Half Yearly Portion Study Materials

11th Standard

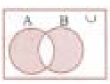
Maths

Multiple Choice Questions

1) The number of constant functions from a set containing m elements to a set containing n elements is
(a) mn (b) m **(c) n** (d) m+n

2) Let f:R→R be defined by f(x)=1-|x|. Then the range of f is
(a) R (b) (1,∞) (c) (-1,∞) **(d) (-∞,1]**

3) The function f:R→R be defined by f(x)=sinx+cosx is
(a) an odd function **(b) neither an odd function nor an even function** (c) an even function (d) both odd function and even function

4) The shaded region in the adjoining diagram represents.

(a) A\B (b) B\A **(c) AΔB** (d) A'

5) Let R be the set of all real numbers. Consider the following subsets of the plane R x R: S = {(x, y) : y =x + 1 and 0 < x < 2} and T = {(x,y) : x - y is an integer} Then which of the following is true?
(a) T is an equivalence relation but S is not an equivalence relation (b) Neither S nor T is an equivalence relation (c) Both S and T are equivalence relation (d) S is an equivalence relation but T is not an equivalence relation.

6) Let A and B be subsets of the universal set N, the set of natural numbers. Then A'∪[(A∩B)∪B'] is
(a) A (b) A' (c) B **(d) N**

7) Let R be the universal relation on a set X with more than one element. Then R is
(a) not reflexive (b) not symmetric **(c) transitive** (d) none of the above

8) The rule f(x) =x^2 is a bijection if the domain and the co-domain are given by
(a) R, R (b) R, (0, ∞) (c) (0, ∞); R **(d) [0, ∞); [0,∞)**

9) $n(p(A))=512, n(p(B))=32, n(A\cup B)=16,$ find $n(A\cap B)$:
(a) 2 (b) 9 (c) 4 (d) 5

10) The natural domain of the function $y=\sqrt{9-x^2}$ is:
(a) $-3\leq x\leq 3$ (b) -3 < x < 3 (c) 0 < x < 3 (d) $(-\infty,-3)\cup(3,\infty)$

11) If A = {1,2}, B = {1,3} then n(A x B) =
(a) 2 **(b) 4** (c) 8 (d) 0

12) The value of $log_3 \frac{1}{81}$ is
(a) -2 (b) -8 **(c) -4** (d) -9

13) If $log_{\sqrt{x}}$ 0.25 =4 ,then the value of x is

(a) 0.5(b) 2.5(c) 1.5(d) 1.25

14) The rationalising factor of $\frac{5}{\sqrt[3]{3}}$ is

(a) $\sqrt[3]{6}$ (b) $\sqrt[3]{3}$ **(c)** $\sqrt[3]{9}$ (d) $\sqrt[3]{27}$

15) The Value of $log_{3/4}^{(4/3)}$ is

(a) -2(b) 1(c) 2**(d) -1**

16) The value of $\log_a^x + \log_{1/a}^x$ is

(a) 1**(b) 0**(c) 2 $\log_a^x$(d) 2 $\log_a^x$

17) Zero of the polynomial p(x) = x^2 - 4x + 4

(a) 1**(b) 2**(c) -2(d) -1

18) If tan α and tan β are the roots of tan x2+atanx+b=0; then $\frac{sin(\alpha+\beta)}{sin\alpha sin\beta}$ is equal to

(a) $\frac{b}{a}$ (b) $\frac{a}{b}$ **(c)** $\frac{a}{b}$ (d) $\frac{b}{a}$

19) The quadratic equation whose roots are tan75° and cot75° is:

(a) x^2+4x+ 1 =0(b) $4x^2$-x+ 1 =0(c) $4x^2$+ 4x - 1 = 0**(d) x^2 - 4x + 1 = 0**

20) There are 10 points in a plane and 4 of them are collinear. The number of straight lines joining any two points is

(a) 45**(b) 40**(c) 39(d) 38

21) a polygon has 44 diagonals, then the number of its sides are

(a) 22(b) 88(c) 8**(d) 11**

22) The remainder when 38^{15} is divided by 13 is

(a) 12**(b) 1**(c) 11(d) 5

23) The coefficient of x^{32} in the expansion of $(x^4 - \frac{1}{x^3})^{15}$

(a) $15C_4$(b) $15C_3$(c) $15C_5$(d) $15C_6$

24) If x, 2x + 2, 3x + 3 . . . are in G.P, then the 4th term is

(a) 27(b) -27(c) 13.5(d) -13.5

25) Which one of the following statements is false?

(a) The image of a point $(\alpha\beta)$ about x-axis $(\alpha, -\beta)$ (b) The image of the line ax+by+c=0 about x-axis is ax-by+c=0 (c) The image of a point (α, β) about y-axis $(-\alpha, \beta)$ **(d) The image of the line ax+by+c=0 about y-axis is ax-by+c=0**

26) Separate equation of lines for a pair of lines whose equation is x^2+xy-12y^2=0 are

(a) x+4y=0 and x+3y=0 (b) 2x-3y=0 and x-4y=0 (c) x-6y=0 and x-3y=0 **(d) x+4y=0 and x-3y=0**

27) Which one of the following is not true about the matrix $\begin{bmatrix}1&0&0\\0&0&0\\0&0&5\end{bmatrix}$?

(a) a scalar matrix **(b) a diagonal matrix** (c) an upper triangular matrix (d) a lower triangular matrix

28) If a≠b,b,c satisfy $\begin{vmatrix}a&2b&2c\\3&b&c\\4&a&b\end{vmatrix} = 0,$ then abc=

(a) a + b + c(b) 0**(c) b^3**(d) ab+bc

29) If $\vec{a},\vec{b}$ are the position vectors A and B, then which one of the following points whose position vector lies on AB, is

(a) $\vec{a}+\vec{b}$ (b) $\frac{2\vec{a}-\vec{b}}{2}$ **(c)** $\frac{2\vec{a}+\vec{b}}{2}$ (d) $\frac{\vec{a}-\vec{b}}{3}$

30) The position vector of the point which divides the join of points $2\vec{a}-3\vec{b}$ and $\vec{a}+\vec{b}$ in the ratio 3:1 is

(a) $\frac{3\vec{a}-2\vec{b}}{2}$ (b) $\frac{7\vec{a}-8\vec{b}}{2}$ (c) $\frac{3\vec{a}}{4}$ **(d)** $\frac{5\vec{a}}{4}$

31) Find the odd one out of the following

(a) matrix multiplication (b) vector cross product (c) Subtraction **(d) Matrix Addition**

32) The value of $lim_{x\to k^-} x-\lfloor x \rfloor$, where k is an integer is

(a) -1 **(b) 1** (c) 0 (d) 2

33) If $f(x)=\begin{cases} x+1, & \text{when } x<2 \\ 2x-1 & \text{when } x\geq 2 \end{cases}$,then f '(2) is

(a) 0 (b) 1 (c) 2 **(d) does not exist**

34) Choose the correct or the most suitable answer from the given four alternatives.

$If \quad y=\sin^{-1}x+\cos^{-1}x$ then $\frac{dy}{dx}$ is

(a) 1 (b) π (c) $\frac{\pi}{2}$ **(d) 0**

35) Choose the correct or the most suitable answer from the given four alternatives.

$If \quad x=a(\theta+\sin\theta), y=a(1+\cos\theta) then \quad \frac{dy}{dx} is$

(a) $\tan\frac{\theta}{2}$ **(b)** $-\tan\frac{\theta}{2}$ (c) $\cot\frac{\theta}{2}$ (d) $-\cot\frac{\theta}{2}$

2 Marks

36) Let X = {a, b, c, d}, and R = {(a, a) (b, b) (a, c)}. Write down the minimum number of ordered pairs to be included to R to make it **Equivalence**

Answer : Given X = {a, b, c, d} and R = {(a, a)(b, b)(a, c)}.
Minimum number of ordered pairs to be included to make R equivalence is (c, c) (d, d) (c, a) (c, d) and (a, d).

37) If U={x:1≤x≤10, x∈N}, A={1,3,5,7,9} and B={2,3,5,9,10} then find A'UB'.

Answer : Given U={1,2,3,4,5,6,7,8,9,10}
A={1,3,5,7,9}
B={2,3,5,9,10}
A'={2,4,6,8,10}
B'={1,4,6,7,8}
A'UB'={1,2,4,6,7,8,10}

38) Show that the relation R on R defined as R={(a,b):a≤b} is reflexive and transitive but not symmetric.

Answer : Give R={(a,b):a≤b} where a,b ∈ R.
Reflexivity: For any a∈R, a≤a
⇒ (a,a)∈R
⇒ R is reflexive
Symmetry: For 2≤3 ⇒ (2,3) ∈ R
but (3,2) ∉ R(∴ 3 ≰ 2)
R is not symmetric.
Transitivity: Let (a,b) ∈ R and (b,c) ∈ R

⇒ Let (a,b) ∈ R and (b,c) ∈ R

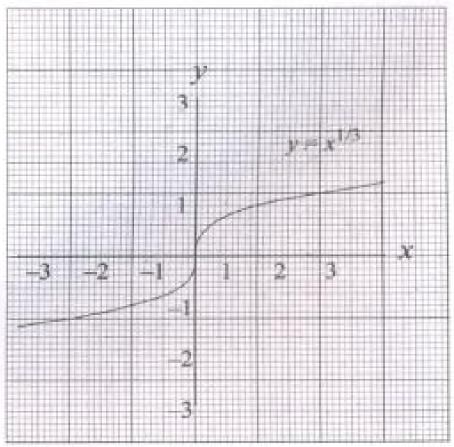
⇒ a≤b and b≤c

⇒ a≤c ⇒ (a,c) ∈ R

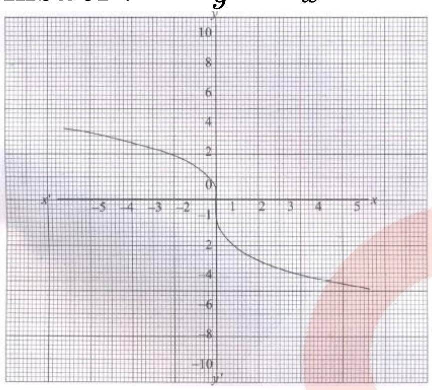
∴ R is transitive.

Hence, R is reflexive and transitive but not symmetric.

39) For the curve $y=-x^{\left(\frac{1}{3}\right)}$ given in figure, draw.



Answer : Let $y=-x^{\left(\frac{1}{3}\right)}$



Then $y=-x^{\frac{1}{3}}$ is the reflection of the graph of $y=-x^{\left(\frac{1}{3}\right)}$ about the x-axis.

40) If x = 1 is one root of two equation. x^3 - 6x + 11x - 6 = 0 find the other roots.

Answer : 2,3

41) Find the principal value of $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

Answer : Let $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ =y ,where $-\frac{\pi}{2}\leq y\leq\frac{\pi}{2}$

$tan\ y=-\frac{1}{\sqrt{3}}\Rightarrow tan\ y=tan(-\frac{\pi}{6})\Rightarrow y=-\frac{\pi}{6}$

Thus the principal value of $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)=-\frac{\pi}{6}$

42) Determine the number of permutations of the letters of the word SIMPLE if all are taken at a time?

Answer : There are 6 letters in the word 'SIMPLE'. So, total number of words is equal to the number of arrangements of these letters, taken all at a time. Sum order of such arrangements is 6 P_6=6!=720

43) Evaluate: $^4P_4$.

Answer : $^4P_4$= 4 x 3 x 1 = 4! = 24

44) Compute 9^7

Answer : $(10-1)^7$ $(a-b)^n = nC_0\, a^n\, b^0 - nC_1\, a^{n-1}\, b^1 + \ldots nC_n\, a^0\, b^n$, n ϵ N

= 10^7 - $7C_1$ 10^6 (1) + $7C_2$ 10^5 $(1)^2$ - $7C_3$ 10^4 $(1)^3$ +$7C_4$ $(10)^3$ $(1)^4$ - $7C_5(10)^2$ $(1)^5$ + $7C_6(10)^1(1)^6$ - $(1)^7$

= 10000000 - 7(1000000) + $\frac{7\times6}{2\times1}$ (100000) - $\frac{7\times6\times5}{3\times2\times1}$ 10000 + $\frac{7\times6\times5}{3\times2\times1}$ 1000

= - $\frac{7\times6}{2\times1}$ (100) + 7(10) - 1

= 10000000 - 7000000 + 21.00000 -350000 + 35000 - 2100 + 70 - 1

= 4782969

45) Write the n^{th} term of the following sequences

2,2,4,4,6,6

Answer : 2,2,4,4,6,6

Given sequences is 2,2,4,4,6,6,

the odd term are 2,4,6 .. and even terms are also 2,4,6

$\therefore \quad a_{n=} \begin{cases} n+1 \\ 1 \end{cases}$

if n is odd

if n is even

46) Find the middle term in $\left(x-\frac{1}{2y}\right)^{10}$

Answer : Given $\left(x-\frac{1}{2y}\right)^{10}$

Here n = 10, x = x and $a=\left(\frac{-1}{2y}\right)$

Middle term = $T_{\frac{10+2}{2}}=T_6$

General term is $T_{r+1}=nCrx^{n-r}a^r$

Putting r = 5 we get,

$T_6=10C_5x^{10-5}\left[-\frac{1}{2y}\right]^5=\frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1}.x^5\left(\frac{-1}{32.y^5}\right)$

$=-225.x^5.\frac{1}{32y^5}T_6=\frac{-63x^5}{8y^5}$

47) Find the locus of P, if for all values of α the co-ordinates of a moving point P is

(i) (9 cos,α 9 sin α)

(ii) (9 cos α, 6 sin α)

Answer : (i) (9 cos α, 9 sin α)

Let P (h, k) be any point on the required path

From the given information, we have

h = 9 cos α and k = 9 sin α

$\Rightarrow$ $\frac{h}{9}$ = cos α and $\frac{k}{9}$ = sin α

$\left(\frac{h}{9}\right)^2+\left(\frac{k}{9}\right)^2=cos^2\alpha+sin^2\alpha$

$\Rightarrow \frac{h^2}{81}+\frac{k^2}{81}=1$ $\quad [\because sin^2\alpha+cos^2\alpha=1]$

$\Rightarrow$ h^2+k^2 = 81

$\therefore$ Locus of (h, k) is x^2+y^2 = 81

(ii) (9 cos α, 6 sin α)

Let P (h, k) be any point on the required path

From the given information, we have

h 9 cos α and k = 6 sin α

$\Rightarrow$ $\frac{h}{9}$ = cos α and $\frac{k}{6}$ = sin α

To eliminate the parameter α,
Squaring and adding we get

$\left(\frac{h}{9}\right)^2+\left(\frac{k}{6}\right)^2=cos^2\alpha+sin^2\alpha$

$\Rightarrow \frac{h^2}{81}+\frac{k^2}{36}=1$ [$\because$ cos2 α + sin2 α = 1]

$\therefore$ Locus of (h, k) is $\frac{x^2}{81}+\frac{y^2}{36}=1$

48) If p is the length of the perpendicular from the origin to the line $\frac{x}{a}+\frac{y}{b}=1$, then prove that $\frac{1}{p_2}=\frac{1}{a^2}+\frac{1}{b^2}$

Answer : Given equation is $\frac{x}{a}+\frac{y}{b}=1$

$\Rightarrow \frac{bx+ay}{ab}=1$

$\Rightarrow$ bx+ay-ab=0....(1)

Given that p= Length of the perpendicular from the origin to the line (1)

$\Rightarrow p=\left|\frac{b(0)+a(0)-ab}{\sqrt{b^2+a^2}}\right|$

$\Rightarrow p^2=\left(\frac{ab}{\sqrt{a^2+b^2}}\right)^2 \Rightarrow P^2=\frac{a^2b^2}{a^2+b^2}$

$\Rightarrow \frac{1}{p^2}=\frac{a^2+b^2}{a^2b^2}$

$\Rightarrow \frac{1}{p^2}=\frac{a^2}{a^2b^2}+\frac{b^2}{a^2b^2}$

$\Rightarrow \frac{1}{p^2}=\frac{1}{b^2}+\frac{1}{a^2}$

Hence proved.

49) Find the values of p, q, r, and s if

$$\begin{bmatrix} p^2-1 & 0 & -31-q^3 \\ 7 & r+1 & 9 \\ -2 & 8 & s-1 \end{bmatrix}=\begin{bmatrix} 1 & 0 & -4 \\ 7 & \frac{3}{2} & 9 \\ -2 & 8 & -\pi \end{bmatrix}$$

Answer : Given $\begin{bmatrix} p^2-1 & 0 & -31-q^3 \\ 7 & r+1 & 9 \\ -2 & 8 & s-1 \end{bmatrix}=\begin{bmatrix} 1 & 0 & -4 \\ 7 & \frac{3}{2} & 9 \\ -2 & 8 & -\pi \end{bmatrix}$

Since the matrices are equal, the corresponding entries on both sides are equal.

$\therefore P^2-1=1 \Rightarrow P^2=2 \Rightarrow P\pm\sqrt{2}$ [Equating a_{11}]

$-31-q^3=-4 \Rightarrow -q^3=-4+31$ [Equating a_{13}]

$\Rightarrow -q^3=27$

$q^3=-27=(-3)^3$

$\Rightarrow q=-3$

Also r+1=3/2 $\Rightarrow$ r =3/2-1=$\frac{3-2}{2}$ =1/2 [Equating a_{22}]

s-1 = - $\pi \Rightarrow s$ =1-π [Equating a_{33}]

p =$\pm\sqrt{2}$,q =-3, r=1/2, s=1-π

50) Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}=4a^2b^2c^2$

Answer : LHS = $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$

Taking out A,b,c common from C_1, C_2 and C_3 respectively we get,

LHS =(abc)$\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

Applying $C_1 \longrightarrow C_1+C_2+C_3$ we get,

=(abc)$\begin{vmatrix} 2(b+c) & c & a+c \\ 2(a+b) & b & a \\ 2(b+c) & +c & c \end{vmatrix} = 2abc \begin{vmatrix} a+c & c & a+c \\ a+b & b & a \\ b+c & b+c & c \end{vmatrix}$

Applying $C_1 \longrightarrow C_1-C_2$ and $C_3 \longrightarrow C_3-C_1$ we get,

=2abc$\begin{vmatrix} a+c & -a & 0 \\ a+b & -a & -b \\ b+c & 0 & -b \end{vmatrix}$

Applying $C_1 \longrightarrow C_1+C_2+C_3$ we get,

LHS = 2abc$\begin{vmatrix} c & -a & 0 \\ 0 & -a & -b \\ c & 0 & -b \end{vmatrix}$

Taking c, a, b common from C_1, C_2 and C_3 respectively.

$=2a_2b_2c_2\begin{vmatrix} 1 & -1 & 0 \\ 0 & -1 & -1 \\ 1 & 0 & -1 \end{vmatrix}$

Expanding along R_1 we get,

$=2a_2b_2c_2\left[1\begin{vmatrix} -1 & -1 \\ 0 & -1 \end{vmatrix}+1\begin{vmatrix} 0 & -1 \\ 1 & -1 \end{vmatrix}\right]$

$=2a^2b^2c^2[(1-0)+(0+1)]=2a^2b^2c^2[2]=4a^2b^2c^2$=RHS

Hence proved.

51) Verify whether the following ratios are direction cosines of some vector or not $\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{2}$

Answer : Let $l=\frac{2}{\sqrt{2}}, m=\frac{1}{2}$ and $n=\frac{1}{2}$

$\therefore l^2+m^2+n^2=(\frac{1}{\sqrt{2}})^2+(\frac{1}{2})^2+(\frac{1}{2})^2 \quad =\frac{1}{2}+\frac{1}{4}+\frac{1}{4}=\frac{2+1+1}{4}=\frac{4}{4}=1$

Hence, the given ratios are direction cosines of some vector.

52) Find the direction cosines and direction ratios for the following vectors.$\hat{i}$ -$\hat{k}$

Answer : The given vector is $\hat{i}$ -$\hat{k}$

The direction ratio are 1,0,-1

x= $\sqrt{x^2+y^2+z^2}=\sqrt{1^2+0^2+(-1)^2}=\sqrt{2}$

Hence, the direction cosines are $\frac{1}{\sqrt{2}}, \frac{0}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}$

53) Complete the table using calculator and use the result to estimate the limit.

$lim_{x \to 2}\frac{x-2}{x^2-4}$

x	1.9	1.99	1.999	2.001	2.01	2.1

f(x)							

Answer : $lim_{x\to2}\frac{x-2}{x^2-4}$ $=lim_{x\to2}\frac{x-2}{(x+2)(x-2)}=lim_{x\to2}\frac{1}{(x+2)}$

x	1.9	1.99	1.999	2.001	2.01	2.1
f(x)	$\frac{1}{1.9+2}=\frac{1}{3.9}$ =0.256	$\frac{1}{1.99+2}=\frac{1}{3.99}$ =0.251	$\frac{1}{1.999+2}=\frac{1}{3.999}$ =0.33	$\frac{1}{2.001+2}=\frac{1}{4.001}$ =0.250	$\frac{1}{2.01+2}=\frac{1}{4.01}$ =0.249	$\frac{1}{2.1+2}=\frac{1}{4.1}$ =0.244

$lim_{x\to2}\frac{x-2}{x^2-4}=0.25$

54) Evaluate the following limits :

$lim_{\sqrt{x}\to3}\frac{x^2-81}{\sqrt{x}-3}$

Answer : $lim_{\sqrt{x}\to3}\frac{x^2-81}{\sqrt{x}-3}$ $=lim_{\sqrt{x}\to3}\frac{(\sqrt{x})^4-3^4}{\sqrt{x}-3}$ $[\because lim_{x\to a}\frac{x^n-a^n}{x-a}=n.a^{n-1}]$

$=4.3^{4-1}=4(3)^3=4(27)=108$

55) Suppose that the diameter of an animal's pupils is given by $f(x)=\frac{160x^{-0.4}+90}{4x^{-0.4}+15}$, where x is the intensity of light and f(x) is in mm. Find the diameter of the pupils with maximum light.

Answer : For maximum light, it is enough to find the limit of the function when x $\to\infty$

$lim_{x\to\infty}f(x)=lim_{x\to\infty}\frac{160x^{-0.4}+90}{4x^{-0.4}+15}=\frac{90}{15}=6mm$

That is, the pupils have a limiting size of 6mm, as the intensity of light is very large.

56) Find the points of discontinuity of the function f, where

f(x)={ $x+2,\quad if\quad x\geq2$
$x^2,\quad if\quad x<2$

Answer : Given f(x)={ $x+2,\quad if\quad x\geq2$
$x^2,\quad if\quad x<2$

$lim_{x\to2^-}f(x)=lim_{x\to2^-}x^2=2^2=4$

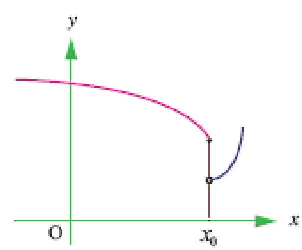
$lim_{x\to2^+}f(x)=lim_{x\to2^+}x+2=2+2=4$

Also f(2)= x+2=2+2=4

$\therefore lim_{x\to2^-}f(x)=lim_{x\to2^+}f(x)=f(2)=4$

$\therefore$ f(x)is continuous in R.

57) State how continuity is destroyed at x= $x_o$for each of the following graphs.



Answer : The left-hand limit and right-hand limit does not coincide at x= x_o.

58) Evaluate$\lim_{x\to0}\frac{a^x-1}{b^x-1}$

Answer : $\lim_{x\to0}\frac{a^x-1}{b^x-1}=\lim_{x\to0}\frac{\frac{a^x-1}{x}}{\frac{b^x-1}{x}}=\frac{\lim_{x\to0}\left(\frac{a^x-1}{x}\right)}{\lim_{x\to0}\left(\frac{b^x-1}{x}\right)}=\frac{\log a}{\log b}$

59) Differentiate the following:$y=e^{\sqrt{x}}$

Answer : Given y=$e^{\sqrt{x}}$

Let u=$\sqrt{x}$ ⇒ y=e^u

$\therefore \frac{du}{dx} = \frac{1}{2}x\frac{1}{2} - 1; \frac{dy}{du}$ =eu

$\Rightarrow \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^u \times \frac{1}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$ [∵ u=$\sqrt{x}$]

60) Differentiate $\sqrt{e^{\sqrt{x}}}, x > 0$.

Answer : Let y=$\sqrt{e^{\sqrt{x}}}$

Differentiating both sides with respect to 'x' we have,

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{e^{\sqrt{x}}} = \frac{1}{2\sqrt{e^{\sqrt{x}}}}.\frac{d}{dx}\left(e^{\sqrt{x}}\right) = \frac{1}{2\sqrt{e^{\sqrt{x}}}}.e^{\sqrt{x}}.\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{e^{\sqrt{x}}}}.e^{\sqrt{x}}.\frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4\sqrt{x}.\sqrt{e^{\sqrt{x}}}}$$

3 Marks

61) By taking suitable sets A, B, C, verify the following results:

(A× B)∩(B× A) = (A∩B) × (B∩A)

Answer : (A× B)={(1,4) (1,5) (1,6) (1,7) (2,4) (2,5) (2,6) (2,7) (3,4) (3,5) (3,6) (3,7)}

(B× A) = {(4,1) (4,2) (4,3) (5,1) (5,2) (5,3) (6,1) (6,2) (6,3) (7,1) (7,2) (7,3)}

LHS = (A× B)∩(B× A)={ } ....(1)

(A∩B) = { }, (B∩A) = { }

∴ RHS = (A∩B) × (B∩A) = { } ....(2)

From (1) and (2), LHS = RHS

62) By taking suitable sets A, B, C, verify the following results:

C-(B-A) = (C∩ A) ∪ (C∩B')

Answer : B-A = {4, 5, 6, 7}

LHS = C-(B-A) = {3, 9} ....(1)

C∩A = {3}

B' = {1,2,3,8,9}

C∩B' = {3, 9}

RHS = (C∩A)∪ (C∩B') = {3, 9} ....(2)

From (1) and (2), LHS = RHS

63) Find the pairs of equal sets from the following sets. A = {0}, B = {x: x > 15 and x < 5}, C = {x: x - 5 = 0}, D = {x: x^2 = 25}, E = {x: x is an integral positive root of the equation x^2 - 2x - 15 = 0}.

Answer : Given A = {0} ...(1)

B = {x: x > 15 and x < 5} ⇒ B = Φ ...(2)

C= {x:x-5=0} ⇒ B= {5} ...(3)

D = {x: x^2 = 25} ⇒ D = {-5, 5} ...(4)

E = {x: x is an integral positive root of x^2 - 2x - 15 = 0}

⇒ E = {5} ...(5)

From (3) and (5), clearly C = E.

Hence C and E are equal sets

64) Which of the following sets are finite and which are infinite?

Set of letters of the English alphabet.

Answer : Set of letters of the English alphabet is a finite set since there are 26 letters.

65) Find the value of $\log_2\left(\frac{\sqrt[3]{4}}{4^2\sqrt{8}}\right)$.

Answer : Given $log_2\left(\frac{\sqrt[3]{4}}{4^2\sqrt{8}}\right)$

= $log_2\sqrt[3]{4} - log_2 4^2(\sqrt{8})$

= $log_2 4^{1/3} - [log_2 4^2 + log_2\sqrt{8}]$

=$\log^2(2^2)^{1/3}$-$\log^2(2^2)^2$-$\log^2(2^3)^{1/2}$

=$\log^2 2^{1/3}$-$\log^2 2^4$-$\log^2 2^{3/2}$

$= \frac{2}{3}(1) - 4(1) - \frac{3}{2}(1)$ $\quad [\because log_2^2 = 1]$

$= \frac{4-24-9}{6} = \frac{-29}{6}$

66) Solve $\log_4 2^{8x}$ =2 $\log_2{}^8$

Answer : Given $\log_4 2^{8x}$ =2$\log_2{}^8$

⇒ 8x $log_4^2 = 2 \times 3 \quad log_2^2$

⇒ 8x log_4^2 = 6 (1) $\quad [\because log_2^2 = 1 \quad]$

⇒ $\frac{8x}{log^4}$ =6

⇒ $\frac{8x}{log_2^{2^2}} = 6$

⇒ $\frac{8x}{2log_2^{2^2}} = 6$ $\quad$ ⇒ $\frac{8x}{2(1)}$ =6

⇒ $\frac{4x}{1}$ =6

⇒ x = $\frac{6}{4}$

⇒ x = $\frac{3}{2}$

67) Given log 2 = 0.310, find the position of the first significant digit in the value of $(0.5)^{10}$.

Answer : Given log 2 = 0.3010

log (0.5)10 = $log\left(\frac{1}{2}\right)^{10} = log\ 2^{-10}$

=-10 log 2 = - 10 (0.3010) = - 3.010

= -3-0.0010-(-3-1)+(-0.010)

=-4+0.990=4.990.

∴ Characteristic of log $(0.5)^{10}$ =4 ie -4

∴ Number of Zeroes immediately after the decimal part = 4 - 1 = 3.

∴ First significant digit is at 4th place after decimal.

68) Prove that $32\left(\sqrt{3}\right)\sin\frac{\pi}{48}\cos\frac{\pi}{48}\cos\frac{\pi}{24}\cos\frac{\pi}{12}\cos\frac{\pi}{6} = 3$.

Answer : LHS=$32\left(\sqrt{3}\right) sin\frac{\pi}{48}cos\frac{\pi}{48}.cos\frac{\pi}{24}cos\frac{\pi}{12}cos\frac{\pi}{6}$

$=\frac{32\sqrt{3}}{2}\left(2sin\frac{\pi}{48}cos\frac{\pi}{48}\right).cos\frac{\pi}{24}cos\frac{\pi}{12}cos\frac{\pi}{6}$

$=16\sqrt{3}\left(sin2\frac{\pi}{48}\right).cos\frac{\pi}{24}cos\frac{\pi}{12}cos\frac{\pi}{6}$

$= \frac{16\sqrt{3}\left[(2)sin\frac{\pi}{24}cos\frac{\pi}{24}\right]cos\frac{\pi}{12}cos\frac{\pi}{6}}{2}$

$=8\sqrt{3}\left(sin\frac{2\pi}{24}\right).cos\frac{\pi}{12}cos\frac{\pi}{6}$

$=8\sqrt{3}.\ sin\frac{\pi}{12}cos\frac{\pi}{12}cos30^\circ$

$=8\sqrt{3}\frac{\sqrt{3}}{2}\frac{2}{2}sin\frac{\pi}{12}cos\frac{\pi}{2}$

$=6\left[sin2\frac{\pi}{12}\right]$

$=6.sin\frac{\pi}{6}$

$=6.sin30^{\circ} = 6\times\frac{1}{2} = 3 = RHS$
Hence proved.
69) Prove that cos $\left(\frac{3\pi}{4}+\pi\right) - cos\left(\frac{3\pi}{4}-4\right) = -\sqrt{2}sin\ x.$
Answer : LHS = cos $\left(\frac{3\pi}{4}+\pi\right) - cos\left(\frac{3\pi}{4}-4\right)$
= cos C - cos D - 2 sin $\left(\frac{C+D}{2}\right).sin\left(\frac{C-D}{2}\right)$
$= -2sin\left(\frac{\frac{3\pi}{4}+x+\frac{3\pi}{4}-x}{2}\right).sin\left(\frac{\frac{3\pi}{4}+x-\frac{3\pi}{4}-x}{2}\right)$ where $C=\frac{3\pi}{4}+x, D=\frac{3\pi}{4}-x$
$= -2\ sin\left(\frac{6\pi}{4(2)}\right).sin\left(\frac{2x}{2}\right)$
$= -2\ sin\left(\frac{3\pi}{4}\right).sin(x)$
$= -2sin\left(\pi-\frac{\pi}{4}\right).sin\ x$
$= -2\ sin\left(\frac{\pi}{4}\right).sin\ x = -2.\frac{1}{\sqrt{2}}.sin\ x = -\sqrt{2}\ sinx$
= RHS.
70) If $tan\alpha = \frac{1}{3}\ and\ tan\beta = \frac{1}{7}$ show that $2\alpha+\beta = \frac{\pi}{4}$.
Answer : $tan2\alpha = \frac{2tan\alpha}{1-tan^2\alpha} = \frac{2(\frac{1}{3})}{1-(\frac{1}{3})^2} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$
$tan(2\alpha+\beta) = \frac{tan2\alpha+tan\beta}{1-tan2\alpha+tan\beta} = \frac{\frac{3}{4}+\frac{1}{7}}{1-(\frac{3}{4})(\frac{1}{7})} = \frac{\frac{21+4}{28}}{\frac{28-3}{28}} = 1$
$\therefore 2\alpha+\beta = 45^0 = \frac{\pi}{4}$
71) How many 3-digit numbers more than 600 can be formed using the digits 2, 3, 4, 6, 7?
Answer : Clearly repetition of digits is allowed. Since, a 3-digit number greater than 600 will have 6 or 7 at hundred's place.
So, hundred's place can be filled in 2 ways. Each of the ten's and one's place can be filled in 5 ways.
Hence, total number of required numbers = 2 x 5 x 5 = 50.
72) Eighteen guests have to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three on the other side. Determine the number of ways in which the seating arrangement can be made?
Answer : Since 4 particular guest want to sit on side A and 3 on the other side B, so we are left with 11 guests out of which we choose 5 for side in $11C_5$ ways and 6 for side B in $6C_6$ ways.
$\therefore$ Number of selections for the two sides is $11C_5\times 6C_6$.
Now, 9 persons on each side of the table can be arranged among themselves in 9! ways.
Hence, the total number of arrangement.
= $11C_5\times\ 6C_6\times\ 9!\times\ 9!$
73) The first three terms in the expansion of $(1+ax)^n$ are $1+12x+64x^2$.Find n and a
Answer : Using binomial theorem, we have
$(1+ax)^n$-$1+nC_1(ax)+nC_2(ax)^2+.......+a^nx^n$
$=1+nax+\frac{n(n-1)}{2}a^2x^2+....a^nx^n$
Given $(1+ax)^n = 1 + 12x + 64x^2 +....$
Conparing the Co-efficient of x and x2, we get

n a = 12

and $\frac{n(n-1)}{2}a^2 = 64$

$(n-1).\frac{na.a}{2} = 64 \Rightarrow (n-1)\frac{(12)a}{2} = 64$

$(n-1)6a = 64 \Rightarrow (n-1)a = \frac{64}{6}$

$[\because na = 12 \Rightarrow a = \frac{12}{n}]$

$\Rightarrow (n-1)\left(\frac{12}{n}\right) = \frac{64}{6}$

$= \frac{n-1}{n} = \frac{64}{6\times12} \Rightarrow \frac{n-1}{n} = \frac{8}{9}$

$\Rightarrow$ 9n-9=8n

$\Rightarrow$ n=9 and $a = \frac{12}{n} = \frac{12}{9} = \frac{4}{3}$

74) If the mth term of a H.P. is n and nth term is m, then show that its pth term is $\frac{mn}{p}$.

Answer : Let the H.P. be $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots$

$\therefore T_m = \frac{1}{a+(m-1)d} = n$ and $\therefore T_n = \frac{1}{a+(n-1)d} = m$

a + (m - 1)d = $\frac{1}{n}(1)$ and a + (n - 1)d = $\frac{1}{m}$

(1) - (2) $\Rightarrow$ (m - 1 - n + 1)d = $\frac{1}{n} - \frac{1}{m}$ $\Rightarrow$ (m - n) d = $\frac{m-n}{mn} \Rightarrow d = \frac{1}{mn}$

$T_p = \frac{1}{a+(p-1)d} = \frac{1}{\frac{1}{mn}+(b-1)\frac{1}{mn}} = \frac{mn}{1+p-1}$

$T_p = \frac{mn}{p}$

75) Sum the series: (1 + x) + (1 + x + x^2) + (1 + x + x^2 +x^3) + ... up to n terms

Answer : (1 + x) + (1 + x + x^2) + (1 + x + x^2 +x^3) + ... up to n terms

= $\frac{1-x^2}{1-x} + \frac{1-x^3}{1-x} + .. \frac{1-x^4}{1-x} + \ldots$ to n terms

= $\frac{1}{1-x}[(1+1+1+\ldots tonterms) - (x^2+x^3+x^4\ldots.tonterms)$

= $\frac{1}{1-x}[n - \frac{x^2(1-x^n)}{1-x}]$

76) Find the equation of the straight line which passes through the point (1, -2) and cuts off equal intercepts from axes.

Answer : Intercept form of straight line $\frac{x}{a} + \frac{y}{b} = 1,$ where a and b are the intercepts on the axis

Given that a = b, $\therefore \frac{x}{a} + \frac{y}{b} = 1$ ....(1)

If equation (1) passes through the point (1, -2) we get

$\frac{1}{a} - \frac{2}{a} = 1 \Rightarrow -\frac{1}{a} = 1 \Rightarrow a = -1$

So, equation of the straight line is

$\frac{x}{-1} + \frac{y}{-1} = 1 \Rightarrow x+y = -1 \Rightarrow x+y+1 = 0$

Hence, the required equation x + y + 1 = 0.

77) Prove that $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$ =4(a+b)(b+c)(c+a). Using factor theorem.

Answer : Let $\triangle$ = $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$

Putting a=-b in (1) we get,

$$\triangle = \begin{vmatrix} 2b & 0 & -b+c \\ 0 & -2b & b+c \\ c-b & c+b & -2c \end{vmatrix}$$

Expanding along R_1 we get,

$\triangle$ =2b (4bc - (b + cp (- b + c) (2b(c - b))

=2b (4bc - b2 - c2 - 2bc) + (c - b) (2bc - 2b2)

=2b (2bc - b2 - c2) + (c - b) (2bc - 2b2)

== $4b^2c - 2b^3 - 2bc^2 + 2bc^2 - 2b^2c - 2b^2c + 2b^3 = 0$

$\therefore$ (a + b) is a factor of A.

Similarly (b + c) and (c + a) are factors of $\triangle$.

Since the leading diagonal is of degree 3, their will be a constant k and 3 factors.

$\therefore \triangle$ =k (a + b)(b + c)(c + a)

$$\triangle = \begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+ & c+b & -2c \end{vmatrix}$$ =k (a + b)(b + c)(c + a)

Put a = 0, b = 1 and c = 2 we get,

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & -2 & 3 \\ 2 & 3 & -4 \end{vmatrix}$$ =k(1)(3)(2)

$\Rightarrow$ -1(-4-6) +2(3+4) =6 k [Expanded along R_1]

$\Rightarrow$ -1(-10)+14= 6k

$\Rightarrow$ 24= 6k

$\Rightarrow$ k=4

$$\therefore \begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix}$$ =4(a+b)(b+c)(c+a)

78) Differentiate the following: $s(t) = \sqrt[4]{\frac{t^3+1}{t^3-1}}$

Answer : Given s(t)= $\sqrt[4]{\frac{t^3+1}{t^3-1}}$ =(t3+1)1/4(t3-1)-1/4

Let u=t^3+1 and v=t^3-1

$\Rightarrow \frac{du}{dt}$ =3t^2 and $\frac{dv}{dt}$ =3t^2

$\therefore$ s(t)=$u^{1/4}$.$v^{-1/4}$

s'(t)= $$u^{1/4}\left(\frac{-1}{4}\right)v^{\frac{-1}{4}-1}.\ \frac{1}{4}u^{\frac{1}{4}-1}.\ \frac{du}{dt} = \frac{-u^{\frac{1}{4}}v^{\frac{-5}{4}}}{4}(3t^2) + \frac{v^{\frac{-1}{4}}.u^{\frac{-3}{4}}}{4}(3t^2)$$

= $$\frac{-(t^3+1)^{\frac{1}{4}}(t^3-1)^{\frac{-5}{4}}}{4}(3t^2) + \frac{(t^3-1)^{\frac{-1}{4}}(t^3+1)^{\frac{-3}{4}}}{4}(3t^2) = \frac{-3t^2(t^3+1)^{\frac{1}{4}}}{(t^3-1)^{\frac{5}{4}}} + \frac{3t^2(t^3+1)^{\frac{-3}{4}}}{(t^3-1)^{\frac{1}{4}}}$$

= $$-\frac{3t^2}{4}\left[\frac{1}{(t^3-1)^{\frac{5}{4}}(t^3+1)^{\frac{-1}{4}}} - \frac{1}{(t^3-1)^{\frac{1}{4}}(t^3+1)^{\frac{3}{4}}}\right] = \frac{3t^2}{4}\left[\frac{(t^3+1)-(t^3-1)}{(t^3-1)^{\frac{5}{4}}(t^3+1)^{\frac{3}{4}}}\right]$$

= $$-\frac{3t^2}{4}\left[\frac{t^3+1-t^3+1}{(t^3-1)^{\frac{5}{4}}(t^3+1)^{\frac{3}{4}}}\right] = \frac{3t^2}{4}\left[\frac{2}{(t^3-1)^{\frac{5}{4}}(t^3+1)^{\frac{3}{4}}}\right] = \frac{-3t^2}{2(t^3-1)^{\frac{5}{4}}(t^3+1)^{\frac{3}{4}}}$$.

79) Differentiate the following: $y = 5^{\frac{-1}{x}}$

Answer : Given y=$5^{\frac{-1}{x}}$

Let u= $\frac{-1}{x}$ and y=5u

$\therefore \frac{du}{dx} = \frac{1}{x^2}$ and $\frac{dy}{du}$ =5u log5

$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 5^u(log5)\left(\frac{1}{x^2}\right) = \frac{5^{-\frac{1}{x}}.log5}{x^2}$

80) If xy = 4, Prove that $x\left(\frac{dy}{dx} + y^2\right) = 3y.$

Answer : Given xy = 4

Differentiating both sides with respect to 'x' we get,

$x.\frac{dy}{dx} = y(1) = 0 \quad \Rightarrow x\frac{dy}{dx} = -y \qquad \ldots(1)$

$LHS \quad = \quad x\left(\frac{dy}{dx} + y^2\right) = x\frac{dy}{dx} + xy^2 = \quad -y + (xy)y = -y + 4y \quad [\because \quad xy = 4]$

$= 3y = RHS \quad HenceProved.$

5 Marks

81) Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 28.

Answer : Let x be the smaller of the two consecutive even positive integers, then the other is x + 2.

According to the given conditions.

x > 5 ,x+2 > 5

and x+(x+2) < 23

⇒ x > 5, x > 3

and 2x < 21

⇒ x > 5 (∵ x > 5 automatically smallest of the lesser than) ...(1)

⇒ x > 3

and x < $\frac{21}{2}$

From (1) and (2), we get

$5 < x < \frac{21}{2}$

Also, x is an even positive integer.

x can take the values 6, 8 and 10.

So, the required possible pairs will be (x,x+2) = (6,8), (8,10), (10,12).

82) find the value of sin $\left(-\frac{11\pi}{3}\right)$

Answer : sin (-11 x 60^0) = sin (-660^0)

=- sin (660^0) = - sin (2 x 360^0 - 60^0)

=- (-sin 60_0) = sin 60_0 = $\frac{\sqrt{3}}{2}$

83) Show that $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) = \pi$

Answer : Let sin^{-1} $\left(\frac{12}{13}\right) + cos^{-1}\left(\frac{4}{5}\right) + tan^{-1}\left(\frac{63}{16}\right) = \pi$

Then $sin \quad x = \frac{12}{13}, cosy\frac{4}{5}, \quad and \quad tanz\frac{63}{16}$

$cos \quad x\sqrt{1 - sin^2x} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$

$and \quad tan \quad x = \frac{sin \quad x}{cos \quad x} = \frac{12}{13}/\frac{5}{13} = \frac{12}{5}$

$sin \quad y\sqrt{1 - cos^2y} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

$tan \quad y = \frac{sin \quad y}{cos \quad y} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$

i.e have tan (x+y)= $\frac{tan \quad x+tan \quad y}{1-tan \quad x.tan \quad y}$

$$\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\times\frac{3}{4}}=\frac{\frac{48+15}{20}}{\frac{20-36}{20}}=-\frac{63}{16}$$

From (1) and (2), tan (x+y) = -tan z

$\Rightarrow \quad tan \quad (x+y)=tan \quad (-z)$

$\Rightarrow \quad tan(x+y)=tan(\pi-z)$

$\Rightarrow \quad x+y \quad = \quad -z \quad or \quad x+y=\pi-z$

Since x,y, and z are positive, x+y $\neq$ -z

$\therefore$ x+y+z = p

$\Rightarrow \quad sin^{-1}\left(\frac{12}{13}\right)+cos^{-1}\left(\frac{4}{5}\right)+tan^{-1}\left(\frac{63}{16}\right)=\pi$

84) There are 11 points in a plane. No three of these lies in the same straight line except 4 points, which are collinear. Find,

(i) the number of straight lines that can be obtained from the pairs of these points?

(ii) the number of triangles that can be formed for which the points are their vertices?

Answer : (i) Assume that no three points out of 11 points are collinear. Then we can draw unique straight through any arbitrary pair of points out of the 11 given points. This is a combination of 2 objects taken at a time from a total of 11 and can be done in $11C_2$ = 55 ways.

However, it is given that 4 points are collinear. Had they not been collinear the number of unique lines that could have been drawn through them is a combination of 2 objects taken at a time from a total of 5 and can be done in $4C_2$ = 6 ways. Since they are collinear we get only one line out of these 4 points instead of 6.

So, the total number of straight lines that can be drawn through 13 points on a plane with 5 of the points being collinear is 55-6+1 =50.

(ii) To form a triangle we need 3 points. The following are the choices.

a) If we take one point from 4 collinear points and 2 from remaining 7 and join the,. So this case will give $4C_1\times 7C_2$ points $=4\times 21=84$

b) If we two points from 4 collinear points and 1 from remaining 7. So this will give $4C_2\times 7C_1=6\times 7=42$

c)If we take all the three points from 7 non collinear points. Which will give $7C_3$ =35

$\therefore$ Total number of triangles are 84 + 42 + 35 = 161.

85) A committee of 12 is to be formed from 9 women and 8 men. In how many ways this can be done if atleast 5 women have to be included in a committee? In how many of these committees the women are in majority?

Answer : There are 9 women and 8 men. A committee of 12, consisting of atleast 5 women can be formed by the following cases:

		Number of ways
(a)	5 women 7 men	$9C_5$ x $8C_7$ = 128 x 8
(b)	6 women and 6 men	$9C_6$ x $8C_6$ = 84 x 28
(c)	7 women and 5 men	$9C_7$ x $8C_5$ = 36 x 56
(d)	8 women and 4 men	$9C_8$ x $8C_4$ = 9 x 70
(e)	9 women and 3 men	$9C_9$ x $8C_3$ = 1 x 56

$\therefore$ Total number of ways of forming the committee

= 128 x 8 + 84 x 28 + 36 x 56+ 9 x 70 + 1 x 56 = 6062.

Clearly, women are majority in cases (c), (d) and (e). $\therefore$ Total number of committees in which women are majority = $9C_7$ x $8C_5$ x $9C_8$ x $8C_4$ x $9C_9$ x $8C_3$
= 36 x 56 + 9 x 70 + 1 x 56
= 2702

86) n^2 - n is divisible by 6, for each natural number n $\geq$ 2.

Answer : Let P(n) : n^3 - n

Step 1: P(2) : 2^3 - 2 = 6 which is divisible by 6. So it is true for P(2).

Step 2 : P(k) :k^3 - k = 6λLet it is be true for k$\geq$ 2

$\Rightarrow$ $k^3 = 6\lambda + k$ ...(i)

Step 3 : P(k+ 1) = $(k+1)^3$-(k+ 1)
=k^3 +1+ $3k^2$+ 3k - k - 1
= k^3 + k + $3(k^2$+ k)
= 6λ + $3(k^2$ + k) [from (i)]

We know that $3(k^2 + k)$ is divisible by 6 for every value of k$\in$ N.
Hence P(k + 1) is true whenever P(k) is true.

87) Find the fourth root of 623 correct to seven places of decimal.

Answer : Fourth root of 623 =$\left(\frac{-2}{625}\right)$

$\left(\frac{-2}{625}\right)$

$= \left[625\left(1-\frac{2}{625}\right)\right]^{\frac{1}{4}} = 5\left[1+\left(-\frac{2}{625}\right)\right]^{\frac{1}{4}}$

$= 5\left[1+\frac{1}{4}\left(\frac{-2}{625}\right)+\frac{\frac{1}{4}\left(-\frac{3}{4}\right)}{1.2}\left(\frac{-2}{625}\right)^2\right]$

Other terms will have more than seven zer0es after the decimal]
=5[1-0.0008-0.0000009]
$\sqrt[4]{623} = 4.9959955$

88) If P(2,-7) is a given point and Q is a point on $(2x^2 + 9y^2 = 18)$, then find the equations of the locus of the mid-point of PQ.

Answer : Let R(h, k) be the locus of the mid-point of PQ where, P is (2, -7) and Q is a point on $(2x^2 + 9y^2 = 18)$

Given equation is $2x^2 + 9y^2 = 18$

Dividing by 18 we get,

$\Rightarrow \quad \frac{x^2}{9}+\frac{y^2}{2}=1$

$\Rightarrow \quad \frac{x^2}{3^2}+\frac{y^2}{(\sqrt{2})^2}=1$

$\Rightarrow \quad a=3 \quad and \quad b=\sqrt{2}$

Any point on the ellipse is (a cos θ, b sin θ)

$\therefore$ Q is (3 cos θ, $\sqrt{2}$ sin θ)

Since R is the mid-point of PQ, we get, (h,k) = $\left(\frac{2+3cos\theta}{2}, \frac{-7+\sqrt{2}sin\theta}{2}\right)$

$\Rightarrow \quad h=\frac{2+3 \quad cos \quad \theta}{2}$

$\Rightarrow \quad 2h=2+3cos\theta$

$\Rightarrow \quad 2h-2=3cos\theta$

$\Rightarrow \quad \frac{2h-2}{3}=cos \quad \theta$

$k=\frac{-7+\sqrt{2}sin\theta}{2}$

$\Rightarrow \quad 2k=-7+\sqrt{2}sin\theta$

$\Rightarrow \quad 2k+7 \quad = \quad \sqrt{2}sin\theta$

$\Rightarrow \quad \frac{2k+7}{\sqrt{2}} = sin \quad \theta$

Squaring and adding we get,

$\left(\frac{2h-2}{3}\right)^2 + \left(\frac{2k+7}{\sqrt{2}}\right)^2 = cos^2\theta + sin^2\theta$

$\Rightarrow \quad \frac{4h^2+4-8h}{9} + \frac{4k^2+49+28k}{2} = 1 \quad [\because cos^2\theta + sin^2\theta = 1]$

$\Rightarrow \quad 2(4h^2+4-8h) = 9(4k^2+49+28k) = 18$

$\Rightarrow \quad 8h^2+8-16h+36k^2+441+252k-18 = 0$

$\Rightarrow \quad 8h^2+36k^2-16h+252k+431 = 0$

$\therefore$ Locus of (h,k) is

$8x^2+36y^2-16x+252y+431=0$

89) Find an equation to the pair of straight lines passing through the origin, perpendicular to the pair of straight lines given by $ax^2+2hxy+by^2=0$

Answer : Given equation of pair of straight lines is $ax^2 + 2hxy + by^2 = 0$

Let m_1 and m_2 be the slopes of the separate lines

Then $m_1 + m_2 = -\frac{2h}{b}$ and $m_1 m_2 = \frac{a}{b}$ ....(1)

Since the pair of straight lines passes through the origin, let their separate equations be $y = m_1x$ and $y = m_2x$.

Any line perpendicular to $y = m_1x$ and passes through the origin is of the form $m_1y + x = 0$.

and any line perpendicular to $y = m_2x$ and passes through the origin is of the form $m_2y + x = 0$.

Hence, their combined equation is

$(m_1y+x)(m_2y+x)=0$

$\Rightarrow m_1m_2y^2+m_1xy+m_2xy+x^2=0$

$\Rightarrow m_1m_2y^2+xy(m_1+m_2)+x^2=0$

$\Rightarrow \left(\frac{a}{b}\right)y^2 + xy\left(\frac{-2h}{b}\right) + x^2 = 0$ [using (1)]

$\Rightarrow \frac{ay^2}{b} - \frac{2hxy}{b} + x^2 = 0$

Multiply by b throughout we get

$ay^2 - 2hxy + bx^2 = 0$

$\Rightarrow$ $bx^2 - 2hxy + ay^2$ which is the required equation.

90) A point moves so that square of its distance from the point (3, -2) is numerically equal to its distance from the line 5x -12y = 3. The equation of its locus is ....

Answer : The given equation of line is 5x - 12y = 3 and the given point is (3, -2).

Let (a, b) be any moving point.

$\therefore$ Distance between (a, b) and the point (3, -2) = $\sqrt{(a-3)^2+(b+2)^2}$ and the distance of (a, b) from the line 5x - 12y = 3 = $\left|\frac{5a-12b-3}{\sqrt{25+144}}\right| = \left|\frac{5a-12b-3}{13}\right|$

According to the question, we have $[\sqrt{(a-3)^2+(b+2)^2}\,] = \left|\frac{5a-12b-3}{13}\right|$

$\Rightarrow (a-3)^2(b+2)^2 = \frac{5a-12b-3}{13}$

Taking numerical values only, we have $(a-3)^2(b+2)^2 = \frac{5a-12b-3}{13}$

$\Rightarrow a^2 - 6a + 9 + b^2 + 4b + 4 = \frac{5a-12b-3}{13}$

$\Rightarrow a^2+b^2-6a+4b+13=\frac{5a-12b-3}{13}$

$\Rightarrow$ $13a^2 + 13b^2 - 78a + 52b + 169 = 5a - 12b - 3$

$\Rightarrow$ $13a^2 + 13b^2 - 83a + 64b + 172 = 0$

So, only locus of the point is $13x^2 + 13y^2 - 83x + 64y + 172 = 0$

Hence, the value of the filter is $13x^2 + 13y^2 - 83x + 64y + 172 = 0$.

91) Using Factor Theorem, prove that $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix}$ $=(x-1)^2(x+9)$

Answer : Let $|A|=\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix}$

Putting x = 1, we get | A | = $\begin{vmatrix} 2 & 3 & 5 \\ 2 & 3 & 5 \\ 2 & 3 & 5 \end{vmatrix}=0$

Since all the three rows are identical, $(x - 1)^2$ is a factor of |A|

Putting x = - 9 in | A |,we get| A | = $\begin{vmatrix} -8 & 3 & 5 \\ 2 & -7 & 5 \\ 2 & 3 & -5 \end{vmatrix}=\begin{vmatrix} 0 & 3 & 5 \\ 0 & -7 & 5 \\ 0 & 3 & -5 \end{vmatrix}=0$

Therefore (x + 9) is a factor of |A| [since $C_1 \rightarrow C_1 + C_2 + C_3$].

The product $(x - 1)^2 (x + 9)$ is a factor of | A |. Now the determinant is a cubic polynomial in x.

Therefore the remaining factor must be a constant ‘k’.

Therefore $\begin{vmatrix} x+1 & 3 & 5 \\ 2 & x+2 & 5 \\ 2 & 3 & x+4 \end{vmatrix}$ $=k(x-1)^2(x+9)$

Equating x^3 term on both sides, we get k = 1. Thus | A | = $(x - 1)^2 (x + 9)$.

92) Show that the points A (1, 1, 1), B(1, 2, 3) and C(2, - 1, 1) are vertices of an isosceles triangle.

Answer : Let the position vector of the points A,B,C be

$\overrightarrow{OA}=\hat{i}+\hat{j}+\hat{k}$

$\overrightarrow{OB}=\hat{i}+2\hat{j}+3\hat{k}$

$\overrightarrow{OC}=2\hat{i}-\hat{j}+\hat{k}$

$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(\hat{i}+2\hat{j}+3\hat{k})-(\hat{i}+\hat{j}+\hat{k})=\hat{j}+2\hat{k}$

$|\overrightarrow{AB}|=\sqrt{1^2+2^2}=\sqrt{5}$

$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=(2\hat{i}-\hat{j}+\hat{k})-(\hat{i}+2\hat{j}+3\hat{k})=\hat{i}-3\hat{j}-2\hat{k}$

$\therefore |\overrightarrow{BC}|=\sqrt{1^2+(-3)^2+(-2)^2}=\sqrt{1+9+4}=\sqrt{14}$

$\overrightarrow{CA}=\overrightarrow{OA}-\overrightarrow{OC}=(\hat{i}+\hat{j}+\hat{k})-(2\hat{i}-\hat{j}+\hat{k})=-\hat{i}+2\hat{j}$

$\therefore |\overrightarrow{CA}|=\sqrt{(-1)^2+2^2}=\sqrt{5}$

Since $|\overrightarrow{AB}|=|\overrightarrow{CA}|$,the given points form an isosceles triangle.

93) Evaluate the following limits :

$lim_{x\to a}\frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}(a>b)$

Answer : $lim_{x\to a}\frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}(a>b)$

Multiplying and dividing by $(\sqrt{x-b}+\sqrt{a-b})$ we get,

$=lim_{x\to a}\frac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}\frac{\sqrt{x-b}+\sqrt{a-b}}{\sqrt{x-b}+\sqrt{a-b}}$

$=lim_{x\to a}\frac{(x-b)-(a-b)}{(x^2-a^2)[\sqrt{x-b}+\sqrt{a-b}]}$

$=\lim_{x\to a}\frac{x-b-a+b}{(x+a)(x-a)\left[\sqrt{x-b}+\sqrt{a-b}\right]}$

$=\lim_{x\to a}\frac{x-a}{(x+a)(x-a)\left[\sqrt{x-b}+\sqrt{a-b}\right]}$

$=\frac{1}{(a+a)[\sqrt{a-b}+\sqrt{a-b}]}=\frac{1}{2a[2\sqrt{a-b}]}$

$=\frac{1}{4a[\sqrt{a-b}]}$

94) Evaluate $\lim_{x\to\frac{1}{\sqrt{2}}}\frac{x-\cos(\sin^{-1}(x))}{1-\tan(\sin^{-1}x)}$

Answer :

$Let \quad \sin^{-1}x=t\Rightarrow x \quad =\sin t$

$Also \quad x\to\frac{1}{\sqrt{2}}\Rightarrow\sin t\to\frac{1}{\sqrt{2}}\Rightarrow t\to\frac{\pi}{4}$

$\therefore\lim_{x\to\frac{1}{\sqrt{2}}}\frac{x-\cos(\sin^{-1}(x))}{1-\tan(\sin^{-1}x)}=\lim_{t\to\frac{\pi}{4}}\frac{\sin t-\cos t}{1-\tan t}=\lim_{t\to\frac{\pi}{4}}\frac{\sin t-\cos t}{1-\frac{\sin t}{\cos t}}=\lim_{t\to\frac{\pi}{4}}\frac{(\sin t-\cos t)\cos t}{(\cos t-\sin t)}$

$=\lim_{t\to\frac{\pi}{4}}-\cos t=-\cos\left(\frac{\pi}{4}\right)=-\frac{1}{\sqrt{2}}$

95) If y = $\sin^{-1}$x then find y''.

Answer : Given y=$\sin^{-1}$x

y'=$\frac{1}{\sqrt{1-x^2}}=(1-x^2)^{-\frac{1}{2}}$

$\therefore$ y''=$-\frac{1}{2}(1-x^2)^{\frac{1}{2}-1}\frac{d}{dx}$ (1-x2)

$y^n=\frac{-1}{2}$ $(1-x^2)^{-3/2}(-2x)=x(1-x^2)^{-3/2}$

y''=$\frac{x}{(1-x^2)^{\frac{3}{2}}}$.