QB365 Question Bank Software

12 Maths case study questions Continuity and Differentiability -2024

12th Standard

Maths

Section - A 2 x 4 = 8

1) Derivative of y = f(x) w.r.t. x (if exists) is denoted by $\frac{dy}{dx}$ .or f(x) and is called the first order derivative of y If we take derivative of $\frac{dy}{dx}$ again,

then we get $\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d^2y}{dx^2}$ or f''(x) and is called the second order derivative of y.

Similarly $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$ is denoted and defined as $\frac{d^3y}{dx^3}$ or $f'''(x)$ and is known as third order derivative of y.

Based on the above information, answer the following questions.

(i) If $y=\tan^{-1}\left(\frac{\log(e/x^2)}{\log(ex^2)}\right)+\tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$, then $\frac{d^2y}{dx^2}$ is equal to

(a) 2 **(b) 1** **(c) 0** **(d) -1**

(ii) If u = $x^2 + y^2$ and x = s + 3t, y = 2s - t, then $\frac{d^2u}{ds^2}$ is equal to

(a) 12 **(b) 32** **(c) 36** **(d) 10**

(iii) If f(x) = 2 log sin x, then f''(x) is equal to

(a) 2cosec^3x **(b) $2\cot^2 x - 4x^2 \text{cosec}^2 x2$** **(c) $2x \cot x^2$** **(d) -2cosec^2x**

(iv) If f(x) = $e^x\sin x$, then f'''(x) =

(a) 2tf(sin x + cos x) **(b) $2e^x$(cos x - sin x)** **(c) $2e^x$(sin x - cos x)** **(d) 0**

(v) If $y^2 = ax^2 + bx + c$, then $\frac{d}{dx}\left(y^3y_2\right)=$

(a) 1 **(b) -1** **(c) $\frac{4ac-b^2}{a^2}$** **(d) 0**

Answer : (i) (c) : Given, $y=\tan^{-1}\left(\frac{\log\left(\frac{e}{x^2}\right)}{\log ex^2}\right)+\tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$

$=\tan^{-1}\left(\frac{1-\log x^2}{1+\log x^2}\right)+\tan^{-1}\left(\frac{3+2\log x}{1-6\log x}\right)$

$=\tan^{-1}(1)-\tan^{-1}(2\log x)+\tan^{-1}(3)+\tan^{-1}(2\log x)$

$\Rightarrow y=\tan^{-1}(1)+\tan^{-1}(3)$

$\Rightarrow \frac{dy}{dx}=0\Rightarrow\frac{d^2y}{dx^2}=0$

(ii) (d) : Given, $x=s+3t, y=2s-t\Rightarrow\frac{dx}{ds}=1,\frac{dy}{ds}=2$

Now, $u=x^2+y^2\Rightarrow\frac{du}{ds}=2x\frac{dx}{ds}+2y\frac{dy}{ds}=2x+4y$

$\Rightarrow\frac{d^2u}{ds^2}=2\left(\frac{dx}{ds}\right)+4\left(\frac{dy}{ds}\right)\Rightarrow\frac{d^2u}{ds^2}=2(1)+4(2)=10$

(iii) (d) : We have, j(x) = 2 log sin x

$\Rightarrow f'(x)=2\cdot\frac{1}{\sin x}\cdot\cos x=2\cot x\Rightarrow f''(x)=-2\operatorname{cosec}^2 x$

(iv) (b) : We have, f(x) = e^xsinx

$\Rightarrow f'(x)=e^x\cos x+e^x\sin x=e^x(\cos x+\sin x)$

$\Rightarrow f''(x)=e^x(\cos x-\sin x)+e^x(\cos x+\sin x)=2e^x\cos x$

$\Rightarrow f'''(x)=2\left[e^x\cos x-e^x\sin x\right]=2e^x[\cos x-\sin x]$

(v) (d) : Given $y^2 = ax^2 + bx + c$

$\Rightarrow 2yy_1=2ax+b$

$\Rightarrow 2yy_2+y_1(2y_1)=2a$

$\Rightarrow yy_2=a-y_1^2\Rightarrow yy_2=a-\left(\frac{2ax+b}{2y}\right)^2$ (Using (i))

$=\frac{4y^2a-(4a^2x^2+b^2+4abx)}{4y^2}$

$\Rightarrow y^3y_2=\frac{4a(ax^2+bx+c)-(4a^2x^2+b^2+4abx)}{4}$

$=\frac{4ac-b^2}{4}$

$\Rightarrow\frac{d}{dx}(y^3y_2)=0$

2) If a relation between x and y is such that y cannot be expressed in terms of x, then y is called an implicit function of x.

When a given relation expresses y as an implicit function of x and we want to find $\frac{dy}{dx}$.then we differentiate every term of the given relation w.r.t. x. remembering that a term in y is first differentiated w.r.t. y and then multiplied by $\frac{dy}{dx}$.

Based on the above information, find the value of $\frac{dy}{dx}$ in each of the following questions

(i) $x^3+x^2y+xy^2+y3=81$

(a) $\frac{(3x^2+2xy+y^2)}{x^2+2xy+3y^2}$ (b) $\frac{-(3x^2+2xy+y^2)}{x^2+2xy+3y^2}$ (c) $\frac{(3x^2+2xy-y^2)}{x^2-2xy+3y^2}$ (d) $\frac{3x^2+xy+y^2}{x^2+xy+3y^2}$

(ii) x^y = c- y

(a) $\frac{x-y}{(1+\log x)}$ (b) $\frac{x+y}{(1+\log x)}$ (c) $\frac{x-y}{x(1+\log x)}$ (d) $\frac{x+y}{x(1+\log x)}$

(iii) $e^{\sin y}$ = xy

(a) $\frac{-y}{x(y\cos y-1)}$ (b) $\frac{y}{y\cos y-1}$ (c) $\frac{y}{y\cos y+1}$ (d) $\frac{y}{x(y\cos y-1)}$

(iv) $\sin^2 x + \cos^2 y = 1$

(a) $\frac{\sin 2y}{\sin 2x}$ **(b)** $-\frac{\sin 2x}{\sin 2y}$ **(c)** $-\frac{\sin 2y}{\sin 2x}$ **(d)** $\frac{\sin 2x}{\sin 2y}$

(v) $y = (\sqrt{x})^{\sqrt{x}}$

(a) $\frac{-y^2}{x(2-y\log x)}$ **(b)** $\frac{y^2}{2+y\log x}$ **(c)** $\frac{y^2}{x(2+y\log x)}$ **(d)** $\frac{y^2}{x(2-y\log x)}$

Answer : (i) (b) : $x^3 + x^2y + xy^2 + y^3 = 81$

$$\Rightarrow 3x^2 + x^2\frac{dy}{dx} + 2xy + 2xy\frac{dy}{dx} + y^2 + 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \left(x^2 + 2xy + 3y^2\right)\frac{dy}{dx} = -3x^2 - 2xy - y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\left(3x^2+2xy+y^2\right)}{x^2+2xy+3y^2}$$

(ii) (c) : $x^y = e^{x-y} \Rightarrow y \log x = x - y$

$$\Rightarrow y \times \frac{1}{x} + \log x \cdot \frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}[\log x + 1] = 1 - \frac{y}{x} \Rightarrow \frac{dy}{dx} = \frac{x-y}{x[1+\log x]}$$

(iii) (d) : $e^{\sin y} = xy \Rightarrow \sin y = \log x + \log y$