Parabola

Exercise 22

Q. 1 A. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola:

$y^2 = 12x$

Answer :

Given equation : $y^2 = 12x$

Comparing given equation with parabola having equation,

$y^2 = 4ax$

$4a = 12$

• $a = 3$

Focus :

$F(a,0) = F(3,0)$

Vertex :

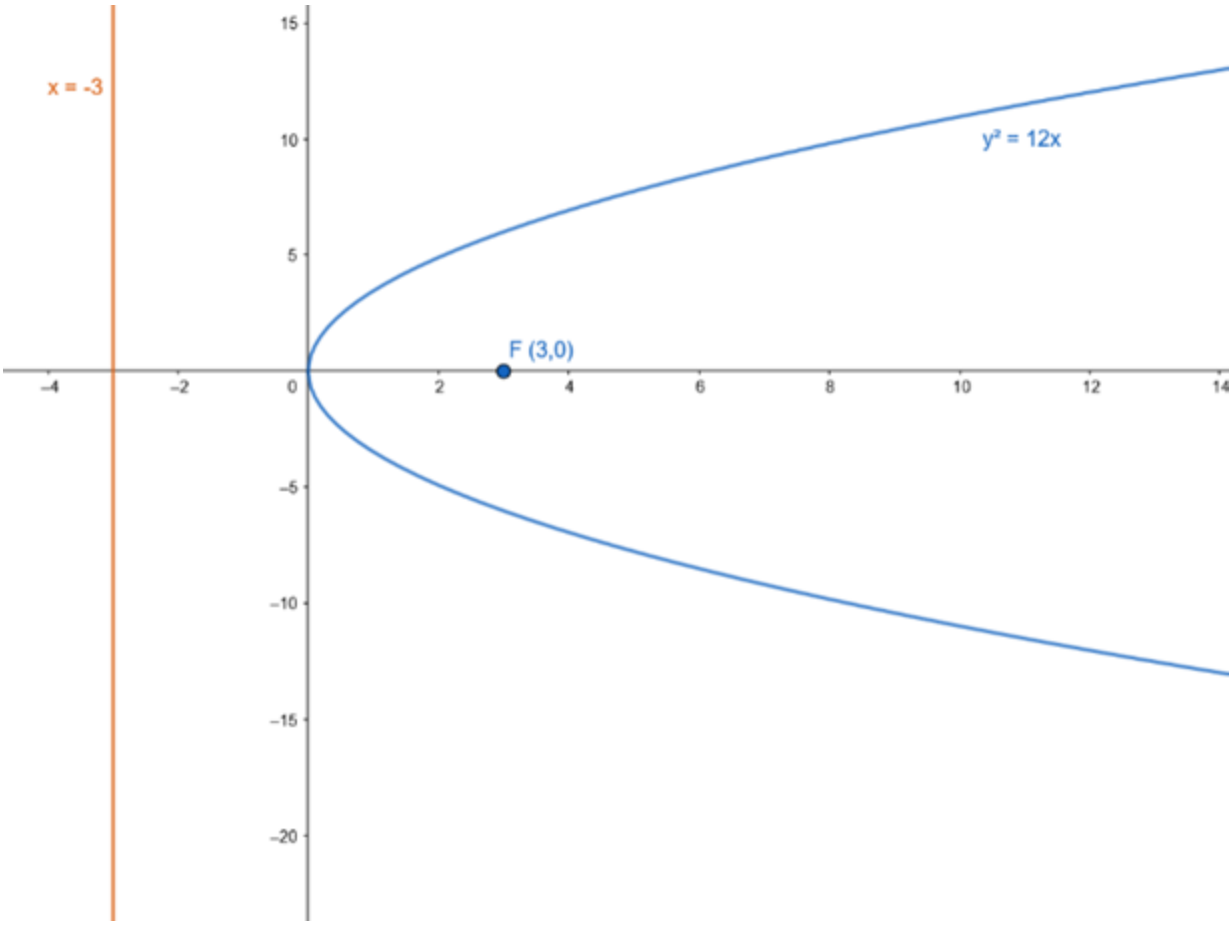
$A(0,0) = A(0,0)$

Equation of the directrix : $x+a=0$

• $x+3=0$

• $x = -3$

Lenth of latusrectum : $4a = 4.(3) = 12$



Q. 1 B. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola:

$y^2 = 10x$

Answer : Given equation : $y^2 = 10x$

Comparing given equation with parabola having equation,

$y^2 = 4ax$

$4a = 10$

• $a = 2.5$

Focus : $F(a,0) = F(2.5,0)$

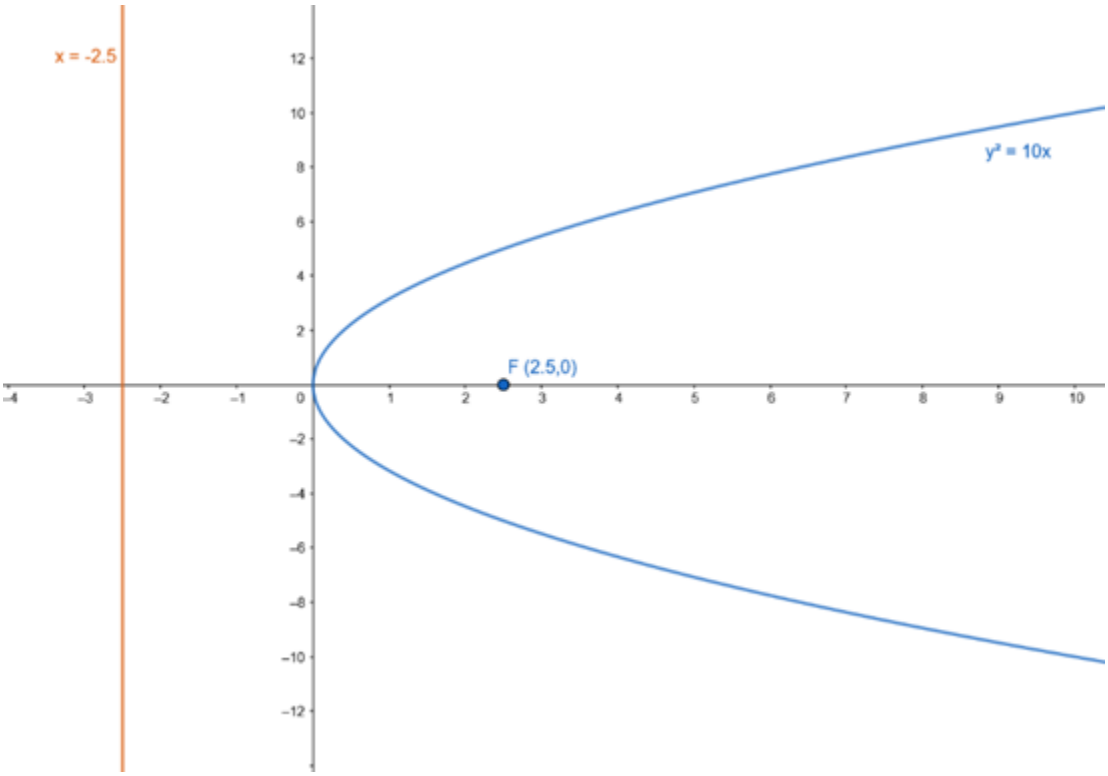
Vertex : $A(0,0) = A(0,0)$

Equation of the directrix : $x+a=0$

• x+2.5=0

• x = -2.5

Lenth of latusrectum : 4a = 4.(2.5) = 10



Q. 1 C. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola:

$3y^2 = 8x$

Answer : Given equation :

$3y^2 = 8x$

- $y^2 = \frac{8}{3}x$

Comparing the given equation with parabola having equation,

$y^2 = 4ax$

$4a = \frac{8}{3}$

- $a = \frac{2}{3}$

Focus : F(a,0) = $F\left(\frac{2}{3}, 0\right)$

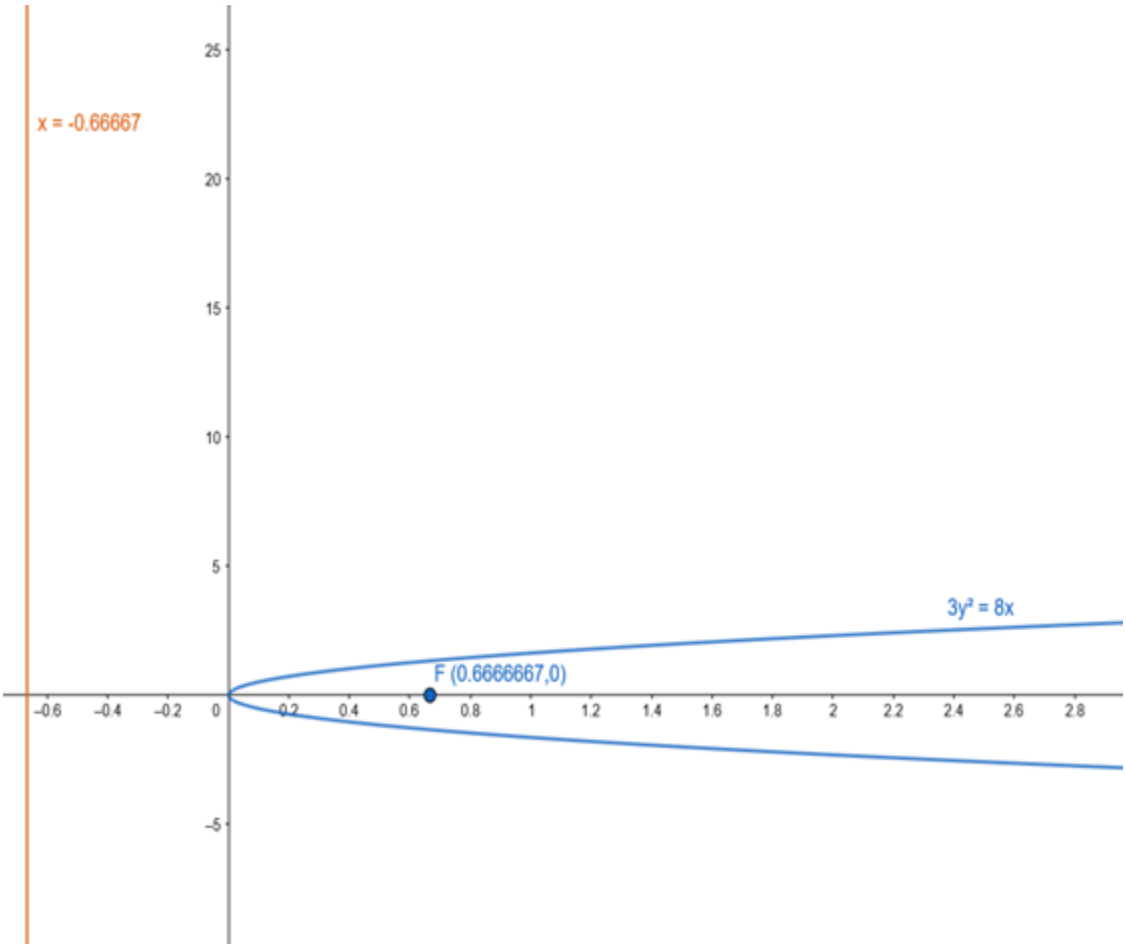
Vertex : A(0,0) = A(0,0)

Equation of the directrix : x+a=0

- $x + \frac{2}{3} = 0$

- $x = -\frac{2}{3}$

Lenth of latusrectum : $4a = \frac{8}{3}$



Q. 2 A. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$y^2 = -8x$

Answer : Given equation :

$y^2 = -8x$

Comparing given equation with parabola having equation,

$y^2 = - 4ax$

$4a = 8$

• $a = 2$

Focus : $F(-a,0) = F(-2,0)$

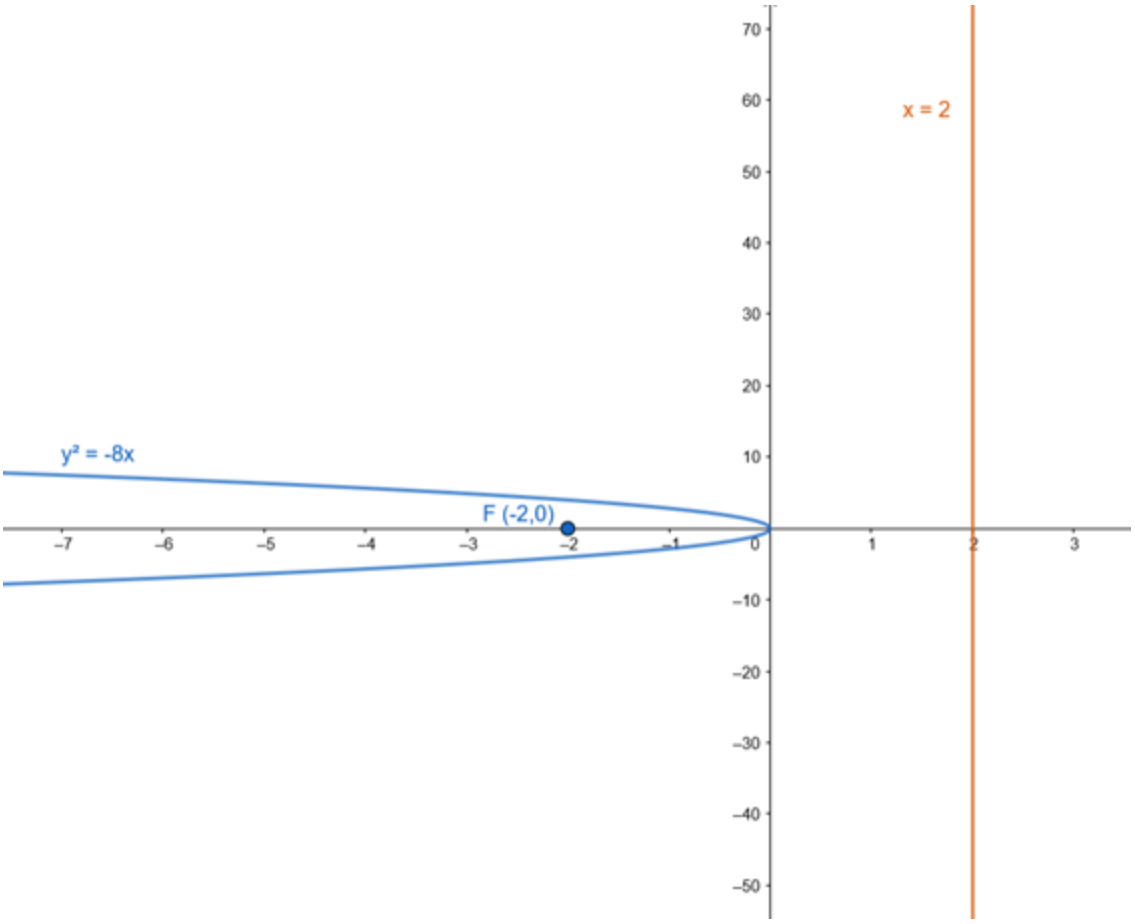
Vertex : $A(0,0) = A(0,0)$

Equation of the directrix : $x – a = 0$

• $x – 2 = 0$

• $x = 2$

Lenth of latusrectum : $4a = 8$



Q. 2 B. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$y^2 = -6x$

Answer : Given equation :

$y^2 = -6x$

Comparing given equation with parabola having equation,

$y^2 = -4ax$

$4a = 6$

- $a = \frac{3}{2}$

Focus : F(-a,0) $= F\left(-\frac{3}{2}, 0\right)$

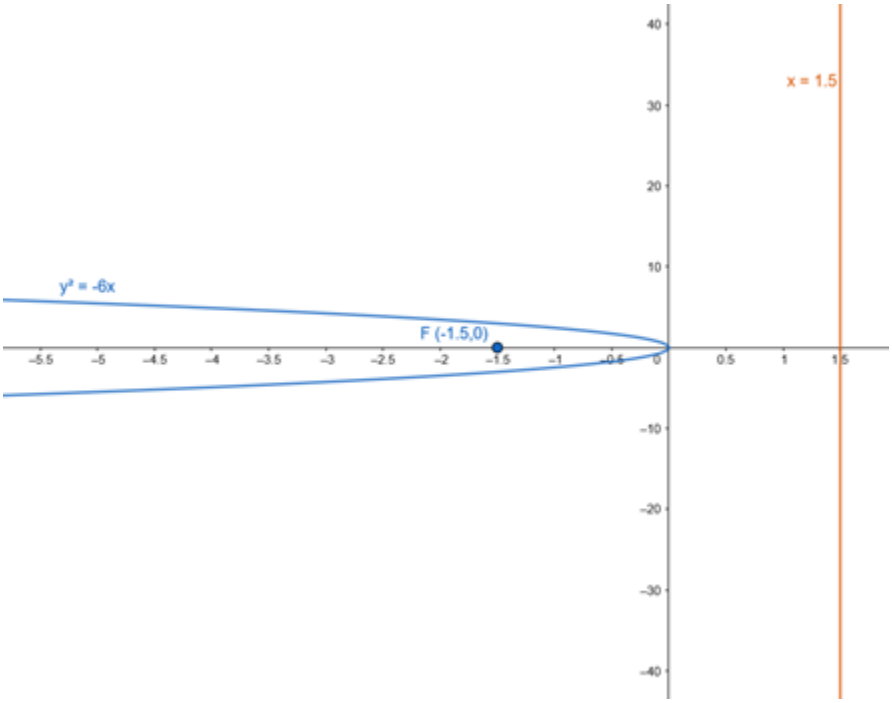
Vertex : A(0,0) = A(0,0)

Equation of the directrix : x – a = 0

- $x - \frac{3}{2} = 0$

- $x = \frac{3}{2}$

Lenth of latusrectum : 4a = 6



Q. 2 C. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$5y^2 = -16x$

Answer : Given equation :

$5y^2 = -16x$

- $y^2 = -\frac{16}{5}x$

Comparing the given equation with parabola having an equation,

$y^2 = -4ax$

- $4a = \frac{16}{5}$
- $a = \frac{4}{5}$

Focus : F(-a,0)

$= F\left(-\frac{4}{5}, 0\right)$

Vertex :

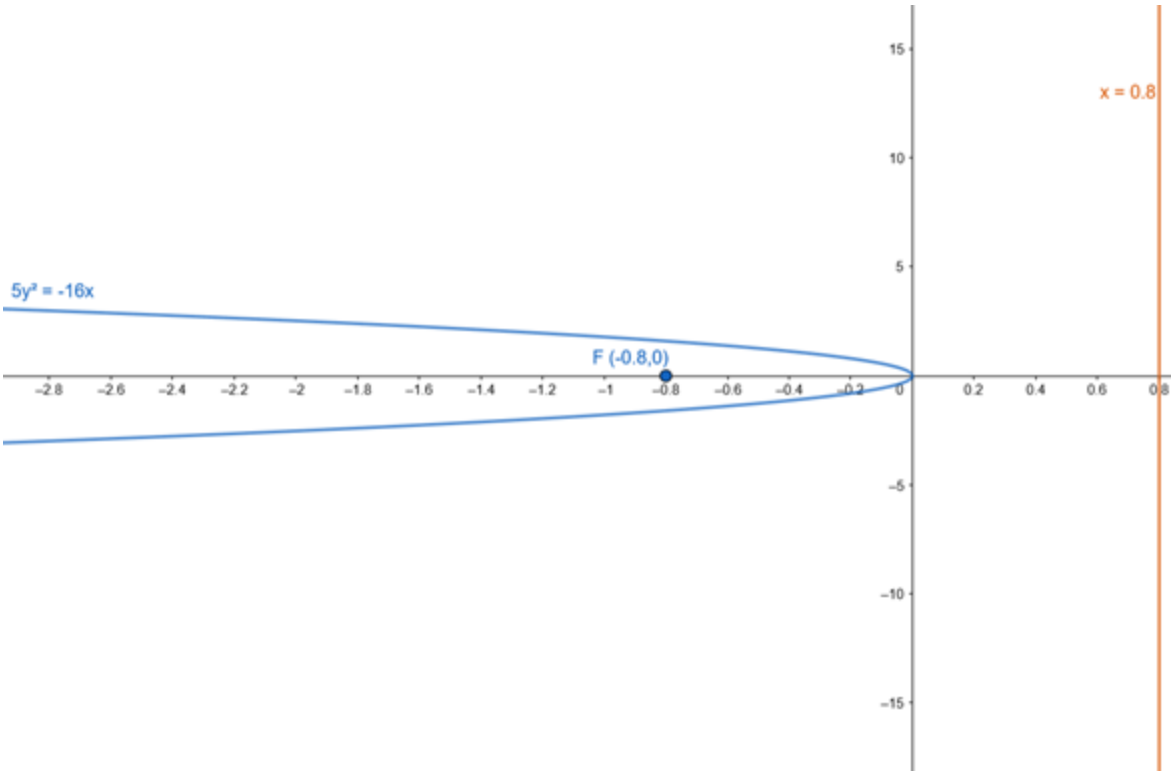
A(0,0) = A(0,0)

Equation of the directrix :

x – a = 0

- $x - \frac{4}{5} = 0$
- $x = \frac{4}{5}$

Lenth of latusrectum : $4a = \frac{16}{5}$



Q. 3 A. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$x^2 = 16y$

Answer : Given equation : $x^2 = 16y$

Comparing given equation with parabola having equation,

$x^2 = 4ay$

$4a = 16$

• $a = 4$

Focus : $F(0,a) = F(0,4)$

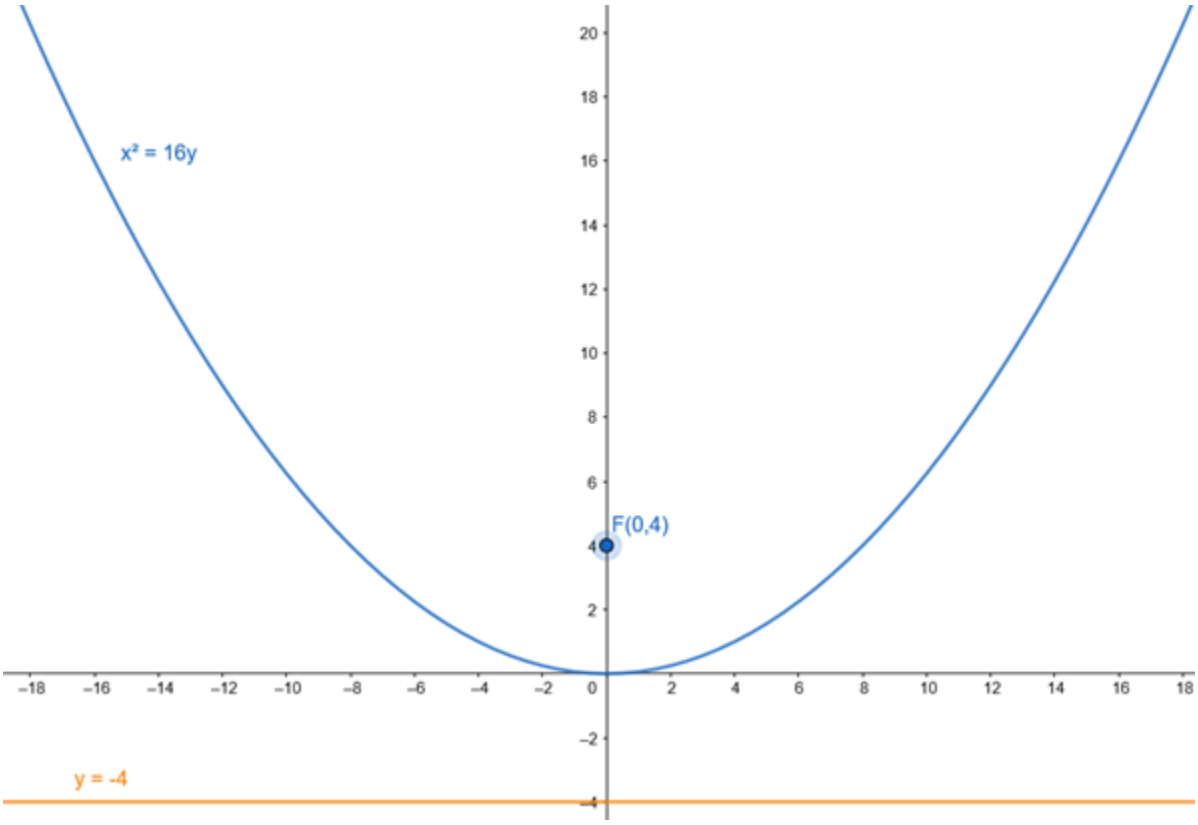
Vertex : $A(0,0) = A(0,0)$

Equation of the directrix : $y+a=0$

• $y + 4=0$

• $y = -4$

Lenth of latusrectum : 4a = 16



Q. 3 B. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$x^2 = 10y$

Answer : Given equation : $x^2 = 10y$

Comparing given equation with parabola having equation,

$x^2 = 4ay$

$4a = 10$

• $a = 2.5$

Focus : F(0,a) = F(0,2.5)

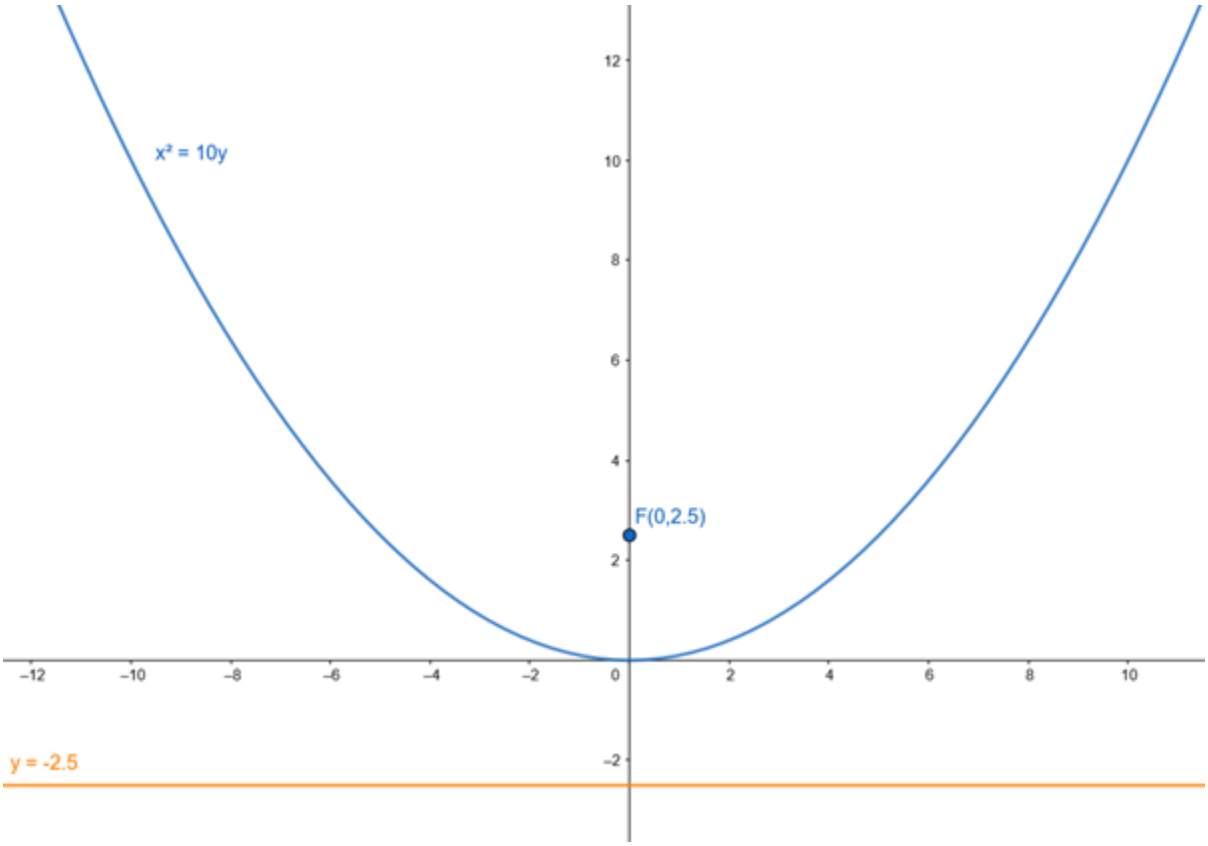
Vertex : A(0,0) = A(0,0)

Equation of the directrix : y+a=0

• y + 2.5=0

• y = -2.5

Lenth of latusrectum : 4a = 10



Q. 3 C. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$3x^2 = 8y$

Answer : Given equation :

$3x^2 = 8y$

- $x^2 = \frac{8}{3}y$

Comparing the given equation with parabola having an equation,

$x^2 = 4ay$

- $4a = \frac{8}{3}$

- $a = \frac{2}{3}$

Focus : F(0,a) = $F\left(0, \frac{2}{3}\right)$

Vertex : A(0,0) = A(0,0)

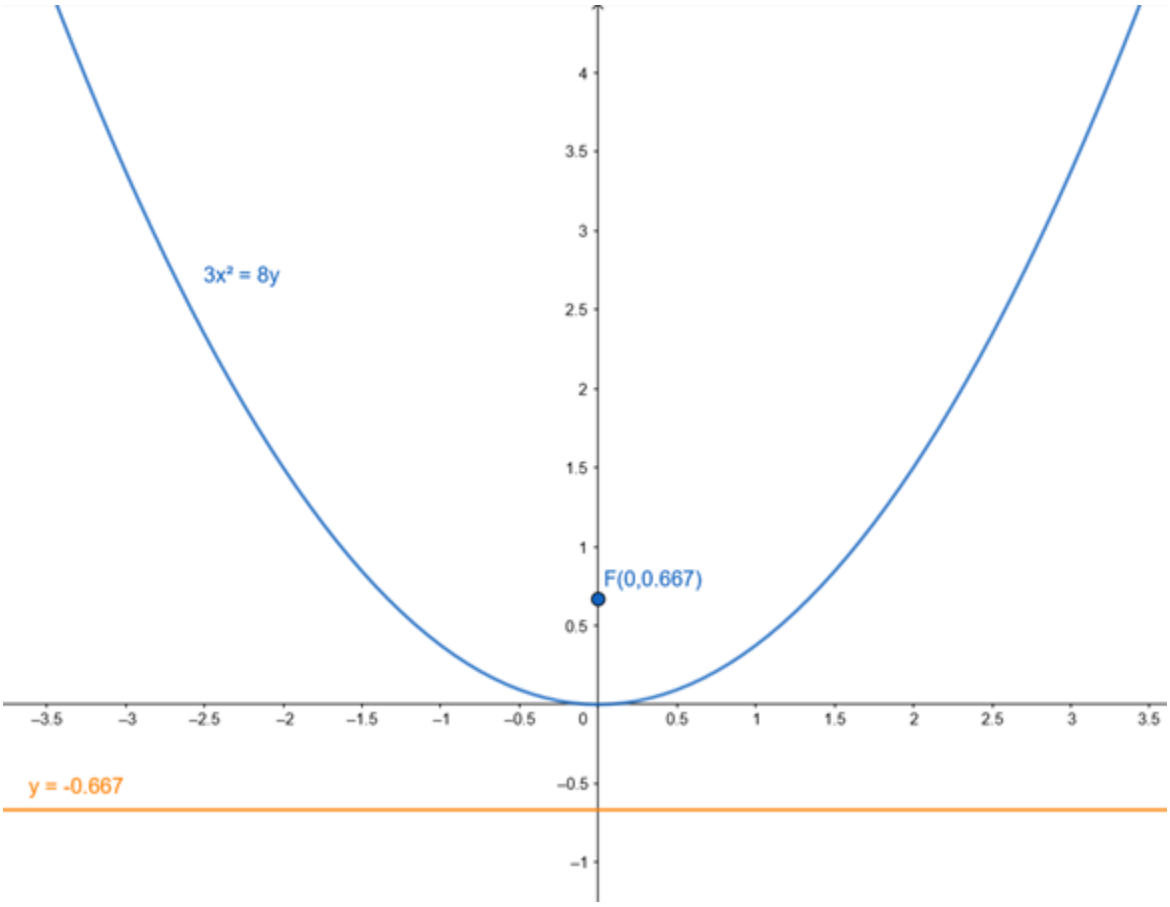
Equation of the directrix : y + a = 0

- $y + \frac{2}{3} = 0$

- $y = -\frac{2}{3}$

Lenth of latusrectum :

$4a = \frac{8}{3}$



Q. 4 A. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$x^2 = -8y$

Answer : Given equation : $x^2 = -8y$

Comparing given equation with parabola having equation,

$x^2 = -4ay$

$4a = 8$

• $a = 2$

Focus : $F(0,-a) = F(0,-2)$

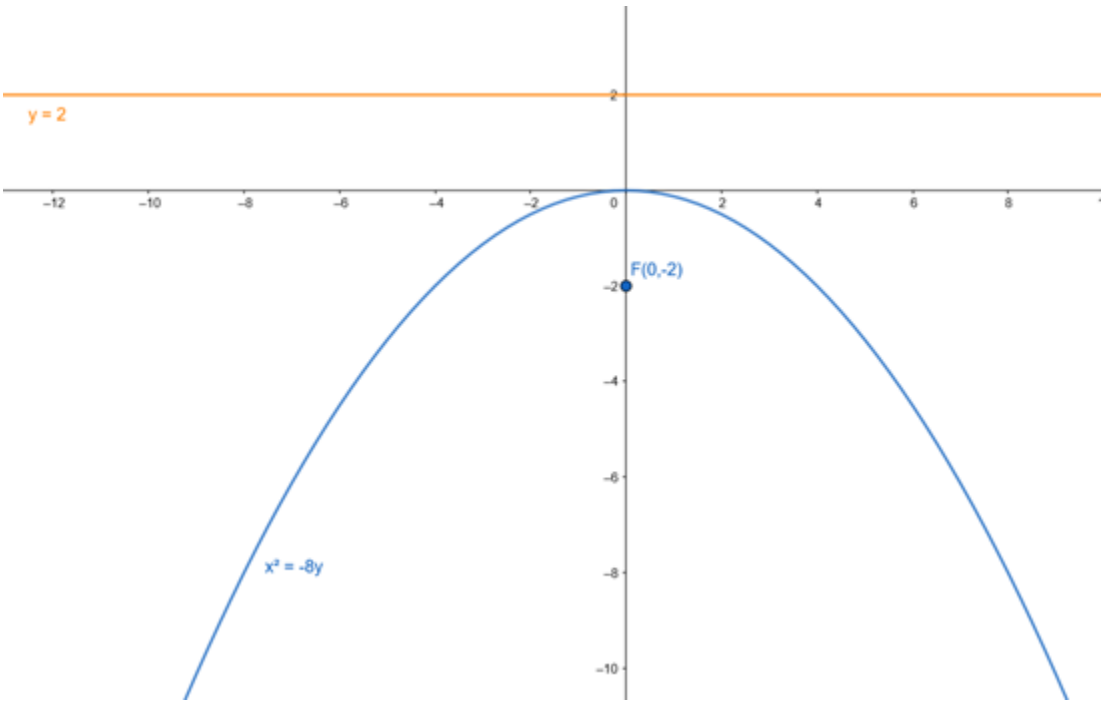
Vertex : $A(0,0) = A(0,0)$

Equation of the directrix : $y - a=0$

• $y - 2=0$

• $y = 2$

Lenth of latusrectum : $4a = 8$



Q. 4 B. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$x^2 = -18y$

Answer :

Given equation : $x^2 = -18y$

Comparing given equation with parabola having equation,

$x^2 = -4ay$

$4a = 18$

- $a = \frac{9}{2}$

Focus : F(0,-a) $= F\left(0, -\frac{9}{2}\right)$

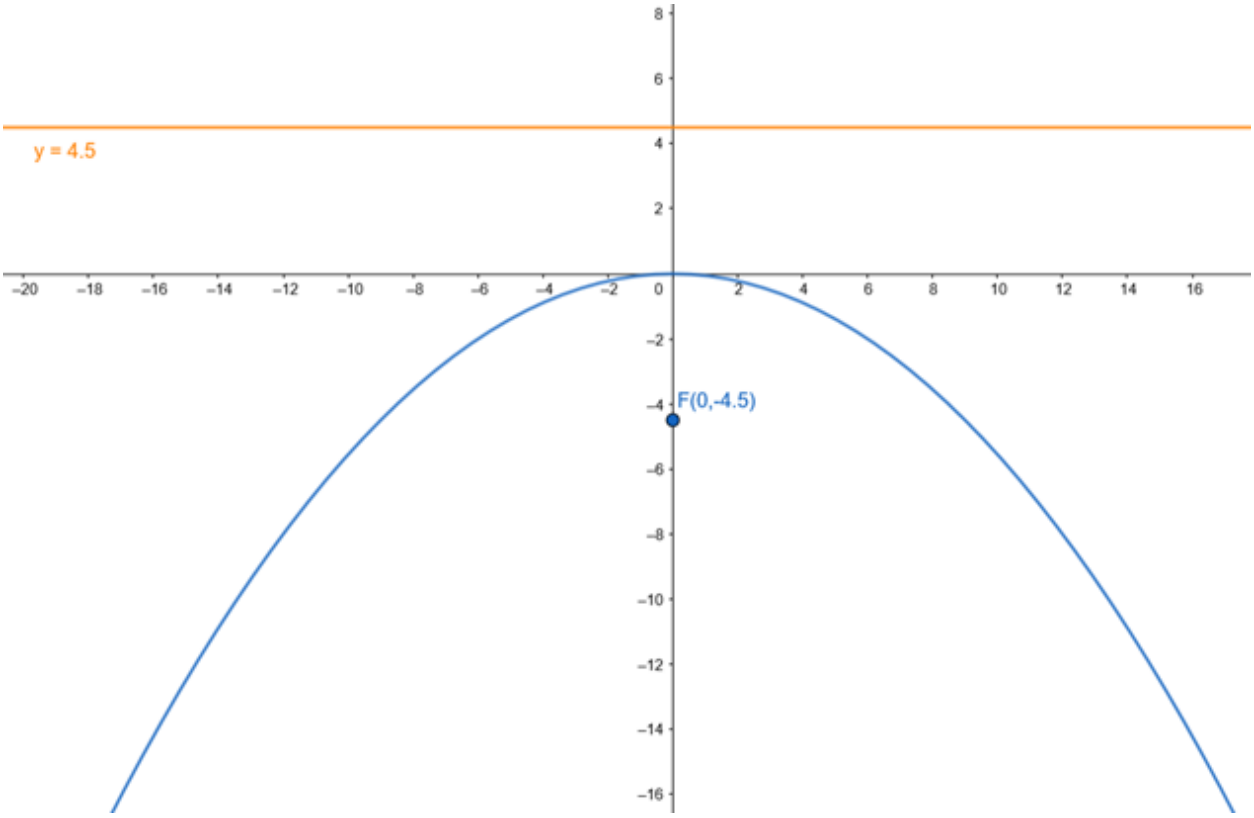
Vertex : A(0,0) = A(0,0)

Equation of the directrix : y - a=0

- $y - \frac{9}{2} = 0$

- $y = \frac{9}{2}$

Lenth of latusrectum : 4a = 18



Q. 4 C. Find the coordinates of the focus and the vertex, the equations of the directrix and the axis, and length of the latus rectum of the parabola :

$3x^2 = -16y$

Answer : Given equation :

$3x^2 = -16y$

- $x^2 = -\frac{16}{3}y$

Comparing the given equation with parabola having an equation,

$x^2 = 4ay$

- $4a = \frac{16}{3}$

- $a = \frac{4}{3}$

Focus : F(0,-a) $= F\left(0, -\frac{4}{3}\right)$

Vertex : A(0,0) = A(0,0)

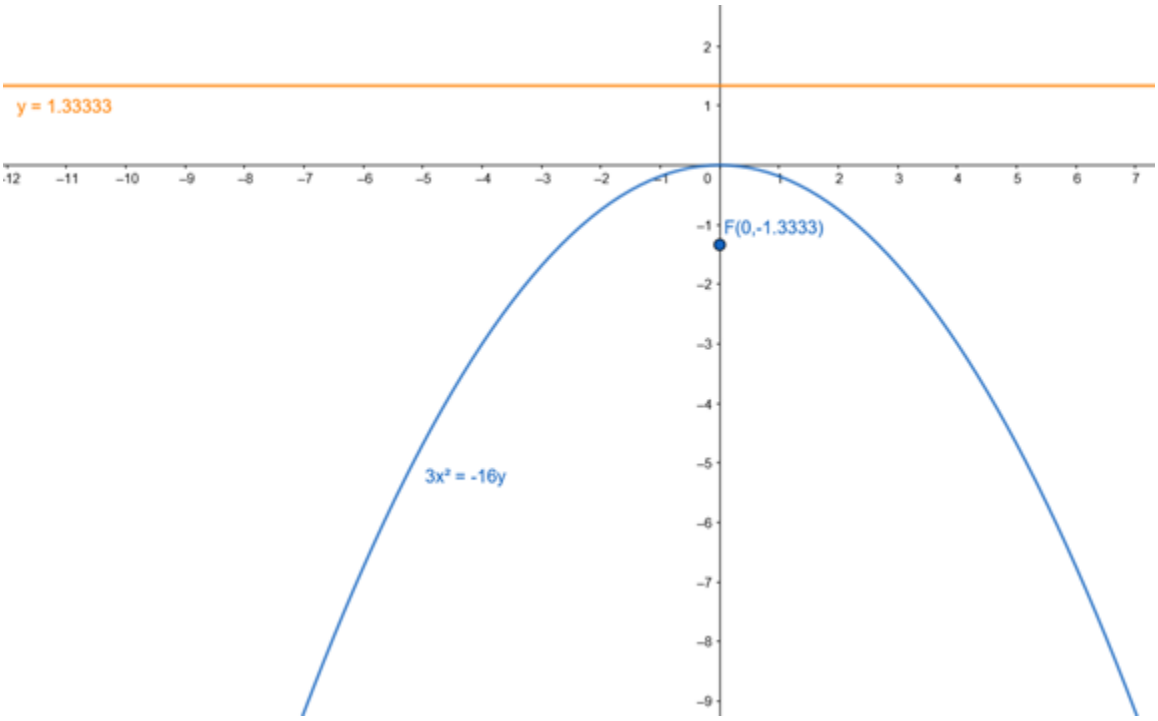
Equation of the directrix : y - a = 0

- $y - \frac{4}{3} = 0$

- $y = \frac{4}{3}$

Lenth of latusrectum :

$4a = \frac{16}{3}$



Q. 5. Find the equation of the parabola with vertex at the origin and focus at F(-2, 0).

Answer : Vertex : A (0,0)

Given focus F(-2,0) is of the form F(-a,0)

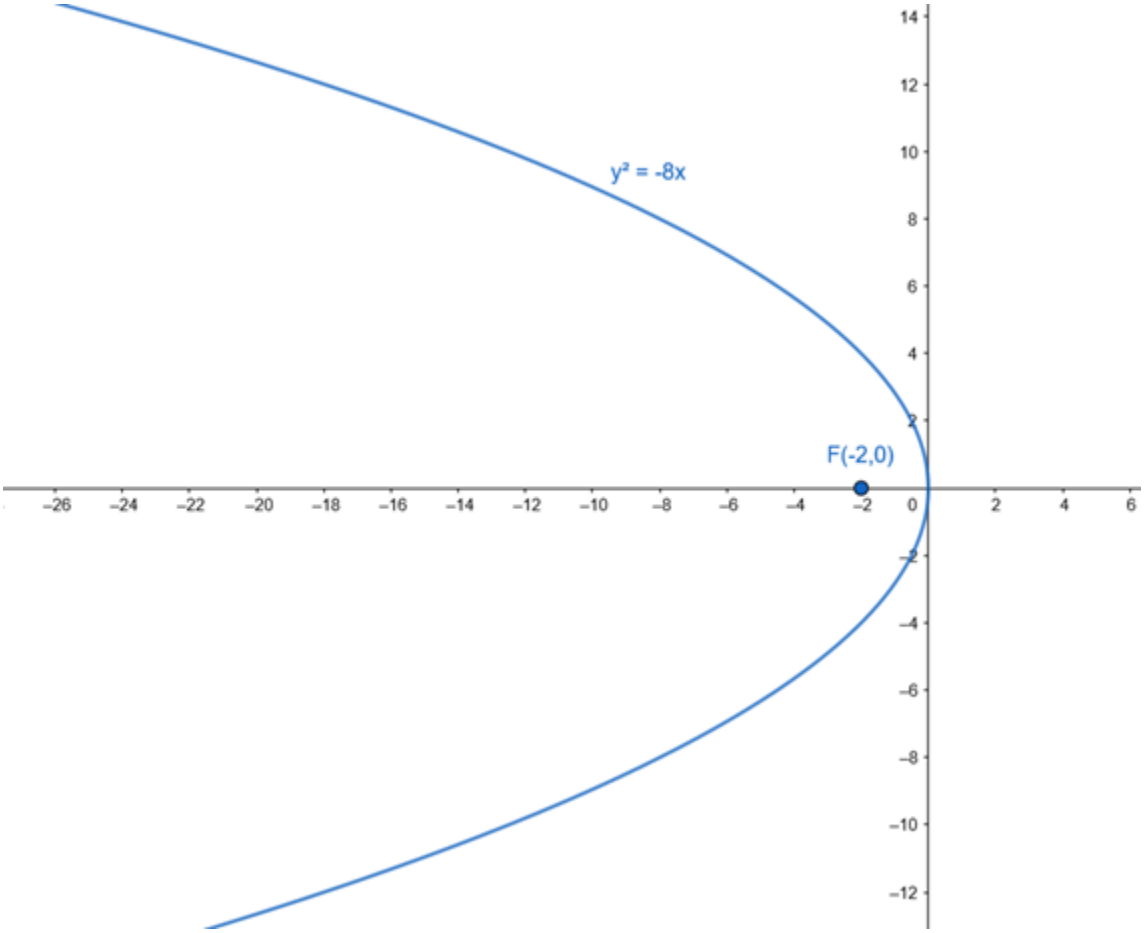
For Vertex A(0,0) and Focus F(-a,0), equation of parabola is

$y^2 = - 4ax$

Here, a = 2

Therefore, equation of parabola,

$y^2 = - 8x$



Q. 6. Find the equation of the parabola with focus F(4, 0) and directrix x = -4.

Answer :

Given equation of directrix : x = -4

• x + 4 = 0

Above equation is of the form, x + a = 0

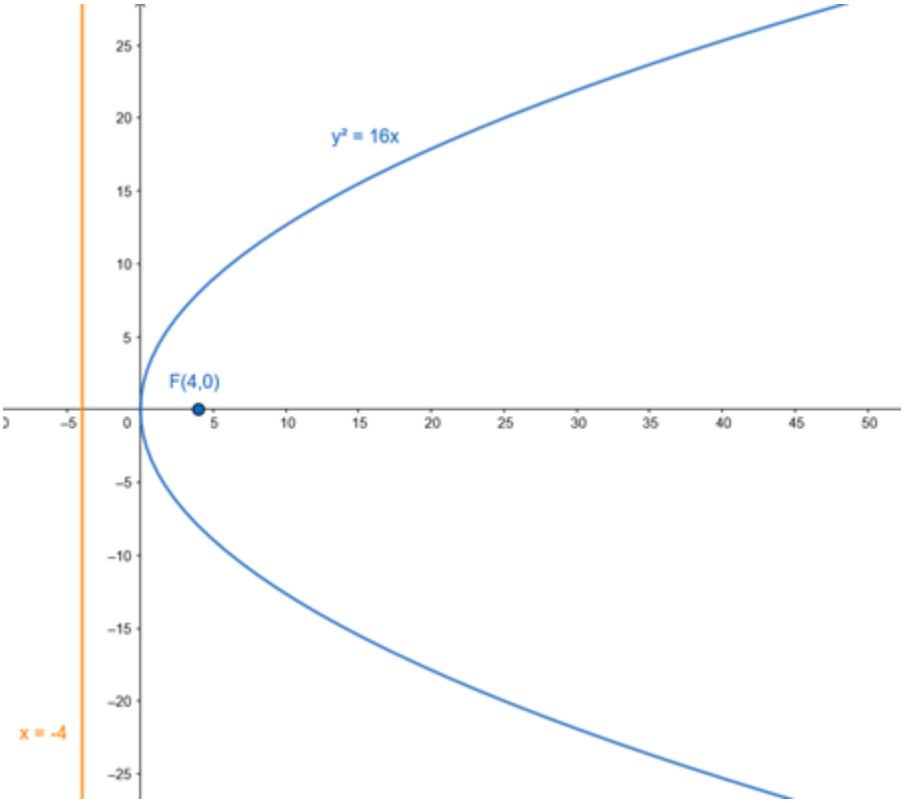
Focus of the parabola F(4,0) is of the form F(a,0)

Therefore, a = 4

For directrix with equation x+a=0 and focus (a,0), equation of the parabola is,

$y^2 = 4ax$

• $y^2 = 16x$



Q. 7. Find the equation of the parabola with focus F(0, -3) and directrix y = 3.

Answer : Given equation of directrix : y = 3

• y - 3 = 0

Above equation is of the form, y - a = 0

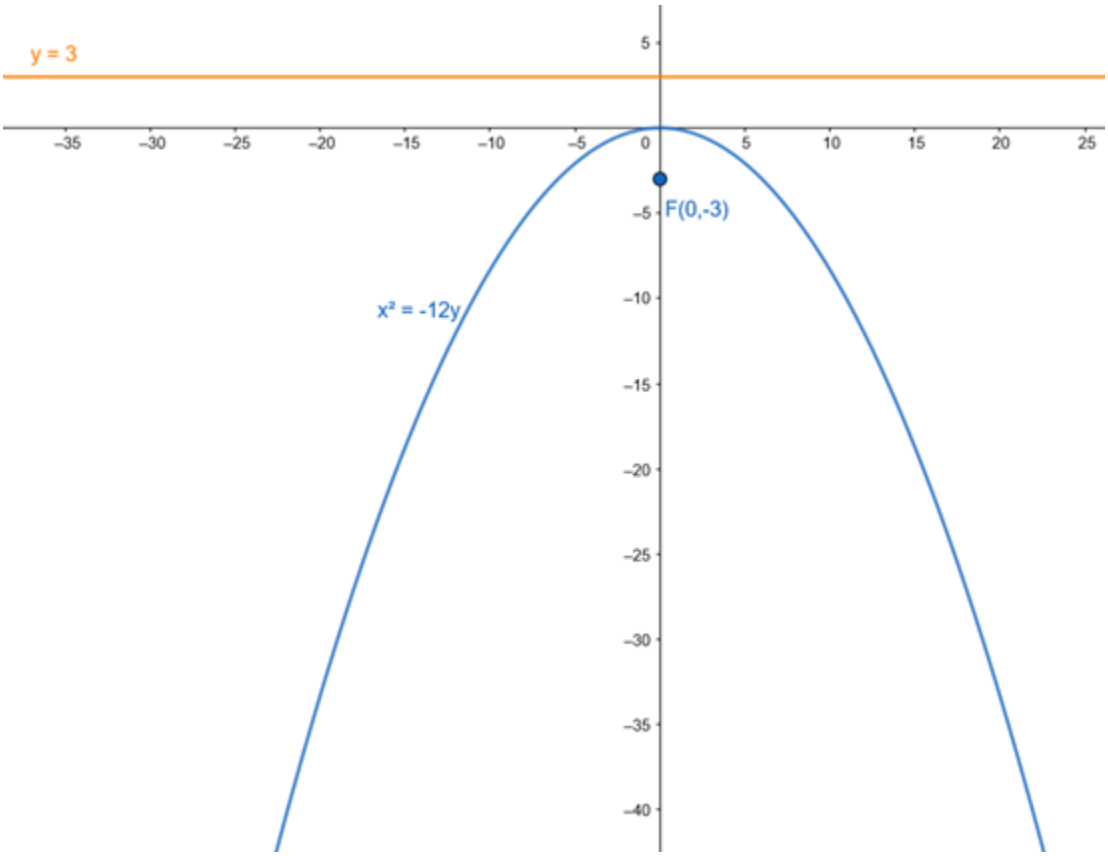
Focus of the parabola F(0,-3) is of the form F(0,-a)

Therefore, a = 3

For directrix with equation y-a=0 and focus (0,-a), equation of the parabola is,

$x^2 = -4ay$

• $x^2 = -12y$



Q. 8. Find the equation of the parabola with vertex at the origin and focus F(0, 5).

Answer : Vertex : A (0,0)

Given focus F(0,5) is of the form F(0,a)

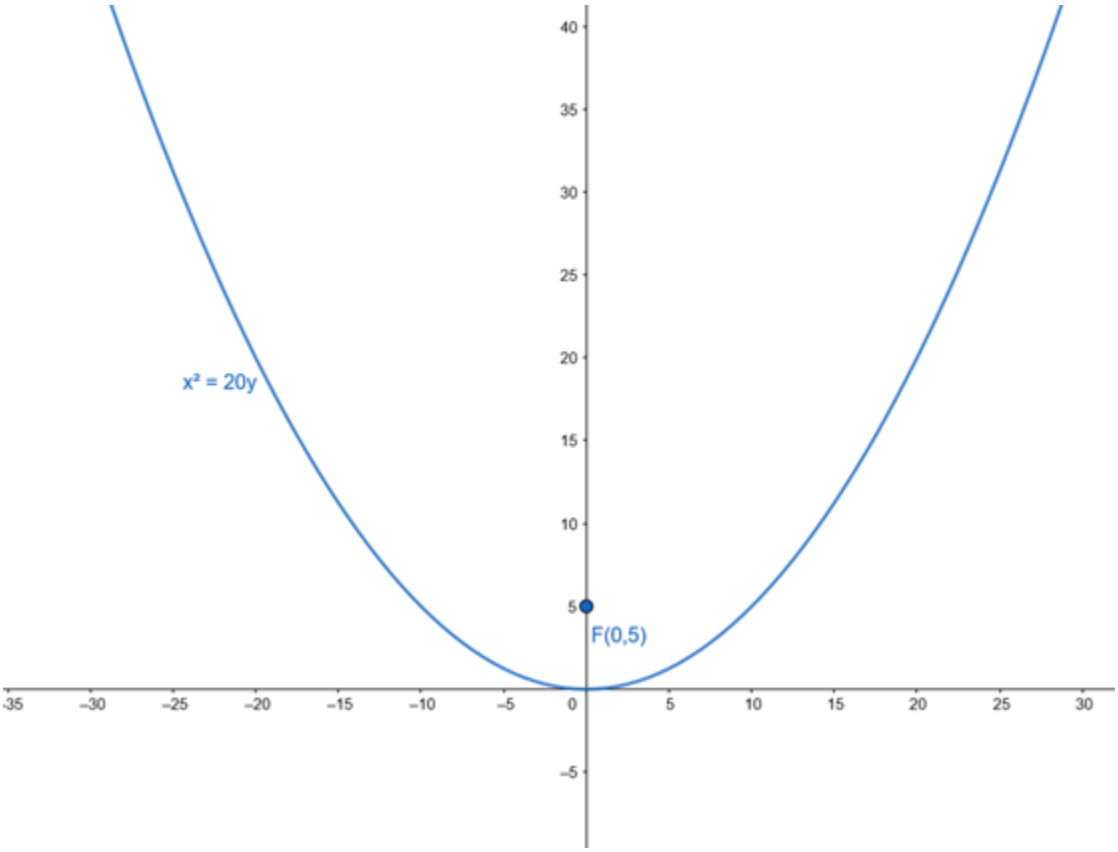
For Vertex A(0,0) and Focus F(0,a), equation of parabola is

$x^2 = 4ay$

Here, a = 5

Therefore, equation of parabola,

$x^2 = 20y$



Q. 9. Find the equation of the parabola with vertex at the origin, passing through the point P(5, 2) and symmetric with respect to the y-axis.

Answer : The equation of a parabola with vertex at the origin and symmetric about the y-axis is

$x^2 = 4ay$

Since point P(5,2) passes through above parabola we can write,

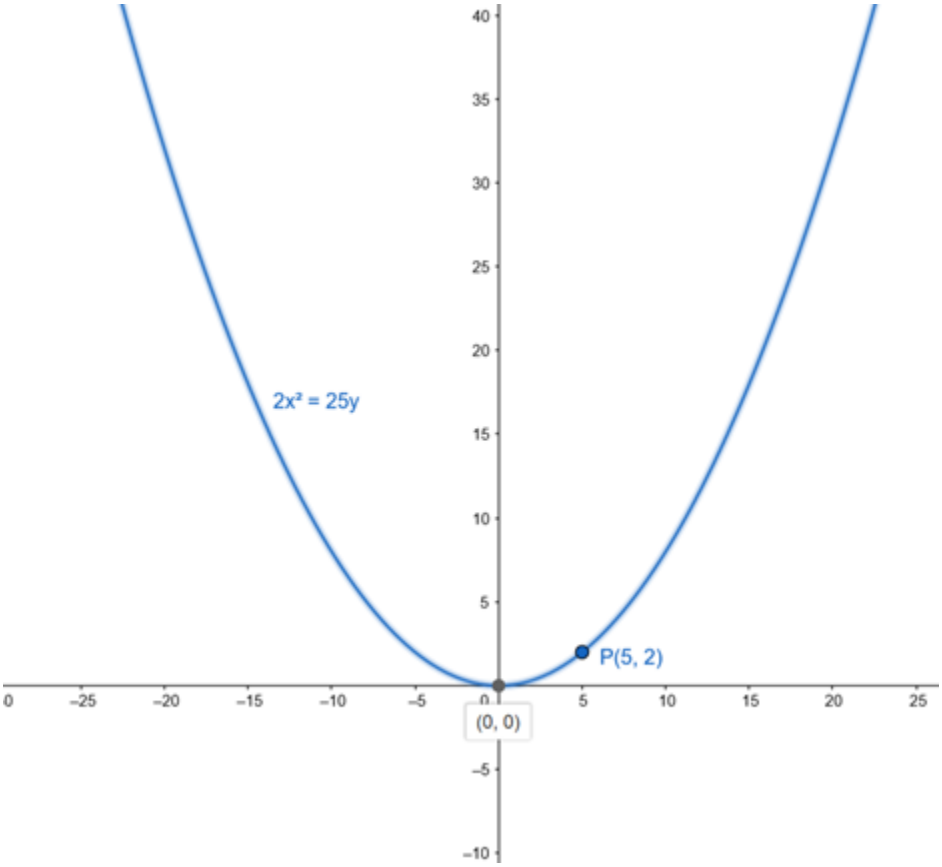
$5^2 = 4a(2)$

• $25 = 8a$

• $a = \frac{25}{8}$

Therefore, the equation of a parabola is

- $x^2 = 4 . \frac{25}{8} y$
- $x^2 = \frac{25}{2} y$
- $2x^2 = 25y$



Q. 10. Find the equation of the parabola, which is symmetric about the y-axis and passes through the point P(2, -3).

Answer : The equation of a parabola with vertex at the origin and symmetric about the y-axis is

$x^2 = 4ay$

Since point P(2,-3) passes through above parabola we can write,

$2^2 = 4a(-3)$

- 4 = -12a
- $a = -\frac{1}{3}$

Therefore, the equation of a parabola is

- $x^2 = 4 . \left(-\frac{1}{3}\right) y$
- $x^2 = -\frac{4}{3} y$
- $3x^2 = -4y$