QB365 Question Bank Software

12th Maths CBSE Case Study Differential Equations Questions For - 2024

12th Standard

Maths

SECTION-A 2 x 4 = 8

1) Order: The order of a differential equation is the order of the highest order derivative appearing in the differential equation.
Degree : The degree of differential equation is the power of the highest order derivative, when differential coefficients are made free from radicals and fractions. Also, differential equation must be a polynomial equation
in derivatives for the degree to be defined.
Based on the above information, answer the following questions.

(i) Find the degree of the differential equation $2\frac{d^2y}{dx^2}+3\sqrt{1-\left(\frac{dy}{dx}\right)^2-y=0}$

(a) 3 **(b) 4** **(c) 2** **(d) 1**

(ii) Order and degree of the differential equation $y\frac{dy}{dx}=\frac{x}{\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3}$ are respectively

(a) 1,1 **(b) 1,2** **(c) 1,3** **(d) 1,4**

(iii) Find order and degree of the equation $y'''+y^2+e^{y'}=0$

(a) order = 3, degree = undefined (b) order = 1, degree = 3 (c) order = 2, degree = undefined (d) order = 1, degree = 2

(iv) Determine degree of the differential equation $\left(\sqrt{a+x}\right)\cdot\left(\frac{dy}{dx}\right)+x=0$

(a) 3 **(b) not defined** **(c) 1** **(d) 2**

(v) Order and degree of the differential equation $\left(1+\left(\frac{dy}{dx}\right)^3\right)^{\frac{7}{3}}=7\frac{d^2y}{dx^2}$ are respectively

(a) 2, 1 **(b) 2,3** **(c) 1,3** **(d)** $1,\frac{7}{3}$

Answer : **(i) (c) :** We have, $2\frac{d^2y}{dx^2} + 3\sqrt{1-\left(\frac{dy}{dx}\right)^2} - y = 0$

$\therefore \quad 2\frac{d^2y}{dx^2} = -3\sqrt{1-\left(\frac{dy}{dx}\right)^2} - y$

Squaring both sides, we get

$4\left(\frac{d^2y}{dx^2}\right)^2 = 9\left[1-\left(\frac{dy}{dx}\right)^2 - y\right]$

Here, highest order derivative is $\frac{d^2y}{dx^2}$ and its power is 2. So, its degree is 2.

(ii) (d) : We have, $y\frac{dy}{dx} = \frac{x}{\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3}$

$\Rightarrow y\left(\frac{dy}{dx}\right)^2 + y\left(\frac{dy}{dx}\right)^4 = x$

$\Rightarrow$ Here, highest order derivative is $\frac{dy}{dx}$ is So , its order is 1 and degree is 4.

(iii) (a) : We have,$y''' + y^2 + e^{y'} = 0$

$\frac{d^3y}{dx^3} + y^2 + e^{(dy/dx)} = 0$

Highest order derivative is $\frac{d^3y}{dx^3}$ .So, its order is 3.

Also, the given differential cannot be expressed as a polynomial. So, its degree is not defined.

(iv) (c) : The given differential equation is,

$\sqrt{a+x}\cdot\left(\frac{dy}{dx}\right) + x = 0 \Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{a+x}}$

Clearly, degree = 1

(v) (b) : We have $y\frac{dy}{dx} = \frac{x}{\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3}$

$\Rightarrow y\left(\frac{dy}{dx}\right)^2 + y\left(\frac{dy}{dx}\right)^4 = x$

$\Rightarrow$ Here, highest order derivative is $\frac{dy}{dx}$,So , its order is 1 and degree is 4.

(iii) (a) : We have, $y''' + y^2 + e^y = 0$

$\frac{d^3y}{dx^3} + y^2 + e^{(dy/dx)} = 0$

Highest order derivative is $\frac{d^3y}{dx^3}$ So, its order is 3.

Also, the given differential cannot be expressed as a polynomial. So, its degree is not defined

(iv) (c) : The given differential equation is,

$\sqrt{a+x}\cdot\left(\frac{dy}{dx}\right) + x = 0 \Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{a+x}}$

Clearly, degree = 1.

(v) (b) : We have $\left(1+\left(\frac{dy}{dx}|\right)^3\right)^{\frac{1}{3}} = 7\frac{d^2y}{dx^2}$

$\therefore$ Order is 2 and degree is 3.

2) If the equation is of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ or $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$,wheref (x, y), g(x, y) are homogeneous functions of the same degree in x and y, then put y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, so that the dependent variable y is changed to another variable v and then apply variable separable method. Based on the above information, answer the following questions.

(i) The general solution of $x^2\frac{dy}{dx} = x^2 + xy + y^2$ is

(a) $\tan^{-1}\frac{x}{y}=\log|x|+c$ **(b)** $\tan^{-1}\frac{y}{x}=\log|x|+c$ **(c)** $y=x\log|x|+cx$ **(d)** $x=y\log|y|+c$

(ii) Solution of the differential equation $2xy\frac{dy}{dx}=x^2+3y^2$ is

(a) $x^3+y^2=cx^2$ **(b)** $\frac{x^2}{2}+\frac{y^3}{3}=y^2+c$ **(c)** $x^2+y^3=cx^2$ **(d)** $x^2+y^2=cx^3$

(iii) Solution of the differential equation $\left(x^2+3xy+y^2\right)dx-x^2dy=0$ is

(a) $\frac{x+y}{x}-\log x=c$ **(b)** $\frac{x+y}{x}+\log x=c$ **(c)** $\frac{x}{x+y}-\log x=c$ **(d)** $\frac{x}{x+y}+\log x=c$

(iv) General solution ofthe differential equation $\frac{dy}{dx}=\frac{y}{x}\left\{\log\left(\frac{y}{x}\right)+1\right\}$ is

(a) $\log(xy)=c$ **(b)** $\log y=cx$ **(c)** $\log\left(\frac{y}{x}\right)=cx$ **(d)** $\log x=cy$

(v) Solution ofthe differential equation $\left(x\frac{dy}{dx}-y\right)e^{\frac{y}{x}}=x^2\cos x$ is

(a) $e^{\frac{y}{x}}-\sin x=c$ **(b)** $e^{\frac{y}{x}}+\sin x=c$ **(c)** $e^{\frac{-y}{x}}-\sin x=c$ **(d)** $e^{\frac{-y}{x}}+\sin x=c$

Answer : (i) (b): We have, $\frac{dy}{dx}=\frac{x^2+xy+y^2}{x^2}$

Put y = vx and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore v+x\frac{dv}{dx}=\frac{x^2+x\cdot vx+v^2x^2}{x^2}=1+v+v^2$

$\Rightarrow x\frac{dv}{dx}=1+v^2\Rightarrow\int\frac{dv}{1+v^2}=\int\frac{dx}{x}+c$

$\Rightarrow\tan^{-1}v=\log|x|+c\Rightarrow\tan^{-1}\frac{y}{x}=\log|x|+c$

(ii) (d): We have, $2xy\frac{dy}{dx}=x^2+3y^2\Rightarrow\frac{dy}{dx}=\frac{x^2+3y^2}{2xy}$

Put y = vx and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore v+x\frac{dv}{dx}=\frac{x^2+3v^2x^2}{2vx^2}\Rightarrow x\frac{dv}{dx}=\frac{1+3v^2}{2v}-v$

$\Rightarrow x\frac{dv}{dx}=\frac{1+v^2}{2v_*}\Rightarrow\int\frac{2v}{1+v^2}dv=\int\frac{dx}{x}+\log c$

$\Rightarrow\log\left|1+v^2\right|=\log|x|+\log|c|\Rightarrow\log\left|v^2+1\right|=\log|xc|$

$\Rightarrow\quad v^2+1=xc\Rightarrow\frac{y^2}{2}+1=xc\Rightarrow x^2+y^2=x^3c$

(iii) (d): We have, $\left(x^2+3xy+y^2\right)dx-x^2dy=0$

$\Rightarrow\frac{x^2+3xy+y^2}{x^2}=\frac{dy}{dx}$

Put y = vx and $\frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore\frac{x^2+3x^2v+x^2v^2}{x^2}=\left(v+x\frac{dv}{dx}\right)$

$\Rightarrow 1+3v+v^2=v+x\frac{dv}{dx}\Rightarrow 1+2v+v^2=x\frac{dv}{dx}$

$\Rightarrow\int\frac{dx}{x}-\int(v+1)^{-2}dv=c\Rightarrow\log x+\frac{1}{v+1}=c$