QB365 - Question Bank Software

Algebra Study Materials

10th Standard

Maths

Multiple Choice Question

1) $\frac{3y-3}{y} \div \frac{7y-7}{3y^2}$ is

(a) $\frac{9y}{7}$ (b) $\frac{9y^2}{(21y-21)}$ (c) $\frac{21y^2-42y+21}{3y^2}$ (d) $\frac{7(y^2-2y+1)}{y^2}$

2) The square root of $\frac{256x^8y^4z^{10}}{25x^6y^6z^6}$ is equal to

(a) $\frac{16}{5}\left|\frac{x^2z^4}{y^2}\right|$ (b) $16\left|\frac{y^2}{x^2z^2}\right|$ (c) $\frac{16}{5}\left|\frac{y}{xz^2}\right|$ **(d)** $\frac{16}{5}\left|\frac{xz^2}{y}\right|$

3) The number of points of intersection of the quadratic polynomial $x^2 + 4x + 4$ with the X axis is

(a) 0 **(b) 1** (c) 0 or 1 (d) 2

4) For the given matrix A = $\begin{pmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 9 & 11 & 13 & 15 \end{pmatrix}$ the order of the matrix A^T is

(a) 2 x 3 (b) 3 x 2 (c) 3 x 4 **(d) 4 x 3**

5) Transpose of a column matrix is

(a) unit matrix (b) diagonal matrix (c) column matrix **(d) row matrix**

2 Marks

6) Solve 3x + y -3z = 1; -2x - y + 2z = 1; -x - y + z = 2.

Answer : Let 3x + y -3z = 1 .... (1) -2x - y + 2z = 1 .... (2) -x - y + z = 2 ....(3)

Adding (1) and (2),
$$3x + y - 3z = 1$$
$$-2x - y + 2z = 1 \quad (+)$$
$$x - z = 2 \quad \dots (4)$$

Adding (1) and (3),
$$3x + y - 3z = 1$$
$$-x - y + z = 2 \quad (+)$$
$$2x - 2z = 3 \quad \dots (5)$$

Now, (5) −2×(4) we get,
$$2x - 2z = 3$$
$$2x - 2z = 4 \quad (-)$$
$$0 = -1$$

Here we arrive at a contradiction as $0 \neq -1$.

This means that the system is inconsistent and has no solution.

7) Solve $\frac{x}{2} - 1 = \frac{y}{6} + 1 = \frac{z}{7} + 2$; $\frac{y}{3} + \frac{z}{2} = 13$

Answer : Considering, $\frac{x}{2} - 1 = \frac{y}{6} + 1$

$\frac{x}{2} - \frac{y}{6}$ = 1 + 1 gives, $\frac{6x-2y}{12}$ = 2 we get, 3x - y = 12 .... (1)

Considering $\frac{x}{2} - 1 = \frac{z}{7} + 2$

$\frac{x}{2} - \frac{z}{7}$ = 1 + 2 gives, $\frac{7x-2z}{14}$ = 3 we get, 7x - 2z = 42 .... (2)

Also, from $\frac{y}{3} + \frac{z}{2}$ = 13 gives, $\frac{2y+3z}{6}$ = 13 we get, 2y + 3z = 78 ....(3)

Eliminating z from (2) and (3)

$$
\begin{array}{lrl}
(2)\times 3 \text{ gives,} & 21x \quad -6z & = 126 \\
(3)\times 2 \text{ gives,} & 4y+6z & = 156 \quad (+) \\
\hline
 & 21x+4y & = 282 \\
(1)\times 4 \text{ gives,} & 12x-4y & = 48 \quad (+) \\
\hline
 & 33x & = 330 \quad \text{so, } x = 10
\end{array}
$$

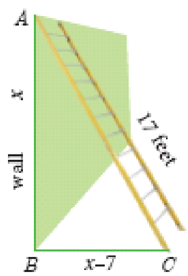
Substituting x = 10 in (1), 30 - y = 12 we get, y = 18

Substituting x = 10 in (2), 70 - 2x = 42 then, z = 14

Therefore, x = 10, y = 18, z = 14.

8) A ladder 17 feet long is leaning against a wall. If the ladder, vertical wall and the floor from the bottom of the wall to the ladder form a right triangle, find the height of the wall where the top of the ladder meets if the distance between bottom of the wall to bottom of the ladder is 7 feet less than the height of the wall?

Answer :



Let the height of the wall AB = x feet

As per the given data BC = (x – 7) feet

In the right triangle ABC, AC =17 ft, BC = (x – 7) feet

By Pythagoras theorem, $AC^2 = AB^2 + BC^2$

$(17)^2 = x^2 + (x - 7)^2$; $289 = x^2 + x^2 - 14x + 49$

$x^2 - 7x - 120 = 0$ hence, $(x - 15)(x + 8) = 0$ then, x = 15 (or) −8

Therefore, height of the wall AB = 15 ft (Rejecting −8 as height cannot be negative)

9) If the difference between the roots of the equation $x^2 - 13x + k = 0$ is 17. find k

Answer : $x^2 - 13x + k = 0$ here, a = 1, b = -13, c = k

Let α, β be the roots of the equation. Then

$\alpha + \beta = \frac{-b}{a} = \frac{-(-13)}{1} = 13$ ...... (1) also α - β = 17 ......(2)

(1) + (2) we get, 2α = 30 gives α = 15

Therefore, 15 + β = 13 (from (1)) gives β = -2

But, $\alpha\beta = \frac{c}{a} = \frac{k}{1}$ gives 15 x (-2) = k we get, k = -30

10) If a matrix has 16 elements, what are the possible orders it can have?

Answer : We know that a matrix of order m x n has mn elements. Thus to find all possible orders of a matrix with 16 elements, we will find all ordered pairs of natural numbers whose product is 16.

Such ordered pairs are (1, 16), (16, 1), (4,4), (8,2), (2,8)

Hence possible orders are 1 x 16, 16 x 1, 4 x 4, 2 x 8, 8 x 2

5 Marks

11) Find the square root of the following expressions

$256(x - a)^2 (x - b)^4 (x - c)^{16} (x - d)^{20}$

Answer : $\sqrt{256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20}} = 16|(x - a)_4(x - b)_2 (x - c)_8 (x - d)_{10}|$

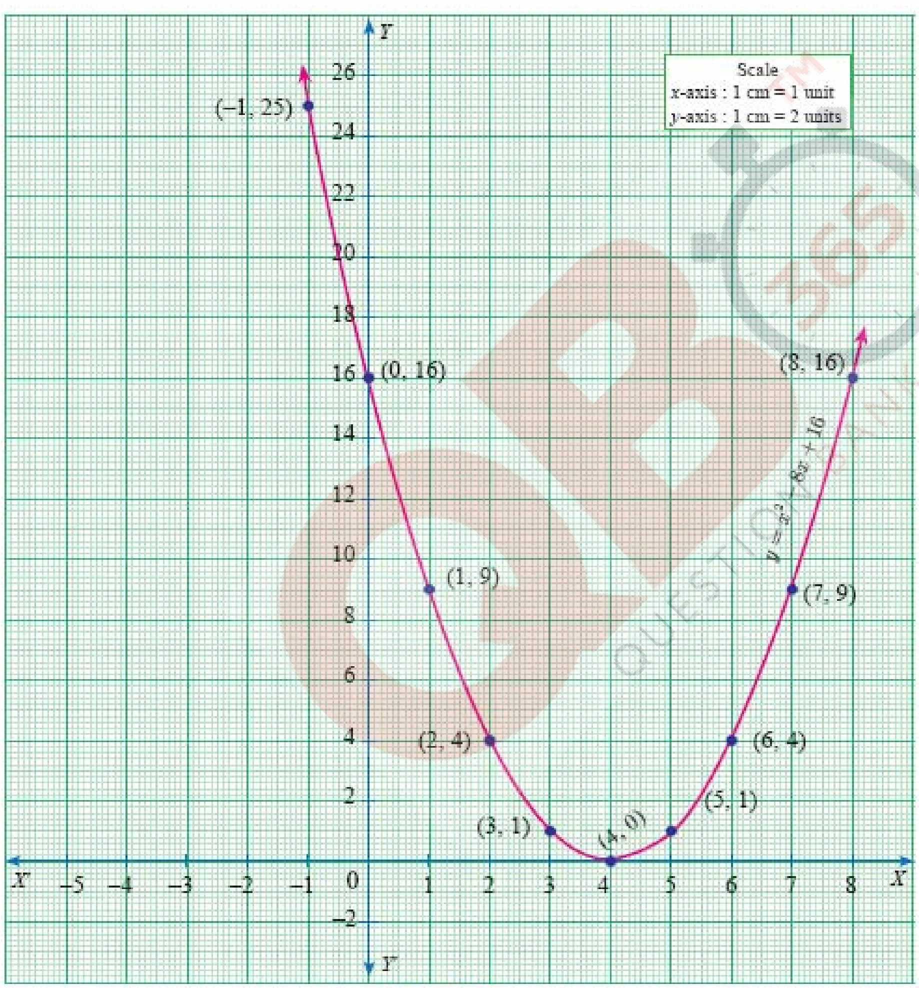
12) Find the zeroes of the quadratic expression $x^2 + 8x + 12$

Answer : Let p(x) = $x^2 + 18x + 12 = (x + 2)(x + 6)$
p(-2) = 4 - 16 + 20 = 0
p(-6) = 36 - 48 + 12 = 0
Therefore -2 and -6 are zero of p(x) = $x^2 + 8x + 12$

13) Discuss the nature of solutions of the following quadratic equations.
$x^2 - 8x + 16 = 0$
Answer : $x^2 - 8x + 16 = 0$
Step 1 Prepare the table of values for the equation $y = x^2 - 8x + 16$

x	-1	0	1	2	3	4	5	6	7	8
y	25	16	9	4	1	0	1	4	9	16

Step 2: Plot the points for the above ordered pairs (x, y) on the graph using suitable scale.



Step 3: Draw the parabola and mark the coordinates of the parabola which intersect with the X axis.
Step 4: The roots of the equation are the x coordinates of the intersecting points of the parabola with the X axis (4,0) which is 4.
Since there is only one point of intersection with X axis, the quadratic equation $x^2 - 8x + 16 = 0$ has real and equal roots.

14) A two digit number is such that the product of its digits is 12. When 36 is added to the number the digits interchange their places. Find the number.
Answer : Let the ten's digit ofthe number be x. It is given that the product of the digits

is 12.
Unit's digit $\frac{12}{x}$
Number =10x+$\frac{12}{x}$
It 36 is added to the number the digits interchange their places.
$\therefore 10x+\frac{12}{x}+36=10\times\frac{12}{x}+x$
$\Rightarrow 10x+\frac{12}{x}+36=\frac{120}{x}+x$
$\Rightarrow 9x-\frac{108}{x}+36=0$
$\Rightarrow 9x^2 - 108 + 36x = 0$
$\Rightarrow X^2 + 4x - 12 = 0$
$\Rightarrow (x + 6)(x - 2) = 0$ ($\because$ $(x + 6) \neq 0$ as $x > 0$)
x=-6,2
But a number can never be (-ve). So, x = 2. The
number is 10x2+$\frac{12}{2}$=26

15) A chess board contains 64 equal squares and the area of each square is 6.25 cm^2, A border round the board is 2 cm wide.

Answer : 

Let the length of the side of the chess board be x cm. Then
Area of 64 squares = $(x - 4)^2$
$(x - 4)^2 = 64 \times 6.25$
$\Rightarrow x^2-8x+16=400$
$\Rightarrow X^2 - 8x - 384 = 0$
$\Rightarrow X^2 - 24x + 16x - 384 = 0$
$\Rightarrow (x - 24)(x + 16) = 0$
$\Rightarrow$ x=24 cm.

8 Marks

16) Find the least common multiple of $xy(k^2 + 1) + k(x^2 + y^2)$ and $xy(k^2 - 1) + k(x^2 - y^2)$
Answer : $xy (k^2 + 1) + k (x^2 +y)$ ....(1)
$xy (k^2 - 1) + k (x2 - y)$ ....(2)
(1) $\Rightarrow xyk^2 + xy + kx^2 + ky^2$
(2) $\Rightarrow xyk^2 - xy + kx^2 - ky^2$
(1) $\Rightarrow yk (xk + y) + x (xk + y)$
$= (xk +y)(x +yk)$
(2) $\Rightarrow yk (xk- y) + x (xk- y)$
$= (x +yk) (xk- y)$
$\therefore$ L.C.M.: $(x +yk) (xk+ y) (xk-y)$
$= (x +yk) (x^2k^2 - y^2)$

17) Find the GCD of the following by division algorithm $2x^4 + 13x^3 + 27x + 7$, $x^3 + 3x^2 + 3x + 1$, $x^2 + 2x + 1$
Answer : $2x^4+13x^3+27x^2+23x+7$,
x^3+3x^2+3x+1, x^2+2x+1

By divistion algorithm, first divide

$$\begin{array}{r|l} & x+1 \\ \hline x^2+2x+1 & x^3+3x^2+3x+1 \\ & x^2+2x+x \\ & (-)\ (-)\ (-) \\ \hline & x^2+2x+1 \\ & (-)\ (-)\ (-) \\ & x^2+2x+1 \\ \hline & 0 \end{array}$$

∴ $(x+1)^2$ is G.C.D of x3+$3x^2$+3x+1 and x^2+2x+1.

Next let us divide

$2x^4+13x^3+27x^2+23x+7$ by x^2+2x+1

$$\begin{array}{r|l} & 2x+7 \\ \hline x^3+3x^2+3x+1 & 2x^4+13x^3+27x^2+23x+7 \\ & (-)\ (-)\ (-)\ (-) \\ & 2x^4+6x^3+6x^2+2x \\ \hline & 7x^3+21x^2+21x+7 \\ & (-)\ (-)\ (-)\ (-) \\ & 7x^3+21x^2+21x+7 \\ \hline & 0 \end{array}$$

∴ x^2+3x^2+3x+1 is the G.C.D of $2x^4+13x^3+27x^2+23x+7$ and x^3+3x+1

∴ G.C.D of $2x^4+13x^3+27x^2+23x+7$, x^3+3x^2+3x+1,x^2+2x+1 is $(x+1)^2$.

18) A = $\begin{pmatrix} 3 & 0 \\ 4 & 5 \end{pmatrix}$, B = $\begin{pmatrix} 6 & 3 \\ 8 & 5 \end{pmatrix}$, C = $\begin{pmatrix} 3 & 6 \\ 1 & 1 \end{pmatrix}$ find the matrix D, such that CD – AB = 0

Answer : $A=\begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}, B=\begin{bmatrix} 6 & 3 \\ 8 & 5 \end{bmatrix}, C=\begin{bmatrix} 3 & 6 \\ 1 & 1 \end{bmatrix}$

CD-AB=0⇒CD=AB

$$AB=\begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 6 & 3 \\ 8 & 5 \end{bmatrix}=\begin{bmatrix} (18+0) & (9+0) \\ (24+40) & (12+25) \end{bmatrix}$$

$$CD=\begin{bmatrix} 18 & 9 \\ 64 & 37 \end{bmatrix}$$

$$Let \quad D=\begin{bmatrix} x & y \\ z & w \end{bmatrix}$$

$$\begin{bmatrix} 3 & 6 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x & y \\ z & w \end{bmatrix}=\begin{bmatrix} 18 & 9 \\ 64 & 37 \end{bmatrix}$$

$$\begin{bmatrix} 3x+6z & 3y+6w \\ x+z & y+w \end{bmatrix}=\begin{bmatrix} 18 & 9 \\ 64 & 37 \end{bmatrix}$$

3x+6z=18 ...(1)

x+z=64 ...(2)

$$\begin{array}{rl} (1)-6(2) \Rightarrow & 3x+6z=18 \\ & 6x+6z=384 \\ \hline & -3x \quad =-366 \\ & x \quad =122 \end{array}$$

Sub.x=122 in(2)

122+z=64

z=64-122=-58

3y+6w=9 ....(3)

y+w=37 ....(4)

$$(3)-3(4) \Rightarrow \begin{array}{r} 3y+6w=9 \\ \underset{(-)}{3y}+\underset{(-)}{3w}=\underset{(-)}{111} \\ \hline 3w=-102 \\ w=-34 \end{array}$$

Sub.w=-34 in (4)

y-34=37

y=37+34=71

∴ Solutions: x=122

y=71

z=-58

w=-34

$$\therefore D=\begin{bmatrix} 122 & 71 \\ -58 & -34 \end{bmatrix}$$

19) Find two consecutive natural numbers whose product is 20.

Answer : Let a natural number be x.

The next number = x + 1

x (x + 1) = 20

X^2+x-20 = 0

(x + 5)(x - 4) = 0

x=-5,4

∴ x=4

(∵ x≠-5, x is a natural number

The next number = 4 + 1= 5

Two consecutive numbers are 4,5.

20) A two digit number is such that the product of its digits is 18, when 63 is subtracted from the number, the digits interchange their places. Find the number.

Answer : Let the tens digits be x. Then the um.ts digits=$\frac{18}{x}$

∴ Number =10x+$\frac{18}{x}$

and number obtained by interchanging the digits =10x+$\frac{18}{x}$

$\therefore\left(10x+\frac{18}{x}\right)-\left(10\times\frac{18}{x}+x\right)=63$

$\Rightarrow 10x+\frac{18}{x}-\frac{180}{x}-x=0$

$\Rightarrow 9x-\frac{162}{x}-63=0$

⇒ $9x^2$-63x-162=0

⇒ x^2-7x-18=0

⇒ (x-9)(x+2)=0⇒ x=9,-2

But a digit can never be (-ve), so x = 9.

So, the required number $=10\times 9+\frac{18}{9}=92$