pi}{6} \Rightarrow x = 2n\pi \pm \alpha, n \in \mathbb{I}$$

$$\text{Put } n=0 \Rightarrow x = 2n\pi \pm \frac{\pi}{6} \Rightarrow x = \frac{\pi}{6}$$

$$\text{Put } n=1 \Rightarrow x = 2\pi \pm \frac{\pi}{6} \Rightarrow x = \frac{11\pi}{6}, \frac{13\pi}{6} \Rightarrow x = \frac{11\pi}{6}, \frac{13\pi}{6}$$

$[\frac{13\pi}{6} > 2\pi$ So it is not include in principal solution]

So principal solution is $x = \frac{\pi}{6}$ and $\frac{11\pi}{6}$

Q. 2. Find the principal solutions of each of the following equations :

(i) $\sin x = \frac{-1}{2}$

(ii) $\sqrt{2} \cos x + 1 = 0$

(iii) $\tan x = -1$

(iv) $\sqrt{3} \operatorname{cosec} x + 2 = 0$

(v) $\tan x = -\sqrt{3}$

(vi) $\sqrt{3} \sec x + 2 = 0$

Answer : To Find: Principal solution.

(i) Given: $\sin x = \frac{-1}{2}$

Formula used: $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in I$

By using above formula, we have

$$\sin x = \frac{-1}{2} = -\sin \frac{\pi}{6} = \sin(\pi + \frac{\pi}{6}) = \sin \frac{7\pi}{6} \Rightarrow x = n\pi + \frac{7\pi}{6}(-1)^n$$

$$\text{Put } n=0 \Rightarrow x = 0 \times \pi + \frac{7\pi}{6}(-1)^0 \Rightarrow x = \frac{7\pi}{6}$$

$$\text{Put } n=1 \Rightarrow x = 1 \times \pi + \frac{7\pi}{6}(-1)^1 \Rightarrow x = 1 \times \pi + \frac{7\pi}{6}(-1) \Rightarrow x = \pi - \frac{7\pi}{6} = -\frac{\pi}{6}$$

[NOTE: $-\frac{\pi}{6} = \frac{11\pi}{6}$]

So principal solution is $x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$

(ii) Given: $\sqrt{2} \cos x + 1 = 0 \Rightarrow \cos x = \frac{-1}{\sqrt{2}}$

Formula used: $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in I$

By using above formula, we have

$$\cos x = \frac{-1}{\sqrt{2}} = \cos \frac{3\pi}{4} \Rightarrow x = 2n\pi \pm \alpha, n \in I$$

$$\text{Put } n = 0 \Rightarrow x = 2 \times 0 \times \pi \pm \frac{3\pi}{4} \Rightarrow x = \frac{3\pi}{4}$$

$$\text{Put } n = 1 \Rightarrow x = 2\pi \pm \frac{3\pi}{4} \Rightarrow x = \frac{5\pi}{4}, \frac{11\pi}{4} \Rightarrow x = \frac{5\pi}{4}, \frac{11\pi}{4}$$

$[\frac{11\pi}{4} > 2\pi$ So it is not include in principal solution]

So principal solution is $x = \frac{3\pi}{4}$ and $\frac{5\pi}{4}$

(iii) Given: $\tan x = -1$

Formula used: $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$

By using above formula, we have

$$\tan x = -1 = \tan\frac{3\pi}{4} \Rightarrow x = n\pi + \alpha, n \in I$$

$$\text{Put } n=0 \Rightarrow x = n\pi + \frac{3\pi}{4} \Rightarrow x = \frac{3\pi}{4}$$

$$\text{Put } n=1 \Rightarrow x = \pi + \frac{3\pi}{4} \Rightarrow x = \frac{7\pi}{4} \Rightarrow x = \frac{7\pi}{4}$$

So principal solution is $x = \frac{3\pi}{4}$ and $\frac{7\pi}{4}$

$$\text{(iv) Given: } \sqrt{3} \operatorname{cosec} x + 2 = 0 \Rightarrow \operatorname{cosec} x = \frac{-2}{\sqrt{3}}$$

We know that $\operatorname{cosec} \theta \times \sin \theta = 1$

$$\text{So } \sin x = \frac{-\sqrt{3}}{2}$$

Formula used: $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in \mathbb{Z}$

By using above formula, we have

$$\sin x = \frac{-\sqrt{3}}{2} = \sin \frac{4\pi}{3} \Rightarrow \theta = n\pi + \frac{4\pi}{3}(-1)^n$$

$$\text{Put } n=0 \Rightarrow x = 0 \times \pi + \frac{4\pi}{3}(-1)^0 \Rightarrow x = \frac{4\pi}{3}$$

$$\text{Put } n=1 \Rightarrow x = 1 \times \pi + \frac{4\pi}{3}(-1)^1 \Rightarrow x = 1 \times \pi + \frac{4\pi}{3}(-1)^1 \Rightarrow x = \pi - \frac{4\pi}{3} = \frac{-\pi}{3}$$

$$[\text{NOTE: } \frac{-\pi}{3} = \frac{5\pi}{3}]$$

So principal solution is $x = \frac{4\pi}{3}$ and $\frac{5\pi}{3}$

(v) Given: $\tan x = -\sqrt{3}$

Formula used: $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi \pm \alpha, n \in \mathbb{I}$

By using above formula, we have

$$\tan x = -\sqrt{3} = \tan \frac{2\pi}{3} \Rightarrow x = n\pi + \alpha, n \in \mathbb{I}$$

$$\text{Put } n=0 \Rightarrow x = n\pi + \frac{2\pi}{3} \Rightarrow x = \frac{2\pi}{3}$$

$$\text{Put } n=1 \Rightarrow x = \pi + \frac{2\pi}{3} \Rightarrow x = \frac{5\pi}{3}$$

So principal solution is $x = \frac{2\pi}{3}$ and $\frac{5\pi}{3}$

$$\text{(vi) Given: } \sqrt{3} \sec x + 2 = 0 \Rightarrow \sec x = \frac{-2}{\sqrt{3}}$$

We know that $\sec \theta \times \cos \theta = 1$

$$\text{So } \cos x = \frac{-\sqrt{3}}{2}$$

Formula used: $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in \mathbb{I}$

By using the above formula, we have

$$\cos x = \frac{\sqrt{3}}{2} = \cos \frac{5\pi}{6} \Rightarrow x = 2n\pi \pm \alpha, n \in \mathbb{I}$$

$$\text{Put } n=0 \Rightarrow x = 2n\pi \pm \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{6}$$

$$\text{Put } n=1 \Rightarrow x = 2\pi \pm \frac{5\pi}{6} \Rightarrow x = \frac{7\pi}{6}, \frac{17\pi}{6} \Rightarrow x = \frac{7\pi}{6}, \frac{17\pi}{6}$$

$[\frac{17\pi}{6} > 2\pi$ So it is not include in principal solution]

So principal solution is $x = \frac{5\pi}{6}$ and $\frac{7\pi}{6}$

Q. 3. Find the general solution of each of the following equations:

(i) $\sin 3x = 0$

(ii) $\sin \frac{3x}{2} = 0$

(iii) $\sin \left(x + \frac{\pi}{5} \right) = 0$

(iv) $\cos 2x = 0$

(v) $\cos \frac{5x}{2} = 0$

(vi) $\cos \left(x + \frac{\pi}{10} \right) = 0$

(vii) $\tan 2x = 0$

(viii) $\tan \left(3x + \frac{\pi}{6} \right) = 0$

(ix) $\tan \left(2x - \frac{\pi}{4} \right) = 0$

Answer : To Find: General solution.

[NOTE: A solution of a trigonometry equation generalized by means of periodicity, is known as general solution]

(i) Given: $\sin 3x = 0$

Formula used: $\sin \theta = 0 \Rightarrow \theta = n\pi, n \in \mathbb{I}$

By using above formula, we have

$$\sin 3x = 0 \Rightarrow 3x = n\pi \Rightarrow x = \frac{n\pi}{3} \text{ where } n \in I$$

So general solution is $x = \frac{n\pi}{3}$ where $n \in I$

(ii) Given: $\sin \frac{3x}{2} = 0$

Formula used: $\sin \theta = 0 \Rightarrow \theta = n\pi, n \in I$

By using above formula, we have

$$\sin \frac{3x}{2} = 0 \Rightarrow \frac{3x}{2} = n\pi \Rightarrow x = \frac{2n\pi}{3} \text{ where } n \in I$$

So general solution is $x = \frac{2n\pi}{3}$ where $n \in I$

(iii) Given: $\sin\left(x + \frac{\pi}{5}\right) = 0$

Formula used: $\sin\theta = 0 \Rightarrow \theta = n\pi, n \in I$

By using the above formula, we have

$$\sin\left(x + \frac{\pi}{5}\right) = 0 \Rightarrow x + \frac{\pi}{5} = n\pi \Rightarrow x = n\pi - \frac{\pi}{5} \text{ where } n \in I$$

So general solution is $x = n\pi - \frac{\pi}{5}$ where $n \in I$

(iv) Given: $\cos 2x = 0$

Formula used: $\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}, n \in I$

By using above formula, we have

$$\cos 2x = 0 \Rightarrow 2x = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{4} \text{ where } n \in \mathbb{I}$$

So general solution is $x = (2n+1)\frac{\pi}{4}$ where $n \in \mathbb{I}$

(v) Given: $\cos \frac{5x}{2} = 0$

Formula used: $\cos \theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}, n \in \mathbb{I}$

By using the above formula, we have

$$\cos \frac{5x}{2} = 0 \Rightarrow \frac{5x}{2} = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{5} \text{ where } n \in \mathbb{I}$$

So general solution is $x = (2n+1)\frac{\pi}{5}$ where $n \in \mathbb{I}$

(vi) Given: $\cos\left(x + \frac{\pi}{10}\right) = 0$

Formula used: $\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}, n \in I$

By using the above formula, we have

$$\cos\left(x + \frac{\pi}{10}\right) = 0 \Rightarrow x + \frac{\pi}{10} = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{2} - \frac{\pi}{10} \Rightarrow x = n\pi + \frac{2\pi}{5}$$

where $n \in I$

So general solution is $x = n\pi + \frac{2\pi}{5}$ where $n \in I$

(vii) Given: $\tan 2x = 0$

Formula used: $\tan\theta = 0 \Rightarrow \theta = n\pi, n \in I$

Formula used: $\tan\theta = 0 \Rightarrow \theta = n\pi, n \in I$

By using above formula, we have

$$\tan 2x = 0 \Rightarrow 2x = n\pi \Rightarrow x = \frac{n\pi}{2} \text{ where } n \in I$$

So general solution is $x = \frac{n\pi}{2}$ where $n \in I$

(viii) Given: $\tan\left(3x + \frac{\pi}{6}\right) = 0$

Formula used: $\tan\theta = 0 \Rightarrow \theta = n\pi, n \in I$

By using above formula, we have

$$\tan\left(3x + \frac{\pi}{6}\right) = 0 \Rightarrow 3x + \frac{\pi}{6} = n\pi \Rightarrow 3x = n\pi - \frac{\pi}{6} \Rightarrow x = \frac{n\pi}{3} - \frac{\pi}{18} \text{ where } n \in I$$

So general solution is $x = \frac{n\pi}{3} - \frac{\pi}{18}$ where $n \in I$

(ix) Given: $\tan\left(2x - \frac{\pi}{4}\right) = 0$

Formula used: $\tan\theta = 0 \Rightarrow \theta = n\pi, n \in I$

By using above formula, we have

$$\tan\left(2x - \frac{\pi}{4}\right) = 0 \Rightarrow 2x - \frac{\pi}{4} = n\pi \Rightarrow 2x = n\pi + \frac{\pi}{4} \Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8} \text{ where } n \in I$$

So general solution is $x = \frac{n\pi}{2} + \frac{\pi}{8}$ where $n \in I$

Q. 4. Find the general solution of each of the following equations:

(i) $\sin x = \frac{\sqrt{3}}{2}$

(ii) $\cos x = 1$

(iii) $\sec x = \sqrt{2}$

Answer : To Find: General solution.

(i) Given: $\sin x = \frac{\sqrt{3}}{2}$

Formula used: $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in I$

By using above formula, we have

$$\sin x = \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3} \Rightarrow x = n\pi + (-1)^n \cdot \frac{\pi}{3}$$

So general solution is $x = n\pi + (-1)^n \cdot \frac{\pi}{3}$ where $n \in I$

(ii) Given: $\cos x = 1$

Formula used: $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in I$

By using above formula, we have

$$\cos x = 1 = \cos(0^\circ) \Rightarrow x = 2n\pi, n \in I$$

So general solution is $x = 2n\pi$ where $n \in I$

(iii) Given: $\sec x = \sqrt{2}$

We know that $\sec\theta \times \cos\theta = 1$

$$\text{So } \cos x = \frac{1}{\sqrt{2}}$$

Formula used: $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in I$

By using above formula, we have

$$\cos x = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4} \Rightarrow x = 2n\pi \pm \frac{\pi}{4}, n \in I$$

So general solution is $x = 2n\pi \pm \frac{\pi}{4}$ where $n \in I$

Q. 5. Find the general solution of each of the following equations:

(i) $\cos x = \frac{-1}{2}$

(ii) $\operatorname{cosec} x = -\sqrt{2}$

(iii) $\tan x = -1$

Answer : To Find: General solution.

Formula used: $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in I$

By using above formula, we have

$$\cos x = \frac{-1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) \Rightarrow x = 2n\pi \pm \frac{2\pi}{3}, n \in I$$

So general solution is $x = 2n\pi \pm \frac{2\pi}{3}$ where $n \in I$

(ii) Given: $\operatorname{cosec} x = -\sqrt{2}$

We know that $\operatorname{cosec}\theta \times \sin\theta = 1$

$$\text{So } \sin x = \frac{-1}{\sqrt{2}}$$

Formula used: $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha, n \in \mathbb{Z}$

By using above formula, we have

$$\sin x = \frac{-1}{\sqrt{2}} = \sin\frac{5\pi}{4} \Rightarrow x = n\pi + (-1)^n \cdot \frac{5\pi}{4}$$

So general solution is $x = n\pi + (-1)^n \cdot \frac{5\pi}{4}$ where $n \in \mathbb{Z}$

(iii) Given: $\tan x = -1$

Formula used: $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha, n \in \mathbb{Z}$

By using above formula, we have

$$\tan x = -1 = \tan\frac{3\pi}{4} \Rightarrow x = n\pi + \frac{3\pi}{4}, n \in \mathbb{Z}$$

So the general solution is $x = n\pi + \frac{3\pi}{4}$ where $n \in \mathbb{Z}$

Q. 6. Find the general solution of each of the following equations:

(i) $\sin 2x = \frac{1}{2}$

(ii) $\cos 3x = \frac{1}{\sqrt{2}}$

(iii) $\tan \frac{2x}{3} = \sqrt{3}$

Answer : To Find: General solution.

(i) Given: $\sin 2x = \frac{1}{2}$

Formula used: $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in I$

By using above formula, we have

$$\sin 2x = \frac{1}{2} = \sin \frac{\pi}{6} \Rightarrow 2x = n\pi + (-1)^n \cdot \frac{\pi}{6} \Rightarrow x = \frac{n\pi}{2} + (-1)^n \cdot \frac{\pi}{12}, n \in I$$

So general solution is $x = \frac{n\pi}{2} + (-1)^n \cdot \frac{\pi}{12}$ where $n \in I$

(ii) Given: $\cos 3x = \frac{1}{\sqrt{2}}$

Formula used: $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in I$

By using above formula, we have

$$\cos 3x = \frac{1}{\sqrt{2}} = \cos\left(\frac{\pi}{4}\right) \Rightarrow 3x = 2n\pi \pm \frac{\pi}{4} \Rightarrow x = \frac{2n\pi}{3} \pm \frac{\pi}{12}, n \in I$$

So the general solution is $x = \frac{2n\pi}{3} \pm \frac{\pi}{12}$ where $n \in I$

(iii) Given: $\tan \frac{2x}{3} = \sqrt{3}$

Formula used: $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha, n \in I$

By using above formula, we have

$$\tan \frac{2x}{3} = \sqrt{3} = \tan \frac{\pi}{3} \Rightarrow \frac{2x}{3} = n\pi + \frac{\pi}{3} \Rightarrow x = \frac{3n\pi}{2} + \frac{\pi}{2}, n \in I$$

So general solution is $x = (3n+1)\frac{\pi}{2}$, where $n \in I$

Q. 7. Find the general solution of each of the following equations:

(i) $\sec 3x = -2$

(ii) $\cot 4x = -1$

(iii) $\operatorname{cosec} 3x = \frac{-2}{\sqrt{3}}$

Answer : To Find: General solution.

(i) Given: $\sec 3x = -2$

We know that $\sec\theta \times \cos\theta = 1$

$$\text{So } \cos 3x = \frac{-1}{2}$$

Formula used: $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in I$

By using above formula, we have

$$\cos 3x = \frac{-1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3} \Rightarrow 3x = 2n\pi \pm \frac{2\pi}{3} \Rightarrow x = \frac{2n\pi}{3} \pm \frac{2\pi}{9},$$

$n \in I$

So the general solution is $x = \frac{2n\pi}{3} \pm \frac{2\pi}{9}$, where $n \in I$

(ii) Given: $\cot 4x = -1$

We know that $\tan\theta \times \cot\theta = 1$

So $\tan 4x = -1$

Formula used: $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$

By using above formula, we have

$$\tan 4x = -1 = \tan\frac{3\pi}{4} \Rightarrow 4x = n\pi + \frac{3\pi}{4} \Rightarrow x = \frac{n\pi}{4} + \frac{3\pi}{16}, n \in I$$

So general solution is $x = (4n+3)\frac{\pi}{16}$, where $n \in I$

(iii) Given: $\operatorname{cosec} 3x = \frac{-2}{\sqrt{3}}$

We know that $\operatorname{cosec} \theta \times \sin \theta = 1$

So $\sin 3x = \frac{-\sqrt{3}}{2}$

Formula used: $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \cdot \alpha, n \in I$

By using above formula, we have

$$\sin 3x = \frac{-\sqrt{3}}{2} = \sin \frac{4\pi}{3} \Rightarrow 3x = n\pi + (-1)^n \cdot \frac{4\pi}{3} \Rightarrow x = \frac{n\pi}{3} + (-1)^n \cdot \frac{4\pi}{9}, n \in I$$

So general solution is $x = \frac{n\pi}{3} + (-1)^n \cdot \frac{4\pi}{9}$, where $n \in I$

Q. 8. Find the general solution of each of the following equations:

(i) $4\cos^2 x = 1$

(ii) $4\sin^2 x - 3 = 0$

(iii) $\tan^2 x = 1$

Answer : To Find: General solution.

(i) Given: $4\cos^2 x = 1 \Rightarrow \cos^2 x = \left(\frac{1}{4}\right)$

$\therefore \cos^2 x = \cos^2 \frac{2\pi}{3}$

Formula used: $\cos^2 \theta = \cos^2 \alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$

By using the above formula, we have

$$x = n\pi \pm \frac{\pi}{3}, n \in I$$

So the general solution is $x = n\pi \pm \frac{\pi}{3}$ where $n \in I$

(ii) Given: $4\sin^2 x - 3 = 0 \Rightarrow \sin^2 x = \frac{3}{4} = \sin^2 \frac{\pi}{3}$

$$\therefore \sin^2 x = \sin^2 \frac{\pi}{3}$$

Formula used: $\sin^2 \theta = \sin^2 \alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$

By using the above formula, we have

$$x = n\pi \pm \frac{\pi}{3}, n \in I$$

So the general solution is $x = n\pi \pm \frac{\pi}{3}$ where $n \in I$

(ii) Given: $\tan^2 x = 1 \Rightarrow \tan^2 x = \tan^2 \frac{\pi}{4}$

$$\therefore \tan^2 x = \tan^2 \frac{\pi}{4}$$

The formula used: $\tan^2 \theta = \tan^2 \alpha \Rightarrow \theta = n\pi \pm \alpha, n \in I$

By using the above formula, we have

$$x = n\pi \pm \frac{\pi}{4}, n \in I$$

So the general solution is $x = n\pi \pm \frac{\pi}{4}$ where $n \in I$

So general solution is $x = \frac{n\pi}{3} + (-1)^n \cdot \frac{4\pi}{9}$, where $n \in I$

Q. 9. Find the general solution of each of the following equations:

(i) $\cos 3x = \cos 2x$

(ii) $\cos 5x = \sin 3x$

(iii) $\cos mx = \sin nx$

Answer : To Find: General solution.

(i) Given: $\cos 3x = \cos 2x \Rightarrow \cos 3x - \cos 2x = 0 \Rightarrow -2\sin\frac{(5x)}{2} \sin\frac{(x)}{2} = 0$

[NOTE: $\cos C - \cos D = -2\sin\frac{(C+D)}{2} \sin\frac{(C-D)}{2}$]

So, $\sin\frac{(5x)}{2} = 0$ or $\sin\frac{(x)}{2} = 0$

Formula used: $\sin\theta = 0 \Rightarrow \theta = n\pi, n \in I$

$\frac{(5x)}{2} = n\pi$ or $\frac{(x)}{2} = m\pi$ where $n, m \in I$

$x = 2n\pi/5$ or $x = 2m\pi$ where $n, m \in I$

So general solution is $x = 2n\pi/5$ or $x = 2m\pi$ where $n, m \in I$

(ii) Given: $\cos 5x = \sin 3x \Rightarrow \cos 5x = \cos\left(\frac{\pi}{2} - 3x\right)$

Formula used: $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha, n \in I$

By using the above formula, we have

$$5x = 2n\pi + \left(\frac{\pi}{2} - 3x\right) \text{ or } 5x = 2n\pi - \left(\frac{\pi}{2} - 3x\right)$$

$$8x = 2n\pi + \frac{\pi}{2} \text{ or } 2x = 2n\pi - \frac{\pi}{2}$$

$$x = \frac{n\pi}{4} + \frac{\pi}{16} \text{ or } x = n\pi - \frac{\pi}{4} \text{ where } n \in I$$

So general solution is $x = \frac{n\pi}{4} + \frac{\pi}{16}$ or $x = n\pi - \frac{\pi}{4}$ where $n \in I$

(iii) Given: $\cos mx = \sin nx \Rightarrow \cos mx = \cos\left(\frac{\pi}{2} - nx\right)$

Formula used: $\cos \theta = \cos a \Rightarrow \theta = 2k\pi \pm a, k \in I$

By using the above formula, we have

$$m x = 2k\pi + \left(\frac{\pi}{2} - n x\right) \text{ or } 5x = 2k\pi - \left(\frac{\pi}{2} - n x\right)$$

$$(m+n)x = 2k\pi + \frac{\pi}{2} \text{ or } (m-n)x = 2k\pi - \frac{\pi}{2}$$

$$x = \frac{2k\pi}{(m+n)} + \frac{\pi}{2(m+n)} \text{ or } x = \frac{2k\pi}{(m-n)} + \frac{\pi}{2(m-n)} \text{ where } k \in I$$

$$x = \frac{(4k+1)\pi}{2(m+n)} \text{ or } x = \frac{(4k-1)\pi}{2(m-n)} \text{ where } k \in I$$

So the general solution is $x = \frac{(4k+1)\pi}{2(m+n)}$ or $x = \frac{(4k-1)\pi}{2(m-n)}$ where $k \in I$

Q. 10. Find the general solution of each of the following equations:

$\sin x = \tan x$

Answer : To Find: General solution.

Given: $\sin x = \tan x \Rightarrow \sin x = \sin x \div \cos x$

So $\sin x = 0$ or $\cos x = 1 = \cos(0)$

Formula used: $\sin\theta = 0 \Rightarrow \theta = n\pi, n \in I$ and $\cos\theta = \cos\alpha \Rightarrow \theta = 2k\pi \pm \alpha, k \in I$

$x = n\pi$ or $x = 2k\pi$ where $n, k \in I$

So general solution is $x = n\pi$ or $x = 2k\pi$ where $n, k \in I$

Q. 11. Find the general solution of each of the following equations:

$$4\sin x \cos x + 2\sin x + 2\cos x + 1 = 0$$

Answer : To Find: General solution.

$$\text{Given: } 4\sin x \cos x + 2\sin x + 2\cos x + 1 = 0 \Rightarrow 2\sin x (2\cos x + 1) + 2\cos x + 1 = 0$$

$$\text{So } (2\cos x + 1) (2\sin x + 1) = 0$$

$$\cos x = \frac{-1}{2} = \cos\left(\frac{2\pi}{3}\right) \text{ or } \sin x = \frac{-1}{2} = \sin\frac{7\pi}{6}$$

Formula used: $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$ or $\sin\theta = \sin\alpha \Rightarrow \theta = m\pi + (-1)^m\alpha$
where $n, m \in I$

$$x = 2n\pi \pm \frac{2\pi}{3} \text{ or } x = m\pi + (-1)^m \cdot \frac{7\pi}{6} \text{ where } n, m \in I$$

So the general solution is $x = 2n\pi \pm \frac{2\pi}{3}$ or $x = m\pi + (-1)^m \cdot \frac{7\pi}{6}$ where $n, m \in I$

Q. 12. Find the general solution of each of the following equations:

$$\sec^2 2x = 1 - \tan 2x$$

Answer : To Find: General solution.

$$\text{Given: } \sec^2 2x = 1 - \tan 2x \Rightarrow 1 + \tan^2 2x + \tan 2x = 1 \Rightarrow \tan 2x (1 + \tan 2x) = 0$$

So, $\tan 2x = 0$ or $\tan 2x = -1 = \tan\left(\frac{3\pi}{4}\right)$

Formula used: $\tan\theta = 0 \Rightarrow \theta = n\pi, n \in I$ and $\tan\theta = \tan\alpha \Rightarrow \theta = k\pi \pm \alpha, k \in I$

By using above formula, we have

$$2x = n\pi \text{ or } 2x = k\pi \pm \frac{3\pi}{4} \Rightarrow x = \frac{n\pi}{2} \text{ or } x = \frac{k\pi}{2} \pm \frac{3\pi}{8}$$

So the general solution is $x = \frac{n\pi}{2}$ or $x = \frac{k\pi}{2} \pm \frac{3\pi}{8}$ where $n, k \in I$

Q. 13. Find the general solution of each of the following equations:

$$\tan^3 x - 3\tan x = 0$$

Answer : To Find: General solution.

$$\text{Given: } \tan^3 x - 3\tan x = 0 \Rightarrow \tan x(\tan^2 x - 3) = 0 \Rightarrow \tan x = 0 \text{ or } \tan x = \pm\sqrt{3}$$

$$\Rightarrow \tan x = 0 \text{ or } \tan x = \tan\left(\frac{\pi}{3}\right) \text{ or } \tan x = \tan\left(\frac{2\pi}{3}\right)$$

$$\Rightarrow \text{Formula used: } \tan\theta = 0 \Rightarrow \theta = n\pi, n \in I, \tan\theta = \tan\alpha \Rightarrow \theta = k\pi \pm \alpha, k \in I$$

$$\text{So } x = n\pi \text{ or } x = k\pi + \frac{\pi}{3} \text{ or } x = p\pi + \frac{2\pi}{3} \text{ where } n, k, p \in I$$

$$\text{So general solution is } x = n\pi \text{ or } x = k\pi + \frac{\pi}{3} \text{ or } x = p\pi + \frac{2\pi}{3} \text{ where } n, k, p \in I$$

Q. 14. Find the general solution of each of the following equations:

$$\sin x + \sin 3x + \sin 5x = 0$$

Answer : To Find: General solution.

$$\text{Given: } \sin x + \sin 3x + \sin 5x = 0 \Rightarrow \sin 3x + 2\sin 3x \cos 2x = 0 \Rightarrow \sin 3x (1 + 2\cos 2x) = 0$$

[NOTE: $\sin C + \sin D = 2\sin \frac{(C+D)}{2} \times \cos \frac{(C-D)}{2}$]

$$\Rightarrow \sin 3x = 0 \text{ or } \cos 2x = \frac{-1}{2} = \cos\left(\frac{2\pi}{3}\right)$$

Formula used: $\sin\theta = 0 \Rightarrow \theta = n\pi, n \in I, \cos\theta = \cos\alpha \Rightarrow \theta = 2k\pi \pm \alpha, k \in I$

$$\Rightarrow 3x = n\pi \text{ or } 2x = 2k\pi \pm \frac{2\pi}{3} \Rightarrow x = \frac{n\pi}{3} \text{ or } x = k\pi \pm \frac{\pi}{3} \text{ where } n, k \in I$$

So general solution is $x = \frac{n\pi}{3}$ or $x = k\pi \pm \frac{\pi}{3}$ where $n, k, \in I$

Q. 15. Find the general solution of each of the following equations:

$$\sin x \tan x - 1 = \tan x - \sin x$$

Answer : To Find: General solution.

$$\text{Given: } \sin x \tan x - 1 = \tan x - \sin x \Rightarrow \sin x(\tan x + 1) = \tan x + 1$$

So $\sin x = 1 = \sin\left(\frac{\pi}{2}\right)$ or $\tan x = -1 = \tan\left(\frac{3\pi}{4}\right)$

Formula used: $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$, $n \in I$ and $\tan\theta = \tan\alpha \Rightarrow \theta = k\pi \pm \alpha$, $k \in I$

$\Rightarrow x = n\pi + (-1)^n\frac{\pi}{2}$ or $x = k\pi \pm \frac{3\pi}{4}$ where $n, k \in I$

So general solution is $x = n\pi + (-1)^n\frac{\pi}{2}$ or $x = k\pi \pm \frac{3\pi}{4}$ where $n, k \in I$

Q. 16. Find the general solution of each of the following equations:

$\cos x + \sin x = 1$

Answer : To Find: General solution.

Given: $\cos x + \sin x = 1 \Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$

[divide $\sqrt{2}$ on both sides and $\cos(x-y) = \cos x \cos y - \sin x \sin y$]

Formula used: $\cos\theta = \cos\alpha \Rightarrow \theta = 2k\pi \pm \alpha$, $k \in I$

$\Rightarrow x - \frac{\pi}{4} = 2k\pi \pm \frac{\pi}{4} \Rightarrow x = 2k\pi \pm \frac{\pi}{4} + \frac{\pi}{4} \Rightarrow x = 2k\pi + \frac{\pi}{4} + \frac{\pi}{4}$ or $\Rightarrow x = 2k\pi - \frac{\pi}{4} + \frac{\pi}{4}$

$\Rightarrow x = 2k\pi + \frac{\pi}{2}$ or $x = 2k\pi$

So general solution is $x = 2n\pi + \frac{\pi}{2}$ or $x = 2n\pi$ where $n \in I$

Q. 17. Find the general solution of each of the following equations:

$$\cos x - \sin x = -1$$

Answer : To Find: General solution.

$$\text{Given: } \cos x - \sin x = 1 \Rightarrow \cos\left(x + \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}} = \cos \frac{3\pi}{4}$$

[divide $\sqrt{2}$ on both sides and $\cos(x-y) = \cos x \cos y - \sin x \sin y$]

So $\sin x = 0$ or $\cos x = 0$

Formula used: $\cos \theta = \cos \alpha \Rightarrow \theta = 2k\pi \pm \alpha, k \in I$

$$\Rightarrow x + \frac{\pi}{4} = 2k\pi \pm \frac{3\pi}{4} \Rightarrow x = 2k\pi \pm \frac{3\pi}{4} - \frac{\pi}{4} \Rightarrow x = 2k\pi + \frac{3\pi}{4} - \frac{\pi}{4} \text{ or } \Rightarrow x = 2k\pi - \frac{3\pi}{4} - \frac{\pi}{4}$$

$$\Rightarrow x = 2k\pi - \pi \text{ or } x = 2k\pi + \frac{\pi}{2}$$

So general solution is $x = 2n\pi + \frac{\pi}{2}$ or $x = (2n-1)\pi$ where $n \in I$

Q. 18. Find the general solution of each of the following equations:

$$\sqrt{3} \cos x + \sin x = 1$$

Answer : To Find: General solution.

$$\text{Given: } \sqrt{3} \cos x + \sin x = 1 \Rightarrow \cos \left(x - \frac{\pi}{6}\right) = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right) \text{ or } \cos\left(\frac{5\pi}{3}\right)$$

[Divide $\sqrt{2}$ on both sides and $\cos(x-y) = \cos x \cos y - \sin x \sin y$]

Formula used: $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha$

By using above formula, we have

$$\Rightarrow x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3} \Rightarrow x = 2n\pi \pm \frac{\pi}{3} + \frac{\pi}{6}$$

$$\Rightarrow x = 2n\pi + \frac{\pi}{2} \text{ or } x = 2n\pi - \frac{\pi}{6} \text{ where } n \in \mathbb{I}$$

So general solution is $x = 2n\pi + \frac{\pi}{2}$ or $x = 2n\pi - \frac{\pi}{6}$ where $n \in \mathbb{I}$

Q. 19. Find the general solution of each of the following equations:

$$2 \tan x - \cot x + 1 = 0$$

Answer : To Find: General solution.

$$\begin{aligned} \text{Given: } 2 \tan x - \cot x + 1 = 0 &\Rightarrow 2 \tan^2 x - 1 + \tan x = 0 \Rightarrow 2 \tan^2 x - 1 + 2 \tan x - \tan x = \\ 0 &\Rightarrow 2 \tan x (\tan x + 1) - (1 + \tan x) = 0 \end{aligned}$$

$$\Rightarrow (2\tan x - 1)(1 + \tan x) = 0 \Rightarrow \tan x = \frac{1}{2} = \tan^{-1} \frac{1}{2} \text{ or } \tan x = -1 = \tan \frac{3\pi}{4}$$

Formula used: $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha, n \in I$

$$x = n\pi + \tan^{-1} \frac{1}{2} \text{ or } x = n\pi + \frac{3\pi}{4}$$

So the general solution is $x = n\pi + \tan^{-1} \frac{1}{2}$ or $x = n\pi + \frac{3\pi}{4}$ where $n \in I$

Q. 20. Find the general solution of each of the following equations:

$$\sin x \tan x - 1 = \tan x - \sin x$$

Answer : To Find: General solution.

$$\text{Given: } \sin x \tan x - 1 = \tan x - \sin x \Rightarrow \sin x(\tan x + 1) = \tan x + 1$$

$$\text{So } \sin x = 1 = \sin \left(\frac{\pi}{2}\right) \text{ or } \tan x = -1 = \tan\left(\frac{3\pi}{4}\right)$$

Formula used: $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha, n \in I$ and $\tan \theta = \tan \alpha \Rightarrow \theta = k\pi + \alpha, k \in I$

$$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{2} \text{ or } x = k\pi + \frac{3\pi}{4} \text{ where } n, k \in I$$

So general solution is $x = n\pi + (-1)^n \frac{\pi}{2}$ or $x = k\pi + \frac{3\pi}{4}$ where $n, k \in I$

Q. 21. Find the general solution of each of the following equations:

$$\cot x + \tan x = 2 \operatorname{cosec} x$$

Answer : To Find: General solution.

Given: $\cot x + \tan x = 2 \operatorname{cosec} x \Rightarrow \cos^2 x + \sin^2 x = 2 \sin x \cos x \operatorname{cosec} x \Rightarrow 1 = \sin 2x$
 $\operatorname{cosec} x$

$$\Rightarrow \operatorname{cosec} 2x = \operatorname{cosec} x \Rightarrow \sin x = \sin 2x \Rightarrow \sin x = 2 \sin x \cos x \Rightarrow \sin x = 0 \text{ or } \cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

Formula used: $\sin \theta = 0 \Rightarrow \theta = n\pi$, $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha$

By using above formula, we have

$$x = n\pi \text{ or } x = 2m\pi \pm \frac{\pi}{3} \text{ where } n, m \in \mathbb{I}$$

So general solution is $x = n\pi$ or $x = 2m\pi \pm \frac{\pi}{3}$ where $n, m \in \mathbb{I}$