QB365 Question Bank Software

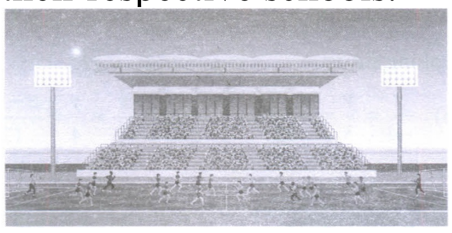
12th Maths CBSE Case Study Three Dimensional Geometry Questions For - 2024

12th Standard

Maths

SECTION-A 2 x 4 = 8

1) A football match is organised between students of class XII of two schools, say school A and school B. For which a team from each school is chosen. Remaining students of class XII of school A and B are respectively sitting on the plane represented by the equation $\vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 6$,to cheer up the team of their respective schools.



Based on the above information, answer the following questions.

(i) The cartesian equation of the plane on which students of school A are seated is

(a) 2x - y +z = 8 **(b) 2x + y + z = 8** **(c) x + y + 2z = 5** **(d) x + y + z = 5**

(ii) The magnitude of the normal to the plane on which students of school B are seated, is

(a) $\sqrt{5}$ **(b)** $\sqrt{6}$ **(c)** $\sqrt{3}$ **(d)** $\sqrt{2}$

(iii) The intercept form of the equation of the plane on which students of school B are seated, is

(a) $\frac{x}{6} + \frac{y}{6} + \frac{z}{6} = 1$ **(b)** $\frac{x}{3} + \frac{y}{(-6)} + \frac{z}{6} = 1$ **(c)** $\frac{x}{3} + \frac{y}{6} + \frac{z}{6} = 1$ **(d)** $\frac{x}{3} + \frac{y}{6} + \frac{z}{3} = 1$

(iv) Which of the following is a student of school B?

(a) Mohit sitting at (1, 2, 1) **(b) Ravi sitting at (0,1,2)** **(c) Khushi sitting at (3, 1, 1)** **(d) Shewta sitting at (2, -1, 2)**

(v) The distance of the plane, on which students of school B are seated, from the origin is

(a) 6 units **(b)** $\frac{1}{\sqrt{6}}$ **units** **(c)** $\frac{5}{\sqrt{6}}$ **units** **(d)** $\sqrt{6}$ **units**

Answer : **(i) (c):** Clearly, the plane for students of school A is $\vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 5,$ which can be rewritten as

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 5$

$\Rightarrow \quad x + y + 2z = 5,$ which is the required cartesian equation.

(ii) (b): Clearly, the equation of plane for students of school B is $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 6,$ which is of the form $\vec{r} \cdot \vec{n} = d$

∴ Normal vector to the plane is, $\vec{n} = 2\hat{i} - \hat{j} + \hat{k}$ and its magnitude is $|\vec{n}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$

(iii) (b): The cartesian form is 2x - y + z = 6, which can be rewritten as

$\frac{2x}{6} - \frac{y}{6} + \frac{z}{6} = 1 \Rightarrow \frac{x}{3} + \frac{y}{(-6)} + \frac{z}{6} = 1$

(iv) (c): Since, only the point (3, I, 1) satisfy the equation of plane representing seating position of students of school B, therefore Khushi is the student of school B.

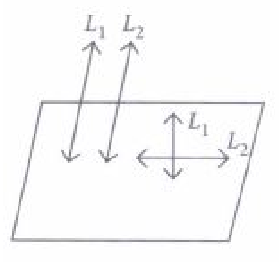
(v) (d): Equation of plane representing students of school B is $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 6,$ which is not in normal form, as $|\vec{n}| \neq 1$

On dividing both sides by $\sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$, we get $\vec{r} \cdot \left(\frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}\right) = \frac{6}{\sqrt{6}}$

which is of the form $\vec{r} \cdot \hat{n} = d$

Thus, the required distance is $\sqrt{6}$ units.

2) If a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratios of two lines say L_1 and L_2 respectively. Then $L_1 \parallel L_2$ iff $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ and $L_1 \perp L_2$ iff $a_1a_2 + b_1b_2 + c_1c_2 = 0$.



Based on the above information, answer the following questions

(i) If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of L, and L2 respectively, then L_1, will be perpendicular to L_2, iff

(a) $l_1l_2 + m_1m_2 + n_1n_2 = 0$ **(b)** $l_1m_2 + m_1l_2 + n_1n_2 = 0$ **(c)** $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$ **(d) none of these**

(ii) If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of L_1, and L_2 respectively, then L_1, will be parallel to L_2, iff

(a) $l_1l_2 + m_1m_2 + n_1n_2 = 0$ **(b)** $l_1m_2 + m_1l_2 + n_1n_2 = 0$ **(c)** $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$ **(d)** $m_1n_2 + m_2n_2 + l_1l_2 = 0$

(iii) The coordinates of the foot of the perpendicular drawn from the point A (1, 2, 1) to the line joining B (1, 4, 6) and C (5, 4, 4), are

(a) (1,2,1) (b) (2,4,5) (c) (3,4,5) (d) (4,3,5)

(iv) The direction ratios of the line which is perpendicular to the lines with direction ratios proportional to (1, -2, -2) and (0, 2, 1) are

(a) < 1,2,1 > (b) < 2, -1, 2 > (c) < -1, 2, 2 > (d) none of these

(v) The lines $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$ and $\frac{x-1}{1} = \frac{y+3/2}{3/2} = \frac{z+5}{2}$ are

(a) parallel (b) perpendicular (c) skew lines (d) non-intersecting

Answer : (i) (a) : Since, D.R.'s are proportional to D.C.'s, therefore L; will be perpendicular to L_2 iff

$l_1l_2 + m_1m_2 + n_1n_2 = 0$

(ii) (c) : Since, D.R.'s are proportional to D.C.'s, therefore L, will be parallel to L_2, iff

$\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

(iii) (c) : Equation of line joining Band C is $\frac{x-1}{4} = \frac{y-4}{0} = \frac{z-6}{-2}$

Let coordinates of foot of perpendicular be D(x, y, z).

∴D.R.'s of AD are < x-1, y-2, z-1 > .

Now, 4(x - 1) + 0(y - 2) -2(z - 1) = 0 ⇒ 4x - 2z = 2

Also, (x, y, z) will satisfy equation of line BC.

Here, (3, 4, 5) satisfy both the conditions.

∴ Required coordinates are (3, 4, 5).

(iv) (b) : Let a, b, c be the direction ratios of the required line. Since it is perpendicular to the lines whose direction ratios are (1, -2, -2) and (0, 2, 1) respectively.

∴ a - 2b - 2c = 0 ..... (i)

0 .a + 2b + c = 0 ....(ii)

On solving (i) and (ii) by cross-multiplication, we get

$\frac{a}{-2+4} = \frac{b}{0-1} = \frac{c}{2} \Rightarrow \frac{a}{2} = \frac{b}{-1} = \frac{c}{2}$

Thus, the direction ratios of the required line are < 2, -1, 2 > .

(v) (b) : D.R 's of given lines are < 3, -2, 0 > and < 1, $-\frac{3}{2}$,2 >

Now, as $3.1 + (-2).(\frac{3}{2}) + 0 \cdot 2 = 3 - 3 + 0 = 0$

∴ Given lines are perpendicular to each other.