A TO Z
MATHEMATICS

XII - MATHEMATICS
QUESTION BANK
FIRST FOUR CHAPTERS

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XII - Mathematics

Question Bank

1. Applications of Matrices and Determinants

Objectives

1. If $|adj(adj\, A)| = |A|^9$, then the order of the square matrix A is

(1) 3 (2) 4 (3) 2 (4) 5

2. If A is a 3×3 non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, then $BB^T =$

(1) A (2) B (3) I (4) B^T

3. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = adj\, A$ and $C = 3A$, then $\dfrac{|adj\, B|}{|C|} =$

(1) $\frac{1}{3}$ (2) $\frac{1}{9}$ (3) $\frac{1}{4}$ (4) 1

4. If $A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then A is

(1) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ (3) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$

5. If $A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I - A =$

(1) A^{-1} (2) $\frac{A^{-1}}{2}$ (3) $3A^{-1}$ (4) $2A^{-1}$

6. If $A = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$ then $|adj(AB)| =$

(1) -40 (2) -80 (3) -60 (4) -20

7. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is

(1) 15 (2) 12 (3) 14 (4) 11

8. If $A=\begin{pmatrix}3 & 1 & -1\\ 2 & -2 & 0\\ 1 & 2 & -1\end{pmatrix}$ and $A^{-1}=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}$ then the value of a_{23} is

(1) 0 (2) -2 (3) -3 (4) -1

9. If A, B and C are invertible matrices of some order, then which one of the following is not true ?

(1) $adj\,A=|A|A^{-1}$ (2) $adj(AB)=(adj\,A)(adj\,B)$

(3) $\det A^{-1}=(\det A)^{-1}$ (4) $(ABC)^{-1}=C^{-1}B^{-1}A^{-1}$

10. If $(AB)^{-1}=\begin{bmatrix}12 & -17\\ -19 & 27\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1 & -1\\ -2 & 3\end{bmatrix}$, then $B^{-1}=$

(1) $\begin{bmatrix}2 & -5\\ -3 & 8\end{bmatrix}$ (2) $\begin{bmatrix}8 & 5\\ 3 & 2\end{bmatrix}$ (3) $\begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix}$ (4) $\begin{bmatrix}8 & -5\\ -3 & 2\end{bmatrix}$

11. If $A^T A^{-1}$ is symmetric, then $A^2=$

(1) A^{-1} (2) $\left(A^T\right)^2$ (3) A^T (4) $\left(A^{-1}\right)^2$

12. If A is a non-singular matrix such that $A^{-1}=\begin{pmatrix}5 & 3\\ -2 & -1\end{pmatrix}$, then $\left(A^T\right)^{-1}=$

(1) $\begin{pmatrix}-5 & 3\\ 2 & 1\end{pmatrix}$ (2) $\begin{pmatrix}5 & 3\\ -2 & -1\end{pmatrix}$ (3) $\begin{pmatrix}-1 & -3\\ 2 & 5\end{pmatrix}$ (4) $\begin{pmatrix}5 & -2\\ 3 & -1\end{pmatrix}$

13. If $A=\begin{bmatrix}\frac{3}{5} & \frac{4}{5}\\ x & \frac{3}{5}\end{bmatrix}$ and $A^T=A^{-1}$, then the value of x is

(1) $-\frac{4}{5}$ (2) $-\frac{3}{5}$ (3) $\frac{3}{5}$ (4) $\frac{4}{5}$

14. If $A=\begin{bmatrix}1 & \tan\frac{\theta}{2}\\ -\tan\frac{\theta}{2} & 1\end{bmatrix}$ and $AB=I$, then $B=$

(1) $\left(\cos^2\frac{\theta}{2}\right)A$ (2) $\left(\cos^2\frac{\theta}{2}\right)A^T$ (3) $\left(\cos^2\theta\right)I$ (4) $\left(\sin^2\frac{\theta}{2}\right)A$

15. If $A=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$ and $A(adj\,A)=\begin{pmatrix}k & 0\\ 0 & k\end{pmatrix}$, then $k=$

(1) 0 (2) $\sin\theta$ (3) $\cos\theta$ (4) 1

16. If $A=\begin{pmatrix}2 & 3\\ 4 & -1\end{pmatrix}$ be such that $\lambda A^{-1}=A$, then λ is

(1) 17 (2) 14 (3) 19 (4) 21

17. If $adj\,A=\begin{bmatrix}2 & 3\\ 4 & -1\end{bmatrix}$ and $adj\,B=\begin{bmatrix}1 & -2\\ -3 & 1\end{bmatrix}$ then $adj(AB)$ is

(1) $\begin{bmatrix}-7 & -1\\ 7 & -9\end{bmatrix}$ (2) $\begin{bmatrix}-6 & 5\\ -2 & -10\end{bmatrix}$ (3) $\begin{bmatrix}-7 & 7\\ -1 & -9\end{bmatrix}$ (4) $\begin{bmatrix}-6 & 2\\ 5 & -10\end{bmatrix}$

18. The rank of the matrix $A=\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 6 & 8\\ -1 & -2 & -3 & -4\end{pmatrix}$ is

(1) 1 (2) 2 (3) 4 (4) 3

19. If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1=\begin{vmatrix}m & b\\ n & d\end{vmatrix}$, $\Delta_2=\begin{vmatrix}a & m\\ c & n\end{vmatrix}$, $\Delta_3=\begin{vmatrix}a & b\\ c & d\end{vmatrix}$, then the value of x and y is

(1) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (2) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$

(3) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ (4) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

20. Which of the following is/are correct ?

(i) Adjoint of a symmetric matrix is also a symmetric matrix

(ii) Adjoint of a diagonal matrix is also a diagonal matrix

(iii) If A is a square matrix of order n and λ is a scalar, then $adj(\lambda A)=\lambda^n\, adj(A)$.

(iv) $A(adj\,A)=(adj\,A)A=|A|I$

(1) only (i) (2) (ii) and (iii) (3) (iii) and (iv) (4) (i), (ii) and (iv)

21. If $\rho(A)=\rho([A\,|\,B])$, then the system $AX=B$ of linear equations is

(1) consistent and has a unique solution (2) consistent

(3) consistent and has infinitely many solution (4) inconsistent

22. If $0\le\theta\le\pi$ and the system of equations $x+(\sin\theta)y-(\cos\theta)z=0$, $(\cos\theta)x-y+z=0$, $(\sin\theta)x+y-z=0$ has non-trivial solution then θ is

(1) $\frac{2\pi}{3}$ (2) $\frac{3\pi}{4}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{4}$

23. The augmented matrix of a system of linear equations is $A = \begin{pmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{pmatrix}$. The system has infinitely many solutions if

(1) $\lambda = 7;\ \mu \neq -5$ (2) $\lambda = -7;\ \mu = 5$ (3) $\lambda \neq 7;\ \mu \neq -5$ (4) $\lambda = 7;\ \mu = -5$

24. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$. If B is the inverse of A, then the value of x is

(1) 2 (2) 4 (3) 3 (4) 1

25. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then $adj(adj\, A)$ is

(1) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ (4) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

Two Marks

1.2 If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is non-singular, find A^{-1}

1.4 If A is a non-singular matrix of odd order, prove that $|adj\, A|$ is positive.

1.6 If $adj(A) = \begin{pmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, find A^{-1}

1.7 If A is symmetric, prove that then $adjA$ is also symmetric.

1.8 Verify $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{pmatrix} 2 & 9 \\ 1 & 7 \end{pmatrix}$

1.11 Prove that $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.

1.(i) Find the adjoint of the matrix $\begin{pmatrix} -3 & 4 \\ 6 & 2 \end{pmatrix}$

1.(ii) Find the adjoint of the matrix $\begin{pmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix}$

2.(i) Find the inverse (if it exists) of the matrix $\begin{pmatrix} -2 & 4 \\ 1 & -3 \end{pmatrix}$

2.(iii) Find the inverse (if it exists) of the matrix $\begin{pmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix}$

9. If $adj(A) = \begin{pmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{pmatrix}$, find A^{-1}

10. Find $adj(adjA)$ if $adj(A) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{pmatrix}$

1.13 Reduce the matrix $\begin{pmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{pmatrix}$ to a row-echelon form.

1.15(i) Find the rank of the matrix $\begin{pmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{pmatrix}$ by minor method

1.16(i) Find the rank of the matrix $\begin{pmatrix} 2 & 0 & -7 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ which is in row-echelon form.

1.16(ii) Find the rank of the matrix $\begin{pmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ which is in row-echelon form.

1.16(iii) Find the rank of the matrix $\begin{pmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ which is in row-echelon form.

1.17 Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{pmatrix}$ by reducing it to a row-echelon form.

1.20 Find the inverse of the non-singular matrix $A = \begin{pmatrix} 0 & 5 \\ -1 & 6 \end{pmatrix}$, by Gauss-Jordan method.

1.(i) Find the rank of the matrix $\begin{pmatrix} 2 & -4 \\ -1 & 2 \end{pmatrix}$ by minor method.

1.(ii) Find the rank of the matrix $\begin{pmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{pmatrix}$ by minor method.

1.(iii) Find the rank of the matrix $\begin{pmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{pmatrix}$ by minor method.

1.(iv) Find the rank of the matrix $\begin{pmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$ by minor method.

1.(v) Find the rank of the matrix $\begin{pmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{pmatrix}$ by minor method.

1.22 Solve the following system of linear equations, using matrix inversion method :

$5x + 2y = 3$; $3x + 2y = 5$

1.(i) Solve the system of linear equations by matrix inversion method :

$2x + 5y = -2$; $x + 2y = -3$

1.(ii) Solve the system of linear equations by matrix inversion method :

$2x - y = 8$; $3x + 2y = -2$

1.(i) Solve the following system of linear equations by Cramer's rule :

$5x - 2y + 16 = 0$; $x + 3y - 7 = 0$

1.(ii) Solve the following system of linear equations by Cramer's rule :

$$\frac{3}{x}+2y=12\ ;\ \frac{2}{x}+3y=13$$

Three Marks

1.3 Find the inverse of the matrix $A=\begin{pmatrix}2 & -1 & 3\\ -5 & 3 & 1\\ -3 & 2 & 3\end{pmatrix}$

1.5 Find a matrix A if $adj(A)=\begin{pmatrix}7 & 7 & -7\\ -1 & 11 & 7\\ 11 & 5 & 7\end{pmatrix}$

1.9 Verify $(AB)^{-1}=B^{-1}A^{-1}$ with $A=\begin{pmatrix}0 & -3\\ 1 & 4\end{pmatrix}$ and $B=\begin{pmatrix}-2 & -3\\ 0 & -1\end{pmatrix}$

1.(iii) Find the adjoint of the matrix $\frac{1}{3}\begin{pmatrix}2 & 2 & 1\\ -2 & 1 & 2\\ 1 & -2 & 2\end{pmatrix}$

2.(ii) Find the inverse (if it exists) of the matrix $\begin{pmatrix}5 & 1 & 1\\ 1 & 5 & 1\\ 1 & 1 & 5\end{pmatrix}$

5. If $A=\frac{1}{9}\begin{pmatrix}-8 & 1 & 4\\ 4 & 4 & 7\\ 1 & -8 & 4\end{pmatrix}$, prove that $A^{-1}=A^{T}$

6. If $A=\begin{pmatrix}8 & -4\\ -5 & 3\end{pmatrix}$, verify that $A(adjA)=(adjA)A=|A|I_2$

7. If $A=\begin{pmatrix}3 & 2\\ 7 & 5\end{pmatrix}$ and $B=\begin{pmatrix}-1 & -3\\ 5 & 2\end{pmatrix}$, verify that $(AB)^{-1}=B^{-1}A^{-1}$

8. If $adj(A)=\begin{pmatrix}2 & -4 & 2\\ -3 & 12 & -7\\ -2 & 0 & 2\end{pmatrix}$, find A.

11. If $A=\begin{pmatrix} 1 & \tan x \\ -\tan x & 1 \end{pmatrix}$, show that $A^T A^{-1}=\begin{pmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{pmatrix}$

12. Find the matrix A for which $A\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix}=\begin{pmatrix} 14 & 7 \\ 7 & 7 \end{pmatrix}$

13. Given $A=\begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$, $B=\begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$ and $C=\begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$, find a matrix X such that $AXB=C$

15. Decrypt the received encoded message $\begin{bmatrix} 2 & -3 \end{bmatrix}\begin{bmatrix} 20 & 4 \end{bmatrix}$ with encryption matrix $\begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1-26 to the letters A-Z respectively, and the number 0 to a blank space.

1.14 Reduce the matrix $\begin{pmatrix} 0 & 3 & 1 & 6 \\ -1 & 0 & 2 & 5 \\ 4 & 2 & 0 & 0 \end{pmatrix}$ to a row-echelon form.

1.15 (ii)Find the rank of the matrix $\begin{pmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{pmatrix}$ by minor method.

1.18 Find the rank of the matrix $\begin{pmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{pmatrix}$ by reducing it to a row-echelon form.

1.19 Show that the matrix $\begin{pmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{pmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformation.

1.21 Find the inverse of $A=\begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ by Gauss-Jordan method.

2.(i) Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{pmatrix}$ by row reduction method.

2.(ii) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{pmatrix}$ by row reduction method.

2.(iii) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{pmatrix}$ by row reduction method.

2.(ii) Find the rank of the matrix $\begin{pmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{pmatrix}$ by row reduction method.

3.(i) Find the inverse of the matrix $\begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}$ by Gauss-Jordan method.

3. A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was Rs.19,800 per month at the end of the first month after 3 years of service and Rs.23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment.
(Use matrix inversion method to solve the problem).

4. 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

2. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ?
(Use Cramer's rule to solve the problem)

3. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution ?
(Use Cramer's rule to solve the problem)

4. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out a same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself ?
(Use Cramer's rule to solve the problem)

1.35 Solve the following system : $x+2y+3z=0$; $3x+4y+4z=0$; $7x+10y+12z=0$.

1.40 If the system of equations $px+by+cz=0$; $ax+qy+cz=0$; $ax+by+rz=0$ has a non-trivial solution and $p \neq q, q \neq r, r \neq c$, prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2$.

Five Marks

1.1 If $A=\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$, verify that $A(adjA)=(adjA)A=|A|I_3$

1.10 If $A=\begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix}$, find x and y such that $A^2+xA+yI_2=O_2$. Hence find A^{-1}

1.12 If $A=\frac{1}{7}\begin{pmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{pmatrix}$ is orthogonal, find a, b and c and hence find A^{-1}

3. If $F(\alpha)=\begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix}$, show that $[F(\alpha)]^{-1}=F(-\alpha)$

4. If $A=\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix}$, show that $A^2-3A-7I_2=O_2$. Hence find A^{-1}

14. If $A=\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, show that $A^{-1}=\frac{1}{2}(A^2-3I)$

3.(ii) Find the inverse of the matrix $\begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{pmatrix}$ by Gauss-Jordan method.

3.(iii) Find the inverse of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$ by Gauss-Jordan method.

1.23 Solve the following system of linear equations, using matrix inversion method :

$2x_1+3x_2+3x_3=5$; $x_1-2x_2+x_3=-4$; $3x_1-x_2-2x_3=3$

1.24 If $A=\begin{pmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{pmatrix}$ and $B=\begin{pmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{pmatrix}$, find the products AB and BA and hence solve the system of equations $x-y+z=4$; $x-2y-2z=9$; $2x+y+3z=1$.

1.(iii) Solve the system of linear equations by matrix inversion method :
$2x+3y-z=9$; $x+y+z=9$; $3x-y-z=-1$

1.(iv) Solve the system of linear equations by matrix inversion method :
$x+y+z-2=0$; $6x-4y+5z-31=0$; $5x+2y+2z=13$

2. If $A=\begin{pmatrix}-5 & 1 & 3\\ 7 & 1 & -5\\ 1 & -1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}1 & 1 & 2\\ 3 & 2 & 1\\ 2 & 1 & 3\end{pmatrix}$, find the products AB and BA and hence solve the system of equations $x+y+2z=1$; $3x+2y+z=7$; $2x+y+3z=2$.

5. The prices of three commodities A, B and C are Rs.x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 units of B and one unit of C. In the process, P, Q and R earn Rs.15,000, Rs.1,000 and Rs.4,000 respectively. Find the prices per unit of A, B and C.
(Use matrix inversion method to solve the problem)

1.25 Solve by Cramer's rule, the system of equations :
$x_1-x_2=3$; $2x_1+3x_2+4x_3=17$; $x_2+2x_3=7$

1.26 In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y=ax^2+bx+c$ with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points $(10,8),(20,16),(40,22)$, can you conclude that Chennai Super Kings won the match ? Justify your answer.(Use Cramer's rule to solve the problem)

1.(iii) Solve the following system of linear equations by Cramer's rule :
$3x+3y-z=11$; $2x-y+2z=9$; $4x+3y+2z=25$

1.(iv) Solve the following system of linear equations by Cramer's rule :
$\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0$; $\frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0$; $\frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$

5. A family of 3 people went out for dinner in a restaurent. The cost of two dosai, three idlies and two vadais is Rs.150. The cost of the two dosai, two idlies and four vadais is Rs. 200. The cost of five dosai, four idlies and two vadais is Rs. 250. The family has Rs. 350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ? (Use Cramer's rule to solve the problem)

1.27 Solve the following system of linear equations by Gaussian elimination method :

$4x+3y+6z=25$; $x+5y+7z=13$; $2x+9y+z=1$

1.28 The upward speed $v(t)$ of a rocket at time t is approximated by $v(t)=at^2+bt+c$, $0\le t\le 100$ where a, b and c are constants. It has been found that the speed at times $t=3$, $t=6$ and $t=9$ seconds are 64, 133 and 208 miles per second respectively. Find the speed at time $t=15$ seconds. (Use Gaussian elimination method)

1.(i) Solve the following system of linear equations by Gaussian elimination method :
$2x-2y+3z=2$; $x+2y-z=3$; $3x-y+2z=1$

1.(ii) Solve the following system of linear equations by Gaussian elimination method :
$2x+4y+6z=22$; $3x+8y+5z=27$; $-x+y+2z=2$

2. If ax^2+bx+c is divided by $x+3$, $x-5$ and $x-1$, the remainders are 21, 61 and 9 respectively. Find a, b and c. (Use Gaussian elimination method)

3. An amount of Rs.65,000 is invested in three bonds at the rates of 6%, 8% and 9% per annum respectively. The total annual income is Rs.4,800. The income from the third bond is Rs.600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method)

4. A boy is walking along the path $y=ax^2+bx+c$ through the points $(-6,8)$, $(-2,-12)$ and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend ?
(Use Gaussian elimination method)

1.29 Test the consistency of the following system of linear equations and if possible solve
$x+2y-z=3$; $3x-y+2z=1$; $x-2y+3z=3$; $x-y+z+1=0$

1.30 Test the consistency of the following system of linear equations and if possible solve
$4x-2y+6z=8$; $x+y-3z=-1$; $15x-3y+9z=21$

1.31 Test the consistency of the following system of linear equations and if possible solve
$x-y+z=-9$; $2x-2y+2z=-18$; $3x-3y+3z+27=0$

1.32 Test the consistency of the following system of linear equations
$x-y+z=-9$; $2x-y+z=4$; $3x-y+z=6$; $4x-y+2z=7$

1.33 Find the condition on a, b and c so that the following system of linear equations has one parameter family of solutions :
$x+y+z=a$; $x+2y+3z=b$; $3x+5y+7z=c$

1.34 Investigate for what values of λ and μ the system of linear equations
$x+2y+z=7$; $x+y+\lambda z=\mu$; $x+3y-5z=5$
has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

1.(i) Test for consistency and if possible, solve the following system of equations by rank method : $x-y+2z=2$; $2x+y+4z=7$; $4x-y+z=4$

1.(ii) Test for consistency and if possible, solve the following system of equations by rank method : $3x+y+z=2$; $x-3y+2z=1$; $7x-y+4z=5$

1.(iii) Test for consistency and if possible, solve the following system of equations by rank method : $3x+2y+z=5$; $x-y+z=1$; $3x+y+2z=4$

1.(iv) Test for consistency and if possible, solve the following system of equations by rank method : $2x-y+z=2$; $6x-3y+3z=6$; $4x-2y+2z=4$

2. Find the value of k for which the equations

$$kx-2y+z=1 \; ; \; x-2ky+z=-2 \; ; \; x-2y+kz=1$$

have (i) no solution (ii) unique solution (iii) infinitely many solutions

3. Investigate the values of λ and μ the system of linear equations

$$2x+3y+5z=9 \; ; \; 7x+3y-5z=8 \; ; \; 2x+3y+\lambda z=\mu$$

have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

1.36 Solve the following system : $x+3y-2z=0$; $2x-y+4z=0$; $x-11y+14z=0$

1.37 Solve the following system :

$$x+y-2z=0 \; ; \; 2x-3y+z=0 \; ; \; 3x-7y+10z=0 \; ; \; 6x-9y+10z=0$$

1.38 Determine the values of λ for which the following system of equations

$$(3\lambda-8)x+3y+3z=0 \; ; \; 3x+(3\lambda-8)y+3z=0 \; ; \; 3x+3y+(3\lambda-8)z=0$$

has a non-trivial solution.

1.39 By using Gaussian elimination method, balance the chemical reaction equation :

$C_5H_8+O_2 \rightarrow CO_2+H_2O$

1.(i) Solve the following system of homogeneous equations

$$3x+2y+7z=0 \; ; \; 4x-3y-2z=0 \; ; \; 5x+9y+23z=0$$

1.(ii) Solve the following system of homogeneous equations

$$2x+3y-z=0 \; ; \; x-y-2z=0 \; ; \; 3x+y+3z=0$$

2. Determine the values of λ for which the following system of equations

$$x+y+3z=0 \; ; \; 4x+3y+\lambda z=0 \; ; \; 2x+y+2z=0$$

has (i) a unique solution (ii) a non-trivial solution

3. By using Gaussian elimination method, balance the chemical reaction equation :

$C_2H_6+O_2 \rightarrow H_2O+CO_2$

Marushikaa Maths Academy

Shevapet, Salem -2.

XII - Mathematics **P.C. Senthil Kumar**

Question Bank

2. Complex Numbers

Objectives

1. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

(i) 0 (2) 1 (3) -1 (4) i

2. The value of $\sum_{i=1}^{13}\left(i^n + i^{n-1}\right)$ is

(1) $1+i$ (2) i (3) 1 (4) 0

3. The area of the triangle formed by the complex numbers z, iz and $z+iz$ in the Argand's diagram is

(1) $\frac{1}{2}|z|^2$ (2) $|z|^2$ (3) $\frac{3}{2}|z|^2$ (4) $2|z|^2$

4. The conjugate of a complex number is $\frac{1}{i-2}$. Then, the complex number is

(1) $\frac{1}{i+2}$ (2) $\frac{-1}{i+2}$ (3) $\frac{-1}{i-2}$ (4) $\frac{1}{i-2}$

5. If $z = \frac{\left(\sqrt{3}+i\right)^3 (3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

(1) 0 (2) 1 (3) 2 (4) 3

6. If z is a non-zero complex number, such that $2iz^2 = \overline{z}$ then $|z|$ is

(1) 0 (2) 1 (3) 2 (4) 3

7. If $|z-2+i| \le 2$, then the greatest value of $|z|$ is

(1) $\sqrt{3}-2$ (2) $\sqrt{3}+2$ (3) $\sqrt{5}-2$ (4) $\sqrt{5}+2$

8. If $\left|z-\frac{3}{2}\right| = 2$, then the least value of $|z|$ is

(1) 1 (2) 2 (3) 3 (4) 5

9. If $|z| = 1$, then the value of $\frac{1+z}{1+\overline{z}}$ is

(1) z (2) $\overline{z}$ (3) $\frac{1}{z}$ (4) 1

10. The solution of the equation $|z|-z=1+2i$ is

(1) $\frac{3}{2}-2i$ (2) $-\frac{3}{2}+2i$ (3) $2-\frac{3}{2}i$ (4) $2+\frac{3}{2}i$

11. If $|z_1|=1$, $|z_2|=2$, $|z_3|=3$ and $|9z_1z_2+4z_1z_3+z_2z_3|=12$, then the value of $|z_1+z_2+z_3|$ is

(1) 1 (2) 2 (3) 3 (4) 4

12. If z is a complex number such that $z \in C \setminus R$ and $z+\frac{1}{z} \in R$, then $|z|$ is

(1) 0 (2) 1 (3) 2 (4) 3

13. z_1, z_2 and z_3 are complex numbers such that $z_1+z_2+z_3=0$ and $|z_1|=|z_2|=|z_3|=1$ then $z_1^2+z_2^2+z_3^2$ is

(1) 3 (2) 2 (3) 1 (4) 0

14. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 3

15. If $z=x+iy$ is a complex number such that $|z+2|=|z-2|$, then the locus of z is

(1) real axis (2) imaginary axis (3) ellipse (4) circle

16. The principal argument of $\frac{3}{-1+i}$ is

(1) $\frac{-5\pi}{6}$ (2) $\frac{-2\pi}{3}$ (3) $\frac{-3\pi}{4}$ (4) $\frac{-\pi}{2}$

17. The principal argument of $(\sin 40^\circ + \cos 40^\circ)^5$ is

(1) -110° (2) -70° (3) 70° (4) 110°

18. If $(1+i)(1+2i)(1+3i)\cdots(1+ni)=x+iy$, then $2\cdot5\cdot10\cdots(1+n^2)$ is

(1) 1 (2) i (3) x^2+y^2 (4) $1+n^2$

19. If $\omega \neq 1$ is a cube root of unity and $(1+\omega)^7 = A+B\omega$, then (A, B) is

(1) $(1, 0)$ (2) $(-1, 1)$ (3) $(0, 1)$ (4) $(1, 1)$

20. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{2}$

21. If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is

(1) -2 (2) -1 (3) 1 (4) 2

22. The product of all four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{3/4}$ is

(1) -2 (2) -1 (3) 1 (4) 2

23. If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to

(1) 1 (2) -1 (3) $\sqrt{3}\,i$ (4) $-\sqrt{3}\,i$

24. The value of $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^{10}$ is

(1) $cis\frac{2\pi}{3}$ (2) $cis\frac{4\pi}{3}$ (3) $-cis\frac{2\pi}{3}$ (4) $-cis\frac{4\pi}{3}$

25. If $\omega = cis\frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is

(1) 1 (2) 2 (3) 3 (4)4

Two Marks

2.1(i) Simplify : i^7

2.1(ii) Simplify : i^{1729}

2.1(iii) Simplify : $i^{-1924}+i^{2018}$

2.1(iv) Simplify : $\sum_{n=1}^{102} i^n$

2.1(v) Simplify : $i.i^2.i^3.......i^{40}$

1. Simplify : $i^{1947}+i^{1950}$

2. Simplify : $i^{1948}-i^{-1869}$

3. Simplify : $\sum_{n=1}^{12} i^n$

4. Simplify : $i^{59}+\frac{1}{i^{59}}$

5. Simplify : $i.i^2.i^3.......i^{2000}$

6. Simplify : $\sum_{n=1}^{10} i^{n+50}$

1.(i) Evaluate : $z+w$, if $z=5-2i$ and $w=-1+3i$.

1.(ii) Evaluate : $z-iw$, if $z=5-2i$ and $w=-1+3i$.

1.(iii) Evaluate : $2z+3w$, if $z=5-2i$ and $w=-1+3i$.

1.(iv) Evaluate : zw, if $z=5-2i$ and $w=-1+3i$.

1.(v) Evaluate : $z^2+2zw+w^2$, if $z=5-2i$ and $w=-1+3i$.

1.(vi) Evaluate : $(z+w)^2$, if $z=5-2i$ and $w=-1+3i$.

2.(i) Given the complex number $z=2+3i$, represent the complex numbers $z, iz, z+iz$ in Argand diagram.

2.(ii) Given the complex number $z=2+3i$, represent the complex numbers $z, -iz, z-iz$ in Argand diagram.

1.(i) If $z_1=1-3i$, $z_2=-4i$ and $z_3=5$, show that $(z_1+z_2)+z_3=z_1+(z_2+z_3)$.

1.(ii) If $z_1=1-3i$, $z_2=-4i$ and $z_3=5$, show that $(z_1 z_2)z_3=z_1(z_2 z_3)$.

2.(i) If $z_1=3$, $z_2=-7i$ and $z_3=5+4i$, show that $z_1(z_2+z_3)=z_1z_2+z_1z_3$.

2.(ii) If $z_1=3$, $z_2=-7i$ and $z_3=5+4i$, show that $(z_1+z_2)z_3=z_1z_3+z_2z_3$.

3. If $z_1=2+5i$, $z_2=-3-4i$ and $z_3=1+i$, find the additive and multiplicative inverse of z_1, z_2 and z_3.

2.3 Write $\frac{3+4i}{5-12i}$ in the $x+iy$ form, hence find its real and imaginary parts.

2.4 Simplify : $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3$

2.5 If $\frac{z+3}{z-5i}=\frac{1+4i}{2}$, find the complex number z.

2.6 If $z_1=3-2i$ and $z_2=6+4i$, find $\frac{z_1}{z_2}$.

2.7 Find z^{-1}, if $z=(2+3i)(1-i)$.

2.8(i) Show that $\left(2+i\sqrt{3}\right)^{10}+\left(2-i\sqrt{3}\right)^{10}$ is real.

1.(i) Write $\overline{(5+9i)+(2-4i)}$ in the rectangular form.

1.(ii) Write $\dfrac{10-5i}{6+2i}$ in the rectangular form.

1.(iii) Write $\overline{3i}+\dfrac{1}{2-i}$ in the rectangular form.

2.(i) If $z=x+iy$, find the rectangular form of $\mathrm{Re}\left(\dfrac{1}{z}\right)$

2.(ii) If $z=x+iy$, find the rectangular form of $\mathrm{Re}\left(i\,\overline{z}\right)$

2.(iii) If $z=x+iy$, find the rectangular form of $\mathrm{Im}\left(3z+4\overline{z}-4i\right)$

5.(i) Prove that z is real if and only if $z=\overline{z}$.

5.(ii) Prove that $\mathrm{Re}(z)=\dfrac{z+\overline{z}}{2}$.

5.(iii) Prove that $\mathrm{Im}(z)=\dfrac{z-\overline{z}}{2i}$.

6. Find the least value of the positive integer n for which $\left(\sqrt{3}+i\right)^{n}$ is real and purely imaginary.

7.(i) Show that $\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary.

2.9 If $z_1=3+4i$, $z_2=5-12i$ and $z_3=6+8i$, find $|z_1|$, $|z_2|$, $|z_3|$, $|z_1+z_2|$, $|z_2-z_3|$ and $|z_1+z_3|$.

2.10(i) Find $\left|\dfrac{2+i}{-1+2i}\right|$.

2.10(ii) Find $\left|\overline{(1+i)}(2+3i)(4i-3)\right|$.

2.10(iii) Find $\left|\dfrac{i(2+i)^3}{(1+i)^2}\right|$.

2.11 Which one of the points i, $-2+i$ and 3 is the farthest from the origin ?

2.17 Find the square root of $6-8i$.

1.(i) Find the modulus of $\frac{2i}{3+4i}$

1.(ii) Find the modulus of $\frac{2-i}{1+i}+\frac{1-2i}{1-i}$

1.(iii) Find the modulus of $(1-i)^{10}$

1.(iv) Find the modulus of $2i(3-4i)(4-3i)$

3. Which one of the points $10-8i$, $11+6i$ is closest to $1+i$.

4. If $|z|=3$, show that $7\le|z+6-8i|\le 13$.

5. If $|z|=1$, show that $2\le|z^2-3|\le 4$.

10.(i) Find the square root of $4+3i$.

10.(ii) Find the square root of $-6+8i$.

10.(iii) Find the square root of $-5-12i$.

2.19 Show that $|3z-5+i|=4$ represents a circle and find its centre and radius.

2.20 Show that $|z+2-i|<2$ represents interior points of a circle. Find its centre and radius.

2.21(i) Obtain the Cartesian form of the locus of *z*, if $|z|=|z-i|$.

2.21(ii) Obtain the Cartesian form of the locus of *z*, if $|2z-3-i|=3$.

3.(i) Obtain the Cartesian form of the locus of $z=x+iy$, if $[\mathrm{Re}(iz)]^2=3$.

3.(ii) Obtain the Cartesian form of the locus of $z=x+iy$, if $\mathrm{Im}[(1-i)z+1]=0$.

3.(iii) Obtain the Cartesian form of the locus of $z=x+iy$, if $|z+i|=|z-1|$.

3.(iv) Obtain the Cartesian form of the locus of $z=x+iy$, if $\overline{z}=z^{-1}$.

4.(i) Show that $|z-2-i|=3$ represents a circle and find its centre and radius.

4.(ii) Show that $|2z+2-4i|=2$ represents a circle and find its centre and radius.

4.(iii) Show that $|3z-6+12i|=8$ represents a circle and find its centre and radius.

5.(i) Obtain the Cartesian form of the locus of $z=x+iy$, if $|z-4|=16$.

2.22(i) Find the modulus and principal argument of the complex number $\sqrt{3}+i$.

2.22(ii) Find the modulus and principal argument of the complex number $-\sqrt{3}+i$.

2.22(iii) Find the modulus and principal argument of the complex number $-\sqrt{3}-i$.

2.22(iv) Find the modulus and principal argument of the complex number $\sqrt{3}-i$.

2.23(i) Represent the complex number $-1-i$ in polar form.

2.23(ii) Represent the complex number $1+i\sqrt{3}$ in polar form.

1.(i) Write the complex number $2+i2\sqrt{3}$ in the polar form.

1.(ii) Write the complex number $3-i\sqrt{3}$ in the polar form.

1.(iii) Write the complex number $-2-i2$ in the polar form.

2.(i) Find the rectangular form of the complex number $\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

2.(ii) Find the rectangular form of the complex number $\dfrac{\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)}{2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}$

2.29 Simplify : $\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{18}$

9.(i) If $z=2-2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when $\theta=\frac{\pi}{3}$.

9.(ii) If $z=2-2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when $\theta=\frac{2\pi}{3}$.

9.(iii) If $z=2-2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when $\theta=\frac{3\pi}{2}$.

Three Marks

2.2 Find the value of the real numbers x and y, if the complex numbers $(2+i)x+(1-i)y+2i-3$ and $x+(-1+2i)y+1+i$ are equal.

3. Find the value of the real numbers x and y, if the complex numbers $(3-i)x-(2-i)y+2i+5$ and $2x+(-1+2i)y+3+2i$ are equal.

2.8(ii) Show that $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

3. If $z_1 = 2-i$ and $z_2 = -4+3i$, find the inverse of $z_1 z_2$ and $\frac{z_1}{z_2}$.

4. The complex numbers u, v and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If $v = 3-4i$ and $w = 4+3i$, find u in rectangular form.

7.(ii) Show that $\left(\frac{19-7i}{9+i}\right)^{12} + \left(\frac{20-5i}{7-6i}\right)^{12}$ is real.

2.12 If z_1, z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1+z_2+z_3| = 1$, find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$.

2.13 If $|z| = 2$, show that $3 \le |z+3+4i| \le 7$.

2.14 Show that the points 1, $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$ and $-\frac{1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

2.16 Show that the equation $z^2 = \overline{z}$ has four solutions.

2. For any two complex numbers z_1 and z_2 such that $|z_1| = |z_2| = 1$ and $z_1 z_2 \neq -1$, then show that $\frac{z_1+z_2}{1+z_1 z_2}$ is a real number.

8. If the area of the triangle formed by the vertices z, iz and $z+iz$ is 50 square units, find the value of $|z|$.

9. Show that the equation $z^3 + 2\overline{z} = 0$ has five solutions.

2.18 Given the complex number $z = 3+2i$, represent the complex numbers z, iz and $z+iz$ in one Argand diagram. Show that these complex numbers form the vertices of an isosceles right triangle.

1. If $z = x+iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$, show that the locus of z is real axis.

5.(ii) Obtain the Cartesian form of the locus of $z = x + iy$, if $|z-4|^2 - |z-1|^2 = 16$.

2.24 Find the principal argument $Arg(z)$, when $z = \frac{-2}{1+i\sqrt{3}}$.

2.25 Find the product of $\frac{3}{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\cdot 6\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$ in rectangular form.

2.26 Find the quotient $\frac{2\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right) + i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form.

1.(iv) Write the complex number $\frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ in the polar form.

3. If $(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3)\ldots\ldots(x_n + iy_n) = a + ib$, show that

(i) $(x_1^2 + y_1^2)(x_2^2 + y_2^2)(x_3^2 + y_3^2)\ldots\ldots(x_n^2 + y_n^2) = a^2 + b^2$

(ii) $\sum_{r=1}^{n} \tan^{-1}\left(\frac{y_r}{x_r}\right) = \tan^{-1}\left(\frac{b}{a}\right) + 2k\pi,\ k \in Z$.

4. If $\frac{1+z}{1-z} = \cos 2\theta + i\sin 2\theta$, show that $z = i\tan\theta$.

5. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, then show that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$ and

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$.

2.28 If $z = \cos\theta + i\sin\theta$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i\sin n\theta$

2.30 Simplify : $\left(\frac{1+\cos 2\theta + i\sin 2\theta}{1+\cos 2\theta - i\sin 2\theta}\right)^{30}$

2.31(i) Simplify : $(1+i)^{18}$

2.31(ii)Simplify : $(-\sqrt{3} + 3i)^{31}$

2.36 Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, find z_2 and z_3.

1. If $\omega \neq 1$ is a cube root of unity, then show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}=-1$.

3. Find the value of $\left(\frac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right)^{10}$.

6. If $\omega \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3+8=0$ are $-1, 1-2\omega, 1-2\omega^2$.

7. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9}+i\sin\frac{2k\pi}{9}\right)$.

8. If $\omega \neq 1$ is a cube root of unity, show that

(i) $(1-\omega+\omega^2)^6+(1+\omega-\omega^2)^6=128$

(ii) $(1+\omega)(1+\omega^2)(1+w^4)(1+w^8)......(1+\omega^{2^{11}})=1$.

Five Marks

2.15 Let z_1, z_2 and z_3 are complex numbers such that $|z_1|=|z_2|=|z_3|=r>0$ and $z_1+z_2+z_3 \neq 0$, prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|=r$.

6. If $\left|z-\frac{2}{z}\right|=2$, show that the greatest and least value of $|z|$ are $\sqrt{3}+1$ and $\sqrt{3}-1$ respectively.

7. If z_1, z_2 and z_3 are three complex numbers, such that $|z_1|=1$, $|z_2|=2$, $|z_3|=3$ and $|z_1+z_2+z_3|=1$, show that $|9z_1z_2+4z_1z_3+z_2z_3|=6$.

2. If $z=x+iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right)=0$, show that the locus of z is $2x^2+2y^2+x-2y=0$.

2.27 If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$, then show that $x^2+y^2=1$.

6. If $z=x+iy$ and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, then show that $x^2+y^2+3x-3y+2=0$.

2.32 Find the cube roots of unity.

2.33 Find the fourth roots of unity.

2.34 Solve the equation $z^3 + 8i = 0$, where $z \in C$.

2.35 Find all cube roots of $\sqrt{3} + i$.

2. Show that $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = -\sqrt{3}$.

4. If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that

(i) $\frac{x}{y} + \frac{y}{x} = 2\cos(\alpha - \beta)$ (ii) $xy - \frac{1}{xy} = 2i\sin(\alpha + \beta)$

(iii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$ (iv) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

5. Solve the equation $z^3 + 27 = 0$.

10. Prove that the values of $\sqrt[4]{-1}$ are $\pm\frac{1}{\sqrt{2}}(1 \pm i)$.

MARUSHIKAA MATHS ACADEMY

Shevapet, Salem -2.

XII - Mathematics

P.C. Senthil Kumar

Question Bank

3. Theory of Equations

Objectives

1. A zero of x^3+64 is

   (1) 0 (2) 4 (3) $4i$ (4) -4

2. If f and g are polynomials of degrees m and n respectively and if $h(x)=(f\circ g)(x)$, then the degree of h is

   (1) mn (2) $m+n$ (3) m^n (4) n^m

3. A polynomial equation in x of degree n always has

   (1) exactly n roots is R (2) exactly n roots in C
   (3) atleast n roots in R (4) exactly n distinct roots in C

4. If α, β and γ are the roots of $x^3+px^2+qx+r=0$, then $\sum\frac{1}{\alpha}$ is

   (1) $-\frac{q}{r}$ (2) $-\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

5. According to the rational root theroem, which numer is not possible rational root of $4x^7+2x^4-10x^3-5$?

   (1) -1 (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4) 5

6. The polynomial x^3-kx^2+9x has three real roots if and only if, k satisfies

   (1) $|k|\le 6$ (2) $k=0$ (3) $|k|>6$ (4) $|k|\ge 6$

7. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x-2\sin^2 x+1$ is

   (1) 2 (2) 4 (3) 1 (4) ∞

8. The equation $x^3+12x^2+10ax+1999$ may have a positive root, if

   (1) $a\ge 0$ (2) $a>0$ (3) $a<0$ (4) $a\le 0$

9. The polynomial x^3+2x+3 has

   (1) one negative and two real roots (2) one negative and two imaginary roots
   (3) one positive and two real roots (4) one positive and two imaginary roots

10. The number of positive roots of the polynomial $\sum_{r=0}^{n} {}^{n}C_r(-1)^r x^r$ is

(1) 0 (2) n (3) $< n$ (4) r

Two Marks

3.3 If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients.

3.4 Find the sum of the squares of the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$.

2.(i) Construct a cubic equation with roots 1, 2 and 3.

2.(ii) Construct a cubic equation with roots 1, 1 and -2.

2.(iii) Construct a cubic equation with roots 2, -2 and 4.

5. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$.

8. If α, β, γ and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$, find the quadratic equation with integer coefficients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$.

9. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.

11. Formalate into a mathematical problem to find a number such that when its cube root is added to it, the result is 6.

12. A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was cut away.

3.8 Find the monic polynomial equation of minimum degree with real coefficients having $2 - \sqrt{3}\,i$ as a root.

3.9 Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}$ as a root.

3.10 Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

3.11 Show that the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x.

3.12 If $x^2 + 2(k+2)x + 9k = 0$ has equal roots, find k.

3.13 Show that if p, q, r are rational, the roots of the equation $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$

are rational.

3.14 Prove that a line cannot intersect a circle at more than two points.

2. Find a polynomial equation of minimum degree with rational coefficients, having $2+\sqrt{3}i$ as a root.

3. Find a polynomial equation of minimum degree with rational coefficients, having $2i+3$ as a root.

5. Prove that a straight line and parabola cannot intersect at more than two points.

3.16 Solve the equation $x^4-9x^2+20=0$.

1. Solve the cubic equation $2x^3-x^2-18x+9=0$ if sum of two of its roots vanishes.

7. Solve the equation : $x^4-14x^2+45=0$.

1. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9-4x^8+4x^7-3x^6+2x^5+x^3+7x^2+7x+2=0$.

2. Discuss the maximum possible number of positive and negative roots of the polynomial equations x^2-5x+6 and $x^2-5x+16$. Also draw rough sketch of the graphs.

Three Marks

3.1 If α and β are the roots of the quadratic equation $17x^2+43x-73=0$, construct a quadratic equation whose roots are $\alpha+2$ and $\beta+2$.

3.2 If α and β are the roots of the quadratic equation $2x^2-7x+13=0$, construct a quadratic equation whose roots are α^2 and β^2.

3.5 Find the condition that the roots of $x^3+ax^2+bx+c=0$ are in the ratio $p:q:r$.

3.7 If p is real, discuss the nature of the roots of the equation $4x^2+4px+p+2=0$ in terms of p.

1. If the sides of the cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.

3.(i) If α, β and γ are the roots of the cubic equation $x^3+2x^2+3x+4=0$, form a cubic equation whose roots are $2\alpha,\ 2\beta,\ 2\gamma$.

3.(ii) If α, β and γ are the roots of the cubic equation $x^3+2x^2+3x+4=0$, form a cubic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$.

3.(iii) If α, β and γ are the roots of the cubic equation $x^3+2x^2+3x+4=0$, form a cubic equation whose roots are $-\alpha,\ -\beta,\ -\gamma$.

7. If α, β and γ are the roots of the equation $ax^3+bx^2+cx+d=0$, find the value of

$\sum \frac{\alpha}{\beta\gamma}$ in terms of the coefficients.

10. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq' - p'q}{q - q'}$ or $\frac{q - q'}{p' - p}$.

1. If k is real, discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$ in terms of k.

3.17 Solve the equation $x^3 - 3x^2 - 33x + 35 = 0$.

3.18 Solve the equation $2x^3 + 11x^2 - 9x - 18 = 0$.

3.19 Obtain the condition that the roots of $x^3 + px^2 + qx + r = 0$ are in A.P.

3.22 It is known that the roots of the equation $x^3 - 6x^2 - 4x + 24 = 0$ are in A.P. Find its roots.

2. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an A.P.

3. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if the roots form a G.P.

6.(i) Solve the cubic equation : $2x^3 - 9x^2 + 10x = 3$.

6.(ii) Solve the cubic equation : $8x^3 - 2x^2 - 7x + 3 = 0$.

3.27 Solve the equation : $7x^3 - 43x^2 = 43x - 7$

3.29 Find solution, if any, of the equation $2\cos^2 x - 9\cos x + 4 = 0$

1.(i) Solve the equation : $\sin^2 x - 5\sin x + 4 = 0$

2.(i) Examine for the rational roots of $2x^3 - x^2 - 1 = 0$

2.(ii) Examine for the rational roots of $x^8 - 3x + 1 = 0$

5.(ii) Solve the equation : $x^4 + 3x^3 - 3x - 1 = 0$

6. Find all real numbers satisfying $4^x - 3\left(2^{x+2}\right) + 2^5 = 0$

3.30 Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has atleast six imaginary roots.

3.31(i) Discuss the nature of the roots of the polynomial $x^{2018} + 1947x^{1950} + 15x^8 + 26x^6 + 2019$

3.31(ii) Discuss the nature of the roots of the polynomial $x^5 - 19x^4 + 2x^3 + 5x^2 + 11$.

3. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.

4. Determine the number of positive and negative roots of the equation

$x^9 - 5x^8 - 14x^7 = 0$.

5. Find the exact number of real roots and imaginary of the equation $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x = 0$.

Five Marks

3.6 Form the quadratic equation whose roots are the squares of the roots of the cubic equation $x^3 + ax^2 + bx + c = 0$.

4. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.

6. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio $3:2$.

4. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root.

3.15 If $2 + i$ and $3 - \sqrt{2}$ are roots of the equation

$$x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0,$$

find all roots.

3.20 Find the condition that the roots of $ax^3 + bx^2 + cx + d = 0$ are in G.P. Assume $a, b, c, d \neq 0$.

3.21 If the roots of $x^3 + px^2 + qx + r = 0$ are in H.P., Prove that $9pqr = 27r^3 + 2p$.

4. Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots.

5. Find all zeroes of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeroes.

3.23 Solve the equation : $(x-2)(x-7)(x-3)(x+2) + 19 = 0$

3.24 Solve the equation : $(2x-3)(6x-1)(3x-2)(x-12) - 7 = 0$

1. Solve : $(x-5)(x-7)(x+6)(x+4) = 504$

2. Solve : $(x-4)(x-7)(x-2)(x+1) = 16$

3. Solve : $(2x-1)(x+3)(x-2)(2x+3) + 20 = 0$

3.25 Solve the equation : $x^3 - 5x^2 - 4x + 20 = 0$

3.26 Find the roots of $2x^3 + 3x^2 + 2x + 3 = 0$

3.28 Solve the equation : $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

1.(ii) Solve the equation : $12x^3 + 8x = 29x^2 - 4$

3. Solve : $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$

4. Solve : $2\sqrt{\frac{x}{a}}+3\sqrt{\frac{a}{x}}=\frac{b}{a}+\frac{6a}{b}$

5.(i) Solve the equation : $6x^4-35x^3+62x^2-35x+6=0$

7. Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution.

MARUSHIKAA MATHS ACADEMY

Shevapet, Salem -2.

XII - Mathematics **P.C. Senthil Kumar**

Question Bank

4. Inverse Trigonometric Functions

Objectives

1. The value of $\sin^{-1}(\cos x),\ 0 \le x \le \pi$ is

(i) $\pi - x$ (2) $x - \frac{\pi}{2}$ (3) $\frac{\pi}{2} - x$ (4) $\frac{3\pi}{2} - x$

2. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$; then $\cos^{-1} x + \cos^{-1} y$ is equal to

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) π

3. $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} + \sec^{-1}\frac{5}{3} - \text{cosec}^{-1}\frac{13}{12}$ is equal to

(1) 2π (2) π (3) 0 (4) $\tan^{-1}\frac{12}{65}$

4. If $\sin^{-1} x = 2\sin^{-1}\alpha$ has a solution, then

(1) $|\alpha| \le \frac{1}{\sqrt{2}}$ (2) $|\alpha| \ge \frac{1}{\sqrt{2}}$ (3) $|\alpha| < \frac{1}{\sqrt{2}}$ (4) $|\alpha| > \frac{1}{\sqrt{2}}$

5. $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for

(1) $-\pi \le x \le 0$ (2) $0 \le x \le \pi$ (3) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ (4) $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$

6. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101} + y^{101} + z^{101}}$ is

(1) 0 (2) 1 (3) 2 (4) 3

7. If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1} x$ is

(1) $-\frac{\pi}{10}$ (2) $\frac{\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $-\frac{\pi}{5}$

8. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(1) $[1, 2]$ (2) $[-1, 1]$ (3) $[0, 1]$ (4) $[-1, 0]$

9. If $x=\frac{1}{5}$, the value of $\cos\left(\cos^{-1}x+2\sin^{-1}x\right)$ is

(1) $-\sqrt{\frac{24}{25}}$ (2) $\sqrt{\frac{24}{25}}$ (3) $\frac{1}{5}$ (4) $-\frac{1}{5}$

10. $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(1) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (2) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (3) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (4) $\tan^{-1}\left(\frac{1}{2}\right)$

11. If the function $f(x)=\sin^{-1}\left(x^2-3\right)$, then x belongs to

(1) $[-1, 1]$ (2) $[\sqrt{2}, 2]$

(3) $[-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$ (4) $[-2, -\sqrt{2}] \cap [\sqrt{2}, 2]$

12. If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle, then the third angle is

(1) $\frac{\pi}{4}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

13. $\sin^{-1}\left(\tan\frac{\pi}{4}\right)-\sin^{-1}\left(\sqrt{\frac{3}{x}}\right)=\frac{\pi}{6}$. Then x is a root of the equation

(1) $x^2-x-6=0$ (2) $x^2-x-12=0$ (3) $x^2+x-12=0$ (4) $x^2+x-6=0$

14. $\sin^{-1}\left(2\cos^2 x-1\right)+\cos^{-1}\left(1-2\sin^{-1}x\right)=$

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

15. If $\cot^{-1}\left(\sqrt{\sin\alpha}\right)+\tan^{-1}\left(\sqrt{\sin\alpha}\right)=u$, then $\cos 2u$ is equal to

(1) $\tan^2\alpha$ (2) 0 (3) -1 (4) $\tan 2\alpha$

16. If $|x|\le 1$, then $2\tan^{-1}x-\sin^{-1}\frac{2x}{1+x^2}$ is equal to

(1) $\tan^{-1}x$ (2) $\sin^{-1}x$ (3) 0 (4) π

17. The equation $\tan^{-1}x-\cot^{-1}x=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

(1) no solution (2) unique solution

(3) two solutions (4) infinite number of solutions

18. If $\sin^{-1}x+\cot^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}$, then x is equal to

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{3}}{2}$

19. If $\sin^{-1}\frac{x}{5}+\operatorname{cosec}^{-1}\frac{5}{4}=\frac{\pi}{2}$, then the value of x is

(1) 4 (2) 5 (3) 2 (4) 3

20. $\sin(\tan^{-1}x),\ |x|<1$ is equal to

(1) $\frac{x}{\sqrt{1-x^2}}$ (2) $\frac{1}{\sqrt{1-x^2}}$ (3) $\frac{1}{\sqrt{1+x^2}}$ (4) $\frac{x}{\sqrt{1+x^2}}$

Two Marks

4.1 Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ (in radians and degrees).

4.2 Find the principal value of $\sin^{-1}(2)$, if it exists.

4.3(i) Find the principal value of $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

4.3(ii) Find the principal value of $\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$

4.3(iii) Find the principal value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$

1.(i) Find all the values of x such that $-10\pi \le x \le 10\pi$ and $\sin x = 0$.

1.(ii) Find all the values of x such that $-8\pi \le x \le 8\pi$ and $\sin x = -1$.

2.(i) Find the period and amplitude of $y=\sin 7x$

2.(ii) Find the period and amplitude of $y=-\sin\left(\frac{1}{3}x\right)$

2.(iii) Find the period and amplitude of $y=4\sin(-2x)$

3.(i) Sketch the graph of $y=\sin\left(\frac{1}{3}x\right)$ for $0 \le x < 6\pi$.

4.(i) Find the value of $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

4.(ii) Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

5. For what value of x does $\sin x = \sin^{-1} x$?

4.5 Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

4.6(i) Find the value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

4.6(ii) Find the value of $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$

4.6(iii) Find the value of $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$

1.(i) Find all the values of x such that $-6\pi \le x \le 6\pi$ and $\cos x = 0$.

1.(ii) Find all the values of x such that $-5\pi \le x \le 5\pi$ and $\cos x = 1$.

2. State the reason for $\cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right) \ne -\frac{\pi}{6}$

4. Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$

5.(i) Find the value of $2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

5.(ii) Find the value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}(-1)$

6.(ii) Find the domain of $g(x) = \sin^{-1} x + \cos^{-1} x$

7. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds ?

4.8 Find the principal value of $\tan^{-1}(\sqrt{3})$

4.9(i) Find the value of $\tan^{-1}(-\sqrt{3})$

4.9(ii) Find the value of $\tan^{-1}\left(\tan\left(\frac{3\pi}{5}\right)\right)$

4.9(iii) Find the value of $\tan\left(\tan^{-1}(2019)\right)$

4.10 Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

1.(i) Find the domain of $\tan^{-1}\left(\sqrt{9-x^2}\right)$

1.(ii) Find the domain of $\frac{1}{2}\tan^{-1}\left(1-x^2\right)-\frac{\pi}{4}$

2.(i) Find the value of $\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$

2.(ii) Find the value of $\tan^{-1}\left(\tan\left(-\frac{\pi}{6}\right)\right)$

3.(i) Find the value of $\tan\left(\tan^{-1}\left(\frac{7\pi}{4}\right)\right)$

3.(ii) Find the value of $\tan\left(\tan^{-1}(1947)\right)$

3.(iii) Find the value of $\tan\left(\tan^{-1}(-0.2021)\right)$

4.(i) Find the value of $\tan\left(\cos^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(-\frac{1}{2}\right)\right)$

4.12(i) Find the principal value of $\text{cosec}^{-1}(-1)$

4.12(ii) Find the principal value of $\sec^{-1}(-2)$

4.13 Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$

4.14 If $\cot^{-1}\left(\frac{1}{7}\right)=\theta$, find the value of $\cos\theta$

1.(i) Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

1.(ii) Find the principal value of $\cot^{-1}\left(\sqrt{3}\right)$

1.(iii) Find the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

2.(i) Find the value of $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)$

2.(ii) Find the value of $\sin^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\cot^{-1}(2)$

2.(iii) Find the value of $\cot^{-1}(1)+\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-\sec^{-1}\left(-\sqrt{2}\right)$

4.16 Prove that $\frac{\pi}{2}\le\sin^{-1}x+2\cos^{-1}x\le\frac{3\pi}{2}$

4.17(i) Simplify : $\cos^{-1}\left(\cos\left(\frac{13\pi}{3}\right)\right)$

4.17(ii) Simplify : $\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

4.17(iii) Simplify : $\sec^{-1}\left(\sec\left(\frac{5\pi}{3}\right)\right)$

4.17(iv) Simplify : $\sin^{-1}(\sin 10)$

4.18(i) Find the value of $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$

4.21(i) Prove that $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=\frac{\pi}{4}$

1.(i) Find the value of $\sin^{-1}(\cos\pi)$ if it exists. If not, give the reason for non-existence.

3.(i) Find the value of $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$

Three Marks

6.(ii) Find the domain of $g(x)=2\sin^{-1}(2x-1)-\frac{\pi}{4}$

7. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9}+\cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$

4.7 Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$

3. Is $\cos^{-1}(-x)=\pi-\cos^{-1}(x)$ true ? Justify your answer.

5.(iii) Find the value of $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17}-\sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$

6.(i) Find the domain of $f(x)=\sin^{-1}\left(\frac{|x|-2}{3}\right)+\cos^{-1}\left(\frac{1-|x|}{4}\right)$

8.(i) Find the value of $\cos\left(\cos^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{4}{5}\right)\right)$

8.(ii) Find the value of $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)+\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$

4.11 Prove that $\tan\left(\sin^{-1}x\right)=\frac{x}{\sqrt{1-x^2}}$, $-1<x<1$.

4.15 Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)=\sec^{-1}x$, $|x|>1$

4.18(ii) Find the value of $\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$

4.18(iii) Find the value of $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$

4.19 Prove that $\tan\left(\sin^{-1}x\right)=\frac{x}{\sqrt{1-x^2}}$ for $|x|<1$.

4.21(ii)Prove that $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

4.26 Show that $\cot\left(\sin^{-1}x\right)=\frac{\sqrt{1-x^2}}{x}$, $-1\le x\le 1$ and $x\ne 0$

4.25 Solve : $\sin^{-1}x>\cos^{-1}x$

1.(ii) Find the value of $\tan^{-1}\left(\sin\left(-\frac{5\pi}{2}\right)\right)$ if it exists. If not, give the reason for non-existence.

1.(iii) Find the value of $\sin^{-1}(\sin 5)$ if it exists. If not, give the reason for non-existence.

2.(i) Find the value of the expression $\sin\left(\cos^{-1}(1-x)\right)$ in terms of x, with the help of a reference triangle.

2.(ii) Find the value of the expression $\cos\left(\tan^{-1}(3x-1)\right)$ in terms of x, with the help of a reference triangle.

4.(i) Prove that $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$

6. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, show that $x+y+z=xyz$.

7. Prove that $\tan^{-1}x+\tan^{-1}\frac{2x}{1-x^2}=\tan^{-1}\frac{3x-x^3}{1-3x^2}$, $|x|<\frac{1}{\sqrt{3}}$.

Five Marks

4.4 Find the domain of $\sin^{-1}\left(2-3x^2\right)$

6.(i) Find the domain of $f(x)=\sin^{-1}\left(\frac{x^2+1}{2x}\right)$

4.(ii) Find the value of $\sin\left(\tan^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{4}{5}\right)\right)$

4.(iii) Find the value of $\cos\left(\sin^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{3}{4}\right)\right)$

4.20 Evaluate : $\sin\left[\sin^{-1}\left(\frac{3}{5}\right)+\sec^{-1}\left(\frac{5}{4}\right)\right]$

4.22 If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and $0<x,y,z<1$, show that $x^2+y^2+z^2+2xyz=1$.

4.23 If $a_1, a_2, a_3, ..., a_n$ is an arithmetic progression with common difference 'd', prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+......+\tan^{-1}\left(\frac{d}{1+a_na_{n-1}}\right)\right]=\frac{a_n-a_1}{1+a_1a_n}$$

4.24 Solve : $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$, for $x>0$.

4.27 Solve : $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$, $6x^2<1$.

4.28 Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

4.29 Solve : $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)=\sin\left(\cot^{-1}\left(\frac{3}{4}\right)\right)$

2.(iii) Find the value of the expression $\tan\left(\sin^{-1}\left(x+\frac{1}{2}\right)\right)$ in terms of x, with the help of a reference triangle.

3.(ii) Find the value of $\cot\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$

3.(iii) Find the value of $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

4.(ii) Prove that $\sin^{-1}\frac{3}{5}-\cos^{-1}\frac{12}{13}=\sin^{-1}\frac{16}{65}$

5. Prove that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

8. Simplify : $\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{x-y}{x+y}$

9.(i) Solve : $\sin^{-1}\frac{5}{x}+\sin^{-1}\frac{12}{x}=\frac{\pi}{2}$

9.(ii) Solve : $2\tan^{-1}x=\cos^{-1}\frac{1-a^2}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}$, $a>0, b>0$

9.(iii) Solve : $2\tan^{-1}(\cos x)=\tan^{-1}(2\,\text{cosec}\,x)$

9.(iv) Solve : $\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{12}$, $x>0$

10. Find the number of solutions of the equation

$\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}(3x)$