Statistics

Exercise 30A

Q. 1. Find the mean deviation about the mean for the following data :

7, 8, 4, 13, 9, 5, 16, 18

Answer : We have, 7, 8, 4, 13, 9, 5, 16, 18

Mean of the given data is

$$\bar{x} = \frac{7+8+4+13+9+5+16+18}{8} = \frac{80}{8} = 10$$

The respective absolute values of the deviations from the mean , i.e.' $|x_i - \bar{x}|$ are

3, 2, 6, 3, 1, 5, 6, 8

Thus, the required mean deviation about the mean is

$$M.D.(\bar{x}) = \frac{\sum_{i=1}^{8} |x_i - \bar{x}|}{8}$$

$$= \frac{3+2+6+3+1+5+6+8}{8} = \frac{34}{8} = 4.25$$

Q. 2. Find the mean deviation about the mean for the following data :

39, 72, 48, 41, 43, 55, 60, 45, 54, 43

Answer : We have, 39, 72, 48, 41, 43, 55, 60, 45, 54, 43

Mean of the given data is

$$\bar{x} = \frac{39+72+48+41+43+55+60+45+54+43}{10} = \frac{500}{10} = 50$$

The respective absolute values of the deviations from mean , i.e' $|x_i - \bar{x}|$ are

11, 22, 2, 9, 7, 5, 10, 5, 4, 7

Thus, the required mean deviation about the mean is

$$\text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{10} |x_i - \bar{x}|}{10}$$

$$= \frac{11 + 22 + 2 + 9 + 7 + 5 + 10 + 5 + 4 + 7}{10} = \frac{82}{10} = 8.2$$

Q. 3. Find the mean deviation about the mean for the following data :

17, 20, 12, 13, 15, 16, 12, 18, 15, 19, 12, 11

Answer : We have, 17, 20, 12, 13, 15, 16, 12, 18, 15, 19, 12, 11

Mean of the given data is

$$\bar{x} = \frac{17 + 20 + 12 + 13 + 15 + 16 + 12 + 18 + 15 + 19 + 12 + 11}{12}$$

$$\bar{x} = \frac{180}{12} = 15$$

The respective absolute values of the deviations from the mean , i.e., $|x_i - \bar{x}|$ are

2, 5, 3, 2, 0, 1, 3, 3, 0, 4, 3, 4

Thus, the required mean deviation about the mean is

$$\text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{12} |x_i - \bar{x}|}{12}$$

$$= \frac{2 + 5 + 3 + 2 + 0 + 1 + 3 + 3 + 0 + 4 + 3 + 4}{12} = \frac{30}{12} = 2.5$$

Q. 4. Find the mean deviation about the median for the following data :

12, 5, 14, 6, 11, 13, 17, 8, 10

Answer : Here the number of observations is 9 which is odd.

Arranging the data into ascending order, we have 5, 6, 8, 10, 11, 12, 13, 14, 17

Now, $\text{Median(M)} = \left(\frac{9+1}{2}\right)^{\text{th}} \text{ or } 5^{\text{th}} \text{ observation} = 11$

The respective absolute values of the deviations from median , i.e., $|x_i - M|$ are

6, 5, 3, 1, 0, 1, 2, 3, 6

Thus, the required mean deviation about the median is

$$\text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{9}|x_i - M|}{9}$$

$$= \frac{6+5+3+1+0+1+2+3+6}{9} = \frac{27}{9} = 3$$

Q. 5. Find the mean deviation about the median for the following data :

4, 15, 9, 7, 19, 13, 6, 21, 8, 25, 11

Answer : Here the number of observations is 11 which is odd.

Arranging the data into ascending order, we have 4, 6, 7, 8, 9, 11, 13, 15, 19, 21, 25

Now, $\text{Median}(M) = \left(\frac{11+1}{2}\right)^{th} \text{ or } 6^{th} \text{ observation} = 11$

The respective absolute values of the deviations from median , i.e., $|x_i - M|$ are

7, 5, 4, 3, 2, 0, 2, 4, 8, 10, 14

Thus, the required mean deviation about the median is

$$\text{M.D.}(\bar{x}) = \frac{\sum_{i=1}^{11}|x_i - M|}{11}$$

$$= \frac{7+5+4+3+2+0+2+4+8+10+14}{11} = \frac{59}{11} = 5.3$$

Q. 6. Find the mean deviation about the median for the following data :

34, 23, 46, 37, 40, 28, 32, 50, 35, 44

Answer : Here the number of observations is 10 which is odd.Arranging the data into ascending order, we have 23, 28, 32, 34, 35, 37, 40, 44, 46, 50

Now, $\text{Median}(M) = \left(\frac{5^{th}\text{ observation} + 6^{th}\text{ observation}}{2}\right) = \frac{35+37}{2} = 36$

The respective absolute values of the deviations from median , i.e., $|x_i - M|$ are

13, 8, 4, 2, 1, 1, 4, 8, 10, 14

Thus, the required mean deviation about the median is

$$M.D.(\bar{x}) = \frac{\sum_{i=1}^{10} |x_i - M|}{10}$$

$$= \frac{13 + 8 + 4 + 2 + 1 + 1 + 4 + 8 + 10 + 14}{10} = \frac{65}{10} = 6.5$$

Q. 7. Find the mean deviation about the median for the following data :

70, 34, 42, 78, 65, 45, 54, 48, 67, 50, 56, 63

Answer : Here the number of observations is 12 which is odd.

Arranging the data into ascending order, we have 34, 42, 45, 48, 50, 54, 56, 63, 65, 67, 70, 78

Now, $\text{Median}(M) = \left(\frac{6^{th}\text{ observation}+7^{th}\text{ observation}}{2}\right) = \frac{54+56}{2} = 55$

The respective absolute values of the deviations from median , i.e' $|x_i - M|$ are

21, 13, 10, 7, 5, 1, 1, 8, 10, 12, 15, 23

Thus, the required mean deviation about the median is

$$M.D.(\bar{x}) = \frac{\sum_{i=1}^{12} |x_i - M|}{12}$$

$$= \frac{21 + 13 + 10 + 7 + 5 + 1 + 1 + 8 + 10 + 12 + 15 + 23}{12} = \frac{126}{12} = 10.5$$

Q. 8. Find the mean deviation about the mean for the following data :

x_i	6	12	18	24	30	36
f_i	5	4	11	6	4	6

Answer : We have,

x_i	f_i	$f_i\, x_i$
6	5	30
12	4	48
18	11	198
24	6	144
30	4	120
36	6	216
	36	756

Therefore, $\bar{x} = \frac{\sum_{i=1}^{6} f_i\, x_i}{\sum_{i=1}^{6} f_i} = \frac{756}{36} = 21$

Now,

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
6	5	30	15	75
12	4	48	9	36
18	11	198	3	33
24	6	144	3	18
30	4	120	9	36
36	6	216	15	90
	36	756		288

Thus, the required mean deviation about the mean is

$$\bar{x} = \frac{\sum_{i=1}^{6} f_i \lvert x_i - \bar{x} \rvert}{\sum_{i=1}^{6} f_i} = \frac{288}{36} = 8$$

Q. 9. Find the mean deviation about the mean for the following data :

x_i	2	5	6	8	10	12
f_i	2	8	10	7	8	5

Answer : We have,

x_i	f_i	$f_i x_i$
2	2	4
5	8	40
6	10	60
8	7	56
10	8	80
12	5	60
	40	300

Therefore, $\bar{x} = \frac{\sum_{i=1}^{6} f_i x_i}{\sum_{i=1}^{6} f_i} = \frac{300}{40} = 7.5$

Now,

x_i	f_i	$f_i\ x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
2	2	4	5.5	11
5	8	40	2.5	20
6	10	60	1.5	15
8	7	56	0.5	3.5
10	8	80	2.5	20
12	5	60	4.5	22.5
	40	300		92

Thus, the required mean deviation about the mean is

$$\bar{x} = \frac{\sum_{i=1}^{6} f_i \lvert x_i - \bar{x}\rvert}{\sum_{i=1}^{6} f_i} = \frac{92}{40} = 2.3$$

Q. 10 Find the mean deviation about the mean for the following data :

x_i	3	5	7	9	11	13
f_i	6	8	15	25	8	4

Answer : We have,

x_i	f_i	$f_i x_i$
3	6	18
5	8	40
7	15	105
9	25	225
11	8	88
13	4	52
	66	528

Therefore, $\bar{x} = \frac{\sum_{i=1}^{6} f_i x_i}{\sum_{i=1}^{6} f_i} = \frac{528}{66} = 8$

Now,

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
3	6	18	5	30
5	8	40	3	24
7	15	105	1	15
9	25	225	1	25
11	8	88	3	24
13	4	52	5	20
	66	528		138

Thus, the required mean deviation about the mean is

$$M.D(\bar{x}) = \frac{\sum_{i=1}^{6} f_i\lvert x_i - \bar{x}\rvert}{\sum_{i=1}^{6} f_i} = \frac{138}{66} = 2.09$$

Q. 11. Find the mean deviation about the median for the following data :

x_i	15	21	27	30	35
f_i	3	5	6	7	8

Answer : The given observations are in ascending order. Adding a row corresponding to cumulative frequencies to the given data, we get,

x_i	15	21	27	30	35
f_i	3	5	6	7	8
c.f	3	8	14	21	29

Now, N=29 which is odd.

Since, 15^{th} observation lie in the cumulative frequency 21, for which the corresponding observation is 30.

$$\text{Median(M)} = \left(\frac{29+1}{2}\right)^{\text{th}} \text{ or } 15^{\text{th}} \text{ observation} = 30$$

Now, absolute values of the deviations from the median,

$\|x_i - M\|$	15	9	3	0	5
f_i	3	5	6	7	8
$f_i\|x_i - M\|$	45	45	18	0	40

We have, $\sum_{i=1}^{5} f_i = 29$ and $\sum_{i=1}^{5} f_i |x_i - M| = 148$

$$\therefore \ M.D\,(M) = \frac{\sum_{i=1}^{5} f_i |x_i - M|}{\sum_{i=1}^{5} f_i}$$

$$= \frac{148}{29} = 5.10$$

Q. 12. Find the mean deviation about the median for the following data :

x_i	5	7	9	11	13	15	17
f_i	2	4	6	8	10	12	8

Answer : The given observations are in ascending order. Adding a row corresponding to cumulative frequencies to the given data, we get,

x_i	5	7	9	11	13	15	17
f_i	2	4	6	8	10	12	8
c.f	2	6	12	20	30	42	50

Now, N=50 which is even.

Median is the mean of the 25th observation and 26th observation. Both of these observations lie in the cumulative frequency 30, for which the corresponding observation is 13.

$$\text{Median(M)} = \frac{25^{\text{th}}\text{ observation} + 26^{\text{th}}\text{ observation}}{2} = \frac{13+13}{2} = 13$$

Now, absolute values of the deviations from the median,

$\|x_i - M\|$	8	6	4	2	0	2	4
f_i	2	4	6	8	10	12	8
$f_i\|x_i - M\|$	16	24	24	16	0	24	32

We have, $\sum_{i=1}^{5} f_i = 50$ and $\sum_{i=1}^{5} f_i|x_i - M| = 136$

$$\therefore \ \text{M.D}\ (\text{M}) = \frac{\sum_{i=1}^{5} f_i |x_i - M|}{\sum_{i=1}^{5} f_i}$$

$$= \frac{136}{50} = 2.72$$

Q. 13 Find the mean deviation about the median for the following data :

X_i	10	15	20	25	30	35	40	45
F_i	7	3	8	5	6	8	4	9

Answer : The given observations are in ascending order. Adding a row corresponding to cumulative frequencies to the given data, we get,

x_i	10	15	20	25	30	35	40	45
f_i	7	3	8	5	6	8	4	9
c.f	7	10	18	23	29	37	41	50

Now, N=50 which is even.

Median is the mean of the 25^{th} observation and 26^{th} observation. Both of these observations lie in the cumulative frequency 29, for which the corresponding observation is 30.

$$\text{Median(M)} = \frac{25^{\text{th}} \text{ observation} + 26^{\text{th}} \text{ observation}}{2} = \frac{30 + 30}{2} = 30$$

Now, absolute values of the deviations from the median,

$\lvert x_i\text{-M}\rvert$	20	15	10	5	0	5	10	15
f_i	7	3	8	5	6	8	4	9
$f_i\lvert x_i\text{-M}\rvert$	140	45	80	25	0	40	40	135

We have, $\sum_{i=1}^{5} f_i = 50$ and $\sum_{i=1}^{5} f_i \lvert x_i - M\rvert = 505$

$$\therefore \text{ M.D (M)} = \frac{\sum_{i=1}^{5} f_i \lvert x_i - M\rvert}{\sum_{i=1}^{5} f_i}$$

$$= \frac{505}{50} = 10.1$$

Q. 14. Find the mean deviation about the mean for the following data :

Mark	0-10	10-20	20-30	30-40	40-50	50-60
Number of Students	6	8	14	16	4	2

Answer : we make the following table from the given data:

Mark	Number of Students f_i	Mid-points x_i	$f_i x_i$
0-10	6	5	30
10-20	8	15	120
20-30	14	25	350
30-40	16	35	560
40-50	4	45	180
50-60	2	55	110
	50		1350

Therefore, $\bar{x} = \frac{\sum_{i=1}^{6} f_i x_i}{\sum_{i=1}^{6} f_i} = \frac{1350}{50} = 27$

Mark	Number of Students f_i	Mid-points x_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
0-10	6	5	30	22	132
10-20	8	15	120	12	96
20-30	14	25	350	2	28
30-40	16	35	560	8	128
40-50	4	45	180	18	72
50-60	2	55	110	28	56
	50		1350		512

Thus, the required mean deviation about the mean is

$$M.D(\bar{x}) = \frac{\sum_{i=1}^{6} f_i |x_i - \bar{x}|}{\sum_{i=1}^{6} f_i} = \frac{512}{50} = 10.24$$

Q. 15. Find the mean deviation about the mean for the following data :

Height (in cm)	95-105	105-115	115-125	125-135	135-145	145-155
Number of boys	9	16	23	30	12	10

Answer : we make the following table from the given data:

Height (in cm)	Number of boys f_i	Mid-points x_i	$f_i x_i$
95-105	9	100	900
105-115	16	110	1760
115-125	23	120	2760
125-135	30	130	3900
135-145	12	140	1680
145-155	10	150	1500
	100		12500

Therefore, $\bar{x} = \frac{\sum_{i=1}^{6} f_i x_i}{\sum_{i=1}^{6} f_i} = \frac{12500}{100} = 125$

Q. 16. Find the mean deviation about the mean for the following data :

class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

Answer : we make the following table from the given data:

class	Frequency f_i	Mid-points x_i	$f_i x_i$
30-40	3	35	105
40-50	7	45	315
50-60	12	55	660
60-70	15	65	975
70-80	8	75	600
80-90	3	85	255
90-100	2	95	190
	50		3100

Therefore, $\bar{x} = \frac{\sum_{i=1}^{7} f_i x_i}{\sum_{i=1}^{7} f_i} = \frac{3100}{50} = 62$

Q. 17. Find the mean deviation about the median for the following data :

class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	6	7	15	16	4	2

Answer : we make the following table from the given data:

class	Frequency f_i	Cumulative frequency c.f	Mid-points x_i
0-10	6	6	5
10-20	7	13	15
20-30	15	28	25
30-40	16	44	35
40-50	4	48	45
50-60	2	50	55
	50		

The class interval containing $\frac{N}{2}^{th}$ or 25th item is 20-30. Therefore, 20–30 is the median class. We know that

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

Here, l = 20, C = 13, f = 15, h = 10 and N = 50

Therefore, $\text{Median} = 20 + \frac{\frac{50}{2} - 13}{15} \times 10 = 20 + 8 = 28$

Now,

class	Frequency f_i	Cumulative frequency c.f	Mid-points x_i	$\|x_i-M\|$	$f_i \|x_i-M\|$
0-10	6	6	5	23	138
10-20	7	13	15	13	91
20-30	15	28	25	3	45
30-40	16	44	35	7	112
40-50	4	48	45	17	68
50-60	2	50	55	27	54
	50				508

We have, $\sum_{i=1}^{6} f_i = 50$ and $\sum_{i=1}^{6} f_i |x_i - M| = 508$

$$\therefore \ M.D\,(M) = \frac{\sum_{i=1}^{6} f_i |x_i - M|}{\sum_{i=1}^{6} f_i}$$

$$= \frac{508}{50} = 10.16$$

Q. 18. Find the mean deviation about the median for the following data :

class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	6	8	11	18	5	2

Answer : we make the following table from the given data:

class	Frequency f_i	Cumulative frequency c.f	Mid-points x_i
0-10	6	6	5
10-20	8	14	15
20-30	11	25	25
30-40	18	43	35
40-50	5	48	45
50-60	2	50	55
	50		

The class interval containing $\frac{N}{2}^{th}$ or 25th item is 20-30. Therefore, 20–30 is the median class. We know that

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

Here, l = 20, C = 14, f = 11, h = 10 and N = 50

Therefore,

$$\text{Median} = 20 + \frac{\frac{50}{2} - 14}{11} \times 10 = 20 + 10 = 30$$

Now,

class	Frequency f_i	Cumulative frequency c.f	Mid-points x_i	$\|x_i\text{-}M\|$	$f_i\ \|x_i\text{-}M\|$
0-10	6	6	5	25	150
10-20	8	14	15	15	120
20-30	11	25	25	5	55
30-40	18	43	35	5	90
40-50	5	48	45	15	75
50-60	2	50	55	25	50
	50				540

We have, $\sum_{i=1}^{6} f_i = 50$ and $\sum_{i=1}^{6} f_i |x_i - M| = 540$

$$\therefore \text{ M.D (M)} = \frac{\sum_{i=1}^{6} f_i |x_i - M|}{\sum_{i=1}^{6} f_i}$$

$$= \frac{540}{50} = 10.8$$

Exercise 30B

Q. 1. Find the mean, variance and standard deviation for the numbers 4, 6, 10, 12, 7, 8, 13, 12.

Answer : Given data: 4, 6, 10, 12, 7, 8, 13, 12

To find: MEAN

We know that,

$$\text{Mean } (\bar{x}) = \frac{\text{Sum of observations}}{\text{Total number of observations}}$$

$$= \frac{4 + 6 + 10 + 12 + 7 + 8 + 13 + 12}{8}$$

$$= \frac{72}{8}$$

$$\bar{x} = 9$$

To find: VARIANCE

x_i	$\mathbf{x_i - \bar{x}}$	$\mathbf{(x_i - \bar{x})^2}$
4	4 - 9 = -5	$(-5)^2 = 25$
6	6 - 9 = -3	$(-3)^2 = 9$
10	10 - 9 = 1	$(1)^2 = 1$
12	12 - 9 = 3	$(3)^2 = 9$
7	7 - 9 = -2	$(-2)^2 = 4$
8	8 - 9 = -1	$(-1)^2 = 1$
13	13 - 9 = 4	$(4)^2 = 16$
12	12 - 9 = 3	$(3)^2 = 9$
		$\sum (x_i - \bar{x})^2 = 74$

$$\text{Variance}, \sigma^2 = \frac{\sum (x_i - \overline{x})^2}{n}$$

$$= \frac{74}{8}$$

= 9.25

To find: STANDARD DEVIATION

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$

$$= \sqrt{9.25}$$

= 3.04

Q. 2. Find the mean, variance and standard deviation for the first six odd natural numbers.

Answer : Odd natural numbers = 1, 3, 5, 7, 9, …

First Six Odd Natural Numbers = 1, 3, 5, 7, 9, 11

To find: MEAN

We know that,

$$\text{Mean } (\overline{x}) = \frac{\text{Sum of observations}}{\text{Total number of observations}}$$

$$= \frac{1 + 3 + 5 + 7 + 9 + 11}{6}$$

$$= \frac{36}{6}$$

$$\overline{x} = 6$$

To find: VARIANCE

x_i	$\mathbf{x_i - \bar{x}}$	$\mathbf{(x_i - \bar{x})^2}$
1	1 - 6 = -5	$(-5)^2 = 25$
3	3 - 6 = -3	$(-3)^2 = 9$
5	5 - 6 = -1	$(-1)^2 = 1$
7	7 - 6 = 1	$(1)^2 = 1$
9	9 - 6 = 3	$(3)^2 = 9$
11	11 - 6 = 5	$(5)^2 = 25$
		$\sum(x_i - \bar{x})^2 = 70$

$$\text{Variance}, \sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n}$$

$$= \frac{70}{6}$$

= 11.67

To find: STANDARD DEVIATION

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$

$$= \sqrt{11.67}$$

= 3.41

Q. 3. Using short cut method, find the mean, variation and standard deviation for the data :

x_i	**4**	**8**	**11**	**17**	**20**	**24**	**32**
f_i	**3**	**5**	**9**	**5**	**4**	**3**	**1**

Answer : To find: MEAN

Now,

(x_i)	(f_i)	$x_i f_i$
4	3	12
8	5	40
11	9	99
17	5	85
20	4	80
24	3	72
32	1	32
Total	$\sum f_i = 30$	$\sum f_i x_i = 420$

Now,

$$\text{Mean } (\overline{x}) = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{420}{30}$$

=14

To find: VARIANCE

(x_i)	(f_i)	$\mathbf{x_i - \bar{x}}$	$\mathbf{(x_i - \bar{x})^2}$	$\mathbf{f_i(x_i - \bar{x})^2}$
4	3	4 - 14 = -10	$(-10)^2 = 100$	3 × 100 = 300
8	5	8 - 14 = -6	$(-6)^2 = 36$	5 × 36 = 180
11	9	11 - 14 = -3	$(-3)^2 = 9$	9 × 9 = 81
17	5	17 - 14 = 3	$(3)^2 = 9$	5 × 9 = 45
20	4	20 - 14 = 6	$(6)^2 = 36$	4 × 36 = 144
24	3	24 - 14 = 10	$(10)^2 = 100$	3 × 100 = 300
32	1	32 - 14 = 18	$(18)^2 = 324$	1 × 324 = 324
	N = 30			$\sum f_i(x_i - \bar{x})^2 = 1374$

$$\text{Variance}, \sigma^2 = \frac{\sum f_i(x_i - \bar{x})^2}{N}$$

$$= \frac{1374}{30}$$

= 45.8

To find: STANDARD DEVIATION

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$

$$= \sqrt{45.8}$$

= 6.77

Q. 4. Using short cut method, find the mean, variation and standard deviation for the data :

x_i	**6**	**10**	**14**	**18**	**24**	**28**	**30**
f_i	**2**	**4**	**7**	**12**	**8**	**4**	**3**

Answer : To find: MEAN

(x_i)	(f_i)	$x_i f_i$
6	2	12
10	4	40
14	7	98
18	12	216
24	8	192
28	4	112
30	3	90
Total	$\sum f_i = 40$	$\sum f_i x_i = 760$

Now, $\text{Mean } (\bar{x}) = \frac{\sum f_i x_i}{\sum f_i}$

$= \frac{760}{40}$

=19

To find: VARIANCE

(x_i)	(f_i)	$\mathbf{x_i - \bar{x}}$	$\mathbf{(x_i - \bar{x})^2}$	$\mathbf{f_i(x_i - \bar{x})^2}$
6	2	6 – 19 = -13	$(-13)^2 = 169$	2 × 169 = 338
10	4	10 – 19 = -9	$(-9)^2 = 81$	4 × 81 = 324
14	7	14 – 19 = -5	$(-5)^2 = 25$	7 × 25 = 175
18	12	18 – 19 = -1	$(-1)^2 = 1$	12 × 1 = 12
24	8	24 – 19 = 5	$(5)^2 = 25$	8 × 25 = 200
28	4	28 – 19 = 9	$(9)^2 = 81$	4 × 81 = 324
30	3	30 – 19 = 11	$(11)^2 = 121$	3 × 121 = 363
Total	N = 40			$\sum f_i(x_i - \bar{x})^2 = 1736$

$$\text{Variance}, \sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{N}$$

$$= \frac{1736}{40}$$

= 43.4

To find: STANDARD DEVIATION

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$

$$= \sqrt{43.4}$$

= 6.58

Q. 5. Using short cut method, find the mean, variation and standard deviation for the data :

x_i	**10**	**15**	**18**	**20**	**25**
f_i	**3**	**2**	**5**	**8**	**2**

Answer : To find: MEAN

(x_i)	(f_i)	x_if_i
10	3	30
15	2	30
18	5	90
20	8	160
25	2	50
Total	$\sum f_i = 20$	$\sum f_ix_i = 390$

Now, $\text{Mean } (\overline{x}) = \frac{\sum f_i x_i}{\sum f_i}$

$= \frac{390}{20}$

=19.5

To find: VARIANCE

(x_i)	(f_i)	$\mathbf{x_i - \bar{x}}$	$\mathbf{(x_i - \bar{x})^2}$	$\mathbf{f_i(x_i - \bar{x})^2}$
10	3	10 – 19.5 = -9.5	$(-9.5)^2$ = 90.25	3 × 90.25 = 270.75
15	2	15 – 19.5 = -4.5	$(-4.5)^2$ = 20.25	2 × 20.25 = 40.5
18	5	18 – 19.5 = -1.5	$(-1.5)^2$ = 2.25	5 × 2.25 = 11.25
20	8	20 – 19.5 = 0.5	$(0.5)^2$ = 0.25	8 × 0.25 = 2
25	2	25 – 19.5 = 5.5	$(5.5)^2$ = 30.25	2 × 30.25 = 60.5
Total	N = 20			$\sum f_i(x_i - \bar{x})^2 = 385$

$$\text{Variance}, \sigma^2 = \frac{\sum f_i (x_i - \overline{x})^2}{N}$$

$$= \frac{385}{20}$$

= 19.25

To find: STANDARD DEVIATION

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$

$$= \sqrt{19.25}$$

= 4.39

Q. 6. Using short cut method, find the mean, variation and standard deviation for the data :

x_i	**92**	**93**	**97**	**98**	**102**	**104**	**109**
f_i	**3**	**2**	**3**	**2**	**6**	**3**	**3**

Answer : To find: MEAN

(x_i)	(f_i)	$x_i f_i$
92	3	276
93	2	186
97	3	291
98	2	196
102	6	612
104	3	312
109	3	327
Total	$\sum f_i = 22$	$\sum f_i x_i = 2200$

Now, $\text{Mean } (\overline{x}) = \frac{\sum f_i x_i}{\sum f_i}$

$$= \frac{2200}{22}$$

=100

To find: VARIANCE

(x_i)	(f_i)	$\mathbf{x_i - \bar{x}}$	$(\mathbf{x_i - \bar{x}})^2$	$\mathbf{f_i(x_i - \bar{x})^2}$
92	3	92 – 100 = -8	$(-8)^2 = 64$	3 × 64 = 192
93	2	93 – 100 = -7	$(-7)^2 = 49$	2 × 49 = 98
97	3	97 – 100 = -3	$(-3)^2 = 9$	3 × 9 = 27
98	2	98 – 100 = -2	$(-2)^2 = 4$	2 × 4 = 8
102	6	102 – 100 = 2	$(2)^2 = 4$	6 × 4 = 24
104	3	104 – 100 = 4	$(4)^2 = 16$	3 × 16 = 48
109	3	109 – 100 = 9	$(9)^2 = 81$	3 × 81 = 243
Total	$\sum f_i = 22$			$\sum f_i(x_i - \bar{x})^2 = 640$

$$\text{Variance}, \sigma^2 = \frac{\sum f_i (x_i - \overline{x})^2}{N}$$

$$= \frac{640}{22}$$

= 29.09

To find: STANDARD DEVIATION

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$

$$= \sqrt{29.09}$$

= 5.39

Q. 7. Using short cut method, find the mean, variation and standard deviation for the data :

Class	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**
Frequency	**5**	**8**	**15**	**16**	**6**

Answer : Here, we apply the step deviation method with A = 25 and h = 10

To find: MEAN

Class	(f_i)	Class Mark (x_i)	$d_i = x_i - A$	$y_i = \frac{d_i}{10}$	$f_i y_i$
0 - 10	5	5	5 – 25 = -20	-2	-10
10 - 20	8	15	15 – 25 = -10	-1	-8
20 - 30	15	25 = A	0	0	0
30 - 40	16	35	35 – 25 = 10	1	16
40 - 50	6	45	45 – 25 = 20	2	12
Total	$\sum f_i = 50$				$\sum f_i y_i = 10$

Now, $\text{Mean } (\bar{x}) = a + h\left(\frac{\sum f_i y_i}{\sum f_i}\right)$

$$\Rightarrow \bar{x} = 25 + 10\left(\frac{10}{50}\right)$$

$$\Rightarrow \bar{x} = 25 + \frac{100}{50}$$

$\Rightarrow \bar{x} = 25 + 2$

$\Rightarrow \bar{x} = 27$

To find: VARIANCE

(f_i)	(x_i)	y_i	y_i^2	f_iy_i	$f_iy_i^2$
5	5	-2	$(-2)^2 = 4$	-10	20
8	15	-1	$(-1)^2 = 1$	-8	8
15	25 = A	0	0	0	0
16	35	1	$(1)^2 = 1$	16	16
6	45	2	$(2)^2 = 4$	12	24
N=50				$\Sigma f_iy_i = 10$	$\Sigma f_iy_i^2 = 68$

$$\text{Variance}, \sigma^2 = \frac{h^2}{N^2}\left[N\sum f_iy_i^2 - \left(\sum f_iy_i\right)^2\right]$$

$$= \frac{(10)^2}{(50)^2}[50 \times 68 - (10)^2]$$

$$= \frac{100}{50 \times 50}[3400 - 100]$$

$$= \frac{1}{25}[3300]$$

= 132

To find: STANDARD DEVIATION

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$

$$= \sqrt{132}$$

= 11.49

Q. 8. **Using short cut method, find the mean, variation and standard deviation for the data :**

Class	**30-40**	**40-50**	**50-60**	**60-70**	**70-80**	**80-90**	**90-100**
Frequency	**3**	**7**	**12**	**15**	**8**	**3**	**2**

Answer :

Here, we apply the step deviation method with A = 65 and h = 10

To find: MEAN

Class	(f_i)	Class Mark (x_i)	$d_i = x_i - A$	$y_i = \frac{d_i}{10}$	$f_i y_i$
30 – 40	3	35	35 – 65 = -30	-3	-9
40 – 50	7	45	45 – 65 = -20	-2	-14
50 – 60	12	55	55 – 65 = -10	-1	-12
60 – 70	15	65 = A	0	0	0
70 – 80	8	75	75 – 65 = 10	1	8
80 – 90	3	85	85 – 65 = 20	2	6
90-100	2	95	95 – 65 = 30	3	6
Total	$\sum f_i = 50$				$\sum f_i y_i = -15$

Now, $\text{Mean } (\bar{x}) = a + h\left(\frac{\sum f_i y_i}{\sum f_i}\right)$

$\Rightarrow \bar{x} = 65 + 10\left(\frac{-15}{50}\right)$

$\Rightarrow \bar{x} = 65 - \frac{150}{50}$

$\Rightarrow \bar{x} = 65 - 3$

$\Rightarrow \bar{x} = 62$

To find: VARIANCE

(f_i)	(x_i)	y_i	y_i^2	f_iy_i	$f_iy_i^2$
3	35	-3	$(-3)^2 = 9$	-9	27
7	45	-2	$(-2)^2 = 4$	-14	28
12	55	-1	$(-1)^2 = 1$	-12	12
15	65 = A	0	0	0	0
8	75	1	$(1)^2 = 1$	8	8
3	85	2	$(2)^2 = 4$	6	12
2	95	3	$(3)^2 = 9$	6	18
N= 50				$\sum f_iy_i$ =-15	$\sum f_iy_i^2 = 105$

$$\text{Variance}, \sigma^2 = \frac{h^2}{N^2}\left[N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right]$$

$$= \frac{(10)^2}{(50)^2}[50 \times 105 - (-15)^2]$$

$$= \frac{100}{50 \times 50}[5250 - 225]$$

$$= \frac{1}{25}[5025]$$

= 201

To find: STANDARD DEVIATION

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$

$$= \sqrt{201}$$

= 14.17

Q. 9. Using short cut method, find the mean, variation and standard deviation for the data :

Class	**25-35**	**35-45**	**45-55**	**55-65**	**65-75**
Frequency	**3**	**7**	**12**	**15**	**8**

Answer : Here, we apply the step deviation method with A = 50 and h = 10

To find: MEAN

Class	(f_i)	Class Mark (x_i)	$d_i = x_i - A$ $d_i = x_i - 50$	$y_i = \frac{d_i}{10}$	$f_i y_i$
25 - 35	3	30	30 - 50 = -20	-2	-6
35 - 45	7	40	40 - 50 = -10	-1	-7
45 - 55	12	50 = A	0	0	0
55 - 65	15	60	60 - 50 = 10	1	15
65 - 75	8	70	70 - 50 = 20	2	16
Total	$\sum f_i = 45$				$\sum f_i y_i = 18$

Now, $\text{Mean } (\bar{x}) = a + h\left(\frac{\sum f_i y_i}{\sum f_i}\right)$

$$\Rightarrow \bar{x} = 50 + 10\left(\frac{18}{45}\right)$$

$$\Rightarrow \bar{x} = 50 + \frac{2 \times 18}{9}$$

$$\Rightarrow \bar{x} = 50 + 4$$

$$\Rightarrow \bar{x} = 54$$

To find: VARIANCE

(f_i)	(x_i)	y_i	y_i^2	f_iy_i	$f_iy_i^2$
3	30	-2	$(-2)^2 = 4$	-6	12
7	40	-1	$(-1)^2 = 1$	-7	7
12	50 = A	0	0	0	0
15	60	1	$(1)^2 = 1$	15	15
8	70	2	$(2)^2 = 4$	16	32
$\sum f_i = 45$				$\sum f_iy_i = 18$	$\sum f_iy_i^2 = 66$

$$\text{Variance}, \sigma^2 = \frac{h^2}{N^2}\left[N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right]$$

$$= \frac{(10)^2}{(45)^2}[45 \times 66 - (18)^2]$$

$$= \frac{10 \times 10}{45 \times 45}[2970 - 324]$$

$$= \frac{4}{81}[2646]$$

= 130.67

To find: STANDARD DEVIATION

$$\text{Standard Deviation } (\sigma) = \sqrt{\text{Variance}}$$

$$= \sqrt{130.67}$$

= 11.43

Exercise 30C

Q. 1. If the standard deviation of the numbers 2, 3, 2x, 11 is 3.5, calculate the possible values of x.

Answer : Given: Standard Deviation, σ = 3.5

and Numbers are 2, 3, 2x, 11

We know that,

$$\text{Mean } (\overline{x}) = \frac{\text{Sum of observations}}{\text{Total number of observations}}$$

$$= \frac{2 + 3 + 2x + 11}{4}$$

$$= \frac{16 + 2x}{4}$$

$\bar{x} = \dfrac{8 + x}{2}$

x_i	$\mathbf{x_i - \bar{x}}$	$\mathbf{(x_i - \bar{x})^2}$
2	$2 - \dfrac{8+x}{2} = \dfrac{4-8-x}{2} = \dfrac{-4-x}{2}$	$\left(\dfrac{-4-x}{2}\right)^2 = \dfrac{16+8x+x^2}{4}$
3	$3 - \dfrac{8+x}{2} = \dfrac{6-8-x}{2} = \dfrac{-2-x}{2}$	$\left(\dfrac{-2-x}{2}\right)^2 = \dfrac{4+4x+x^2}{4}$
2x	$2x - \dfrac{8+x}{2} = \dfrac{4x-8-x}{2} = \dfrac{3x-8}{2}$	$\left(\dfrac{3x-8}{2}\right)^2 = \dfrac{64-48x+9x^2}{4}$
11	$11 - \dfrac{8+x}{2} = \dfrac{22-8-x}{2} = \dfrac{14-x}{2}$	$\left(\dfrac{14-x}{2}\right)^2 = \dfrac{196-28x+x^2}{4}$

Variance, $\sigma^2 = \dfrac{1}{n}\sum (x_i - \bar{x})^2$

$$(3.5)^2 = \frac{1}{4}\left[\frac{16+8x+x^2}{4} + \frac{4+4x+x^2}{4} + \frac{64-48x+9x^2}{4} + \frac{9-6x+x^2}{4}\right]$$

$$\Rightarrow 12.25 = \frac{1}{16}[16 + 8x + x^2 + 4 + 4x + x^2 + 64 - 48x + 9x^2 + 196 - 28x + x^2]$$

$\Rightarrow 12.25 \times 16 = 280 - 64x + 12x^2$

$\Rightarrow 196 = 280 - 64x + 12x^2$

$\Rightarrow 12x^2 - 64x + 280 - 196 = 0$

$\Rightarrow 12x^2 - 64x + 84 = 0$

$\Rightarrow 3x^2 - 16x + 21 = 0$

$\Rightarrow 3x^2 - 9x - 7x + 21 = 0$

$\Rightarrow 3x(x - 3) - 7(x - 3) = 0$

$\Rightarrow (3x - 7)(x - 3) = 0$

Putting both the factors equal to 0, we get

$3x - 7 = 0$ and $x - 3 = 0$

$\Rightarrow 3x = 7$ and $x = 3$

$$\Rightarrow x = \frac{7}{3}$$

Hence, the possible values of x are $\frac{7}{3}$ & 3

Q. 2. **The variance of 15 observations is 6. If each observation is increased by 8, find the variance of the resulting observations.**

Answer : Let the observations are $x_1, x_2, x_3, x_4, \ldots, x_{15}$

and Let mean = $\bar{x}$

Given: Variance = 6 and n = 15

We know that,

$$\text{Variance}, \sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$$

Putting the given values, we get

$$6 = \frac{1}{15}\sum(x_i - \bar{x})^2$$

$$\Rightarrow 6 \times 15 = \sum(x_i - \bar{x})^2$$

$$\Rightarrow 90 = \sum(x_i - \bar{x})^2$$

or $\sum(x_i - \bar{x})^2 = 90$ …(i)

It is given that each observation is increased by 8, we get new observations

Let the new observation be $y_1, y_2, y_3, \ldots, y_{15}$

where $y_i = x_i + 8$ …(ii) or $x_i = y_i - 8$ …(iii)

Now, we find the variance of new observations

$$\text{i. e. New Variance } = \frac{1}{n}\sum(y_i - \bar{y})^2$$

Now, we calculate the value of $\bar{y}$

We know that,

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Total number of observations}}$$

$$\Rightarrow \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

$\Rightarrow \bar{y} = \frac{\sum_{i=1}^{15} x_i + 8}{15}$ [from eq. (ii)]

$$\Rightarrow \bar{y} = \left(\frac{1}{15}\right)\left\{\sum_{i=1}^{15}(x_i + 8)\right\}$$

$$\Rightarrow \bar{y} = \frac{1}{15}\left[\sum_{i=1}^{15} x_i + 8\sum_{i=1}^{15} 1\right]$$

$$\Rightarrow \bar{y} = \frac{1}{15}\sum_{i=1}^{15} x_i + 8 \times \frac{15}{15}$$

$$\Rightarrow \bar{y} = \bar{x} + 8$$

$$\Rightarrow \bar{x} = \bar{y} - 8$$ …(iv)

Putting the value of eq. (iii) and (iv) in eq. (i), we get

$$\sum (x_i - \bar{x})^2 = 90$$

$$\sum (y_i - 8 - (\bar{y} - 8))^2 = 90$$

$$\Rightarrow \sum (y_i - 8 - \bar{y} + 8)^2 = 90$$

$$\Rightarrow \sum (y_i - \bar{y})^2 = 90$$ So,

$$\text{New Variance } = \frac{1}{n}\sum (y_i - \bar{y})^2$$

$$= \frac{1}{15} \times 90$$

= 6

Q. 3. The variance of 20 observations is 5. If each observation is multiplied by 2. Find the variance of the resulting observations

Answer : Let the observations are $x_1, x_2, x_3, x_4, \ldots, x_{20}$

and Let mean = $\bar{x}$

Given: Variance = 5 and n = 20

We know that,

$$\text{Variance}, \sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$$

Putting the given values, we get

$$5 = \frac{1}{20}\sum(x_i - \bar{x})^2$$

$$\Rightarrow 5 \times 20 = \sum(x_i - \bar{x})^2$$

$$\Rightarrow 100 = \sum(x_i - \bar{x})^2$$

or $\sum(x_i - \bar{x})^2 = 100$ …(i)

It is given that each observation is multiplied by 2, we get new observations

Let the new observation be $y_1, y_2, y_3, \ldots, y_{20}$

where $y_i = 2(x_i)$ …(ii)

or $x_i = \frac{1}{2}y_i$ …(iii)

Now, we find the variance of new observations

$$\text{i.e. New Variance } = \frac{1}{n}\sum(y_i - \bar{y})^2$$

Now, we calculate the value of $\bar{y}$

We know that,

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Total number of observations}}$$

$$\Rightarrow \bar{y} = \frac{\sum y_i}{n}$$

$\Rightarrow \bar{y} = \frac{\sum(2x_i)}{20}$ [from eq. (ii)]

$$\Rightarrow \bar{y} = 2\left(\frac{\sum x_i}{20}\right)$$

$$\Rightarrow \bar{y} = 2\bar{x}$$

$$\Rightarrow \bar{x} = \frac{1}{2}\bar{y}$$ ...(iv)

Putting the value of eq. (iii) and (iv) in eq. (i), we get

$$\sum (x_i - \bar{x})^2 = 100$$

$$\sum \left(\frac{1}{2}y_i - \frac{1}{2}\bar{y}\right)^2 = 100$$

$$\Rightarrow \sum \left(\frac{1}{2}\right)^2 (y_i - \bar{y})^2 = 100$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 \sum (y_i - \bar{y})^2 = 100$$

$$\Rightarrow \sum (y_i - \bar{y})^2 = 100 \times 4$$

$$\Rightarrow \sum (y_i - \bar{y})^2 = 400$$

So,

$$\text{New Variance } = \frac{1}{n}\sum (y_i - \bar{y})^2$$

$$= \frac{1}{20} \times 400$$

= 20

Q. 4. **The mean and variance of five observations are 6 and 4 respectively. If three of these are 5, 7 and 9, find the other two observations.**

Answer : Given: Mean of 5 observations = 6 and Variance of 5 observations = 4

Let the other two observations be x and y

∴, our observations are 5, 7, 9, x and y

Now, we know that,

$$\text{Mean } (\bar{x}) = \frac{\text{Sum of observations}}{\text{Total number of observations}}$$

$$6 = \frac{5 + 7 + 9 + x + y}{5}$$

⇒ 6 × 5 = 21 + x + y

⇒ 30 – 21 = x + y

⇒ 9 = x + y

or x + y = 9 …(i)

Also,

Variance = 4

$$\text{Variance}, \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

x_i	$\mathbf{x_i - \bar{x}}$ = x_i - 6	$\mathbf{(x_i - \bar{x})^2}$
5	5 - 6 = -1	$(-1)^2 = 1$
7	7 - 6 = 1	$(1)^2 = 1$
9	9 - 6 = 3	$(3)^2 = 9$
x	x - 6	$(x - 6)^2$
y	y - 6	$(y - 6)^2$
		$\sum (x_i - \bar{x})^2 = 11 + (x - 6)^2 + (y - 6)^2$

So, $\text{Variance}, \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$

$$4 = \frac{11 + (x - 6)^2 + (y - 6)^2}{5}$$

⇒ 20 = 11 + (x^2 + 36 – 12x) + (y^2 + 36 – 12y)⇒ 20 – 11 = x^2 + 36 – 12x + y^2 + 36 – 12y

⇒ 9 = x^2 + y^2 + 72 – 12(x + y)

$\Rightarrow x^2 + y^2 + 72 - 12(9) - 9 = 0$ [from (i)]

$\Rightarrow x^2 + y^2 + 63 - 108 = 0$

$\Rightarrow x^2 + y^2 - 45 = 0$

$\Rightarrow x^2 + y^2 = 45$ …(ii)

From eq. (i)

$x + y = 9$

Squaring both the sides, we get

$(x + y)^2 = (9)^2$

$\Rightarrow x^2 + y^2 + 2xy = 81$

$\Rightarrow 45 + 2xy = 81$ [from (ii)]

$\Rightarrow 2xy = 81 - 45$

$\Rightarrow 2xy = 36$

$\Rightarrow xy = 18$

$\Rightarrow x = \frac{18}{y}$ …(iii)

Putting the value of x in eq. (i), we get

$x + y = 9$

$$\Rightarrow \frac{18}{y} + y = 9$$

$$\Rightarrow \frac{18 + y^2}{y} = 9$$

$\Rightarrow y^2 + 18 = 9y$

$\Rightarrow y^2 - 9y + 18 = 0$

$\Rightarrow y^2 - 6y - 3y + 18 = 0$

⇒ y(y – 6) – 3(y – 6)= 0

⇒ (y – 3)(y – 6) = 0

⇒ y – 3 = 0 and y – 6 = 0

⇒ y = 3 and y = 6

For y = 3

$$x = \frac{18}{y} = \frac{18}{3} = 6$$

Hence, x = 6, y = 3 are the remaining two observations

For y = 6

$$x = \frac{18}{y} = \frac{18}{6} = 3$$

Hence, x = 3, y = 6 are the remaining two observations

Thus, remaining two observations are 3 and 6.

Q. 5. The mean and variance of five observations are 4.4 and 8.24 respectively. If three of these are 1, 2 and 6, find the other two observations.

Answer : Given: Mean of 5 observations = 4.4

and Variance of 5 observations = 8.24

Let the other two observations be x and y

∴, our observations are 1, 2, 6, x and y

Now, we know that,

$$\text{Mean } (\bar{x}) = \frac{\text{Sum of observations}}{\text{Total number of observations}}$$

$$4.4 = \frac{1 + 2 + 6 + x + y}{5}$$

⇒ 5 × 4.4 = 9 + x + y

⇒ 22 – 9= x + y

⇒ 13 = x + y

or x + y = 13 …(i)

Also,

Variance = 8.24

$$\text{Variance}, \sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n}$$

x_i	$\mathbf{x_i - \bar{x}}$ $= x_i - 4.4$	$(\mathbf{x_i - \bar{x}})^2$
1	1 - 4.4 = -3.4	$(-3.4)^2 = 11.56$
2	2 - 4.4 = -2.4	$(-2.4)^2 = 5.76$
6	6 - 4.4 = 1.6	$(1.6)^2 = 2.56$
x	x - 4.4	$(x - 4.4)^2$
y	y - 4.4	$(y - 4.4)^2$
		$\sum (x_i - \bar{x})^2 = 19.88 + (x - 4.4)^2 + (y - 4.4)^2$

So,

$$\text{Variance}, \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$8.24 = \frac{19.88 + (x - 4.4)^2 + (y - 4.4)^2}{5}$$

$\Rightarrow 41.2 = 19.88 + (x^2 + 19.36 – 8.8x) + (y^2 + 19.36 – 8.8y)$

$\Rightarrow 41.2 - 19.88 = x^2 + 19.36 - 8.8x + y^2 + 19.36 - 8.8y$

$\Rightarrow 21.32 = x^2 + y^2 + 38.72 - 8.8(x + y)$

$\Rightarrow x^2 + y^2 + 38.72 - 8.8(13) - 21.32 = 0$ [from (i)]

$\Rightarrow x^2 + y^2 + 17.4 - 114.4 = 0$

$\Rightarrow x^2 + y^2 - 97 = 0$

$\Rightarrow x^2 + y^2 = 97$ …(ii)

From eq. (i)

$x + y = 17.4$

Squaring both the sides, we get

$(x + y)^2 = (13)^2$

$\Rightarrow x^2 + y^2 + 2xy = 169$

$\Rightarrow 97 + 2xy = 169$ [from (ii)]

$\Rightarrow 2xy = 169 - 97$

$\Rightarrow 2xy = 72$

$\Rightarrow xy = 36$

$\Rightarrow x = \frac{36}{y}$ …(iii)

Putting the value of x in eq. (i), we get

$x + y = 13$

$$\Rightarrow \frac{36}{y} + y = 13$$

$$\Rightarrow \frac{36 + y^2}{y} = 13$$

$\Rightarrow y^2 + 36 = 13y$

$\Rightarrow y^2 - 13y + 36 = 0$

$\Rightarrow y^2 - 4y - 9y + 36 = 0$

$\Rightarrow y(y - 4) - 9(y - 4)= 0$

$\Rightarrow (y - 4)(y - 9) = 0$

$\Rightarrow y - 4 = 0$ and $y - 9 = 0$

$\Rightarrow y = 4$ and $y = 9$

For $y = 4$

$$x = \frac{36}{y} = \frac{36}{4} = 9$$

Hence, $x = 9$, $y = 4$ are the remaining two observations

For $y = 9$

$$x = \frac{36}{y} = \frac{36}{9} = 4$$

Hence, $x = 4$, $y = 9$ are the remaining two observations

Thus, remaining two observations are 4 and 9.

Q. 6. The mean and standard deviation of 18 observations are found to be 7 and 4 respectively. On rechecking it was found that an observation 12 was misread as 21. Calculate the correct mean and standard deviation.

Answer : Given that number of observations (n) = 18

Incorrect Mean $(\bar{x}) = 7$

and Incorrect Standard deviation, $(\sigma) = 4$

We know that,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\Rightarrow 7 = \frac{1}{18}\sum_{i=1}^{18} x_i$$

$$\Rightarrow 7 \times 18 = \sum_{i=1}^{18} x_i$$

$$\Rightarrow 126 = \sum_{i=1}^{18} x_i$$

$\Rightarrow \sum_{i=1}^{18} x_i = 126$ …(i)

∴ Incorrect sum of observations = 126

Finding correct sum of observations, 12 was misread as 21

So, Correct sum of observations = Incorrect Sum – 21 + 12

= 126 – 21 + 12

= 117

Hence,

$$\text{Correct Mean} = \frac{\text{Correct Sum of Observations}}{\text{Total number of observations}}$$

$$= \frac{117}{18}$$

= 6.5

Now, Incorrect Standard Deviation (σ)

$$= \frac{1}{N}\sqrt{N \times \left(\text{Incorrect} \sum x_i^{\,2}\right) - (\text{Incorrect} \sum x_i)^2}$$

$$4 = \frac{1}{18}\sqrt{18 \times \left(\text{Incorrect} \sum x_i^{\ 2}\right) - (126)^2}$$

$$4 \times 18 = \sqrt{18 \times \left(\text{Incorrect} \sum x_i^{\ 2}\right) - (126)^2}$$

$$72 = \sqrt{18 \times \left(\text{Incorrect} \sum x_i^{\ 2}\right) - (126)^2}$$

Squaring both the sides, we get

$$(72)^2 = 18 \times \left(\text{Incorrect} \sum x_i^{\ 2}\right) - (126)^2$$

$$\Rightarrow 5184 = 18 \times \left(\text{Incorrect} \sum x_i^{\ 2}\right) - 15876$$

$$\Rightarrow 5184 + 15876 = 18 \times \text{Incorrect} \sum x_i^{\ 2}$$

$$\Rightarrow 21060 = 18 \times \text{Incorrect} \sum x_i^{\ 2}$$

$$\Rightarrow \frac{21060}{18} = \text{Incorrect} \sum x_i^{\ 2}$$

$$\Rightarrow 1170 = \text{Incorrect} \sum x_i^{\ 2}$$

Since, 12 was misread as 21

So,

$$\text{Correct} \sum_{i=1}^{18} x_i^2 = 1170 - (21)^2 + (12)^2$$

= 1170 – 441 + 144

= 873

Now,

Correct Standard Deviation

$$= \sqrt{\frac{(\text{Correct} \sum x_i^2)}{N} - \left(\frac{\text{Correct } \sum x_i}{N}\right)^2}$$

$$= \sqrt{\frac{873}{18} - (6.5)^2} \left[\because \overline{x} = \frac{\text{Correct } \sum x_i}{N} = 6.5\right]$$

$$= \sqrt{48.5 - 42.25}$$

$$= \sqrt{6.25}$$

= 2.5

Hence, Correct Mean = 6.5

and Correct Standard Deviation = 2.5

Q. 7. For a group of 200 candidates, the mean and standard deviations of scores were found to be 40 and 15 respectively. Later on it was discovered that the score of 43 was misread as 34. Find the correct mean and standard deviation.

Answer : Given that number of observations (n) = 200

Incorrect Mean ($\overline{x}$) = 40

and Incorrect Standard deviation, (σ) = 15

We know that,

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\Rightarrow 40 = \frac{1}{200} \sum x_i$$

$\Rightarrow 40 \times 200 = \sum x_i$

$\Rightarrow 8000 = \sum x_i$

$\Rightarrow \sum x_i = 8000$ …(i)

∴ Incorrect sum of observations = 8000

Finding correct sum of observations, 43 was misread as 34

So, Correct sum of observations = Incorrect Sum – 34 + 43

= 8000 – 34 + 43

= 8009

Hence,

$$\text{Correct Mean} = \frac{\text{Correct Sum of Observations}}{\text{Total number of observations}}$$

$$= \frac{8009}{200}$$

= 40.045

Now, Incorrect Standard Deviation (σ)

$$= \frac{1}{N}\sqrt{N \times \left(\text{Incorrect} \sum x_i^{\,2}\right) - (\text{Incorrect} \sum x_i)^2}$$

$$15 = \frac{1}{200}\sqrt{200 \times \left(\text{Incorrect} \sum x_i^{\,2}\right) - (8000)^2}$$

$$15 \times 200 = \sqrt{200 \times \left(\text{Incorrect} \sum x_i^{\,2}\right) - 64000000}$$

$$3000 = \sqrt{200 \times \left(\text{Incorrect} \sum x_i^{2}\right) - 64000000}$$

Squaring both the sides, we get

$$(3000)^2 = 200 \times \left(\text{Incorrect} \sum x_i^{2}\right) - 64000000$$

$$\Rightarrow 9000000 = 200 \times \left(\text{Incorrect} \sum x_i^{2}\right) - 64000000$$

$$\Rightarrow 9000000 + 64000000 = 200 \times \left(\text{Incorrect} \sum x_i^{2}\right)$$

$$\Rightarrow 73000000 = 200 \times \left(\text{Incorrect} \sum x_i^{2}\right)$$

$$\Rightarrow \frac{73000000}{200} = \left(\text{Incorrect} \sum x_i^{2}\right)$$

$$\Rightarrow 365000 = \left(\text{Incorrect} \sum x_i^{2}\right)$$

Since, 43 was misread as 34

So,

$$\text{Correct} \sum x_i^2 = 365000 - (34)^2 + (43)^2$$

= 365000 – 1156 + 1849

= 125000 + 693

= 365693

Now,

Correct Standard Deviation

$$= \sqrt{\frac{(\text{Correct}\sum x_i^2)}{N} - \left(\frac{\text{Correct } \sum x_i}{N}\right)^2}$$

$$= \sqrt{\frac{365693}{200} - (40.045)^2}$$

$$\left[\because \bar{x} = \frac{\text{Correct } \sum x_i}{N} = 40.045\right]$$

$$= \sqrt{1828.465 - 1603.602025}$$

$$= \sqrt{224.862975}$$

= 14.995

Hence, Correct Mean = 40.045

and Correct Standard Deviation = 14.995

Q. 8. The mean and standard deviations of a group of 100 observations were found to be 20 and 3 respectively. Later on it was found that three observations 21, 12 and 18 were incorrect. Find the mean and standard deviation if the incorrect observations were omitted.

Answer : Given that number of observations (n) = 100

Incorrect Mean ($\bar{x}$) = 20

and Incorrect Standard deviation, (σ) = 3

We know that,

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\Rightarrow 20 = \frac{1}{100}\sum x_i$$

$\Rightarrow 20 \times 100 = \sum x_i$

$\Rightarrow 2000 = \sum x_i$

$\Rightarrow \sum x_i = 2000$ …(i)

∴ Incorrect sum of observations = 2000

Finding correct sum of observations, incorrect observations 21, 12 and 18 are removed

So, Correct sum of observations = Incorrect Sum – 21 – 12 – 18

= 2000 – 51

= 1949

Hence,

$$\text{Correct Mean} = \frac{\text{Correct Sum of Observations}}{\text{Total number of observations}}$$

$$= \frac{1949}{100 - 3}$$

$$= \frac{1949}{97}$$

= 20.09

Now, Incorrect Standard Deviation (σ)

$$= \frac{1}{N}\sqrt{N \times \left(\text{Incorrect} \sum {x_i}^2\right) - (\text{Incorrect} \sum x_i)^2}$$

$$3 = \frac{1}{100}\sqrt{100 \times \left(\text{Incorrect} \sum {x_i}^2\right) - (2000)^2}$$

$$3 \times 100 = \sqrt{100 \times \left(\text{Incorrect} \sum {x_i}^2\right) - 4000000}$$

$$300 = \sqrt{100 \times \left(\text{Incorrect} \sum {x_i}^2\right) - 4000000}$$

Squaring both the sides, we get

$$(300)^2 = 100 \times \left(\text{Incorrect} \sum {x_i}^2\right) - 4000000$$

$$\Rightarrow 90000 = 100 \times \left(\text{Incorrect} \sum {x_i}^2\right) - 4000000$$

$$\Rightarrow 90000 + 4000000 = 100 \times \left(\text{Incorrect} \sum {x_i}^2\right)$$

$$\Rightarrow 4090000 = 100 \times \left(\text{Incorrect} \sum {x_i}^2\right)$$

$$\Rightarrow \frac{4090000}{100} = \left(\text{Incorrect} \sum {x_i}^2\right)$$

$$\Rightarrow 40900 = \left(\text{Incorrect} \sum {x_i}^2\right)$$

Since, 21, 12 and 18 are removed

So,

$$\text{Correct} \sum x_i^2 = 40900 - (21)^2 - (12)^2 - (18)^2$$

= 40900 – 441 – 144 – 324

= 40900 – 909

= 39991

Now,

Correct Standard Deviation

$$= \sqrt{\frac{(\text{Correct} \sum x_i^2)}{N} - \left(\frac{\text{Correct} \sum x_i}{N}\right)^2}$$

$$= \sqrt{\frac{39991}{97} - (20.09)^2}$$

$$\left[\because \bar{x} = \frac{\text{Correct} \sum x_i}{N} = 20.09\right]$$

$$= \sqrt{412.27 - 403.60}$$

$$= \sqrt{8.67}$$

= 2.94

Hence, Correct Mean = 20.09

and Correct Standard Deviation = 2.94

Exercise 30D

Q. 1. The following results show the number of workers and the wages paid to them in two factories F_1 and F_2.

Factory	**A**	**B**
Number of workers	**3600**	**3200**
Mean wages	**Rs. 5300**	**Rs. 5300**
Variance of distribution of wages	**100**	**81**

Which factory has more variation in wages?

Answer : Mean wages of both the factories are the same, i.e., Rs. 5300.

To compare variation, we need to find out the coefficient of variation (CV).

We know, CV = $\frac{\text{SD}}{\text{Mean}} \times 100$, where SD is the standard deviation.

The variance of factory A is 100 and the variance of factory B is 81.

Now, SD of factory A = $\sqrt{100} = 10$

And, SD of factory B = $\sqrt{81} = 9$

Therefore,

The CV of factory A = $\frac{10}{5300} \times 100 = 0.189$

The CV of factory B = $\frac{9}{5300} \times 100 = 0.169$

Here, the CV of factory A is greater than the CV of factory B.

Hence, factory A has more variation.

Q. 2. Coefficient of variation of the two distributions are 60% and 80% respectively, and their standard deviations are 21 and 16 respectively. Find their arithmetic means.

Answer : Given: Coefficient of variation of two distributions are 60% and 80% respectively, and their standard deviations are 21 and 16 respectively.

Need to find: Arithmetic means of the distributions.

For the first distribution,

Coefficient of variation (CV) is 60%, and the standard deviation (SD) is 21.

We know that,

$\Rightarrow \text{CV} = \frac{\text{SD}}{\text{Mean}} \times 100$

$\Rightarrow \text{Mean} = \frac{\text{SD}}{\text{CV}} \times 100$

$\Rightarrow \text{Mean} = \frac{21}{60} \times 100$

$\Rightarrow \text{Mean} = 35$

For the first distribution,

Coefficient of variation (CV) is 80%, and the standard deviation (SD) is 16.

We know that,

$\Rightarrow \text{CV} = \frac{\text{SD}}{\text{Mean}} \times 100$

$\Rightarrow \text{Mean} = \frac{\text{SD}}{\text{CV}} \times 100$

$\Rightarrow \text{Mean} = \frac{16}{80} \times 100$

$\Rightarrow \text{Mean} = 20$

Therefore, the arithmetic mean of 1st distribution is 35 and the arithmetic mean of 2nd distribution is 20.

Q. 3. The mean and variance of the heights and weights of the students of a class are given below:

	Heights	**Weights**
Mean	**63.2 inches**	**63.2 kg**
SD	**11.5 inches**	**5.6 kg**

Which shows more variability, heights or weights?

Answer : In case of heights,

Mean = 63.2 inches and SD = 11.5 inches.

So, the coefficient of variation,

$$CV = \frac{SD}{Mean} \times 100$$

$\Rightarrow$ $CV = \frac{11.5}{63.2} \times 100 = 18.196$

In case of weights,

Mean = 63.2 inches and SD = 5.6 inches.

So, the coefficient of variation,

$$CV = \frac{SD}{Mean} \times 100$$

$\Rightarrow$ $CV = \frac{5.6}{63.2} \times 100 = 8.86$

CV of heights > CV of weights

So, heights show more variability.

Q. 4. The following results show the number of workers and the wages paid to them in two factories A and B of the same industry.

Firms	**A**	**B**
Number of workers	**560**	**650**
Mean monthly wages	**Rs. 5460**	**Rs. 5460**
The variance of distribution of wages	**100**	**121**

(i) Which firm pays a larger amount as monthly wages?
(ii) Which firm shows greater variability in individual wages?

Answer : (i) Both the factories pay the same mean monthly wages.

For factory A there are 560 workers. And for factory B there are 650 workers.

So, factory A totally pays as monthly wage = (5460 x 560) Rs.

= 3057600 Rs.

Factory B totally pays as monthly wage = (5460 x 650) Rs.

= 3549000 Rs.

That means, factory B pays a larger amount as monthly wages.

(ii) Mean wages of both the factories are the same, i.e., Rs. 5460.

To compare variation, we need to find out the coefficient of variation (CV).

We know, CV = $\frac{\text{SD}}{\text{Mean}} \times 100$, where SD is the standard deviation.

The variance of factory A is 100 and the variance of factory B is 121.

Now, SD of factory A =

$$\sqrt{100} = 10$$

And, SD of factory B =

$$\sqrt{121} = 11$$

Therefore,

The CV of factory A =

$$\frac{10}{5460} \times 100 = .183$$

The CV of factory B =

$$\frac{11}{5460} \times 100 = .201$$

Here,the CV of factory B is greater than the CV of factory A.

Hence, factory B shows greater variability.

Q. 5. The sum and the sum of squares of length x (in cm) and weight y (in g) of 50 plant products are given below:

$$\sum_{i=1}^{50} x_i = 212, \sum_{i=1}^{50} x_i^2 = 902.8, \sum_{i=1}^{50} y_i = 261 \text{ and } \sum_{i=1}^{50} y_i^2 = 1457.6$$

Which is more variable, the length or weight?

Answer : To find the more variable, we again need to compare the coefficients of variation (CV).

Here the number of products are n = 50 for length and weight both.

For length,

Mean = $\frac{\sum x_i}{n} = \frac{212}{50} = 4.24$

Variance = $\frac{1}{n^2}[n\sum x_i^2 - (\sum x_i)^2]$

= $\frac{1}{50^2}[(50 \times 902.8) - (212)^2]$

= $\frac{1}{2500}[45140 - 44944]$

= $\frac{196}{2500} = 0.0784$

So, standard deviation, SD = $\sqrt{\text{Variance}} = \sqrt{0.0784} = 0.28$

Therefore, the coefficient of variation of length,

$CV_L = \frac{0.28}{4.24} \times 100 = 6.603$

For weight,

Mean = $\frac{\sum y_i}{n} = \frac{261}{50} = 5.22$

Variance = $\frac{1}{n^2}[n\sum y_i^2 - (\sum y_i)^2]$

$= \frac{1}{50^2}[(50 \times 1457.6) - (261)^2]$

$= \frac{1}{2500}[72880 - 68121]$

$= \frac{4759}{2500} = 1.9036$

So, standard deviation, SD = $\sqrt{\text{Variance}} = \sqrt{1.9036} = 1.37$

Therefore, the coefficient of variation of length,

$CV_W = \frac{1.37}{5.22} \times 100 = 26.245$

Now, $CV_W > CV_L$

Therefore, the weight is more variable than height.