Unit 1 to 3 Three Marks Questions with Answer

12th Standard

Business Maths

1) Find the rank of the matrix $\begin{pmatrix} -5 & -7 \\ 5 & 7 \end{pmatrix}$

Let A = $\begin{pmatrix} -5 & -7 \\ 5 & 7 \end{pmatrix}$

Order of A is 2×2 ∴ρ(A)≤2

Consider the second order minor $\begin{vmatrix} -5 & -7 \\ 5 & 7 \end{vmatrix}$=0

Since the second order minor vanishes,$\rho(A) \neq 2$

Consider a first order minor $|-5| \neq 0$

There is a minor of order 1, which is not zero $\therefore \rho(A) = 1$

2) Find the rank of the matrix A = $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{pmatrix}$

The order of A is 3×4.

$\therefore \rho(A) \leq 3.$

Let us transform the matrix A to an echelon form

Matrix A	Elementary Transformation
$A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 1 & 2 & -2 \end{pmatrix}$	$R_2 \rightarrow R_2 - 3R_1$ $R_3 \rightarrow R_3 - 2R_1$
$\sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	$R_3 \rightarrow R_3 - R_2$

The number of non zero rows is 3.

$\therefore \rho(A) = 3.$

3) Show that the equations 2x+y=5,4x+2y=10 are consistent and solve them.

The matrix equation corresponding to the system is

$\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$

A X=B

Matrix A	Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$ $\sim \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$	$\begin{pmatrix} 2 & 1 & 5 \\ 4 & 2 & 10 \end{pmatrix}$ $\sim \begin{pmatrix} 2 & 1 & 5 \\ 4 & 2 & 10 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$
$\rho(A) = 1$	$\rho([A,B]) = 1$	

$\rho(A) = \rho([A,B]) = 1$ <number of unknowns

∴The given system is consistent and has infinitely many solutions.

Now, the given system is transformed into the matrix equation.

$\begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

$\Rightarrow 2x + y = 5$

Let us take y=k,k∈R

$\Rightarrow 2x + k = 5$

$x = \frac{1}{2}(5-k)$

$x = \frac{1}{2}(5-k), y = k \quad for \quad all \quad k \in R$

Thus by giving different values for k, we get different solution. Hence the system has infinite number of solutions.

4) Show that the equations 3x−2y=6, 6x−4y=10 are inconsistent

The matrix equation corresponding to the given system is

$\begin{pmatrix} 3 & -2 \\ 6 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$

AX=B

Matrix A	Augmented matrix [A,B]	Elementary Transformation
$\begin{pmatrix} 3 & -2 \\ 6 & -4 \end{pmatrix}$ $\sim \begin{pmatrix} 3 & -2 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & -2 & 6 \\ 6 & -4 & 10 \end{pmatrix}$ $\sim \begin{pmatrix} 3 & -2 & 6 \\ 0 & 0 & -2 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$
$\rho(A) = 1$	$\rho([A,B]) = 2$	

$\therefore \rho([A,B]) = 2, \rho(A) = 1$

$\rho(A) \neq \rho([A,B])$

∴ The given system is inconsistent and has no solution.

5) Consider the matrix of transition probabilities of a product available in the market in two brands A and B.

$\begin{matrix} A \\ B \end{matrix}\overset{\begin{matrix} A & \;\; B \end{matrix}}{\begin{pmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix}}$

Determine the market share of each brand in equilibrium position.

Transition probability matrix

$$T = \begin{matrix} A \\ B \end{matrix}\begin{pmatrix} \overset{A}{0.9} & \overset{B}{0.1} \\ 0.3 & 0.7 \end{pmatrix}$$

At equilibrium, (A B) T=(AB) where A+B=1

$$(A\ B)\begin{pmatrix} 0.9 & 0.1 \\ 0.3 & 0.7 \end{pmatrix} = (A\ B)$$

0.9A+ 0.3B = A

0.9A+0.3(1−A) = A

0.9A−0.3A+0.3 = A

0.6A + 0.3 = A

0.4A = 0.3

$A=\frac{0.3}{0.4}=\frac{3}{4}$

$B=1-\frac{3}{4}=\frac{1}{4}$

Hence the market share of brand A is 75% and the market share of brand B is 25%

6) Akash bats according to the following traits. If he makes a hit (S), there is a 25% chance that he will make a hit his next time at bat. If he fails to hit (F), there is a 35%
chance that he will make a hit his next time at bat. Find the transition probability matrix
for the data and determine Akash's long- range batting average.

The Transition probability matrix is $T = \begin{pmatrix} 0.25 & 0.75 \\ 0.35 & 0.65 \end{pmatrix}$

At equilibrium, $(S\ F)\begin{pmatrix} 0.25 & 0.75 \\ 0.35 & 0.65 \end{pmatrix} = (S\ F)$ where S + F = 1

0.25 S + 0.35 F = S

0.25 S + 0.35 (1 – S) = S

On solving this, we get $S=\frac{0.35}{1.10}$

∴ Akash's batting average is 31.8%

7) Show that the equations x- 3y + 4z = 3, 2x - 5y + 7z = 6, 3x - 8y + 11z = 1 are inconsistent

Given non-homogeneous equations are

x- 3y + 4z = 3, 2x - 5y +·7z = 6, 3x - 8y + 11z = 1

Augmented matrix [A, B]	**Elementary Transformation**
$\begin{pmatrix} 1 & -3 & 4 & 3 \\ 2 & -5 & 7 & 6 \\ 3 & -8 & 11 & 1 \end{pmatrix}$	
$\sim\begin{pmatrix} 1 & -3 & 4 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & -8 \end{pmatrix}$	$R_2 \to R_2 - 2R_1$ $R_3 \to R_3 - 3R_1$

Augmented matrix [A, B]	Elementary Transformation
$\sim\begin{pmatrix}1 & -3 & 4 & 3\\0 & 1 & -1 & 0\\0 & 0 & -0 & -8\end{pmatrix}$	$R_3 \rightarrow R_3 - R_2$

Clearly $\rho(A) = 2$ and $\rho(A, B) = 3$

$\rho(A, B) \neq \rho(A)$

Hence, the given system is inconsistent and has no solution.

8) If $\begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}2\\-1\\3\end{bmatrix}$ find x,y and z

$$\begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}2\\-1\\3\end{bmatrix}$$

$$\Rightarrow \begin{pmatrix}x0+0\\0+0+z\\0+y+0\end{pmatrix} = \begin{pmatrix}2\\-1\\3\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2\\-1\\3\end{pmatrix}$$

$\Rightarrow x = 2 \quad z = -1 \quad y = 3$

$\therefore$ Solution set is {2,3, -I}

9) If $A = \begin{pmatrix}2 & 4\\4 & 3\end{pmatrix}, X = \begin{pmatrix}n\\1\end{pmatrix} B = \begin{pmatrix}8\\11\end{pmatrix}$ and AX = B then find n.

GivenAX B

$$\begin{pmatrix}2 & 4\\4 & 3\end{pmatrix}, \begin{pmatrix}n\\1\end{pmatrix}\begin{pmatrix}8\\11\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}2n+4\\4n+3\end{pmatrix} = \begin{pmatrix}8\\11\end{pmatrix}$$

Equating the corresponding entries on both sides, we get

2n +4 = 8

2n = 8-4

2n=4

$$n = \frac{4}{2}$$

2 = 2

10) Evaluate $\int \frac{ax^2+bx+v}{\sqrt{x}}dx$

$\int \frac{ax^2+bx+v}{\sqrt{x}}dx = \int\left(ax^{\frac{2}{3}} + bx^{\frac{2}{3}} + cx^{-\frac{1}{2}}\right)dx$

$=a\int x^{\frac{3}{2}}dx + b\int x^{\frac{1}{2}} + c\int x^{-\frac{1}{2}}dx$

$=\frac{2ax^{\frac{5}{2}}}{5} + \frac{2bx^{\frac{3}{2}}}{3} + 2cx^{\frac{1}{2}} + k$

11) Evaluate $\int \sqrt{2x+1dx}$

$\int \sqrt{2x+1dx} = \int (2x+1)^{\frac{1}{2}} dx$

$\frac{(2x+1)^{\frac{3}{2}}}{3} + c$

12) Evaluate $\int \left(x^3+7\right)(x-4)\,dx$

$\int \left(x^3+7\right)(x-4)\,dx$ =$\int \left(x^4-4x^3+7x-28\right)dx$

$\frac{x^5}{5} - x^4 \frac{7x^2}{2} - 25x + c$

13) Evaluate $\int \frac{2x^2-14x+24}{x-3}dx$

$\int \frac{2x^2-14x+24}{x-3}dx = \int \frac{(x-3)(2x-8)}{x-3}dx$

$\int (2x-8)dx$

$x^2 - 8x + c$

14) Evaluate $\int (logx)^2 dx$

$\int (logx)^2 dx \quad = \quad \int udv$

= uv − $\int$ vdu

= x (log x)^2 − 2$\int$ logxdx...(*)

$= x(logx)^2 - 2\int udv$

$= x(logx)^2 - 2[uv - \int udv]$

$= x(logx)^2 - 2[xlogx - \int dx]$

$= x(logx)^2 - 2xlogx + x + c$

$= x[(log)^2 - logx^2 + 2] + c$

For $\int$ log x dx in (*)	
Take u = (log x) Differentiate $du = \frac{1}{x}dx$	and dv = dx Integrate v = x

15) Evaluate $\int \left(x^2-2x+5\right)e^{-x}dx$

$\int \left(x^2-2x+5\right)e^{-x}dx \quad = \int udv$

= uv − u'v1 + u''v2 − u'''v3 + ...

= $(x^2 - 2x + 5)(-e^{-x})$− $(2x - 2)e^{-x}$ + $2(-e^{-x})$+ c

= $e^{-x}(-x^2 - 5)$+ c

Successive derivatives	Repeated integrals
Take u = x2 − 2x + 5 u' = 2x − 2 u'' = 2	and dv = e^{-x}dx v = − e^{-x} $v_1 = e^{-x}$ $v_2 = e^{-x}$

16) Evaluate $\int \sqrt{x^2+5}$ dx

$\int \sqrt{x^2+5}$ dx = $\int \sqrt{x^2+\left(\sqrt{5}\right)^2}$dx

$= \frac{x}{2}\sqrt{x^2+\left(\sqrt{5}\right)^2} + \frac{\left(\sqrt{5}\right)^2}{2}log\left|x+\sqrt{x^2+\left(\sqrt{5}\right)}\right|$ + c

= $\frac{x}{2}\sqrt{x^2+5} + \frac{5}{2}log\left|x+\sqrt{x^2+5}\right| + c$

17) Evaluate the integral as the limit of a sum: $\int_1^2 x^2$ dx

$\int_a^b f(x)dx = \lim_{n\to\infty h\to 0} \sum_{r=1}^{n} hf(a+rh)$

Here a =1, b = 2, h = $\frac{b-a}{n} = \frac{2-1}{n} = \frac{1}{n}$ and f (x) = x^2

Now, f (a + rh) = f$\left(1+\frac{r}{n}\right) = \left(1+\frac{r}{n}\right)^2 = 1+\frac{2r}{n}+\frac{r^2}{n^2}$

$\therefore \int_1^2 x^2 dx = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(1+\frac{2r}{n}+\frac{r^2}{n^2}\right)$

= $\lim_{n\to\infty} \sum_{r=1}^{n}\left(\frac{1}{n}+\frac{2r}{n^2}+\frac{r^2}{n^3}\right)$

= $\lim_{n\to\infty}\left(\frac{1}{n}\sum_{r=1}^{n} 1+\frac{2}{n^2}\sum_{r=1}^{n} r+\frac{1}{n^3}\sum_{r=1}^{n} r^2\right)$

= $\lim_{n\leftarrow\infty}\left(\frac{1}{n}(n)+\frac{2}{n^2}\frac{n(n+1)}{2}+\frac{1}{n^3}\frac{n(n+1)(2n+1)}{6}\right)$

= $\lim_{n\to\infty}\left[1+\left(1+\frac{1}{n}\right)+\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}\right]$

= $\left[1+1+\frac{(1)(2)}{6}\right]$

$\therefore \int_1^2 x^2 = \frac{7}{3}$

18) Evaluate $\int \frac{(a^x+b^x)^2}{a^x b^x} dx$

$\int \frac{(a^x+b^x)^2}{a^x b^x} dx$ = $\int \frac{a^{2x}+b^{2x}+2a^x b^x}{2a^x b^x}$

[$\because$ (a + b)^2 = a^2 + 2ab + b^2]

= $\int\left(\frac{a^2 x}{a^x b^x}+\frac{b^{2x}}{a^x b^x}+\frac{2a^x b^x}{a^x b^x}\right)dx$

= $\int\left(\frac{a^x}{b^x}+\frac{b^x}{a^x}+2\right)dx$

= $\int\left(\left(\frac{a}{b}\right)^x+\left(\frac{b}{a}\right)^x+2\right)dx$

= $\frac{\left(\frac{a}{b}\right)^x}{log_e\left(\frac{a}{b}\right)}+\frac{\left(\frac{b}{a}\right)^2}{log_e\left(\frac{b}{a}\right)}+2x+c$

19) Evaluate $\int$ sin^3 x cos x dx

Let I = $\int$ sin^3 x cos x dx

Put t = sin x

$\Rightarrow$ dt = cos x dx

$\because$ I = t3.dt = $\frac{t^4}{4}+c$

= $\frac{sin^4 x}{4}+c$

20) Evaluate $\int \frac{1}{\sqrt{16x^2+25}} dx$

$\int \frac{1}{\sqrt{16x^2+25}} dx$= $\int \frac{1}{\sqrt{16\left(x^2+\frac{25}{16}\right)}} dx$

= $\frac{1}{4}\int \frac{dx}{\sqrt{x^2+\left(\frac{5}{4}\right)^2}}$

= $\frac{1}{4}\log\left|x+\sqrt{x+\left(\frac{5}{4}\right)^2}\right|+c$

= $\frac{1}{4}\log\left|\frac{4x+\sqrt{16x^2+25}}{4}\right|+c$

= $\frac{1}{4}\log\left|4x+\sqrt{16x^2+25}\right|-\frac{1}{4}log4+c$

= $\frac{1}{4} log\left|4x + \sqrt{16x^2 + 25}\right| + c_1$

where $c_1 = -\frac{1}{4} log4 + c$

21) Sketch the graph $y = |x+3|$ and evaluate $\int_{-6}^{0} |x+3|$ dx.

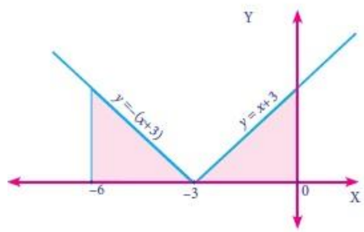
$y = |x+3| = \begin{cases} x+3 & if \quad x \geq -3 \\ -(x+3) & if \quad x < -3 \end{cases}$

Required area = $\int_b^a ydx = \int_{-6}^0 ydx$

= $\int_b^a ydx = \int_{-6}^0 ydx$

= $\int_{-6}^{-3} (x+3)dx + \int_{-3}^{0} (x+3)dx$

= $-\left[\frac{(x+3)^2}{2}\right]_{-6}^{-3} + \left[\frac{(x+3)^2}{2}\right]_{-3}^{0} = \left[0 - \frac{9}{2}\right] + \left[\frac{9}{2} - 0\right]$ = 9 sq. units



22) Using integration find the area of the circle whose center is at the origin and the radius is a units.

Equation of the required circle is $x^2 + y^2 = a^2$ (1)

put $y = 0$, $x^2 = a^2$

⇒ $x = \pm a$

Since equation (1) is symmetrical about both the axes

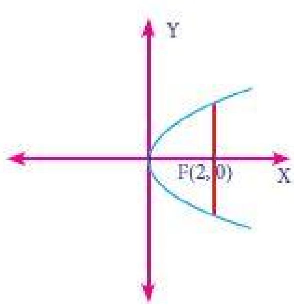
The required area = 4 [Area in the first quadrant between the limit 0 and a.]

$= 4\int_0^a ydx$

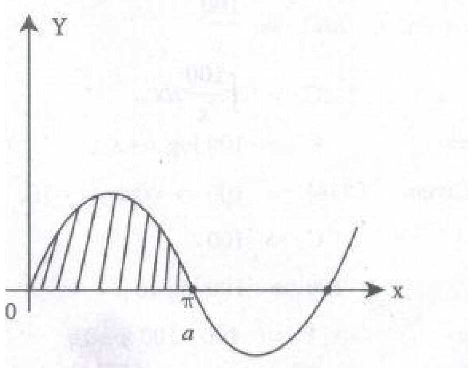
$= 4\int_0^a ydx = 4\left[\frac{x}{2} + \sqrt{a^2 - x^2}dx + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

$= 4\left[\frac{x}{2} + \sqrt{a^2 - x^2}dx + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$

π^2 sq. units



23) Find the area bounded by one arc of the curve y = sin ax and the x-axis.



The limits for one arch of the curve y = sin ax When y = 0 ⇒Sin ax = 0

⇒ sin ax=sin 0, sinπ

⇒ ax=0 or ax= π

⇒ x=0, x=$\frac{\pi}{a}$

∴ The limits are from x=0 to x=$\frac{\pi}{a}$

∴ Area =$\int_a^b ydx$

$= \int_0^a sin \quad ax \quad dx$

$= \left[-\frac{cos \quad ax}{a}\right]_0^{\frac{\pi}{a}}$

$= -\frac{1}{a}\left[cos \quad a \times \frac{\pi}{a} - cos(a)(0)\right]$

$= -\frac{1}{a}[cos \quad \pi - cos0]$

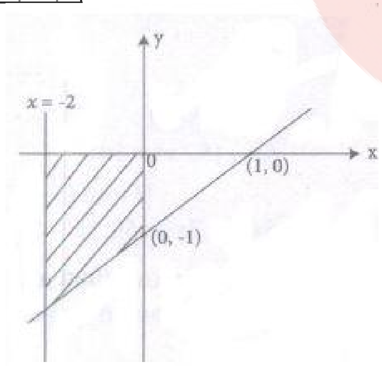
$= -\frac{1}{a}(-1-1)[\because cos0 = 1\&cos\pi = -1]$

$A = \frac{2}{a}$ sq.units.

24) Find the area of the region bounded by the line x - y =1, x-axis and the lines x = -2 and x = 0.

x-y=1

x	0	1
y	-1	0



Here, the area lies below the x-axis

$\therefore \quad A = \int_{-2}^{0}(-y)dx = \int_0^{-2} ydx$

$\left[\because \int_a^b f()dx = -\int_b^a f(x)dx\right]$

$= \int_0^{-2}(x-1)dx = \left[\frac{x^2}{2} - x\right]_0^{-2}$

$\left(\frac{(-2)^2}{2} - (2)\right) - 0$

$= \frac{4}{2} + 2 = 2 + 2$

A=4 sq.units

25) The marginal revenue function is given by $R'(x) = \frac{3}{x^2} - \frac{2}{x}$. Find the revenue function and demand function if R(1)=6

Given $R'(x) = \frac{3}{x^2} - \frac{2}{x}$

$\Rightarrow \int R'(x) = \int \left(\frac{3}{x^2} - \frac{2}{x}\right) dx$

$\Rightarrow R(x) = \frac{-3}{x} - 2log \quad x + k$

Given R(1) = 6 ⇒ when x = 1, R = 6

$\Rightarrow 6 = \frac{-3}{1} - 2log1 + k$

⇒6+3=k [∵ log 1=0]

⇒ k=9

$\therefore R(x) = -\frac{3}{x} - 2log \quad x + 9$

Demand function $P = \frac{R}{x}$

$= \frac{3}{x^2} - \frac{2log \quad x}{x} + \frac{9}{x}$