ST.ANNE'S MAT.HR.SEC.SCHOOL,THATTANCHAVADI

Mid- TERM answer key

12th Standard 2019 EM

Maths

Hardwork Never Fails...

Time : 01:30:00 Hrs

Total Marks : 50

PART-A

10 x 1 = 10

1) (a) n distinct roots
2) (a) $-\frac{q}{r}$
3) (d) |k|≥6
4) (a) 2
5) (c) a<0
6) (b) $0\pi \leq x \leq 0$
7) (c) $\frac{\pi}{10}$
8) (a) [1,2]
9) (d) $-\frac{1}{5}$
10) (d) $tan^{-1}\left(\frac{1}{2}\right)$

PART-B

8 x 2 = 16

11) The roots of x^3+2x2+3x+4=0 are ∝,β,ɣ

∴ ∝+β+ɣ=-co-efficient of x^2=-2 ...(1)

∝β+βɣ+ɣ∝=co-effficient of x=3 ...(2)

-∝βɣ=+4⇒∝βɣ=-4 ...(3)

From the cubic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \gamma\alpha + \alpha\beta}{\alpha\beta\gamma} = \frac{3}{-4} = \frac{-3}{4}$$

$$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \frac{\gamma + \alpha + \beta}{\alpha\beta\gamma} = \frac{-2}{-4} = \frac{1}{2}$$

$$\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)\left(\frac{1}{\gamma}\right) = \frac{1}{\alpha\beta\gamma} = \frac{1}{-4} = -\frac{1}{4}$$

∴ The required cubic equation is

$$x^3 - \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)x^2 + \left(\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}\right)x - \left(\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}\right)$$

$$\Rightarrow x^3 + \frac{3}{4}x^2 + \frac{1}{2}x + \frac{1}{4} = 0$$

Multiplying by 4 we get,

$4x^3+3x^2+2x+1=0$

12) Let $y=sin^{-1}x$

Wheny == 0,0'= sin^{-1}Ix

⇒ sin(O) = sin(sin-1)(x))

⇒ sin 0 =x

$\Rightarrow$ x=0

$\therefore$ only who x = 0, siri x = $\sin^{-1}$x

13) Let P(x) be the polynomial under consideration.

(i) The number of sign changes for P(x) and P(−x) are zero and hence it has no positive roots and no negative roots. Clearly zero is not a root. Thus the polynomial has no real roots and hence all roots of the polynomial are imaginary roots.

14) Let $\sin^{-1}\left(-\frac{1}{2}\right)$=y. Then sin y=-$\frac{1}{2}$

The range of the principal value of $\sin^{-1}$ x is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and hence, Let us find y $\in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ Such that sin y =-$\frac{1}{2}$. Clearly, y=-$\frac{\pi}{6}$

Thus, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$is-$\frac{\pi}{6}$. This corresponds to-30^{o}.

15) By definition, the domain of yx=$\cos^{-1}$ x is -1 This leads to $-1 \le \frac{2+sinx}{3} \le 1$ which is same as -3 $\le$ 2+sinx $\le$ 3

so, -5 $\le \sin x \le 1$ reduces to -1 $\le \sin x \le 1$, which gives

-$\sin^{-1}(1)$ $\le x \le sin^{-}1(1) or -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Thus, the domain of cos-1$(\frac{2+\sin x}{3}) is[-\frac{\pi}{2}, \frac{\pi}{2}]$.

PART-C 4 x 3 = 12

16) $\sin^{-1}$x+2$\cos^{-1}$x=$\sin^{-1}$x+xos-1x=$\frac{\pi}{2}$+$\cos^{-1}$x

We know that 0 $\le$ cos-1x $\le \pi$.Thus, $\frac{\pi}{2} + 0 \le cos^{-1}x$x+$\frac{\pi}{2} \le \pi + \frac{\pi}{2}$

Thus, $\frac{\pi}{2} \le sin^{-}1x + 2cos^{-1}x \le \frac{3\pi}{2}$

17) Given p,q are the roots of lx^2+nx+n=000

$p + q = \frac{-b}{a} = \frac{-n}{l} \qquad ...(1)$

$pq = \frac{c}{a} = \frac{n}{l} \qquad ...(2)$

and $\left(\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}}\right)$

consider $\left(\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}}\right)$

$= \frac{p}{q} + \frac{q}{p} + \frac{n}{l} + 2\sqrt{\frac{pq}{qb}} + 2\sqrt{\frac{qn}{pl}} + 2\sqrt{\frac{np}{ql}}$

$[\because (a+bc)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca]$

$\Rightarrow \frac{p^2+q^2}{pq} + \frac{n}{l} + 2 + 2\sqrt{\frac{p}{q}.pq} + 2\sqrt{\frac{q}{p}.pq} + 2\sqrt{\frac{p}{q}.pq}$

$= \frac{(p+q)^2 - 2pq}{pq} + \frac{n}{l} + 2 + 2\sqrt{p^2} + 2\sqrt{q^2}$

$= \frac{(p+q)^2}{pq} - \frac{2pq}{pq} + \frac{n}{l} + 2 + 2p + 2q$

$\Rightarrow \frac{\left(\frac{-n}{l}\right)^2}{\frac{n}{l}} - \cancel{2} + \frac{n}{l} + \cancel{2} + 2(p+q)$

$\Rightarrow \frac{n^2}{l^2.\frac{n}{l}} + \frac{n}{l} - \frac{2n}{l} \qquad [\because p + q = \frac{-n}{l}]$

$$\Rightarrow \frac{n}{l} + \frac{n}{l} - \frac{2n}{l} = 0 \therefore \left(\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}}\right)^2 = 0$$

Talking square root both sides, we get

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$$

18) Since $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ is a root, x- $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ is a factor. To remove the outermost square root, we take x+ $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as another factor and find their product.

$$\left(x + \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right)\left(x - \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right) = x^2 - \frac{\sqrt{2}}{\sqrt{3}}$$

Still we didn't achieve our goal. So we include another factor $x_2 + \frac{\sqrt{2}}{\sqrt{3}}$ and get the product.

$$\left(x^2 - \frac{\sqrt{2}}{\sqrt{3}}\right)\left(x^2 + \frac{\sqrt{2}}{\sqrt{3}}\right) = x^4 - \frac{2}{3}$$

So, $3x^4$-2=0 is a required polynomial equation with the integer coefficients.

Now we identify the nature of roots of the given equation without solving the equation. The idea comes from the negativity, equality to 0, positivity of $\Delta=b^2-4ac$.

19) f(x)=sin-1 $(\frac{|x| - 2}{3}) + cos^{-}1(\frac{1 - |x|}{4})$

From the definition of $\sin^{-1}$

$$-1 \leq \frac{|x| - 2}{3} \leq 1$$

$$\Rightarrow -3 \leq |x| - 2 \leq 3$$

$$\Rightarrow -3 + 2 \leq |x| \leq |x| \leq 3 + 2$$

$$\Rightarrow -1 \leq |x| \leq 5$$

It reduces to

$$0 \leq |x| \leq 5$$

$$\Rightarrow 0 \leq |x| and |x| \leq 5$$

$$\Rightarrow |x| \geq 0 and - 5 \leq x \leq 5$$

From the definition of $\cos^{-1}x$.

$$-1 \leq \frac{1 - |x|}{4} \leq 1$$

$$\Rightarrow -4 \leq 1 - |x| \leq 4$$

$$\Rightarrow -4 - 1 \leq |x| \leq 4 - 1$$

$$\Rightarrow -5 \leq -|x| \leq 3$$

$$\Rightarrow -3 \leq |x| > 5$$

It reduces to

$$0 \leq |x| \leq 5$$

$$-5 \leq |x| \leq 5$$

From (1) & (2),

Domain is [-5, 5]

20) $8x^{\frac{3}{2x}} - 8x^{\frac{-3}{2x}}=63$

$\Rightarrow 8\left[\left(x^{\frac{1}{2n}}\right)^3 - \left(x^{\frac{-1}{2n}}\right)^3\right] = 63$

Put $x^{\frac{1}{2n}} = y$

$\Rightarrow 8\left(y^2 - \frac{1}{y^3}\right) = 63$

$\Rightarrow y^3 - \frac{1}{y^3} = \frac{63}{8} \Rightarrow \frac{y^6 - 1}{y^3} = \frac{63}{8}$

$\Rightarrow 8y^6 - 8 = 63y^3$

$\Rightarrow 8y^6 - 63y^3 - 8 = 0$

$\Rightarrow 8t^2 - 63t - 8 = 0 \quad [where \quad t = y^3]$

$\Rightarrow (8t - 1)(t - 8) = 0$

$\Rightarrow t = \frac{1}{8}, 8$

Case (i)when $t = 8, \Rightarrow y^3 = 8 \Rightarrow y^2 = 2^3$

$\Rightarrow y = 2$

Case (ii)when $t = \frac{1}{8}, y^3 = \frac{1}{8} \Rightarrow y = \frac{1}{2}$

When $y = 2, x^{\frac{1}{2n}} = 2$

$\Rightarrow x = (2)^{2n} \quad \Rightarrow x = (2^2)^n$

$\Rightarrow x = 4^n$

When

$y = \frac{1}{2}, x^{\frac{1}{2n}} = \frac{1}{2} \Rightarrow x = \left(\frac{1}{2}\right)^{2n}$

$\Rightarrow x = \left(\frac{1}{2^2}\right)^n = \frac{1}{4^n}$

Hence the roots are 4^n.

PART-D

6 x 5 = 30

21) a)

Given $(\sqrt{5} - \sqrt{3})$ is a root

$\Rightarrow \sqrt{5} + \sqrt{3}$

$\therefore$ Sum of the roots $= \sqrt{5} - \sqrt{3} + \sqrt{5} + \sqrt{3} = 2\sqrt{5}$

Product of the roots

$= (\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$

$= (\sqrt{5})^2 - (\sqrt{3})^2 = 5 - 3 = 2$

$\therefore$ One of the factor is X^2 -x (sum of the roots) + product of the roots

$\Rightarrow x^2 - 2x\sqrt{5} + 2$

The other factor also will be $x^2 - 2x\sqrt{5} + 2$

$(x^2 - 2x\sqrt{5} + 2)(x^2 + 2x\sqrt{5} + 2) = 0$

$\Rightarrow (x^2 + 2 - 2\sqrt{5}x)(x^2 + 2 + 2\sqrt{5}x) = 0$

$\Rightarrow \left(x^2 + 2\right)^2 - (2\sqrt{5}x)^2 = 0$

$[\because (a + b)(a - b) = a^2 - b^2]$

$\Rightarrow x^4 + 4x^2 + 4(5)x^2 = 0$

$$\Rightarrow x^4 + 4x^2 + 4 - 20x^2 = 0$$
$$\Rightarrow x^4 - 16x^2 + 4 = 0$$

(OR)

b)

Given cubic equation is $3x^3$-$26x^2$+52x-24=0

Here, a = 3, b = -2b, c = 52,d = -24.

Since the roots form an geometric progression,

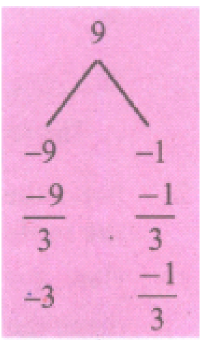
The roots are $\frac{a}{r}$ a, ar, sum of the roots=$\frac{-b}{a}$

$$\Rightarrow \frac{a}{r} + a + ar = \frac{26}{3} \quad ...(1)$$

and product of the roots $\frac{-d}{a}$

$$\Rightarrow \frac{a}{\cancel{r}} + a + a\cancel{r} = \frac{24}{3} = 8$$
$$a^3 = 8 = 2^3$$

a=2



∴ (1) becomes $\frac{2}{r}$+2+2r=$\frac{26}{3}$

$$\Rightarrow \frac{2+2r+2r^2}{r} = \frac{26}{3} \Rightarrow \frac{1+r+r^2}{r} = \frac{13}{3}$$
$$\Rightarrow 3 + 3r + 3r^2 = 13r \Rightarrow 3r^2 - 10r + 3 = 0$$
$$\Rightarrow (r - 3)(3r - 1) = 0 \Rightarrow r = 3$$

∴ The roots are $\frac{a}{r}$,a, and ar

$$\Rightarrow \frac{2}{3}, 2 \quad and \quad 2(3) \Rightarrow \frac{2}{3}m2 \quad and \quad 6$$

22) a)

The given equation is same as

(2x-3)(3x-2)(6x-1)(x-12)-7=0

After a computation, the above equation becomes

($6x^2$-13x+6)($6x^2$-13x+12)-7=0

By taking y=$6x^2$-13x, the above equation becomes

(y+6)(y+12)-7=0

which is same as

y^2+18y+65=0

Solving this equation, we get y=−13 and y=−5.

Substituting the values of y in y=−$6x^2$-13x we get

$6x^2$-13x+5 =0

$6x^2$-13x+13 =0

Solving these two equations, we get

x=$\frac{1}{2}$,x=$\frac{5}{3}$,x = $\frac{13 \pm \sqrt{143i}}{12}$ and x=$\frac{13-\sqrt{143i}}{12}$

as the roots of the given equation.

(OR)

b)

By choosing the co-ordinate axes suitably, we take the equation of the straight line as

y=mx+c ...(1)

and equation of parabola as $y^2=4ax$ ...(2)

Substituting (1) in (2),we get

$(mx + c)^2 = 4ax$

$\Rightarrow m^2x^2 + c^2 + 2mcx = 4ax$

$\Rightarrow m^2x^2+x(2mc - 4a) + c^2 = 0$

Which is a quadratic equation in x.

This equation cannot have more than two solution. Hence, a straight line and a parabola cannot intersect at more than two points.

23) a)

Let $\sin^{-1})x=\theta$.Then, x=sinθ and-1 $\leq x \leq 1$

Now, tan(sin-1 x)=tanθ=$\frac{sin\theta}{cos\theta} = \frac{sin\theta}{\sqrt{1-sin^2\theta}} = \frac{x}{\sqrt{1-x2}}$, $|x| < 1$

(OR)

b)

Consider $tan^{-1}(x-1) + tan^{-1}(x+1)$

$= tan^{-1}\left(\frac{x-1+x+1}{1-(x-1)(x+1)}\right) = tan^{-1}\left(\frac{2x}{1-\left(x^2-1\right)}\right)$

$= tan^{-1}\left(\frac{x-1+x+1}{1-(x-1)(x+1)}\right) = tan^{-1}\left(\frac{2x}{1-\left(x^2-1\right)}\right)$

$= tan^{-1}\left(\frac{2x}{1-x^2+1}\right)$

$= tan^{-1}\left(\frac{2x}{2-x^2}\right)$

$\therefore tan^{-1}\left(\frac{2x}{2-x^2}\right) + tan^{-1}(x) = tan^{-1}(3x)$

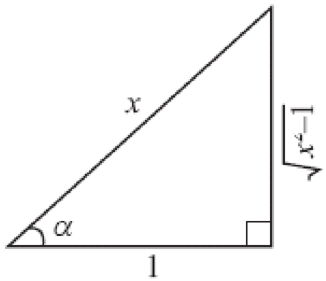
$\Rightarrow tan^{-1}\left(\frac{2x}{2-x^2}\right) = tan^{-1}(3x) - tan^{-1}(x)$

$\Rightarrow tan^{-1}\left(\frac{2x}{2-x^2}\right) = tan^{-1}\left(\frac{3xx}{1+3x^2}\right)$

$\Rightarrow \frac{2x}{2-x^2} = \frac{2x}{1+3x^2}$

Cross multiply, we get a cubic equations,• Hence, there are 3 solutions for the equation,

24) a)



Let cot−1 $\left(\frac{1}{\sqrt{x^2-1}}\right) = \alpha$. Then, cot $\alpha = \frac{1}{\sqrt{x^2-1}}$

We construct a right triangle with the given data.

From the triangle, sec$\alpha = \frac{x}{1} = x$. thus, α=sec-1x

Hence, cot $^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = sec^{-1}x, \ |x| > 1$

(OR)

b)

Let sec-1 $\frac{5}{4} = \theta$. Then, sec$\theta = \frac{5}{4}$ and hence. cos$\theta = \frac{4}{5}$

Also, sin $\theta = \sqrt{1 - cos^2\theta} = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$, which gives $\theta = sin^{-1}(\frac{3}{5})$

Thus, sec $^{-1}\left(\frac{5}{4}\right) = sin^{-1}\left(\frac{3}{5}\right) and \quad sin^{-1}\frac{3}{5} + sec^{-1}\left(\frac{5}{4}\right) = 2sin^{-1}\left(\frac{3}{5}\right)$

We know that sin $^{-1}\left(2x\sqrt{1 - x^2}\right) = 2sin^{-1}x, \quad if\,|x| \le \frac{1}{\sqrt{2}}$

Since $\frac{3}{5} < \frac{1}{\sqrt{2}}$, we have 2 $sin^{-1}\left(\frac{3}{5}\right) = sin^{-1}\left(2 \times \frac{3}{5}\sqrt{1 - \left(\frac{3}{5}\right)^2}\right) = sin^{-1}\left(\frac{24}{25}\right)$

Hence, $sin\left[sin^{-1}\left(\frac{3}{5}\right) + sec^{-1}\left(\frac{5}{4}\right)\right] = sin\left(sin^{-1}\left(\frac{24}{25}\right)\right) = \frac{24}{25}, \quad since\frac{24}{25} \in [-1, 1]$