Circle

Exercise 21A

Q. 1. Find the equation of a circle with

Centre (2, 4) and radius 5

Answer : The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

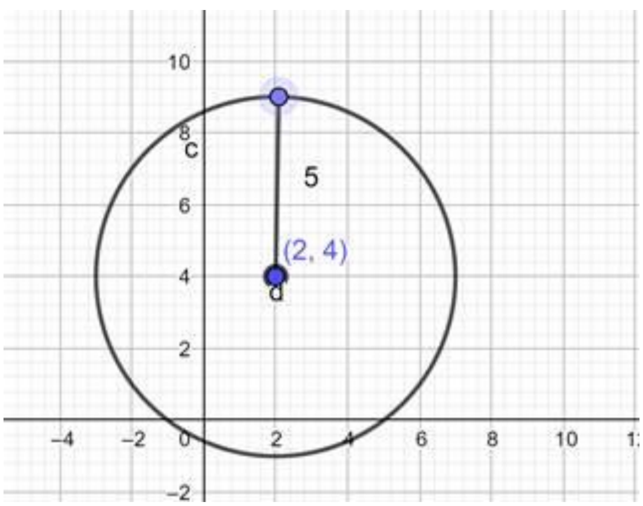
Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in he general form:

$\Rightarrow (x - 2)^2 + (y - 4)^2 = 5^2$

$\Rightarrow (x - 2)^2 + (y - 4)^2 = 25$



Ans; equation of a circle with Centre (2, 4) and radius 5 is:

$\Rightarrow (x - 2)^2 + (y - 4)^2 = 25$

Q. 2. Find the equation of a circle with

Centre (- 3, - 2) and radius 6

Answer : The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

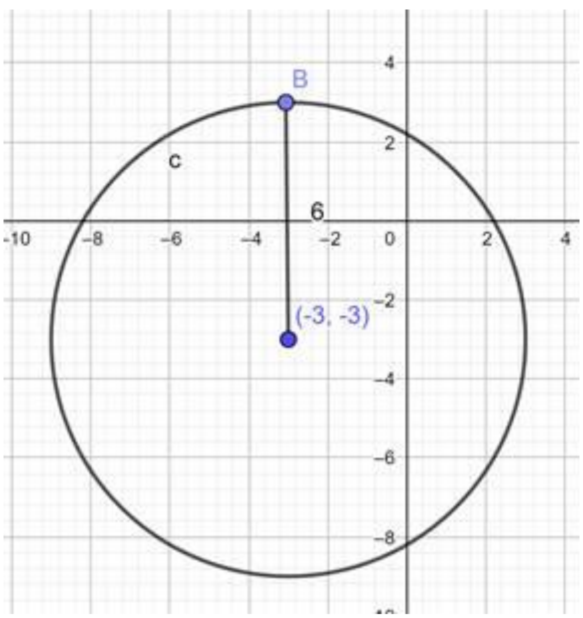
Substituting the centre and radius of the circle in he general form:

$\Rightarrow (x - (- 3))^2 + (y - (- 2))^2 = 6^2$

$\Rightarrow (x + 3)^2 + (y + 2)^2 = 36$

Ans; equation of a circle with Centre (- 3, - 2) and radius 6 is:

$\Rightarrow (x + 3)^2 + (y + 2)^2 = 36$



Q. 3. Find the equation of a circle with

Centre (a, a) and radius $\sqrt{2}$

Answer : The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in he general form:

$\Rightarrow (x - a)^2 + (y - a)^2 = (\sqrt{2})^2$

$\Rightarrow (x - a)^2 + (y - a)^2 = 2$

Ans; equation of a circle with Centre (a, a) and radius √**2**

is:

$(x - a)^2 + (y - a)^2 = 2$

Q. 4. Find the equation of a circle with

Centre (a cos ∝, a sin ∝) and radius a

Answer : The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in he general form:

$(x - (a \cos \propto))^2 + (y - (a \sin \propto))^2 = a^2$

$\Rightarrow (x - a \cos \propto)^2 + (y - a \sin \propto)^2 = a^2$

$\Rightarrow x^2 - 2xa\cos\alpha + a^2 \cos^2 \alpha + y^2 - 2ya\sin\alpha + a^2 \sin^2 \alpha = a^2$

$\Rightarrow x^2 + y^2 + a^2 (\cos^2 \alpha + \sin^2 \alpha) - 2a(x\cos\alpha + y\sin\alpha) = a^2$

$\Rightarrow x^2 + y^2 + a^2 - 2a(x\cos\alpha + y\sin\alpha) = a^2 \ldots((\cos^2\alpha + \sin^2 \alpha) = 1)$

$\Rightarrow x^2 + y^2 - 2a(x\cos\alpha + y\sin\alpha) = 0$

Ans: equation of a circle with Centre (a cos ∝, a sin ∝) and radius a is:

$x^2 + y^2 - 2a(x\cos\alpha + y\sin\alpha) = 0$

Q. 5. Find the equation of a circle with

Centre (- a, - b) and radius $\sqrt{a^2 - b^2}$

Answer : The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in he general form:

$\Rightarrow (x - (- a))^2 + (y - (- b))^2 = \sqrt{(a^22 - b^22)}^2$

$\Rightarrow (x + a)^2 + (y + b)^2 = a^2 - b^2$

$\Rightarrow x^2 + 2xa + a^2 + y^2 + 2y\alpha + b^2 = a^2 - b^2$

$\Rightarrow x^2 + 2xa + y^2 + 2y\alpha = a^2 - 2b^2$

$\Rightarrow x^2 + y^2 + 2a(x + y) = a^2 - 2b^2$

$\Rightarrow x^2 + y^2 + 2a(x + y) = a^2 - 2b^2$

Ans; equation of a circle with Centre (- a, - b) and radius

is:

$\Rightarrow x^2 + y^2 + 2a(x + y) = a^2 - 2b^2$

Q. 6. Find the equation of a circle with

Centre at the origin and radius 4

Answer : The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

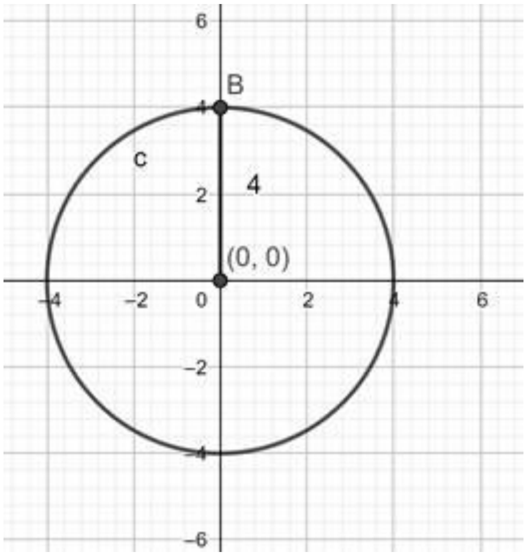
Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Substituting the centre and radius of the circle in he general form:

⇒ $(x - 0)^2 + (y - 0)^2 = 4^2$

⇒ $x^2 + y^2 = 16$



Ans; equation of a circle with . Centre at the origin and radius 4 is:

$x^2 + y^2 = 16$

Q. 7 A. Find the centre and radius of each of the following circles :

$(x – 3)^2 + (y – 1)^2 = 9$

Answer : The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Comparing the given equation of circle with general form we get:

$h = 3$, $k = 1$, $r^2 = 9$

⇒ centre = (3, 1) and radius = 3 units.

Ans: centre = (3, 1) and radius = 3 units.

Q. 7 B. Find the centre and radius of each of the following circles :

$$\left(x-\frac{1}{2}\right)^2+\left(y+\frac{1}{3}\right)^2=\frac{1}{16}$$

Answer : The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Comparing the given equation of circle with general form we get:

h = 1/2 , k = - 1/3, r^2 = 1/16

⇒ centre = (1/2, - 1/3) and radius = 1/4 units.

Ans: centre = (1/2, - 1/3) and radius = 1/4 units.

Q. 7 C. Find the centre and radius of each of the following circles :

$(x + 5)^2 + (y – 3)^2 = 20$

Answer : The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Comparing the given equation of circle with general form we get:

h = - 5 , k = 3, r^2 = 20

⇒ centre = (- 5, 3) and radius = $\sqrt{20} = 2\sqrt{5}$ units.

Ans: centre = (- 5, 3) and radius = $2\sqrt{5}$ units.

Q. 7 D. Find the centre and radius of each of the following circles :

$x^2 + (y – 1)^2 = 2$

Answer : The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Comparing the given equation of circle with general form we get:

$h = 0$, $k = 1$, $r^2 = 2$

⇒ centre = (0, 1) and radius = √2units.

Ans: centre = (0, 1) and radius = √2units.

Q. 8. Find the equation of the circle whose centre is (2, - 5) and which passes through the point (3, 2).

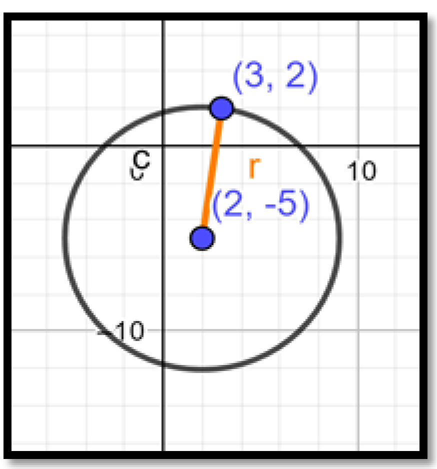
Answer : The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

In this question we know that (h, k) = (2, - 5), so for determining the equation of the circle we need to determine the radius of the circle.



Since the circle passes through (3, 2), that pair of values for x and y must satisfy the equation and we have:

$\Rightarrow (3 - 2)^2 + (2 - (- 5))^2 = r^2$

$\Rightarrow 1^2 + 7^2 = r^2$

$\Rightarrow r^2 = 49 + 1 = 50$

$\therefore r^2 = 50$

$\Rightarrow$ Equation of circle is:

$(x - 2)^2 + (y - (- 5))^2 = 50$

$\Rightarrow (x - 2)^2 + (y + 5)^2 = 50$

Ans:$(x - 2)^2 + (y + 5)^2 = 50$

Q. 9. Find the equation of the circle of radius 5 cm, whose centre lies on the y - axis and which passes through the point (3, 2).

Answer : The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

Since, centre lies on Y - axis, ∴ it's X - coordinate = 0, i.e.h = 0

Hence, (0, k) is the centre of the circle.

Substituting the given values in general form of the equation of a circle we get,

$\Rightarrow (3 - 0)^2 + (2 - k)^2 = 5^2$

$\Rightarrow (3)^2 + (2 - k)^2 = 25$

$\Rightarrow 9 + (2 - k)^2 = 25$

$\Rightarrow (2 - k)^2 = 25 - 9 = 16$

Taking square root on both sides we get,

$\Rightarrow 2 - k = \pm 4$

$\Rightarrow 2 - k = 4$ & $2 - k = - 4$

⇒ k = 2 - 4 & k = 2 + 4

⇒ k = - 2 & k = 6

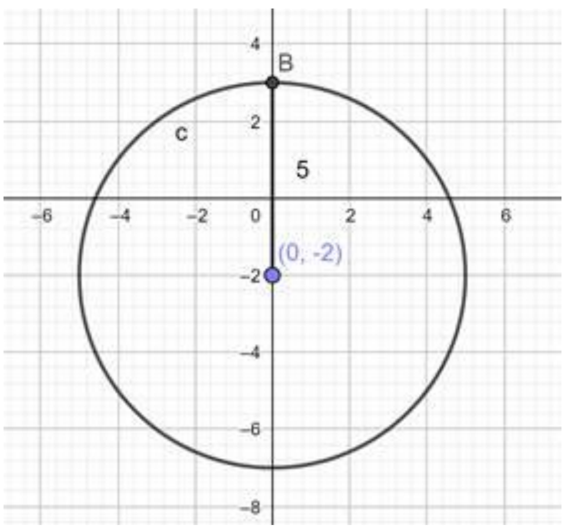
∴ Equation of circle when k = - 2 is: $x^2 + (y + 2)^2 = 25$

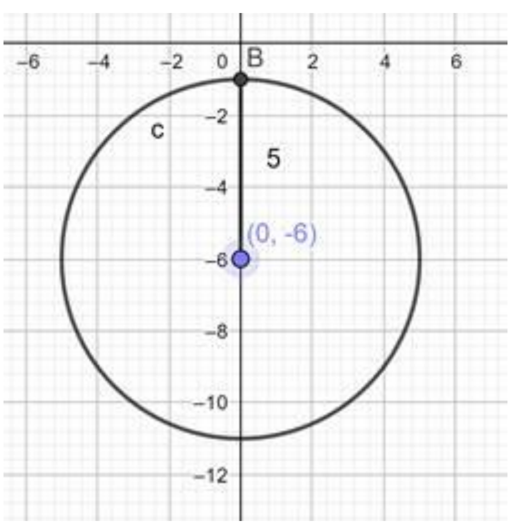
Equation of circle when k = 6 is: $x^2 + (y - 6)^2 = 25$

Ans: Equation of circle when k = - 2 is:

$x^2 + (y + 2)^2 = 25$

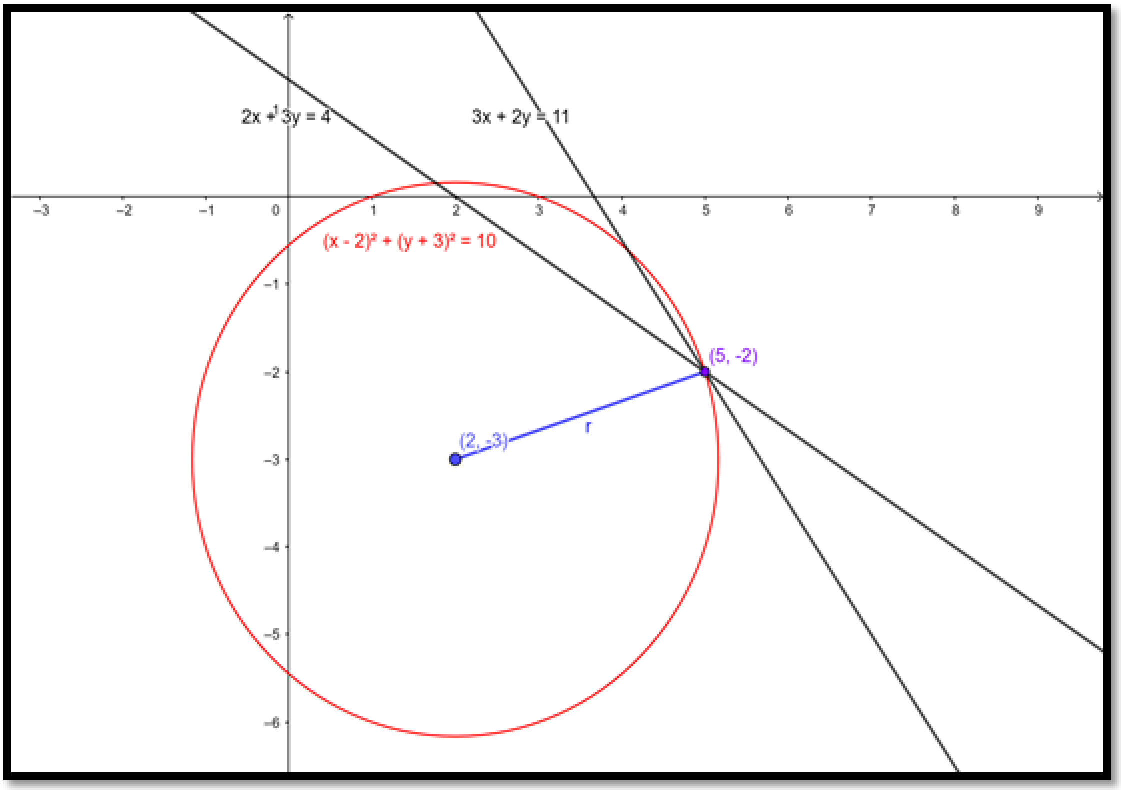
Equation of circle when k = 6 is: $x^2 + (y - 6)^2 = 25$





Q. 10. Find the equation of the circle whose centre is (2, - 3) and which passes through the intersection of the lines 3x + 2y = 11 and 2x + 3y = 4.

Answer :



The intersection of the lines: 3x + 2y = 11 and 2x + 3y = 4

Is (5, - 2)

∴ This problem is same as solving a circle equation with centre and point on the circle given.

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

In this question we know that (h, k) = (2, - 3), so for determining the equation of the circle we need to determine the radius of the circle.

Since the circle passes through (5, - 2), that pair of values for x and y must satisfy the equation and we have:

$\Rightarrow (5 - 2)^2 + (- 2 - (- 3))^2 = r^2$

$\Rightarrow 3^2 + 1^2 = r^2$

$\Rightarrow r^2 = 9 + 1 = 10$

$\therefore r^2 = 10$

$\Rightarrow$ Equation of circle is:

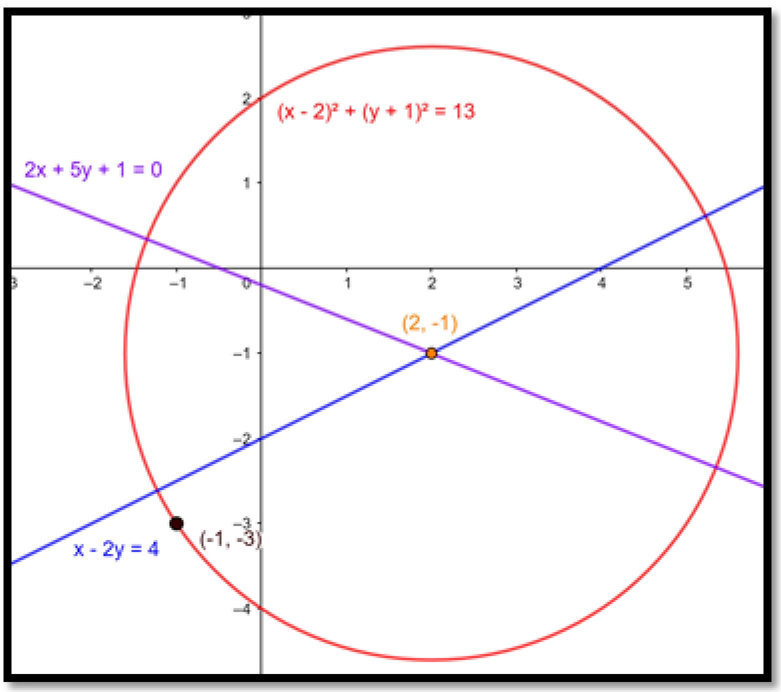
$(x - 2)^2 + (y - (- 3))^2 = 10$

$\Rightarrow (x - 2)^2 + (y + 3)^2 = 10$

Ans:$(x - 2)^2 + (y + 5)^2 = 10$

Q. 11. Find the equation of the circle passing through the point (- 1, - 3) and having its centre at the point of intersection of the lines x – 2y = 4 and 2x + 5y + 1 = 0.

Answer :



The intersection of the lines: x – 2y = 4 and 2x + 5y + 1 = 0.

is (2, - 1)

$\therefore$ This problem is same as solving a circle equation with centre and point on the circle given.

The general form of the equation of a circle is:

$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

In this question we know that (h, k) = (2, - 1), so for determining the equation of the circle we need to determine the radius of the circle.

Since the circle passes through (- 1, - 3), that pair of values for x and y must satisfy the equation and we have:

$\Rightarrow (- 1 - 2)^2 + (- 3 - (- 1))^2 = r^2$

$\Rightarrow (- 3)^2 + (- 2)^2 = r^2$

$\Rightarrow r^2 = 9 + 4 = 13$

$\therefore r^2 = 13$

⇒ Equation of circle is:

$(x - 2)^2 + (y - (- 1))^2 = 13$

$\Rightarrow (x - 2)^2 + (y + 1)^2 = 13$

Ans: $(x - 2)^2 + (y + 1)^2 = 13$

Q. 12. If two diameters of a circle lie along the lines x – y = 9 and x – 2y = 7, and the area of the circle is 38.5 sq cm, find the equation of the circle.

Answer : The point of intersection of two diameters is the centre of the circle.

∴ point of intersection of two diameters x – y = 9 and x – 2y = 7 is (11, 2).

∴ centre = (11, 2)

Area of a circle = πr^2

$38.5 = \pi r^2$

$\Rightarrow r^2 = \frac{38.5}{\pi}$

$\Rightarrow r^2 = 12.25$ sq.cm

the equation of the circle is:

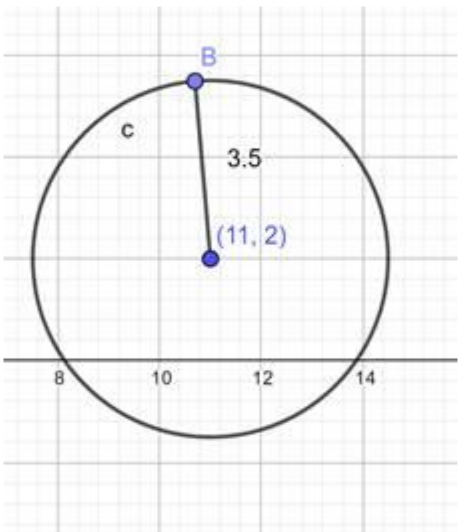
$(x - h)^2 + (y - k)^2 = r^2$

Where, (h, k) is the centre of the circle.

r is the radius of the circle.

$\Rightarrow (x - 11)^2 + (y - 2)^2 = 12.25$

Ans: $(x - 11)^2 + (y - 2)^2 = 12.25$



Q. 13 A. Find the equation of the circle, the coordinates of the end points of one of whose diameters are

A(3, 2) and B(2, 5)

Answer : The equation of a circle passing through the coordinates of the end points of diameters is:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Substituting, values:$(x_1, y_1) = (3, 2)$ & $(x_2, y_2) = (2, 5)$

We get:

$(x - 3)(x - 2) + (y - 2)(y - 5) = 0$

$\Rightarrow x^2 - 2x - 3x + 6 + y^2 - 5y - 2y + 10 = 0$

$\Rightarrow x^2 + y^2 - 5x - 7y + 16 = 0$

Ans: $x^2 + y^2 - 5x - 7y + 16 = 0$

Q. 13 B. Find the equation of the circle, the coordinates of the end points of one of whose diameters are

A(5, - 3) and B(2, - 4)

Answer : The equation of a circle passing through the coordinates of the end points of diameters is:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Substituting, values:$(x_1, y_1) = (5, - 3)$ & $(x_2, y_2) = (2, - 4)$

We get:

$(x - 5)(x - 2) + (y + 3)(y + 4) = 0$

$\Rightarrow x^2 - 2x - 5x + 10 + y^2 + 3y + 4y + 12 = 0$

$\Rightarrow x^2 + y^2 - 7x + 7y + 22 = 0$

Ans: $x^2 + y^2 - 7x + 7y + 22 = 0$

Q. 13 C. Find the equation of the circle, the coordinates of the end points of one of whose diameters are

A(- 2, - 3) and B(- 3, 5)

Answer : The equation of a circle passing through the coordinates of the end points of diameters is:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Substituting, values:$(x_1, y_1) = (- 2, - 3)$ & $(x_2, y_2) = (- 3, 5)$

We get:

$(x + 2)(x + 3) + (y + 3)(y - 5) = 0$

$\Rightarrow x^2 + 3x + 2x + 6 + y^2 - 5y + 3y - 15 = 0$

$\Rightarrow x^2 + y^2 + 5x - 2y - 9 = 0$

Ans: $x^2 + y^2 + 5x - 2y - 9 = 0$

Q. 13 D. Find the equation of the circle, the coordinates of the end points of one of whose diameters are

A(p, q) and B(r, s)

Answer : The equation of a circle passing through the coordinates of the end points of diameters is:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Substituting, values:$(x_1, y_1) = (p, q)$ & $(x_2, y_2) = (r, s)$

We get:

$(x - p)(x - r) + (y - q)(y - s) = 0$

$\Rightarrow x^2 - rx - px + pr + y^2 - sy - qy + qs = 0$

$\Rightarrow x^2 + y^2 - (r + p)x - (s + q)y + (pr + qs) = 0$

Ans: $x^2 + y^2 - (r + p)x - (s + q)y + (pr + qs) = 0$

Q. 14. The sides of a rectangle are given by the equations x = - 2, x = 4, y = - 2 and y = 5. Find the equation of the circle drawn on the diagonal of this rectangle as its diameter.

Answer : The intersection points in clockwise fashion are:(- 2, 5), (4, 5), (4, - 2), (- 2, - 2).

The equation of a circle passing through the coordinates of the end points of diameters is:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

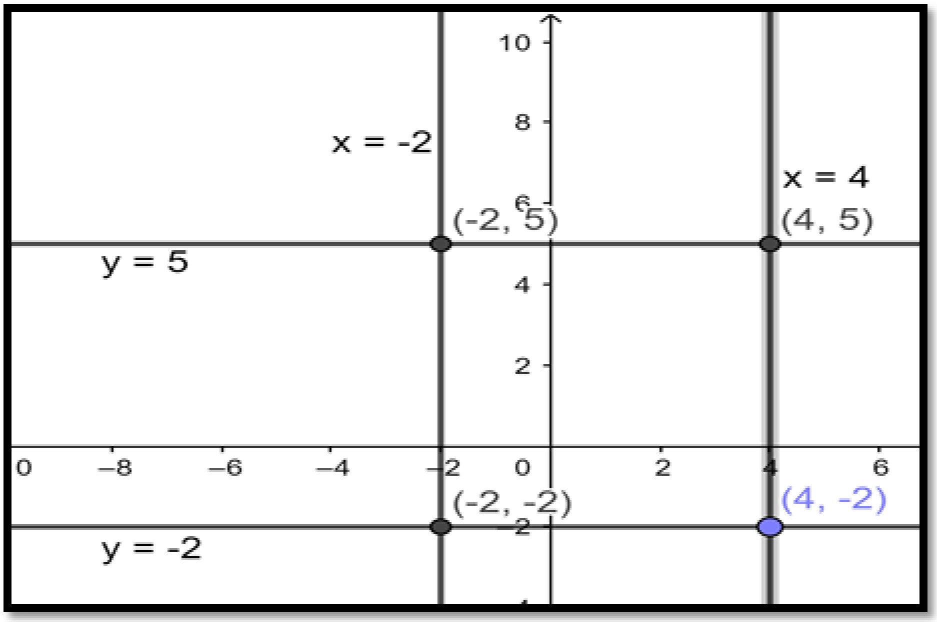
Substituting, values:$(x_1, y_1) = (-2, 5)$ & $(x_2, y_2) = (4, -2)$

We get:

$(x + 2)(x - 4) + (y - 5)(y + 2) = 0$

$\Rightarrow x^2 - 4x + 2x - 8 + y^2 + 2y - 5y - 10 = 0$

$\Rightarrow x^2 + y^2 - 2x - 3y - 18 = 0$



Ans: $x^2 + y^2 - 2x - 3y - 18 = 0$

Exercise 21B

Q. 1. Show that the equation $x^2 + y^2 – 4x + 6y – 5 = 0$ represents a circle. Find its centre and radius.

Answer : The general equation of a conic is as follows

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ where a, b, c, f, g, h are constants

For a circle, a = b and h = 0.

The equation becomes:

$x^2 + y^2 + 2gx + 2fy + c = 0$…(i)

Given, $x^2 + y^2 – 4x + 6y – 5 = 0$

Comparing with (i) we see that the equation represents a circle with 2g = - 4 $\Rightarrow$ g = - 2, 2f = 6$\Rightarrow$f = 3 and c = - 5.

Centre (- g, - f) = { - (- 2), - 3}

= (2, - 3).

Radius = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{(-2)^2 + 3^2 - (-5)}$

$= \sqrt{4 + 9 + 5} = \sqrt{18} = 3\sqrt{2}.$

Q. 2. Show that the equation $x^2 + y^2 + x - y = 0$ represents a circle. Find its centre and radius.

Answer : The general equation of a conic is as follows

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ where a, b, c, f, g, h are constants

For a circle, a = b and h = 0.

The equation becomes:

$x^2 + y^2 + 2gx + 2fy + c = 0$…(i)

Given, $x^2 + y^2 + x - y = 0$

Comparing with (i) we see that the equation represents a circle with 2g = 1

$\Rightarrow g = \frac{1}{2}$, 2f = - 1$\Rightarrow f = -\frac{1}{2}$ and c = 0.

Centre (- g, - f) = $\{-\frac{1}{2}, -(-\frac{1}{2})\}$

$= (-\frac{1}{2}, \frac{1}{2}).$

Radius = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{\frac{1}{2}^2 + (-\frac{1}{2}^2) - 0}$

$= \sqrt{\frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{1}{2}}.$

Q. 3

Show that the equation $3x^2 + 3y^2 + 6x - 4y - 1 = 0$ represents a circle. Find its centre and radius.

Answer : The general equation of a conic is as follows

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ where a, b, c, f, g, h are constants

For a circle, a = b and h = 0.

The equation becomes:

$x^2 + y^2 + 2gx + 2fy + c = 0$…(i)

Given, $3x^2 + 3y^2 + 6x - 4y - 1 = 0 \Rightarrow x^2 + y^2 + 2x - \frac{4}{3}y - \frac{1}{3} = 0$

Comparing with (i) we see that the equation represents a circle with $2g = 2 \Rightarrow g = 1$, $2f = -\frac{4}{3} \Rightarrow f = -\frac{2}{3}$ and $c = -\frac{1}{3}$.

Centre $(-g, -f) = \{-1, -(-\frac{2}{3})\}$

$= (-1, \frac{2}{3})$.

Radius = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{1^2 + (-\frac{2}{3})^2 - (-\frac{1}{3})}$

$= \sqrt{1 + \frac{4}{9} + \frac{1}{3}} = \sqrt{\frac{16}{9}} = \frac{4}{3}.$

Q. 4. Show that the equation $x^2 + y^2 + 2x + 10y + 26 = 0$ represents a point circle. Also, find its centre.

Answer : The general equation of a circle:

$x^2 + y^2 + 2gx + 2fy + c = 0$…(i) where c, g, f are constants.

Given, $x^2 + y^2 + 2x + 10y + 26 = 0$

Comparing with (i) we see that the equation represents a circle with $2g = 2 \Rightarrow g = 1$, $2f = 10 \Rightarrow f = 5$ and $c = 26$.

Centre (- g, - f) = (- 1, - 5).

Radius = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{1^2 + 5^2 - 26}$

$= \sqrt{26 - 26} = 0.$

Thus it is a point circle with radius 0.

Q. 5. Show that the equation $x^2 + y^2 - 3x + 3y + 10 = 0$ does not represent a circle.

Answer : Radius =

$\sqrt{g^2 + f^2 - c}$

$= \sqrt{(-\frac{3}{2})^2 + (-\frac{3}{2}^2) - 10}$

$= \sqrt{\frac{9}{2} - 10} = \sqrt{-\frac{11}{2}},$

which implies that the radius is negative. (not possible)

Therefore, $x^2 + y^2 - 3x + 3y + 10 = 0$ does not represent a circle.

Q. 6. Find the equation of the circle passing through the points

(i) (0, 0), (5, 0) and (3, 3)
(ii) (1, 2), (3, - 4) and (5, - 6)
(iii) (20, 3), (19, 8) and (2, - 9)
Also, find the centre and radius in each case.

Answer : (i) The required circle equation

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 0^2+0^2 & 0 & 0 & 1 \\ 5^2+0^2 & 5 & 0 & 1 \\ 3^2+3^2 & 3 & 3 & 1 \end{vmatrix} = 0$$

Using Laplace Expansion

$$(x^2 + y^2)\begin{vmatrix} 0 & 0 & 1 \\ 5 & 0 & 1 \\ 3 & 3 & 1 \end{vmatrix} - x\begin{vmatrix} 0 & 0 & 1 \\ 25 & 0 & 1 \\ 18 & 3 & 1 \end{vmatrix}$$

$$+ y\begin{vmatrix} 0 & 0 & 1 \\ 25 & 5 & 1 \\ 18 & 3 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 0 & 0 \\ 25 & 5 & 0 \\ 18 & 3 & 3 \end{vmatrix} = 0$$

$$\Rightarrow 15(x^2 + y^2) - 75x - 15y = 0$$

$\Rightarrow x^2 + y^2 - 5x - y = 0$ is the equation with centre = (2.5, 0.5)

Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(-2.5^2) + (-0.5)^2 - 0} = 2.549$

(ii) The required circle equation

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ 1^2+2^2 & 1 & 2 & 1 \\ 3^2+(-4)^2 & 3 & -4 & 1 \\ 5^2+(-6)^2 & 5 & -6 & 1 \end{vmatrix} = 0$$

Using Laplace Expansion

$$(x^2 + y^2)\begin{vmatrix} 1 & 2 & 1 \\ 3 & -4 & 1 \\ 5 & -6 & 1 \end{vmatrix} - x\begin{vmatrix} 5 & 2 & 1 \\ 25 & -4 & 1 \\ 61 & -6 & 1 \end{vmatrix} + y\begin{vmatrix} 5 & 1 & 1 \\ 25 & 3 & 1 \\ 61 & 5 & 1 \end{vmatrix} - \begin{vmatrix} 5 & 1 & 2 \\ 25 & 3 & -4 \\ 61 & 5 & -6 \end{vmatrix} = 0$$

$$\Rightarrow 8(x^2 + y^2) - 176x - 32y - 200 = 0$$

$\Rightarrow x^2 + y^2 - 22x - 4y - 25 = 0$ is the equation with centre = (11, 2)

Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(-11)^2 + (-2)^2 - 25} = 10$

(iii) The required circle equation

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ 20^2 + 3^2 & 20 & 3 & 1 \\ 19^2 + 8^2 & 19 & 8 & 1 \\ 2^2 + (-9)^2 & 2 & -9 & 1 \end{vmatrix} = 0$$

Using Laplace Expansion

$$(x^2 + y^2)\begin{vmatrix} 20 & 3 & 1 \\ 19 & 8 & 1 \\ 2 & -9 & 1 \end{vmatrix} - x\begin{vmatrix} 409 & 3 & 1 \\ 425 & 8 & 1 \\ 85 & -9 & 1 \end{vmatrix} + y\begin{vmatrix} 409 & 20 & 1 \\ 425 & 19 & 1 \\ 85 & 2 & 1 \end{vmatrix} - \begin{vmatrix} 409 & 20 & 3 \\ 425 & 19 & 8 \\ 85 & 2 & -9 \end{vmatrix} = 0$$

$\Rightarrow 102(x^2 + y^2) - 1428x - 612y - 11322 = 0$

$\Rightarrow x^2 + y^2 - 14x - 6y - 111 = 0$ is the equation with centre = (7, 3)

Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{(-7)^2 + (-3)^2 - (-111)} = 13$

Q. 7. Find the equation of the circle which is circumscribed about the triangle whose vertices are A(- 2, 3), b(5, 2) and C(6, - 1). Find the centre and radius of this circle.

Answer : The general equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$

…(i), where (h, k) is the centre and r is the radius.

Putting A(- 2, 3), B(5, 2) and c(6, - 1) in (i) we get

$h^2 + k^2 + 4h - 6k + 13 = r^2$ …(ii)

$h^2 + k^2 - 10h - 4k + 29 = r^2$ …(iii)and

$h^2 + k^2 - 12h + 2k + 37 = r^2$ …(iv)

subtracting (ii) from (iii) and also from (iv),

$- 14h + 2k + 16 = 0 \Longrightarrow - 7h + k + 8 = 0$

$- 16h + 8k + 24 = 0 \Longrightarrow - 2h + k + 3 = 0$

Subtracting,

$5h - 5 = 0 \Longrightarrow h = 1$

$k = - 1$

Centre = (1, - 1)

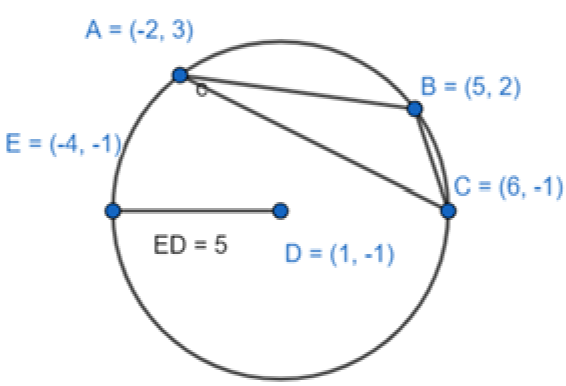
Putting these values in (ii) we get, radius

$$= \sqrt{1 + 1 + 4 + 6 + 13} = \sqrt{25} = 5$$

Equation of the circle is

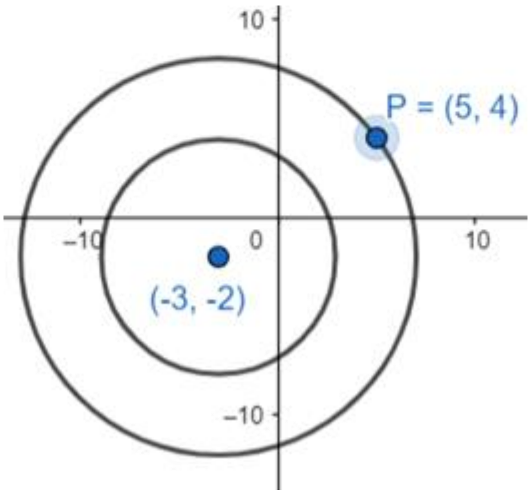
$(x - 1)^2 + \{y - (- 1)\}^2 = 5^2$

$(x - 1)^2 + (y + 1)^2 = 25.$



Q. 8. Find the equation of the circle concentric with the circle $x^2 + y^2 + 4x + 6y + 11 = 0$ and passing through the point P(5, 4).

Answer : 2 or more circles are said to be concentric if they have the same centre and different radii.



Given, $x^2 + y^2 + 4x + 6y + 11 = 0$

The concentric circle will have the equation

$x^2 + y^2 + 4x + 6y + c' = 0$

As it passes through P(5, 4), putting this in the equation

$$5^2 + 4^2 + 4 \times 5 + 6 \times 4 + c' = 0$$

$$\Rightarrow 25 + 16 + 20 + 24 + c' = 0$$

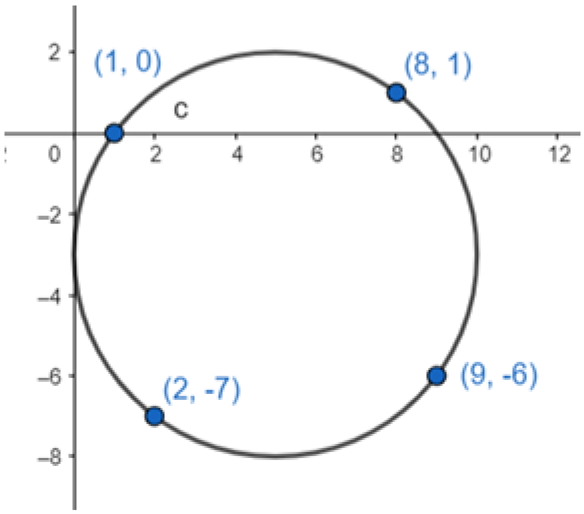
$$\Rightarrow c' = -85$$

The required equation is

$x^2 + y^2 + 4x + 6y - 85 = 0$

Q. 9. Show that the points A(1, 0), B(2, - 7), c(8, 1) and D(9, - 6) all lie on the same circle. Find the equation of this circle, its centre and radius.

Answer :



The general equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$

…(i), where (h, k) is the centre and r is the radius.

Putting (1, 0) in (i)

$(1 - h)^2 + (0 - k)^2 = r^2$

$\Rightarrow h^2 + k^2 + 1 - 2h = r^2$ ..(ii)

Putting (2, - 7) in (i)

$(2 - h)^2 + (- 7 - k)^2 = r^2$

$\Rightarrow h^2 + k^2 + 53 - 4h + 14k = r^2$

$\Rightarrow (h^2 + k^2 + 1 - 2h) + 52 - 2h + 14k = r^2$

$h - 7k - 26 = 0$..(iii) [from (ii)]

Similarly putting (8, 1)

$7h + k - 32 = 0$..(iv)

Solving (iii)&(iv)

h = 5 and k = - 3

centre(5, - 3)

Radius = 25

To check if (9, - 6) lies on the circle, $(9 - 5)^2 + (- 6 + 3)^2 = 5^2$

Hence, proved.

Q. 10. Find the equation of the circle which passes through the points (1, 3) and (2, - 1), and has its centre on the line 2x + y – 4 = 0.

Answer : The equation of a circle: $x^2 + y^2 + 2gx + 2fy + c = 0$…(i)

Putting (1, 3) & (2, - 1) in (i)

2g + 6f + c = - 10..(ii)

4g - 2f + c = - 5..(iii)

Since the centre lies on the given straight line, (- g, - f) must satisfy the equation as

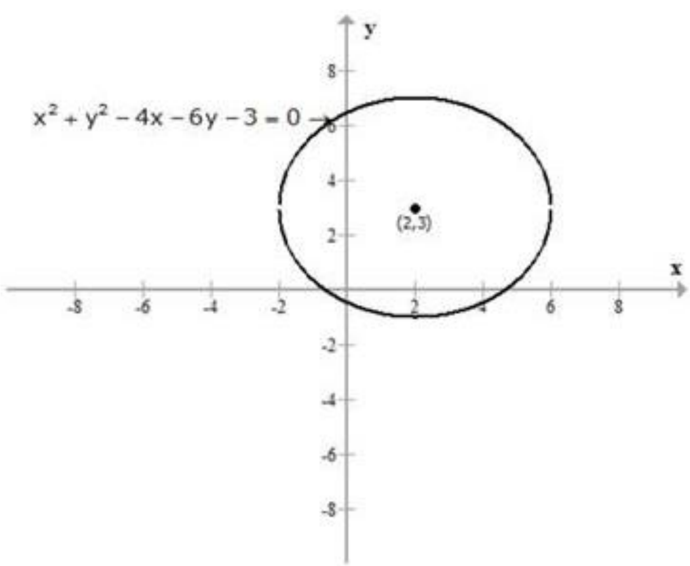
- 2g –f - 4 = 0…(iv)

Solving, f = - 1, g = - 1.5, c = - 1

The equation is $x^2 + y^2 - 3x - 2y - 1 = 0$

Q. 11. Find the equation of the circle concentric with the circle $x^2 + y^2 – 4x – 6y – 3 = 0$ and which touches the y-axis.

Answer : The given image of the circle is:



We know that the general equation of the circle is given by:

$x^2 + y^2 + 2gx + 2fy + c = 0$

Also,

Radius r =

$$\sqrt{g^2 + f^2 - c}$$

Now,

$$r = \sqrt{(2)^2 + (3)^2 - (-3)}$$

$$r = \sqrt{4 + 9 + 3}$$

r = 4 units.

We need to the find the equation of the circle which is concentric to the given circle and touches y-axis.

The centre of the circle remains the same.

Now, y-axis will be tangent to the circle.

Point of contact will be (0, 3)

Therefore, radius = 2

Now,

Equation of the circle:

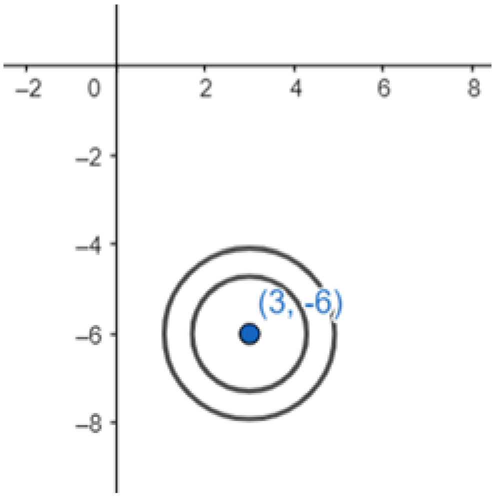
$(x – 2)^2 + (y – 3)^2 = (2)^2$

$x^2 + 4 – 4x + y^2 + 9 – 6y = 4$

$x^2 + y^2 – 4x – 6y + 9 = 0$

Q. 12. Find the equation of the circle concentric with the circle $x^2 + y^2 – 6x + 12y + 15 = 0$ and of double its area.

Answer : 2 or more circles are said to be concentric if they have the same centre and different radii.



Given, $x^2 + y^2 - 6x + 12y + 15 = 0$

Radius r =

$$\sqrt{g^2 + f^2 - c} = \sqrt{(-3^2) + 6^2 - 15} = \sqrt{30}$$

The concentric circle will have the equation

$x^2 + y^2 - 6x + 12y + c' = 0$

Also given area of circle = 2×area of the given circle.

$\Rightarrow r'^2 = 2 \times r^2 = 2 \times 30 = 60$

We can get c' = 45 - 60 = - 15

The required equation is $x^2 + y^2 - 6x + 12y - 15 = 0$.

Q. 13. Prove that the centres of the three circles $x^2 + y^2 – 4x – 6y – 12 = 0$, $x^2 + y^2 + 2x + 4y – 5 = 0$ and $x^2 + y^2 – 10x – 16y + 7 = 0$ are collinear.

Answer : Given,

$x^2 + y^2 – 4x – 6y – 12 = 0$

centre $(-g_1, -f_1) = (2, 3)$

$x^2 + y^2 + 2x + 4y - 5 = 0$

centre $(-g_2, -f_2) = (-1, -2)$

$x^2 + y^2 - 10x - 16y + 7 = 0$

centre $(-g_3, -f_3) = (5, 8)$

to prove that the centres are collinear,

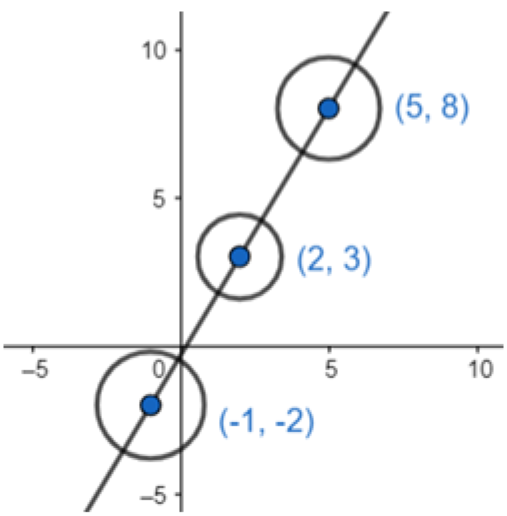
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

Where x_1, y_1 are the coordinates of the ist centre and so on.

$$\Rightarrow \begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

$= 2(-2 - 8) - 3(-1 - 5) + 1(-8 + 10)$

$= -20 + 18 + 2 = 0$



The centres are collinear.

Q. 14. Find the equation of the circle which passes through the points A(1, 1) and B(2, 2) and whose radius is 1. Show that there are two such circles.

Answer : The general equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$

…(i), where (h, k) is the centre and r is the radius.

Putting A(1, 1) in (i)

$(1 - h)^2 + (1 - k)^2 = 1^2$

$\Rightarrow h^2 + k^2 + 2 - 2h - 2k = 1$

$\Rightarrow h^2 + k^2 - 2h - 2k = - 1$..(ii)

Putting B(2, 2) in (i)

$(2 - h)^2 + (2 - k)^2 = 1^2$

$\Rightarrow h^2 + k^2 + 8 - 4h - 4k = 1$

$\Rightarrow h^2 + k^2 - 4h - 4k = - 7$

$\Rightarrow (h^2 + k^2 - 2h - 2k) - 2h - 2k = - 7$

$\Rightarrow - 1 - 2h - 2k = - 7$ [from (ii)]

$\Rightarrow - 2h - 2k = - 6$

$\Rightarrow h + k = 3 \Rightarrow h = 3 - k$

Putting it in (ii)

$\Rightarrow (3 - k)^2 + k^2 - 2(3 - k) - 2k = - 1$

$\Rightarrow 9 + 2k^2 - 6k - 6 + 2k - 2k = - 1$

$\Rightarrow 2k^2 + 4 - 6k = 0$

$\Rightarrow k^2 - 3k + 2 = 0$

$\Rightarrow k = 2$ or $k = 1$

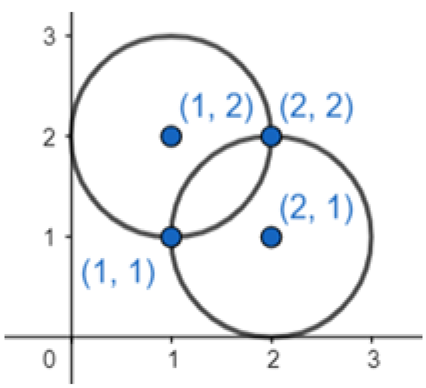
When k = 2, h = 3 - 2 = 1

Equation of 1 circle

$(x - 1)^2 + (y - 2)^2 = 1$

When k = 1, h = 3 - 1 = 2

$(x - 2)^2 + (y - 1)^2 = 1$

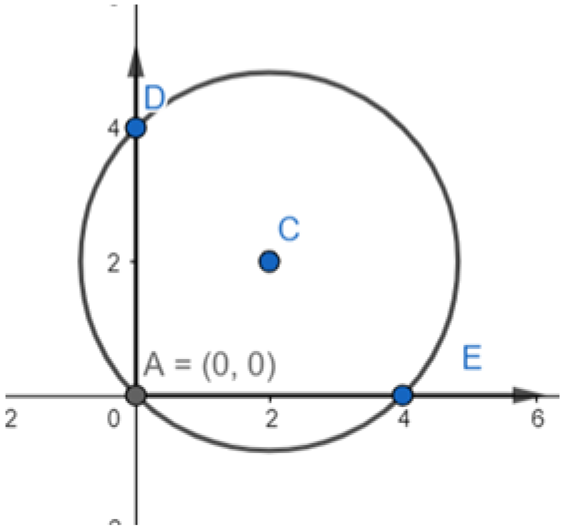


Q. 15. Find the equation of a circle passing through the origin and intercepting lengths a and b on the axes.

Answer : From the figure

AD = b units and AE = a units.

D(0, b), E(a, 0) and A(0, 0) lies on the circle. C is the centre.



The general equation of a circle: $(x - h)^2 + (y - k)^2 = r^2$

…(i), where (h, k) is the centre and r is the radius.
Putting A(0, 0) in (i)

$(0 - h)^2 + (0 - k)^2 = r^2$

$\Rightarrow h^2 + k^2 = r^2$ ...(ii)

Similarly putting D(0, b) in (i)

$(0 - h)^2 + (b - k)^2 = r^2$

$\Rightarrow h^2 + k^2 + b^2 - 2kb = r^2$

$\Rightarrow r^2 + b^2 - 2kb = r^2$

$\Rightarrow b^2 - 2kb = 0$

⇒ $(b - 2k)b = 0$

Either $b = 0$ or $k = \frac{b}{2}$

Similarly putting E(a, 0) in (i)

$(a - h)^2 + (0 - k)^2 = r^2$

$\Rightarrow h^2 + k^2 + a^2 - 2ha = r^2$

$\Rightarrow r^2 + a^2 - 2ha = r^2$

$\Rightarrow a^2 - 2ha = 0$

⇒ $(a - 2h)a = 0$

Either $a = 0$ or $h = \frac{a}{2}$

Centre = $C\left(\frac{a}{2}, \frac{b}{2}\right)$

$r^2 = h^2 + k^2$

$$\Rightarrow r^2 = \frac{a^2 + b^2}{4}$$

Putting the value of r^2, h and k in equation (i)

$(x - h)^2 + (y - k)^2 = r^2$

$$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \frac{a^2 + b^2}{4}$$

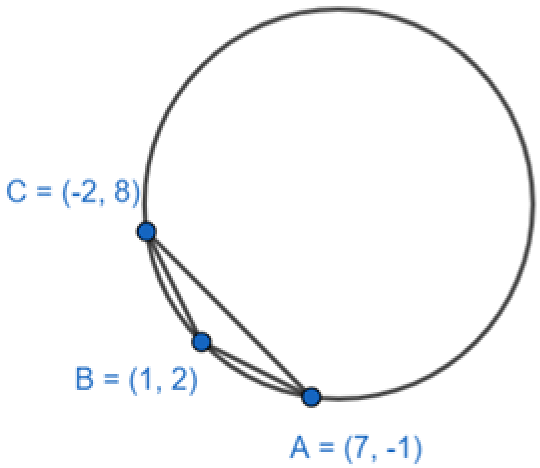
$$\Rightarrow x^2 + y^2 + \frac{a^2}{4} + \frac{b^2}{4} - xa - yb = \frac{a^2 + b^2}{4}$$

$$\Rightarrow x^2 + y^2 - xa - yb = 0$$

which is the required equation.

Q. 16. Find the equation of the circle circumscribing the triangle formed by the lines $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$.

Answer : Solving the equations we get the coordinates of the triangle:



The required circle equation

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ (-2)^2+8^2 & -2 & 8 & 1 \\ 1^2+2^2 & 1 & 2 & 1 \\ 7^2+(-1)^2 & 7 & -1 & 1 \end{vmatrix} = 0$$

Using Laplace Expansion

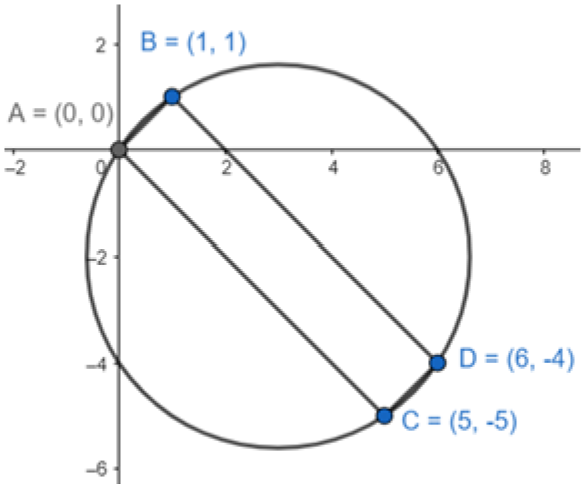
$$(x^2 + y^2)\begin{vmatrix} -2 & 8 & 1 \\ 1 & 2 & 1 \\ 7 & -1 & 1 \end{vmatrix} - x\begin{vmatrix} 68 & 8 & 1 \\ 5 & 2 & 1 \\ 50 & -1 & 1 \end{vmatrix} + y\begin{vmatrix} 68 & -2 & 1 \\ 5 & 1 & 1 \\ 50 & 7 & 1 \end{vmatrix} - \begin{vmatrix} 68 & -2 & 8 \\ 5 & 1 & 2 \\ 50 & 7 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 27(x^2 + y^2) - 459x - 513y + 1350 = 0$$

$$\Rightarrow x^2 + y^2 - 17x - 19 + 50 = 0$$

Q. 17. Show that the quadrilateral formed by the straight lines x – y = 0, 3x + 2y = 5, x – y = 10 and 2x + 3y = 0 is cyclic and hence find the equation of the circle.

Answer : Solving the euations we get the coordinates of the quadrilateral.



Slope of AB $= \frac{1-0}{1-0} = 1$

Slope of CD = 1

AB||CD

Slope of BD = AC = - 1

AC||BD

So they form a rectangle with all sides = 90°

The quadrilateral is cyclic as sum of opposite angles = 180°.

Now, AD = diameter of the circle equation of the circle with extremities A(0, 0)&D(6, - 4) is

$(x - 0)(x - 6) + (y - 0)(y + 4) = 0$

$x^2 + y^2 - 6x + 4y = 0$

Q. 18. If (- 1, 3) and (∝, β) are the extremities of the diameter of the circle $x^2 + y^2 - 6x + 5y - 7 = 0$, find the coordinates (∝, β).

Answer : Given $x^2 + y^2 - 6x + 5y - 7 = 0$

Centre$\left(3, -\frac{5}{2}\right)$

As (- 1, 3) & (∝, β) are the 2 extremities of the diameter, using mid - point formula we can write

$$\frac{\propto -1}{2} = 3$$

$$\Rightarrow \propto = 7$$

and $\frac{\beta + 3}{2} = -\frac{5}{2}$

$$\Rightarrow \beta = -8$$

(∝, β) = (7, - 8)