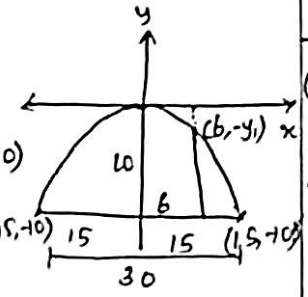
Exercise : 5.5

Eqn of Parabola is

$x^2 = -4ay \rightarrow ①$



It Passes through (15,-10)

$(15)^2 = -4a(-10)$

$225 = 40a$

$\Rightarrow a = \frac{225}{40}$

$① \Rightarrow x^2 = -4\left(\frac{225}{40}\right)y$

It passes through $(6, -y_1)$

$36 = -4\left(\frac{225}{40}\right)(-y_1)$

$\frac{36 \times 10}{225} = y_1$

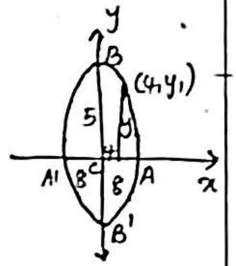
$1.6 = y_1$

The required height $= 10 - y_1 = 10 - 1.6$

$= 8.4m$.

②

$CA = a = 8, \ CB = 5 = b$



Eqn of ellipse is

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$\frac{x^2}{25} + \frac{y^2}{64} = 1$

It passes through $(4, y_1)$

$\frac{16}{25} + \frac{y_1^2}{64} = 1$

$\frac{y_1^2}{64} = 1 - \frac{16}{25}$

$y_1^2 = 64\left(\frac{9}{25}\right)$

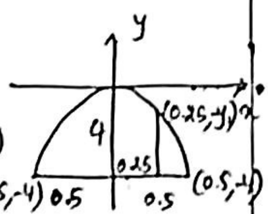
$y_1 = \frac{8 \times 3}{5} = \frac{24}{5} = 4.8m$.

∴ The required wide for the opening is $2y = 2(4.8) = 9.6m$.

③

Eqn of the Parabola is

$x^2 = -4ay \rightarrow ①$



It passes through (0.5,-4)

$(0.5)^2 = -4a(-4)$

$a = \frac{0.25}{16}$

$① \Rightarrow x^2 = -4\left[\frac{0.25}{16}\right]y$

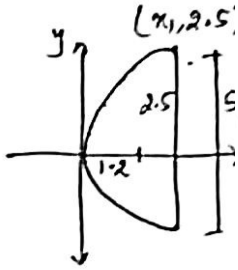
It passes through $(0.25, -y_1)$

$(0.25)^2 = -4\left[\frac{0.25}{16} \times (-y_1)\right]$

$y_1 = \frac{4 \times (0.25)^2}{0.25} = 4 \times 0.25 = 1m$

The required height $= 4 - y_1 = 4 - 1 = 3m$

④



(a) Eqn of Parabola is

$y^2 = 4ax$

$a = 1.2$

$y^2 = 4.8x$

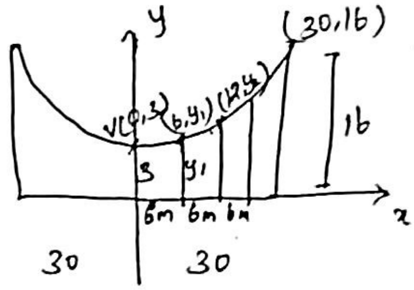
(b) It Passes through $(x_1, 2.5)$

$(2.5)^2 = 4.8x_1$

$x_1 = \frac{2.5 \times 2.5}{4.8} = 1.3m$

The depth of the satellite dish at the vertex is 1.3m.

⑤



Eqn of the parabola is

$(x-h)^2 = 4a(y-k)$

$(x-0)^2 = 4a(y-3)$

It passes through (30, 16)

$(30)^2 = 4a(16-3)$

$a = \frac{30 \times 30}{13 \times 4}$

$x^2 = 4\left(\frac{30 \times 30}{13 \times 4}\right)(y-3)$

It passes through $(6, y_1)$

$6^2 = \frac{900}{13}(y_1 - 3)$

$(y_1 - 3) = \frac{36 \times 13}{900} = 0.52$

$y_1 = 3 + 0.52 = 3.52m$.

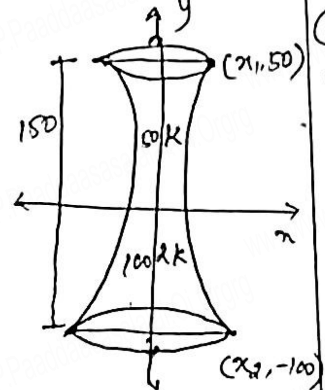
It also passes through $(12, y_2)$

$12^2 = \frac{900}{13}(y_2 - 3)$

$y_2 - 3 = \frac{144 \times 13}{900} = 2.08$

$y_2 = 3 + 2.08 = 5.08m$

$3k = 150$
$k = 50$



$(x_1, 50)$ lies on the hyperbola

$$\frac{x_1^2}{30^2} - \frac{50^2}{44^2} = 1$$

$$\frac{x_1^2}{30^2} = 1 + \frac{50^2}{44^2}$$

$$x_1^2 = \frac{30^2}{44^2}[1936 + 2500]$$

$$x_1 = \frac{30}{44}\sqrt{4436} = \frac{30}{44}(66.60)$$

$$x_1 = 45.40$$

Radius of the top of the tower is 45.40m

$(x_2, -100)$ lies on the hyperbola

$$\frac{x_2^2}{30^2} - \frac{100^2}{44^2} = 1$$

$$\frac{x_2^2}{30^2} = 1 + \frac{100^2}{44^2}$$

$$x_2^2 = \frac{30^2}{44^2}(1936 + 10000)$$

$$x_2 = \frac{30}{44}\sqrt{11936}$$

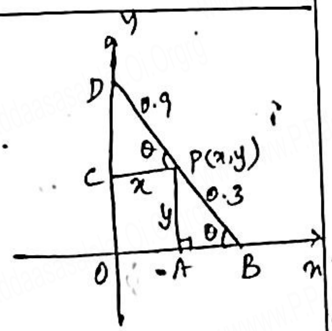
$$= \frac{30}{44} \times 109.25$$

$$x_2 = \frac{3277.5}{44} = 74.48\text{ m}$$

Radius of the base of the tower is 74.48 m.

(7) BAP $\cong$ PCD

Let $\angle ABP = \angle CPD = \theta$.



In ΔPAB

$$\sin\theta = \frac{y}{0.3}$$

In ΔPCD

$$\cos\theta = \frac{x}{0.9}$$

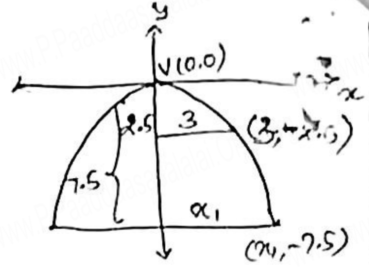
$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{x^2}{(0.9)^2} + \frac{y^2}{(0.3)^2} = 1$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{0.3^2}{0.9^2}} = \sqrt{1 - \left(\frac{1}{3}\right)^2}$$

$$e = \sqrt{8/9} = \frac{2\sqrt{2}}{3}\text{ m.}$$

(8) The eqn of the parabola is



$$x^2 = -4ay$$

It passes through $(3, -2.5)$

$$3^2 = -4a(-2.5)$$

$$a = \frac{9}{10}$$

$$x^2 = -4\left(\frac{9}{10}\right)y$$

$$x^2 = -18/5\, y$$

It passes through $(x_1, -7.5)$

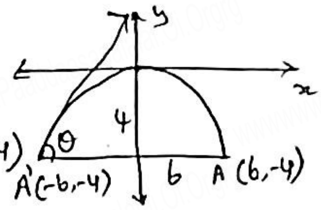
$$x_1^2 = -18/5\,(-7.5)$$

$$x_1^2 = 27$$

$$x_1 = 3\sqrt{3}\text{ m}$$

The water strikes the ground $3\sqrt{3}$ m beyond the vertical line.

(9) The eqn of parabola is $x^2 = -4ay$



It passes through $(6, -4)$

$$36 = 16a$$

$$\Rightarrow a = \frac{36}{16} = 9/4$$

$$x^2 = -4(9/4)y$$

$$x^2 = -9y$$

Diff w.r.to x

$$2x = -9\frac{dy}{dx}$$

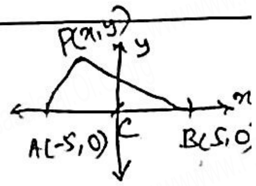
$$\frac{dy}{dx} = -2x/9$$

At $(-6, -4)$

$$\tan\theta = \frac{dy}{dx} = \frac{-2}{9}(-6) = 4/3$$

$$\theta = \tan^{-1} 4/3.$$

(10) $ae = 5$



$$AP \sim BP = 2a = 6$$

$$a = 3$$

$$b^2 = a^2(e^2 - 1) = (ae)^2 - (a)^2 = 25 - 9$$

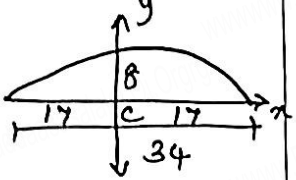
$$b^2 = 16$$

The eqn of the required hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

The locus of P is hyperbola.

Eg : 5.37



Sol:

Let $a = 17$, $b = 8$

$c^2 = a^2 - b^2$

$= 289 - 64$

$c^2 = 225$

$c = 15$

Foci are $F_1(15, 0)$, $F_2(-15, 0)$

Eg : 5.38

Eqn of ellipse is $\frac{(x-11)^2}{484} + \frac{y^2}{64} = 1$

$a^2 = 484$, $b^2 = 64$

$c^2 = a^2 - b^2$

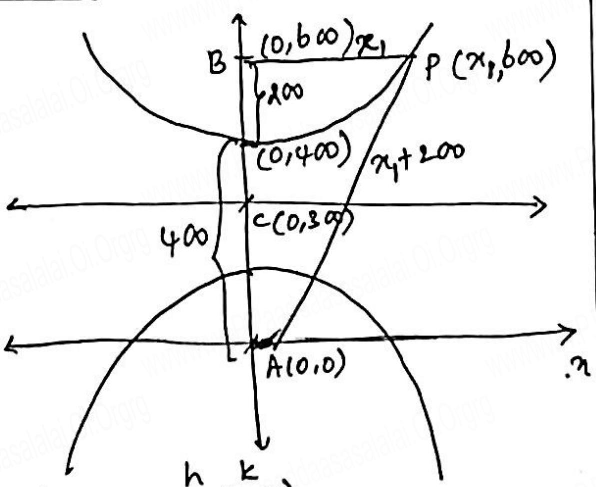
$= 484 - 64$

$c^2 = 420$

$c \simeq 20.5$

$\therefore$ The patient's kidney stone should be placed 20.5 cm from the centre of the ellipse.

Eg : 5.39.



Centre : $C(\overset{h}{0}, \overset{k}{300})$

Eqn of hyperbola is

$$\frac{(y-300)^2}{a^2} - \frac{(x-0)^2}{b^2} = 1 \rightarrow (1)$$

It passes through (0, 400)

$$\frac{(400-300)^2}{a^2} - 0 = 1$$

$a^2 = (100)^2$

$a = 100$

In ΔABP

$AP^2 = AB^2 + BP^2$

$(x_1 + 200)^2 = 600^2 + x_1^2$

$\cancel{x_1^2} + 400x_1 + (200)^2 = (600)^2 + \cancel{x_1^2}$

$400x_1 = (600)^2 - (200)^2$

$= 360000 - 40000$

$x_1 = \frac{\cancel{32}0000}{\cancel{400}}$

$x_1 = 800$

P(800, 600) lies on the hyperbola

$(1) \Rightarrow \frac{(y-300)^2}{a^2} - \frac{x^2}{b^2} = 1$

$\frac{(600-300)^2}{a^2} - \frac{(800)^2}{b^2} = 1$

$\frac{(300)^2}{(100)^2} - 1 = \frac{(800)^2}{b^2}$

$\frac{90000}{10000} - 1 = \frac{(800)^2}{b^2}$

$b^2 = \frac{\cancel{64}0000}{\cancel{8}}$

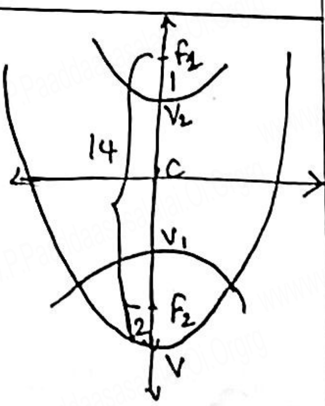
$b^2 = 80000$

The required eqn of hyperbola is

$$\frac{(y-300)^2}{10000} - \frac{x^2}{80000} = 1$$

The exact location can be determined using data from a 3rd station.

Eg : 5.40



$F_1F_2 = 14 - 2 = 12$

$2ae = 12$

$ae = 6$

$V_2F_1 = CF_2 - CV_2$

$1 = ae - a$

$1 = 6 - a$

$\boxed{a = 5}$

$b^2 = a^2(e^2 - 1)$

$= a^2e^2 - a^2$

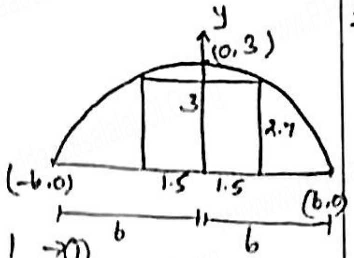
$= 36 - 25$

$b^2 = 11$

The eqn of the hyperbola is

$$\frac{y^2}{25} - \frac{x^2}{11} = 1$$

Eg: 5.30



Let $a = 6$, $b = 3$

Eqn of the ellipse is $\frac{x^2}{36} + \frac{y^2}{9} = 1 \rightarrow (1)$

When $x = 1.5$, $y = ?$

or $x = \frac{3}{2}$

$(1) \Rightarrow \frac{\left(\frac{3}{2}\right)^2}{36} + \frac{y^2}{9} = 1$

$y^2 = 9\left[1 - \frac{9}{144}\right]$

$= \frac{9 \times 135}{144} = \frac{135}{16}$

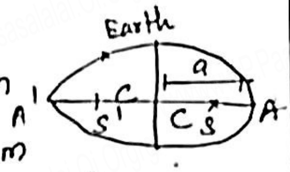
$y = \frac{\sqrt{135}}{16} = \frac{11.62}{16}$

$y = 2.90$

The truck will clear the archway.

Eg: 5.31

Sol:



Let $SA = 94.5 \times 10^6$ km

$SA' = 152 \times 10^6$ km

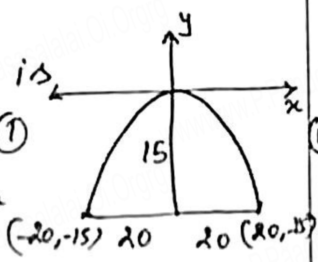
$SA = a - c = 94.5 \times 10^6 \rightarrow (1)$

$SA' = a + c = 152 \times 10^6 \rightarrow (2)$

$(2) - (1) \Rightarrow 2c = 57.5 \times 10^6 = 575 \times 10^5$ km

$SS' = 575 \times 10^5$ km.

Eg: 5.32



The eqn of parabola is

$x^2 = -4ay \rightarrow (1)$

It passes through $(20, -15)$

$(20)^2 = -4a(-15)$

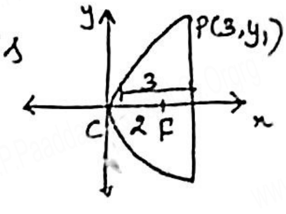
$\Rightarrow 4a = \frac{400}{15} = \frac{80}{3}$

$(1) \Rightarrow x^2 = -\frac{80}{3}y$

The required eqn is

$3x^2 = -80y$

Eg: 5.33



Eqn of Parabola is

$y^2 = 4ax$

$a = 2$

$y^2 = 8x$

It passes through $P(3, y_1)$

$y_1^2 = 8 \times 3$

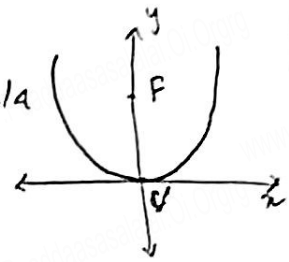
$y_1 = 2\sqrt{2} \times \sqrt{3}$

$y_1 = 2\sqrt{6}$

Required width $2y_1 = 4\sqrt{6}$ m.

Eg: 5.34

Sol:



Eqn of the Parabola

$y = \frac{1}{32}x^2$

$x^2 = 32y$

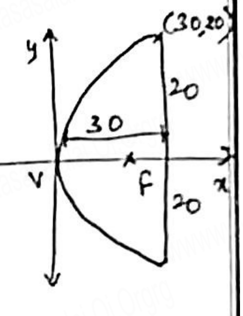
$x^2 = 4(8)y$

$\Rightarrow a = 8$

The Heating tube needs to be Placed 8 units above the vertex of the parabola.

Eg: 5.35.

Sol



Eqn of Parabola is

$y^2 = 4ax \rightarrow (1)$

(1) It passes through $(30, 20)$

$(20)^2 = 4a(30)$

$\frac{400}{30} = 4a$.

$(1) \rightarrow y^2 = \frac{40}{3}x$

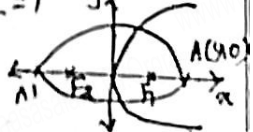
(2) $4a = \frac{40}{3}$

$a = 10/3$

The bulb is at focus $(10/3, 0)$

Eg: 5.36

Sol:



Eqn of ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$a^2 = 16$, $b^2 = 9$

$c^2 = a^2 - b^2 = 7$

$c = \pm\sqrt{7}$

Foci: $F_1(\sqrt{7}, 0)$, $F_2(-\sqrt{7}, 0)$. Focus of the Parabola $(\sqrt{7}, 0) \Rightarrow a = \sqrt{7}$

Eqn of the parabola is $y^2 = 4\sqrt{7}x$

$$\frac{2x}{3} - \frac{y}{2} = 1.$$

$$\boxed{4x - 3y - 6 = 0}$$

Normal equation at 'θ'

$$\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$$

$\theta = \pi/3$

$$\frac{3x}{2} + \frac{2\sqrt{3}y}{\sqrt{3}} = 9 + 12.$$

$$\frac{3x}{2} + 2y = 21.$$

$$\boxed{3x + 4y - 42 = 0}$$

7. Solu:

Tangent equation at 't_1' is

$$yt_1 = x + at_1^2 \longrightarrow ①$$

at 't_2' is

$$yt_2 = x + at_2^2 \longrightarrow ②$$

$$① - ② \Rightarrow y(t_1 - t_2) = a(t_1^2 - t_2^2)$$

$$y = \frac{a(t_1+t_2)(t_1 - t_2)}{t_1 - t_2}.$$

$$y = a(t_1 + t_2)$$

Using $y = a(t_1+t_2)$ in ①, we get

$$a(t_1+t_2)t_1 = x + at_1^2$$

$$at_1^2 + at_1t_2 = x + at_1^2$$

$$x = at_1t_2.$$

$\therefore$ The point of intersection is

$$[at_1t_2,\ a(t_1+t_2)]$$

8. Solu:

Normal equation of a parabola at $P(at_1^2, 2at_1)$ is

$$y + xt_1 = at_1^3 + 2at_1.$$

It meets the parabola again at $Q(at_2^2, 2at_2)$

$$2at_2 + (at_2^2)t_1 = at_1^3 + 2at_1$$

$$2at_2 + at_1t_2^2 = at_1^3 + 2at_1$$

$$2at_2 - 2at_1 = at_1^3 - at_1t_2^2$$

$$2a(t_2 - t_1) = at_1(t_1^2 - t_2^2)$$

$$2a(t_2 - t_1) = at_1(t_1 - t_2)(t_1 + t_2)$$

$$2a(t_2 - t_1) = -at_1(t_2 - t_1)(t_1 + t_2)$$

$$-2 = t_1(t_1 + t_2)$$

$$t_1 + t_2 = -\frac{2}{t_1}$$

$$t_2 = -\frac{2}{t_1} - t_1$$

$$t_2 = -\left(t_1 + \frac{2}{t_1}\right)$$

Hence proved.

2. Solu:-

$$\frac{x^2}{16} - \frac{y^2}{64} = 1$$

$a^2 = 16$, $b^2 = 64$.

slope of the tangent $m = \frac{-10}{-3}$

$$m = \frac{10}{3}.$$

Any tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is of the form,

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

$$y = \frac{10}{3}x \pm \sqrt{16\left(\frac{100}{9}\right) - 64}$$

$$y = \frac{10}{3}x \pm \sqrt{\frac{1024}{9}}$$

$$y = \frac{10}{3}x \pm \frac{32}{3}$$

$$3y = 10x \pm 32.$$

$$10x - 3y \pm 32 = 0.$$

Tangent equations are

$$10x - 3y + 32 = 0$$

$$10x - 3y - 32 = 0.$$

3. Solu:-

$x - y + 4 = 0$

$y = x + 4.$

$m = 1$, $c = 4$

$x^2 + 3y^2 = 12$

$\frac{x^2}{12} + \frac{y^2}{4} = 1$

$a^2 = 12$, $b^2 = 4$

Condition for tangency

$$c^2 = a^2m^2 + b^2$$

$c^2 = 16$, $a^2m^2 + b^2 = 12(1)^2 + 4 = 16.$

$\therefore$ The line is a tangent to the ellipse.

point of contact:

$$\left(-\frac{a^2m}{c}, \frac{b^2}{c}\right) = \left[-\frac{12(1)}{4}, \frac{4}{4}\right]$$

$$= (-3, 1)$$

4. Solu:

$$y^2 = 16x$$

$$4a = 16 \Rightarrow a = 4$$

Any line $\perp r$ to $2x + 2y + 3 = 0$ is $2x - 2y + k = 0$

$$x - y + \frac{k}{2} = 0 \rightarrow (1)$$

$$y = x + \frac{k}{2}.$$

$$m = 1, \quad c = \frac{k}{2}.$$

The condition for the line $y = mx + c$ be a tangent to the parabola $y^2 = 4ax$ is $c = \frac{a}{m}$

$$c = \frac{4}{1}$$

$$\frac{k}{2} = 4 \Rightarrow \boxed{k = 8}$$

Substitute $k = 8$ in (1)

Required $\perp r$ line is $\boxed{x - y + 4 = 0}$

5. Solu:

$$y^2 = 8x$$

$$4a = 8 \Rightarrow a = 2$$

Here $t = 2$, $a = 2$.

The equation of tangent at 't' on the parabola is

$$yt = x + at^2$$

$$y(2) = x + 2(4)$$

$$2y = x + 8.$$

$$\boxed{x - 2y + 8 = 0}$$

6. Solu: $12x^2 - 9y^2 = 108$

$\div$ by $108 \Rightarrow \frac{x^2}{9} - \frac{y^2}{12} = 1$

$a^2 = 9$, $b^2 = 12$

$a = 3$, $b = 2\sqrt{3}$.

Tangent equation at 'θ'

$$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1.$$

$\theta = \pi/3$, $\frac{x\sec(\pi/3)}{3} - \frac{y\tan\pi/3}{2\sqrt{3}} = 1$

$$\frac{2x}{3} - \frac{\sqrt{3}y}{2\sqrt{3}} = 1$$

Note: Tangent equation at (x_1, y_1) for all the curves,
Replace x^2 by xx_1, y^2 by yy_1
x by $\frac{1}{2}(x+x_1)$, y by $\frac{1}{2}(y+y_1)$
and xy by $\frac{1}{2}(xy_1+yx_1)$.

Example: 5.27.

Soln:-

$$x^2+6x+4y+5=0.$$

Tangent equation at (x_1, y_1) is

$$xx_1+6\left(\frac{x+x_1}{2}\right)+4\left(\frac{y+y_1}{2}\right)+5=0.$$

$$xx_1+3(x+x_1)+2(y+y_1)+5=0.$$

$$(x_1, y_1)=(1,-3).$$

$$x(1)+3(x+1)+2(y-3)+5=0$$

$$x+3x+3+2y-6+5=0$$

$$4x+2y+2=0$$

$\div$ by 2, $\boxed{2x+y+1=0}$.

Normal Equation: at $(1,-3)$

$$x-2y+k=0$$

$$1-2(-3)+k=0$$

$$1+6+k=0$$

$$7+k=0$$

$$k=-7.$$

$$\boxed{\therefore\ x-2y-7=0}$$

Example: 5.28.

Soln:-

$$x^2+4y^2=32$$

$$\frac{x^2}{32}+\frac{y^2}{8}=1.$$

$a^2=32$, $b^2=8$
$a=4\sqrt{2}$, $b=2\sqrt{2}$

Tangent equation at 'θ'

$$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$$

$\theta=\pi/4$,

$$\frac{x\cos\pi/4}{4\sqrt{2}}+\frac{y\sin\pi/4}{2\sqrt{2}}=1$$

$$\frac{x}{8}+\frac{y}{4}=1 \Rightarrow \boxed{x+2y-8=0}$$

Normal Equation at 'θ'

$$\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$$

$\theta=\pi/4$,

$$\frac{4\sqrt{2}\,x}{\cos\pi/4}-\frac{2\sqrt{2}\,y}{\sin\pi/4}=32-8$$

$$8x-4y=24.$$

i.e., $\boxed{2x-y-6=0}$

Exercise: 5.4.

1. Soln: $2x^2+7y^2=14$, $(5,2)$

$\div$ by 14, $\frac{x^2}{7}+\frac{y^2}{2}=1.$

$$a^2=7,\quad b^2=2.$$

Any tangent to the Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is of the form,

$$y=mx\pm\sqrt{a^2m^2+b^2}$$

$$2=m(5)\pm\sqrt{7m^2+2}.$$

$$2-5m=\pm\sqrt{7m^2+2}$$

Squaring on both sides, we get

$$4-20m+25m^2=7m^2+2.$$

$$18m^2-20m+2=0$$

$\div$ by 2, $9m^2-10m+1=0$

$$(9m-1)(m-1)=0.$$

$$\therefore m=\frac{1}{9} \text{ (or) } m=1.$$

Tangent Equations:

when $m=1$, $(x_1, y_1)=(5,2)$

$$\boxed{y-y_1=m(x-x_1)}$$

$$y-2=(1)(x-5)$$

$$\boxed{x-y-3=0}$$

when $m=\frac{1}{9}$, $(x_1, y_1)=(5,2)$

$$y-2=\frac{1}{9}(x-5)$$

$$9y-18=x-5$$

$$\boxed{x-9y+13=0}$$

Exercise : 5.3.

1. $2x^2 - y^2 = 7$.

Solu:

$2x^2 - y^2 = 7$

Compared with

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$A = 2,\ B = 0,\ C = -1$

$B^2 - 4AC = 0 - 4(2)(-1) = 8 > 0$

$\therefore$ The given equation is a Hyperbola.

2. Solu:

$3x^2 + 3y^2 - 4x + 3y + 10 = 0$

$A = 3,\ B = 0,\ C = 3,$

$A = C = 3$ & $B = 0$.

The given equation is a circle.

3. Solu:

$3x^2 + 2y^2 = 14$.

$A = 3,\ B = 0,\ C = 2,$

$B^2 - 4AC = 0 - (4)(3)(2) = -12 < 0$

$\therefore$ The given equation is an Ellipse.

4. Solu:

$x^2 + y^2 + x - y = 0$

$A = 1,\ B = 0,\ C = 1,\ D = 1,\ E = -1,\ F = 0$

Here $A = C = 1,\ B = 0$.

$\therefore$ The given equation is a circle.

5. Solu:

$11x^2 - 25y^2 - 44x + 50y - 256 = 0$

$A = 11,\ B = 0,\ C = -25,\ D = -44,\ E = 50,\ F = -256$.

$B^2 - 4AC = 0 - 4(11)(-25) = 1100 > 0$

The given equation is a Hyperbola.

6. Solu:

$y^2 + 4x + 3y + 4 = 0$.

$A = 0,\ B = 0,\ C = 1,\ D = 4,\ E = 3,\ F = 4$.

$B^2 - 4AC = 0 - 4(0)(1) = 0$.

The given equation is a Parabola.

Example : 5.26.

(i) Solu:

$16y^2 = -4x^2 + 64$.

$4x^2 + 16y^2 - 64 = 0$.

$A = 4,\ B = 0,\ C = 16,$

$B^2 - 4AC = 0 - 4(4)(16) = -256 < 0$

$\therefore$ The given equation is an Ellipse.

(ii) Solu:

$x^2 + y^2 = -4x - y + 4$

$x^2 + y^2 + 4x + y - 4 = 0$

$A = 1,\ B = 0,\ C = 1,\ D = 4,\ E = 1,\ F = -$

$A = C = 1,\ B = 0$

$\therefore$ The given equation is a Circle.

(iii) $x^2 - 2y = x + 3$

$x^2 - x - 2y - 3 = 0$

$A = 1,\ B = 0,\ C = 0,\ D = -1,\ E = -2,\ F = -3$

$B^2 - 4AC = 0 - 4(1)(0)$

$= 0$.

$\therefore$ The given equation is a Parabola.

(iv) Solu:

$4x^2 - 9y^2 - 16x + 18y - 29 = 0$

$A = 4,\ B = 0,\ C = -9,\ D = -16,\ E = 18,\ F = -2$

$B^2 - 4AC = 0 - 4(4)(-9)$

$= 144 > 0$

$\therefore$ The given equation is a Hyperbola.

Centre $(h,k) = (0,0)$

Focus $(h, k\pm ae) = (0, \pm 5)$

Vertices $(h, k\pm a) = (0, \pm 4)$.

$e = \sqrt{\frac{a^2+b^2}{a^2}} = \frac{5}{4}$

Directrix $y = \pm \frac{a}{e} = \frac{(4)}{(5/4)}$

$y = \pm \frac{16}{5}$.

EXE 5.2 (8)

(i) $\frac{(x-3)^2}{225} + \frac{(y-4)^2}{289} = 1$.

$b^2 = 225 \quad a^2 = 289$

$(ae)^2 = a^2 - b^2 = 289 - 225$

$(ae)^2 = 64$

$\boxed{ae = 8}$

Centre $(h,k) = (3,4)$

Focus $(h, k\pm ae) = (3, 4\pm 8)$

$F_1(3,12) \quad F_2(3,-4)$

Vertices $(h, k\pm a) = (3, 4\pm 17)$

$A(3,21)$, $A'(3,-13)$.

$e = \sqrt{\frac{a^2-b^2}{a^2}} = \frac{8}{17}$

Directrix $y-4 = \pm \frac{a}{e}$

$y-4 = \pm \frac{17}{(8/17)}$

$y-4 = \pm \frac{289}{8}$

$y = 4 \pm \frac{289}{8}$.

(ii) $\frac{(x+1)^2}{100} + \frac{(y-2)^2}{64} = 1$.

$a^2 = 100, \ b^2 = 64$.

$(ae)^2 = a^2 - b^2 = 100 - 64 = 36$

$\boxed{ae = 6}$.

Centre, $(h,k) = (-1,2)$.

Focus $(h\pm ae, k) = (-1\pm 6, 2)$

$F_1(5,2)$ & $F_2(-7,2)$.

Vertices $(h\pm a, k) = (-1\pm 10, 2)$

$A(9,2)$, $A'(-11,2)$.

$e = \sqrt{\frac{a^2-b^2}{a^2}} = \frac{6}{10} = \frac{3}{5}$

Equation of Directrices $x = \pm \frac{a}{e}$

$x+1 = \pm \frac{a}{e}$

$x+1 = \pm \frac{10}{(3/5)}$

$x+1 = \pm \frac{50}{3}$

$x = -1 \pm \frac{50}{3}$.

(iii) $\frac{(x+3)^2}{225} - \frac{(y-4)^2}{64} = 1$.

$a^2 = 225, \ b^2 = 64$

$(ae)^2 = a^2 + b^2 = 225 + 64 = 289$

$\boxed{ae = 17}$

Centre $(h,k) = (-3,4)$

Focus $(h\pm ae) = (-3\pm 17, 4)$

$F_1(14,4)$, $F_2(-20,4)$

Vertices $(h\pm a, k) = (-3\pm 15, 4)$

$A(12,4) \quad A'(-18,4)$

Directrix $x = \pm a/e = \frac{15}{17/15} = \frac{225}{17}$

$x+3 = \pm \frac{225}{17}$

$x = -3 \pm \frac{225}{17}$

(iv) $y^2 - 4y - 8x + 12 = 0$

$y^2 - 4y + 4 = +8x - 8$

$(y-2)^2 = +8(x-1)$

$y^2 = +4ax \quad \therefore a = 2$

The parabola open ~~leftward~~ Right

vertices $(h,k) = (1,+2)$

Focus $(h+a, 0+k) = (3,2)$

Lactus rectum $x = +a$

$x - 1 = +2$

$x = +1+2$

$\boxed{x = 3}$

Directrix $x = -a$

$x - 1 = -2$

$\boxed{x = -1}$

length of the lactus rectum.

$4a = 4(2) = 8.$

EXE 5.2 (5).

(i) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

$a^2 = 25 \Rightarrow a = \pm 5$

$b^2 = 9 \Rightarrow b = \pm 3.$

$(ae)^2 = a^2 - b^2 = 25 - 9 = 16$

$\boxed{ae = 4}$

centre $(h,k) = (0,0)$

Focus $(h \pm ae, k) = (\pm 4, 0).$

vertices $(h \pm a, k) = (\pm 5, 0).$

Directrix $x = \pm \frac{a}{e}$

$e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

$x = \pm \frac{5}{(3/5)} = \pm \frac{25}{3}.$

(ii) $\frac{x^2}{3} + \frac{y^2}{10} = 1.$

$a^2 = 10, \quad b^2 = 3.$

$(ae)^2 = a^2 - b^2 = 10 - 3 = 7$

$ae = \sqrt{7}$

Centre $(h,k) = (0,0).$

vertices $(h, \pm a + k) = (0, \pm\sqrt{10}).$

Focus $(h, \pm ae + k) = (0, \pm\sqrt{7}$

$e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{10-3}{10}} = \frac{\sqrt{7}}{\sqrt{10}}$

directrix $= \pm \frac{a}{e} = \pm \frac{\sqrt{10}}{(\sqrt{7}/\sqrt{10})}$

$y = \frac{10}{\sqrt{7}}$

(iii) $\frac{x^2}{25} - \frac{y^2}{144} = 1.$

$a^2 = 25, \quad b^2 = 144.$

$(ae)^2 = a^2 + b^2 = 25 + 144 = 169$

$ae = 13.$

Centre $= (h,k) = (0,0)$

Focus $(h \pm ae, k) = (\pm 13, 0).$

vertices $(h \pm a, k) = (\pm 5, 0).$

$e = \sqrt{\frac{a^2 + b^2}{a^2}} = \frac{13}{5}$

Directrix $x = \pm \frac{a}{e} = \pm \frac{5}{(13/5)}$

$x = \pm \frac{25}{13}.$

(iv) $\frac{y^2}{16} - \frac{x^2}{9} = 1$

$a^2 = 16, \quad b^2 = 9$

$(ae)^2 = a^2 + b^2 = 16 + 9 = 25$

$\boxed{ae = 5}$

Equation of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

At $(5,-2)$ & $a=4$

$$\frac{25}{16} - \frac{4}{b^2} = 1.$$

$$\frac{25}{16} - 1 = \frac{4}{b^2}$$

$$\frac{25-16}{16} = \frac{4}{b^2}$$

$$\frac{9}{16} = \frac{4}{b^2}$$

$$\frac{b^2}{4} = \frac{16}{9}$$

$$b^2 = \frac{64}{9}$$

Equation of the hyperbola.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{16} - \frac{y^2}{64/9} = 1$$

$$\frac{x^2}{16} - \frac{9y^2}{64} = 1.$$

EXE 5.2 (4).

(i) $y^2 = 16x \Rightarrow y^2 = 4ax$

$y^2 = 4(4)x$ $\boxed{a=4}$

The parabola open rightward

vertices $(0,0)$

Focus $(a,0) = (4,0)$

Lactus rectum $x = a \Rightarrow x = 4$

Directrix $x = -a \Rightarrow x = -4$

length of L.R $4a = 4(4) = 16$

(ii) $x^2 = 24y$. $x^2 = 4ay$.

$x^2 = 4(6)y$ $a = 6$.

The parabola open upward

vertices $(0,0) = (h,k)$

focus $(0,a) = (0,6)$

Lactus rectum $y = a \Rightarrow y = 6$

Directrix $y = -a \Rightarrow y = -6$

length of L.R $4a = 4(6) = 24$.

(iii) $y^2 = -8x$ $y^2 = -4ax$

$y^2 = -4(2)x$ $a = 2$.

The parabola open leftward

vertices $(0,0) = (h,k)$

focus $(-a,0) = (-2,0)$

Lactus rectum $x = -a \Rightarrow x = -2$

Directrix $x = a \Rightarrow x = 2$.

length of the Lactus rectum

$4a = 4(2) = 8$

iv) $x^2 - 2x + 8y + 17 = 0$

$x^2 - 2x + 17 = -8y$

$x^2 - 2x + 1 = -8y - 16$

$(x-1)^2 = -8(y+2)$

The parabola open downward

Centre $(h,k) = (1,-2)$

Vertices $(0+h, -a+k) = (1,-4)$

Lactus rectum $y = -a$

$y+2 = -2$

$\boxed{y = -4}$

Directrix $y = a$

$y+2 = +2$

$\boxed{y = 0}$

length of the lactus rectum

$4a = 4(2) = 8$

EXAMPLE : 5.25.

Astronomical unit long $2a = 36.18$
wide (or) short $2b = 9.12$

$$b^2 = (ae)^2 - a^2$$

$$a^2+b^2 = a^2e^2$$

$$e^2 = \frac{a^2+b^2}{a^2}$$

$$e = \frac{\sqrt{a^2+b^2}}{a}$$

$$= \frac{\sqrt{(18.09)^2-(4.56)^2}}{18.09}$$

$$e \simeq 0.97.$$

EXE 5.2 (3)(i)

Foci $(\pm ae, 0) = (\pm 2, 0)$

$\boxed{ae = 2}$

$e = 3/2$.

$a(3/2) = 2$

$\boxed{a = \frac{4}{3}}$

$$\therefore b^2 = (ae)^2 - a^2$$

$$= 4 - \frac{16}{9}$$

$$= \frac{36-16}{9}$$

$$b^2 = \frac{20}{9}$$

Equation of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

$$\frac{x^2}{16/9} - \frac{y^2}{20/9} = 1$$

$$\frac{9x^2}{16} - \frac{9y^2}{20} = 1.$$

EXE 5.2 (3)(ii)

Centre $(h,k)(2,1)$, Fous $(8,1)$

The distance between centre & focus

$$ae = \sqrt{(8-2)^2+(1-1)^2}$$

$$= \sqrt{6^2}$$

$$ae = 6.$$

corresponding directrix $x = 4$
$(4,1)$

The distance between centre & directrix

$$\frac{a}{e} = \sqrt{(4-2)^2+(1-1)^2}$$

$$= \sqrt{2^2}$$

$$\frac{a}{e} = 2$$

$$\Rightarrow (ae)\left(\frac{a}{e}\right) = 6 \times 2$$

$$\boxed{a^2 = 12}$$

$$(ae)^2 = 36.$$

$$b^2 = (ae)^2 - a^2$$

$$= 36 - 12$$

$$b^2 = 24.$$

Equation of the hyperbola.

$$\frac{(x-2)^2}{a^2} - \frac{(y-1)^2}{b^2} = 1$$

$$\frac{(x-2)^2}{12} - \frac{(y-1)^2}{24} = 1.$$

EXE 5.2 (3)(iii).

passing through $(5,2) = (x,y)$.

length of tranverse axis, long x-axis length 8 units.

$$2a = 8$$

$$\boxed{a = 4}$$

centre $(h,k) = (0,0)$

EXAMPLE 5·20

$$4x^2+36y^2+40x-288y+532=0$$

$$4[x^2+10x]+36[y^2-8y]=-532$$

$$4[(x+5)^2-25]+36[(y-4)^2-16]=-532$$

$$4(x+5)^2+36(y-4)^2=-532+100+576$$

$$4(x+5)^2+36(y-4)^2=144$$

$$\frac{(x+5)^2}{36}+\frac{(y-4)^2}{4}=1.$$

Centre $(-5,4)=(h,k)$

$a^2=36,\quad b^2=4$

$a=6 \qquad b=2.$

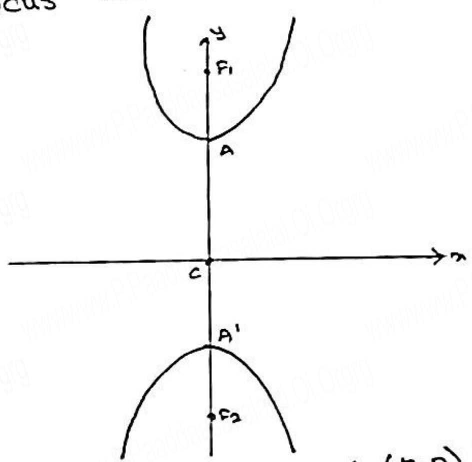
Length of the major axis $2a=12$

Length of the minor axis $2b=4$

EXAMPLE 5·22.

Vertices $(0,\pm a)=(0,\pm 4)$

Focus $(0,\pm ae)=(0,\pm 6)$



The midpoint of the foci $(0,0)$

$2a=8 \qquad 2ae=12$

$a=4 \qquad ae=6.$

$$b^2=(ae)^2-a^2$$

$$=36-(4)^2=36-16$$

$$b^2=20$$

Equation of the hyperbola

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1.$$

$$\frac{y^2}{16}-\frac{x^2}{20}=1.$$

EXAMPLE 5·23.

$$9x^2-16y^2=144$$

$$\frac{x^2}{16}-\frac{y^2}{9}=1.$$

$a^2=16 \Rightarrow a=\pm 4$

$b^2=9 \Rightarrow b=\pm 3.$

Vertices $(\pm a,0)=(\pm 4,0).$

$$b^2=(ae)^2-a^2$$

$$(ae)^2=a^2+b^2$$

$$=16+9$$

$$(ae)^2=25$$

$$ae=\pm 5$$

Focus $(\pm ae,0)=(\pm 5,0).$

EXAMPLE : 5.24.

$$11x^2-25y^2-44x+50y-256=0$$

$$11[x^2-4x]-25[y^2-2y]=256$$

$$11[(x-2)^2-4]-25[(y-1)^2-1]=256$$

$$11(x-2)^2-25(y-1)^2=256+44-25$$

$$11(x-2)^2-25(y-1)^2=275$$

$$\frac{(x-2)^2}{25}-\frac{(y-1)^2}{11}=1.$$

$a^2=25,\quad a=\pm 5$

$b^2=11,\quad b=\pm\sqrt{11}$

$\therefore\ b^2=(ae)^2-a^2$

$(ae)^2=a^2+b^2=25+11=36$

$ae=\pm 6.$

$ae=6$ $\qquad$ Focus $(\pm ae,0)=(\pm 5,0)$

$\boxed{e=6/5}$ $\qquad x-2=\pm 6 \mid y-1=0$

$x=\pm 6+2 \qquad y=1.$

Focus $(\pm 6+2,1)$

$(8,1)\ (-4,1)$

Foci $ae = 2$, vertices $a = 3$.

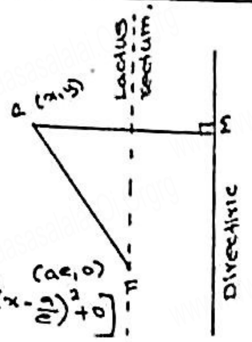
$$b^2 = a^2 - (ae)^2$$
$$= 9 - 4$$
$$b^2 = 5$$

Equation of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{5} = 1.$$

EXAMPLE: 5.19.



$$\frac{FM}{PM} = e$$

$$FM^2 = e^2 PM^2$$

$$(x-ae)^2 + (y-0)^2 = e^2\left[\left(x - \frac{a}{e}\right)^2 + 0\right]$$

Foci $(ae, 0) = (2, 3)$. $e = 1/2$.

directrix $x = \frac{a}{e} = 7$.

$$(x-2)^2 + (y-3)^2 = (1/2)^2\left[(x-7)^2 - 0\right]$$

$$(x-2)^2 + (y-3)^2 = \frac{1}{4}(x-7)^2$$

$$x^2 - 4x + 4 + y^2 - 6y + 9 = \frac{1}{4}(x^2 - 14x + 49)$$

$$4x^2 + 4y^2 - 16x - 24y + 52 = x^2 - 14x + 49$$

$$3x^2 + 4y^2 - 2x - 24y + 3 = 0.$$

$$3\left(x - 1/3\right)^2 + 4\left(y-3\right)^2 = \frac{1}{3} + 36 - 3$$

$$3\left(x - \frac{1}{3}\right)^2 + 4\left(y-3\right)^2 = \frac{100}{3}.$$

$$\frac{(x - 1/3)^2}{\frac{100}{9}} + \frac{4(y-3)^2}{\frac{100}{3}} = 1$$

$$\frac{(x-1/3)^2}{100/9} + \frac{(y-3)^2}{100/12} = 1.$$

length of the major axis $= 2a \Rightarrow 2\sqrt{\frac{100}{9}}$

length of the minor axis $= 2b \Rightarrow 2\sqrt{\frac{100}{12}}$

EXAMPLE 5.20.

$$4x^2 + y^2 + 24x - 2y + 21 = 0$$

$$4[x^2 + 6x] + 1[y^2 - 2y] = -21$$

$$4[(x+3)^2 - 9] + 1[(y-1)^2 - 1] = -21$$

$$4(x+3)^2 + 1(y-1)^2 = -21 + 36 + 1$$

$$4(x+3)^2 + (y-1)^2 = 16$$

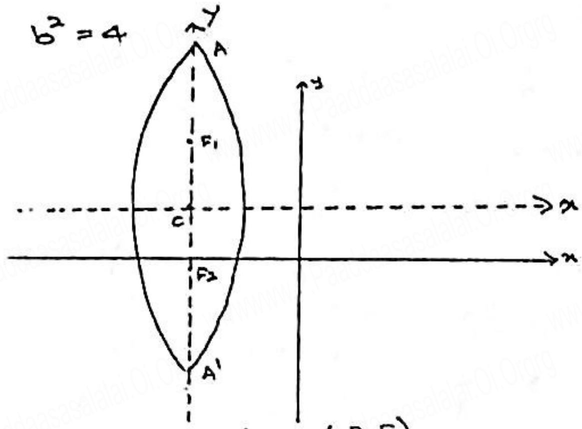
$$\frac{4(x+3)^2}{16} + \frac{(y-1)^2}{16} = 1.$$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{16} = 1.$$

$(h,k) = (-3, 1)$ centre

$a^2 = 16$

$b^2 = 4$



vertices $(0, \pm a)$ $A(-3, 5)$

$A'(-3, -3)$

$$b^2 = a^2 - (ae)^2$$
$$4 = 16 - 16e^2$$
$$-12 = -16e^2$$
$$\frac{3}{4} = e^2 \quad \boxed{e = \frac{\sqrt{3}}{2}}$$

Foci $(0, \pm ae)$. $ae = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

$x + 3 = 0$, $\quad y - 1 = \pm 2\sqrt{3}$

$x = -3 \quad y = \pm 2\sqrt{3} + 1$

Foci $(0, \pm ae) = (-3, \pm 2\sqrt{3} + 1)$

$ae = 4$, $a = 5$

$5e = 4$

$e = 4/5$

$\therefore b^2 = a^2 - (ae)^2$

$= 25 - 16$

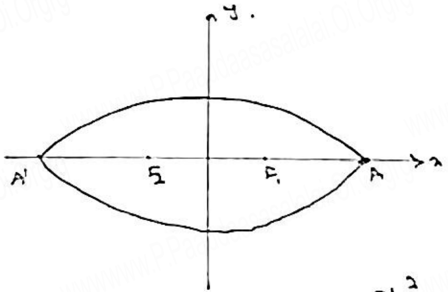
$b^2 = 9$

Equation of the ellipses.

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$$

$$\frac{x^2}{9} + \frac{y^2}{25} = 1.$$

EXE (5.2) (2) (iii).



Length of the Lactus rectum $\frac{2b^2}{a} = 8$

$2b^2 = 8a$

$b^2 = 4a$

$e = 3/5.$

$\therefore b^2 = a^2 - (ae)^2$

$4a = a^2[1 - e^2]$

$4 = a[1 - 9/25]$

$4 = a\left[\frac{16}{25}\right]$

$\boxed{a = \frac{25}{4}}$

$\therefore b^2 = 4a$

$b^2 = 4\left(\frac{25}{4}\right) = 25$

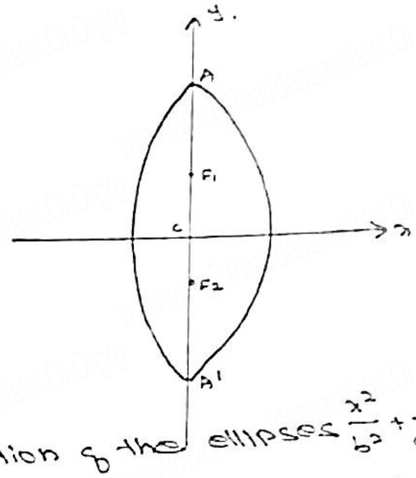
Equation of the ellipses

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

$$\frac{x^2}{\left(\frac{625}{16}\right)} + \frac{y^2}{25} = 1.$$

$$\frac{16x^2}{625} + \frac{y^2}{25} = 1.$$

EXE 5.2 (2) (iv).



Equation of the ellipses $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Length of the lactus rectum $\frac{2b^2}{a} = 4$

$2b^2 = 4a$

$\boxed{b^2 = 2a}$

distance between foci

$2ae = 4\sqrt{2}$

$ae = 2\sqrt{2}$

$\therefore b^2 = a^2 - (ae)^2$

$2a = a^2[1 - e^2]$

$2a = a^2 - (2\sqrt{2})^2$

$2a = a^2 - 8$

$a^2 - 2a - 8 = 0.$

$(a-4)(a+2) = 0$

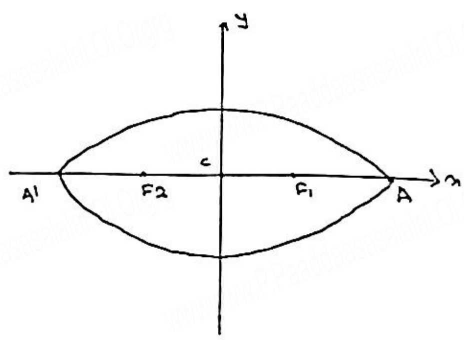
$\therefore a = 4$ and $a = -2$ (not possible)

$b^2 = 2(4) = 8.$

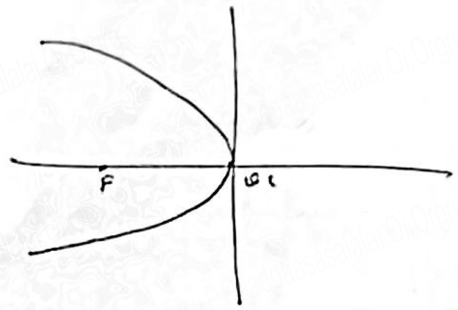
Equation of the ellipses.

$$\frac{x^2}{8} + \frac{y^2}{16} = 1.$$

EXAMPLE : 5.18 .



EXAMPLE : 5.15



The parabola open leftward

$$(y-k)^2 = -4a(x-h)$$

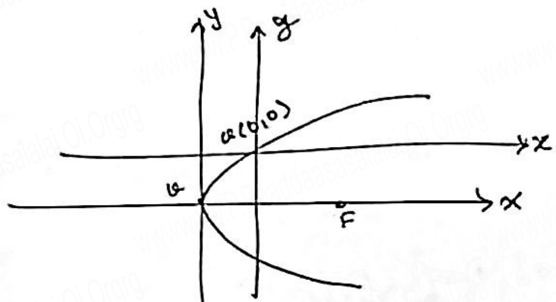
$V(h,k) = (5,-2)$, $F(-a,0) = (2,-2)$

$\boxed{a=3}$

$$(y+2)^2 = -4(3)(x-5)$$

$$(y+2)^2 = -12(x-5)$$

EXAMPLE : 5.16



The parabola open ward (up).

$$(x-h)^2 = 4a(y-k).$$

$V(h,k) = (-1,-2)$, passes $(3,6)$.

$$(3+1)^2 = 4a(6+2)$$

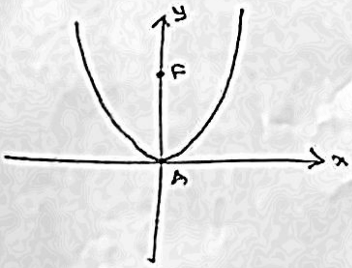
$$16 = 4a(8)$$

$$\boxed{a = 1/2}$$

$$(x+1)^2 = 4(1/2)(y+2)$$

$$(x+1)^2 = 2(y+2).$$

EXAMPLE : 5.17



The parabola $x^2 - 4x - 5y - 1 = 0$

$$x^2 - 4x - 1 + 5 = 5y + 5$$

$$x - 4x + 4 = 5y + 5$$

$$(x-2)^2 = 5(y+1).$$

$4a = 5$, length of lactus rectum is 5

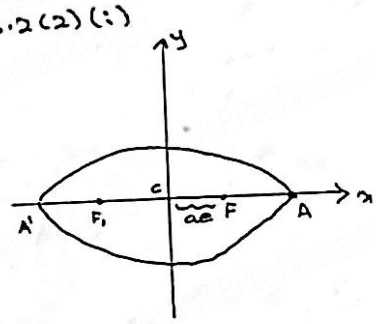
$\boxed{a = 5/4}$

Focus $(0,a)$, $x - 2 = 0$, $x = 2$ | $y + 1 = 5/4$, $y = 1/4$

Focus $(0,a) = (2, 1/4)$.

vertices $(h,k) = (2,-1)$.

EXE 5.2(2)(i)



The Equation of the ellipses.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

$ae = 3$, $e = 1/2$.

$a(1/2) = 3$

$\boxed{a = 6}$ $\therefore b^2 = a^2 - (ae)^2$

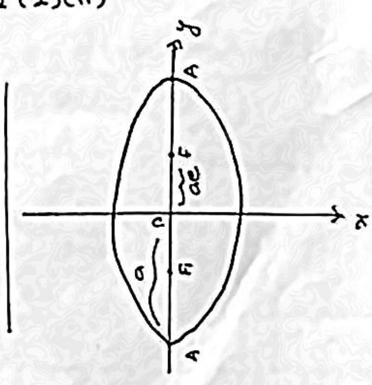
$$= 36 - 9$$

$$b^2 = 27$$

Equation $\dfrac{x^2}{36} + \dfrac{y^2}{27} = 1.$

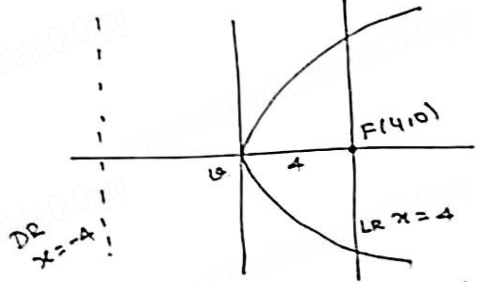
EXE 5.2(2)(ii)

$C = (0,0)$
$F(0,3)$
$F_1(0,-3)$
$A(0,4)$
$A'(0,-4)$.



EXE 5.2 (1) (i)

Focus (4,0) and directrix $x=-4$



The parabola open right ward.

$$(y-k)^2 = 4a(x-h)$$

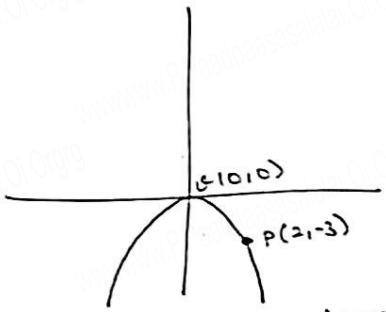
$(h,k) = (0,0)$

$(a,0) = (4,0)$

$$(y-0)^2 = 4(4)(x-0)$$

$$y^2 = 16x.$$

EXE 5.2 (1) (ii)



The parabola open downward.

$$(x-h)^2 = -4a(y-k)$$

$V(h,k) = (0,0)$

$(x,y) = (2,-3)$

$$(2-0)^2 = -4a(-3-0)$$

$$4 = 4a(3)$$

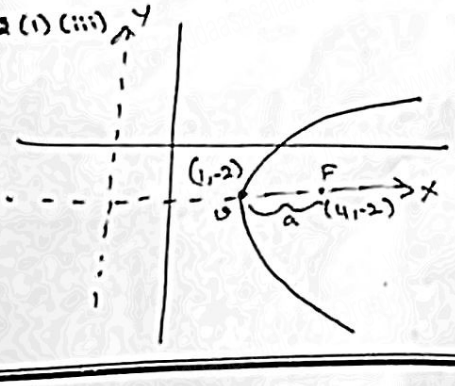
$$\boxed{a = 1/3}$$

Equation. $(x-0)^2 = -4(1/3)(y-0)$

$$x^2 = -\frac{4}{3}y.$$

$$3x^2 = -4y.$$

EXE 5.2 (1) (iii)



The parabola open rightward.

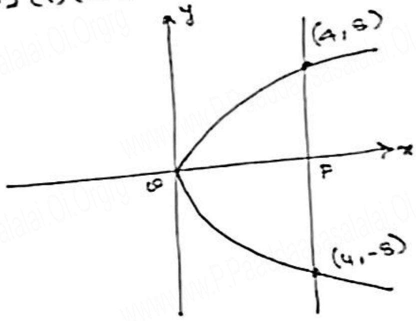
$$(y-k)^2 = 4a(x-h)$$

$\therefore a = 3, \quad V(h,k) = (1,-2)$

$$(y+2)^2 = 4(3)(x-1)$$

$$(y+2)^2 = 12(x-1)$$

EXE 5.2 (1) (iv).



The parabola open rightward.

$$(y-k)^2 = 4a(x-h)$$

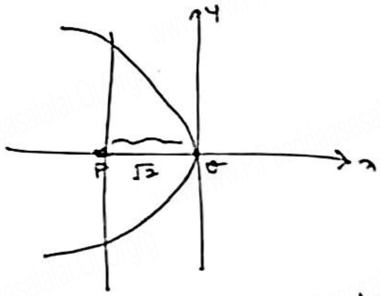
$\therefore 8a = 8$

$V(h,k) = (0,0)$, $\boxed{a = 4}$

$$(y-0)^2 = 4(4)(x-0)$$

$$y^2 = 16x.$$

EXAMPLE 5.14.



The parabola open leftward.

Symmetric as x-axis.

$F(a,0) = (-\sqrt{2},0)$, $\quad \therefore a = \sqrt{2}$.

$V(h,k)$, $(0,0)$.

$$(y-k)^2 = -4a(x-h)$$

$$(y-0)^2 = -4(\sqrt{2})(x-0)$$

$$y^2 = -4\sqrt{2}\,x.$$

CONICS:

A conic is the locus of a point which moves in a plane, so that its distance from a fixed point bears a constant ratio to its distance from a fixed line not containing the fixed point.

fixed point $\longrightarrow$ focus

fixed line $\longrightarrow$ directrix

constant ratio $\longrightarrow$ eccentricity.

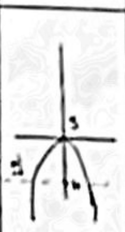
$e = 1$, parabola $\Rightarrow B^2-4AC=0$

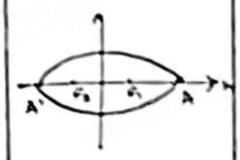
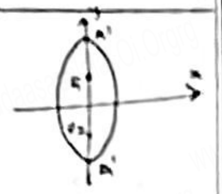
$e<1$, ellipse $\Rightarrow B^2-4AC<0$

$e>1$, hyperbola, $B^2-4AC>0$

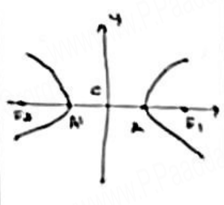
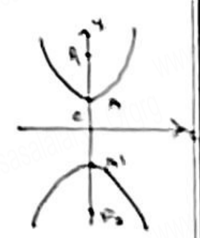
General second grade equation.

$$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$$

Equation	Graph	Vertex	focus	Axis	Equation of Directrix	Length of the Latus rectum
$y^2=4ax$		$(0,0)$	$(a,0)$	$y=0$	$x=-a$	$4a$
$y^2=-4ax$		$(0,0)$	$(-a,0)$	$y=0$	$x=+a$	$4a$
$x^2=4ay$		$(0,0)$	$(0,a)$	$x=0$	$y=-a$	$4a$
$x^2=-4ay$		$(0,0)$	$(0,-a)$	$x=0$	$y=a$	$4a$

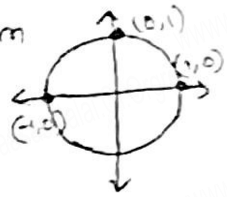
ELLIPSE	$a^2>b^2$	$b^2>a^2$
Equation	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
Diagram		
centre	$(0,0)$	$(0,0)$
Focus	$(\pm ae,0)$	$(0,\pm ae)$
vertices	$(\pm a,0)$	$(0,\pm a)$
eccentricity	$e=\sqrt{1-\frac{b^2}{a^2}}$	$e=\sqrt{1-\frac{b^2}{a^2}}$
Directrix	$x=\pm\frac{a}{e}$	$y=\pm\frac{a}{e}$

Hyperbola.

Equation	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Diagram		
Focus	~~$(0,\pm ae)$~~ $(\pm ae,0)$	$(0,\pm ae)$
Vertices	$(\pm a,0)$	$(0,\pm a)$
eccentricity	$e=\sqrt{1+\frac{b^2}{a^2}}$	$e=\sqrt{1+\frac{b^2}{a^2}}$
directrix	$x=\pm\frac{a}{e}$	$y=\pm\frac{a}{e}$

(6)

From the diagram



Centre : (0,0)

radius : r = 1

Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

$x^2+y^2=1$

---

(7) Area of the circle $=9\pi$

$\pi r^2 = 9\pi \Rightarrow \boxed{r^2=9}$

Two diameters are

$x+y=5 \to ①$ and $x-y=1 \to ②$

$①+② \Rightarrow 2x=6 \Rightarrow \boxed{x=3}$

$① \Rightarrow 3+y=5 \Rightarrow \boxed{y=2}$

$\therefore$ Centre : $(h,k)=(3,2)$.

Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

$(x-3)^2+(y-2)^2=9$

$x^2+9-6x+y^2+4-4y=9$

$x^2+y^2-6x-4y+4=0$.

---

(8) $y=2\sqrt{2}x+c$ | $x^2+y^2=16$

$m=2\sqrt{2}$ | $a^2=16$

$m^2=(2\sqrt{2})^2=8$

Condition : $c^2=a^2(1+m^2)$

$c^2=16(1+8)=16(9)$

$c^2=144$

$\boxed{c=\pm 12}$

---

(9) $x^2+y^2-6x+6y-8=0$

Eqn. of the tangent

$xx_1+yy_1-6\left(\frac{x+x_1}{2}\right)+6\left(\frac{y+y_1}{2}\right)-8=0$

$xx_1+yy_1-3(x+x_1)+3(y+y_1)-8=0$

Here $(x_1,y_1)=(2,2)$

$2x+2y-3(x+2)+3(y+2)-8=0$

$2x+2y-3x-6+3y+6-8=0$

$\boxed{-x+5y-8=0}$

$\Rightarrow x-5y+8=0$

Eqn. of the Normal

$5x+y+k=0$

It passes through (2,2)

$5(2)+2+k=0$

$10+2+k=0 \Rightarrow \boxed{k=-12}$

Eqn. of the normal is

$5x+y-12=0$

---

(10) $x^2+y^2-5x+2y-5=0$

At $(-2,1)$

$① \Rightarrow 4+1+10+2-5=12>0$

$\therefore (-2,1)$ lies out side the circle

At $(0,0)$

$① \Rightarrow 0+0-0+0-5=-5<0$

$\therefore (0,0)$ lies inside the circle.

At $(-4,-3)$

$① \Rightarrow 16+9+20-6-5=34>0$

$\therefore (-4,-3)$ lies outside the circle

---

(11) (i) $x^2+(y+2)^2=0$ $\quad [(x-h)^2+(y-k)^2=r^2]$

$\therefore$ centre $=(0,-2)$ and $r=0$

(ii) $x^2+y^2+2gx+2fy+c=0$

$x^2+y^2+6x-4y+4=0$

$\therefore 2g=6 \Rightarrow \boxed{g=3}$ and $2f=-4 \Rightarrow \boxed{f=-2}$

$\boxed{c=4}$

$\therefore$ Centre $=(-g,-f)=(-3,2)$

radius $r=\sqrt{g^2+f^2-c}=\sqrt{9+4-4}=\sqrt{9}=3$

(iii) $x^2+y^2-x+2y-3=0$

$2g=-1$, $g=-1/2$ | $2f=2$, $f=1$ | $c=-3$

Centre : $(-g,-f)=(1/2,-1)$

radius $r=\sqrt{g^2+f^2-c}=\sqrt{\frac{1}{4}+1+3}=\sqrt{\frac{1+4+12}{4}}$

$\boxed{r=\sqrt{17}/2}$

(iv) $2x^2+2y^2-6x+4y+2=0$

$\div 2 \Rightarrow x^2+y^2-3x+2y+1=0$

$2g=-3$, $g=-3/2$ | $2f=2$, $f=1$ | $c=1$

Centre $=(-g,-f)=(3/2,-1)$

radius $=\sqrt{g^2+f^2-c}=\sqrt{9/4+1-1}=\sqrt{\frac{9}{4}}$

$r=3/2$

---

(12) Sol: $3x^2+(3-p)xy+qy^2-2px=8pq \to ①$

$\therefore$ coeff. of x^2 = coeff. of y^2

$3=q \Rightarrow \boxed{q=3}$

Again, coeff. of $xy=0 \Rightarrow 3-p=0 \Rightarrow \boxed{p=3}$

$① \Rightarrow 3x^2+3y^2-6x-72=0$

$\div 3 \Rightarrow x^2+y^2-2x-24=0$

$2g=-2$, $g=-1$ | $2f=0$, $f=0$ | $c=-24$

Centre $=(-g,-f)=(1,0)$

radius $r=\sqrt{g^2+f^2-c}=\sqrt{1+0+24}=\sqrt{25}=5$

Example 5.13

Let $O_1(12,0)$ and $O_2(34,0)$ be the centres of the semicircle and radius $r=10$.

Equation of the semicircle

$(x-h)^2+(y-k)^2=r^2$

$(x-h)^2+(y-k)^2=100 \quad [r=10]$

case(i) : centre $(12,0)=(h,k)$

$(x-12)^2+(y-0)^2=100$

$x^2+144-24x+y^2-100=0$

$x^2+y^2-24x+44=0, \; y>0.$

Case(ii) : centre $(h,k)=(34,0)$

$(x-34)^2+(y-0)^2=100$

$x^2+1156-68x+y^2-100=0$

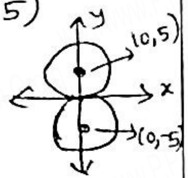
$x^2+y^2-68x+1056=0, \; y>0.$

Ex 5.1

1) Solution:

$r=5$, centre $=(0,\pm 5)$

Equation of the circle



$(x-h)^2+(y-k)^2=r^2$

Here $(h,k)=(0,\pm 5)$, $r=5$

$(x-0)^2+(y\pm 5)^2=25$

$x^2+y^2+25\pm 10y=25$

$\boxed{x^2+y^2\pm 10y=0}$

(2) Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

Here $(h,k)=(2,-1)$

$(x-2)^2+(y+1)^2=r^2$

It passes through $(3,6)$

$(3-2)^2+(6+1)^2=r^2$

$1+49=r^2 \Rightarrow \boxed{r^2=50}$

Equation of the circle

$(x-2)^2+(y+1)^2=50.$

(3) Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

Centre is $(h,k)=(-r,-r)$

$(x+r)^2+(y+r)^2=r^2 \rightarrow (1)$

But it passes through $(-4,-2)$

$(-4+r)^2+(-2+r)^2=r^2$

$16+r^2-8r+4+r^2-4r=r^2$

$r^2-12r+20=0$

$(r-10)(r-2)=0$

$r=10, \; r=2$

case(i) $r=10$

(1) $\rightarrow (x+10)^2+(y+10)^2=10^2$

$x^2+100+20x+y^2+\cancel{100}+20y=\cancel{100}$

$x^2+y^2+20x+20y+100=0$

Case (ii) $r=2$

(1) $\Rightarrow (x+2)^2+(y+2)^2=2^2$

$x^2+4+4x+y^2+\cancel{4}+4y=\cancel{4}$

$x^2+y^2+4x+4y+4=0.$

(4) Centre $(h,k)=(2,3)$

Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

$(x-2)^2+(y-3)^2=r^2 \rightarrow (1)$

Given lines : $3x-2y-1=0, \rightarrow (2)$

$4x+y-27=0 \rightarrow (3)$

$\therefore (2)\times 1 \Rightarrow 3x-2y-1=0$

$(3)\times 2 \Rightarrow 8x+2y-54=0$

$11x-55=0$

$11x=55 \Rightarrow \boxed{x=5}$

(3) $\Rightarrow 4(5)+y-27=0$

$20+y-27=0 \Rightarrow \boxed{y=7}$

$\therefore$ point of intersection $(5,7)$.

$\therefore (1) \Rightarrow (5-2)^2+(7-3)^2=r^2$

$9+16=r^2 \Rightarrow \boxed{r^2=25}$

$\therefore (1) \Rightarrow (x-2)^2+(y-3)^2=25$

$x^2+4-4x+y^2+9-6y-25=0$

$x^2+y^2-4x-6y-12=0$

(5) Equation of the circle

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

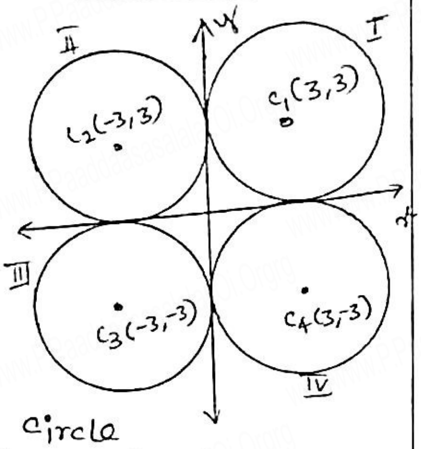
Here $(x_1,y_1)=(3,4)$, $(x_2,y_2)=(2,-7)$

$(x-3)(x-2)+(y-4)(y+7)=0$

$x^2-2x-3x+6+y^2+7y-4y-28=0$

$x^2+y^2-5x+3y-22=0$

Example 5.8



Equation of circle

$(x-h)^2+(y-k)^2=r^2$

i) Centre : $(h,k)=(3,3)$

$(x-3)^2+(y-3)^2=3^2$

$x^2+9-6x+y^2+9-6y=9$

$x^2+y^2-6x-6y+9=0$

(ii) centre : $(h,k)=(-3,3)$

$(x+3)^2+(y-3)^2=3^2$

$x^2+9+6x+y^2+9-6y=9$

$x^2+y^2+6x-6y+9=0$

(iii) centre : $(h,k)=(-3,-3)$

$(x+3)^2+(y+3)^2=3^2$

$x^2+9+6x+y^2+9+6y=9$

$x^2+y^2+6x+6y+9=0$

(iv) centre : $(h,k)=(3,-3)$

$(x-3)^2+(y+3)^2=3^2$

$x^2+9-6x+y^2+9+6y=9$

$x^2+y^2-6x+6y+9=0$

$\therefore$ The req. equations of the circle

$x^2+y^2\pm 6x\pm 6y+9=0$.

Example 5.9

Circle : $3x^2+(a+1)y^2+6x-9y+a+4=0$

$\because$ Coeff. of x^2 = coeff. of y^2

$3=a+1 \Rightarrow \boxed{a=2}$

$\therefore$ Equation of the circle

$3x^2+3y^2+6x-9y+6=0$

$\div 3 \Rightarrow x^2+y^2+2x-3y+2=0$

Here $2g=2$, $g=1$ | $2f=-3$, $f=-3/2$

$\therefore$ Centre $=(-g,-f)=(-1,3/2)$

Radius $r=\sqrt{g^2+f^2-c}=\sqrt{1+\frac{9}{4}-2}$

$=\sqrt{\frac{4+9-8}{4}}=\sqrt{5}/2$.

Example : 5.10

General equation of the circle

$x^2+y^2+2gx+2fy+c=0 \rightarrow (1)$

It passes through points $(1,1)$, $(2,-1)$ and $(3,2)$

$(1,1) \Rightarrow 1+1+2g+2f+c=0$

$2g+2f+c=-2 \rightarrow (2)$

$(2,-1) \Rightarrow 4+1+4g-2f+c=0$

$4g-2f+c=-5 \rightarrow (3)$

$(3,2) \Rightarrow 9+4+6g+4f+c=0$

$6g+4f+c=-13 \rightarrow (4)$

$(2)-(3) \Rightarrow -2g+4f=3 \rightarrow (5)$

$(4)-(3) \Rightarrow 2g+6f=-8 \rightarrow (6)$

$\therefore (5)+(6) \Rightarrow 10f=-5$

$f=\frac{-5}{10}=-\frac{1}{2}$

$\boxed{f=-1/2}$

$(6) \Rightarrow 2g+6(-1/2)=-8$

$2g-3=-8 \Rightarrow 2g=-5$

$\boxed{g=-5/2}$

$(2) \Rightarrow 2(-5/2)+2(-1/2)+c=-2$

$-5-1+c=-2$

$c=-2+6$

$\boxed{c=4}$

$\therefore (1) \Rightarrow x^2+y^2+2(-5/2)x+2(-1/2)y+4=0$

$x^2+y^2-5x-y+4=0$.

Example 5.11

$x^2+y^2=25$ at $P(-3,4)$

Equation of the tangent

$xx_1+yy_1=25$

$(x_1,y_1)=(-3,4)$

$x(-3)+y(4)=25$

$\boxed{-3x+4y=25}$

Equation of the normal

$4x+3y=k$

It passes through $(-3,4)$

$4(-3)+3(4)=k$

$-12+12=k \Rightarrow k=0$

$\therefore$ Equation of the normal is

$\boxed{4x+3y=0}$

Example 5.12

$y=4x+c$, $m=4$ | $x^2+y^2=9$, $a^2=9$

The condition is $c^2=a^2(1+m^2)$

$c^2=9(1+16)$

$c^2=9(17) \Rightarrow \boxed{c=\pm 3\sqrt{17}}$

5. TWO DIMENSIONAL ANALYTICAL GEOMETRY-II

Example : 5.1

Equation of the circle

$(x-h)^2+(y-k)^2=r^2$

$(h,k)=(-3,-4)$ and $r=3$

$(x+3)^2+(y+4)^2=3^2$

$x^2+9+6x+y^2+16+8y=9$

$x^2+y^2+6x+8y+16=0$

Example 5.2

Chord : $3x+y+5=0$

Circle : $x^2+y^2=16 \Rightarrow x^2+y^2-16=0$

The req. equation of the circle

$(x^2+y^2-16)+\lambda(3x+y+5)=0$

$x^2+y^2-16+3\lambda x+\lambda y+5\lambda=0 \quad \text{①}$

Here $2g=3\lambda$, $g=\frac{3\lambda}{2}$ | $2f=\lambda$, $f=\frac{\lambda}{2}$

Centre $=(-g,-f)=\left(\frac{-3\lambda}{2},-\frac{\lambda}{2}\right)$

$\therefore$ Centre lies on the chord $3x+y+5=0$.

$\therefore 3\left(\frac{-3\lambda}{2}\right)+\left(\frac{-\lambda}{2}\right)+5=0$

$\frac{-9}{2}-\frac{-\lambda}{2}+5=0$

$\frac{-10}{2}\lambda+5=0$

$-5\lambda=-5 \Rightarrow \boxed{\lambda=1}$

① $\Rightarrow x^2+y^2-16+3x+y+5=0$

$x^2+y^2+3x+y-11=0$

Example : 5.3

Circle : $x^2+y^2-6x+4y+c=0$

Here $2g=-6$, $g=-3$ | $2f=4$, $f=2$

Centre $(-g,-f)=(3,-2)$ which lies on $x+y-1=0$.

$\therefore x+y-1=0$ passes through the centre.

$\therefore x+y-1=0$ is a diameter of the circle for all possible value of c.

Example : 5.4.

Equation of the circle [diameter form]

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

$(x_1,y_1)=(-4,-2)$, $(x_2,y_2)=(1,1)$

$(x+4)(x-1)+(y+2)(y-1)=0$

$x^2-x+4x-4+y^2-y+2y-2=0$

$x^2+y^2+3x+y-6=0$.

Example 5.5

point $(2,3)$

$x^2+y^2-6x-8y+12$

$=4+9-6(2)-8(3)+12$

$=4+9-12-24+12$

$=-11<0$.

$\therefore (2,3)$ lies inside the circle.

Example 5.6.

$3x+4y=12$

$\div 12 \Rightarrow \frac{x}{4}+\frac{y}{3}=1$

$\therefore A(4,0)$, $B(0,3)$

Equation of the circle

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

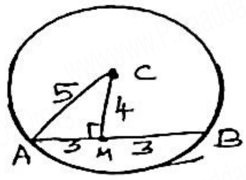
$(x_1,y_1)=(4,0)$, $(x_2,y_2)=(0,3)$

$(x-4)(x-0)+(y-0)(y-3)=0$

$x^2-4x+y^2-3y=0$

$\Rightarrow x^2+y^2-4x-3y=0$.

Example 5.7



Centre $=(2,1)$

$3x+4y+10=0$ cuts a chord AB on the circle.

Let M be the midpoint of AB.

$\therefore AM=MB=3 \quad [\because AB=6]$

$CM=\frac{|3(2)+4(1)+10|}{\sqrt{3^2+4^2}}$

$=\frac{|6+4+10|}{\sqrt{9+16}}=\frac{20}{5}=4$

ΔBMC, $AC^2=AM^2+MC^2=3^2+4^2=9+16=2$

$AC^2=25 \Rightarrow AC=5$ (radius)

Equation of the circle $(x-h)^2+(y-k)^2=$

$(x-2)^2+(y-1)^2=25 \Rightarrow x^2+4-2x+y^2+1-2y$