

+2 PHYSICS**STUDY MATERIAL****Unit – 1 Electrostatics 3m & 5m important****1. Derive an expression for Torque experienced by a dipole due to a uniform electric field**

- ❖ Consider an electric dipole of dipole moment \vec{p} placed in a uniform electric field.
- ❖ The charge $+q$ will experience a force $q\vec{E}$ in the direction of the field & charge $-q$ will experience a force $-q\vec{E}$ in a direction opposite to the field.
- ❖ Since the external field is uniform, the total force acting on the dipole is zero. These two forces acting at different points will constitute a couple and the dipole experience a torque.
- ❖ This torque tends to rotate the dipole

The magnitude of torque is

$\tau = \text{one of the forces} \times \text{perpendicular distances between the forces}$

$$= F \times 2d \sin \theta$$

$$= qE \times 2d \sin \theta$$

$$= pE \sin \theta \quad (p = 2qd)$$

In vector notation $\vec{\tau} = \vec{p} \times \vec{E}$

a) When $\theta = 0$ $\tau = 0$

The dipole moment of dipole parallel to electric field – No torque

b) When $\theta = 90$ $\tau = pE$

Dipole moment perpendicular to electric field, torque is maximum

c) $\theta = 180^\circ$ $\tau = 0$

Dipole moment anti parallel to electric field, torque is zero

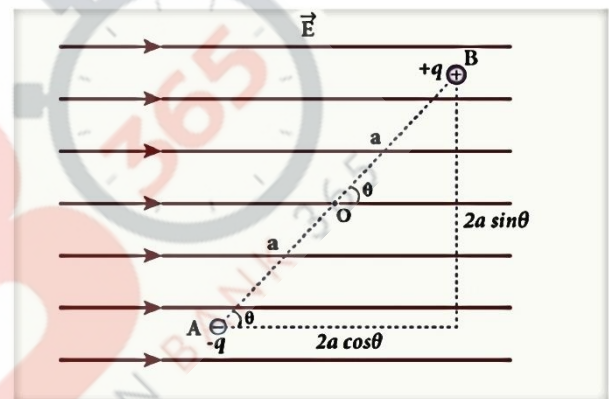


Figure 1.31 The dipole in a uniform electric field

2. Derive an expression for electrostatic potential energy of the dipole in a uniform electric field

- ❖ Consider a dipole placed in the uniform electric field \vec{E} . A dipole experiences a torque when kept in a uniform electric field \vec{E} .
- ❖ This torque rotates the dipole to align it with the direction of the electric field.
- ❖ To rotate the dipole (at constant angular velocity) from its initial angle θ' to another angle θ against the torque exerted by the electric field, an equal and opposite external torque must be applied on the dipole.

The work done by the external torque to rotate the dipole from angle θ' to θ at constant angular velocity

$$W = \int_{\theta'}^{\theta} \tau_{ext} d\theta$$

$$\tau = pE \sin \theta$$

substituting τ in above equation

+2 PHYSICS**STUDY MATERIAL**

$$W = \int_{\theta}^{\theta'} pE \sin \theta \, d\theta$$

$$W = pE (\cos \theta' - \cos \theta)$$

If $\theta' = 90^\circ$

The potential energy stored in the system of dipole kept in the uniform electric field is given by

$$U = -pE \cos \theta = -\vec{p} \cdot \vec{E}$$

$\theta = 180^\circ$ dipole aligned antiparallel to field U is maximum

$\theta = 0^\circ$ dipole aligned parallel to field U is minimum

3. Derive an expression for electrostatic potential due to point charge

Consider a positive charge q kept fixed at the origin. Let P be a point at distance r from the charge q .

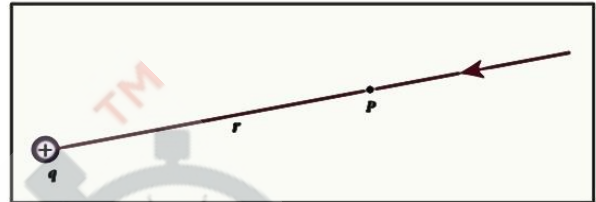


Figure 1.23 Electrostatic potential at a point P

The electric potential at the point P is

$$V = \int_{\infty}^r (-\vec{E}) \cdot d\vec{r} = -\int_{\infty}^r \vec{E} \cdot d\vec{r} \quad (1.32)$$

Electric field due to positive point charge q is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$V = \frac{-1}{4\pi\epsilon_0} \int_{\infty}^r \frac{q}{r^2} \hat{r} \cdot d\vec{r}$$

The infinitesimal displacement vector, $d\vec{r} = dr \hat{r}$ and using $\hat{r} \cdot \hat{r} = 1$, we have

$$V = -\frac{1}{4\pi\epsilon_0} \int_{\infty}^r \frac{q}{r^2} \hat{r} \cdot dr \hat{r} = -\frac{1}{4\pi\epsilon_0} \int_{\infty}^r \frac{q}{r^2} dr$$

After the integration,

$$V = -\frac{1}{4\pi\epsilon_0} q \left[-\frac{1}{r} \right]_{\infty}^r = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

Hence the electric potential due to a point charge q at a distance r is

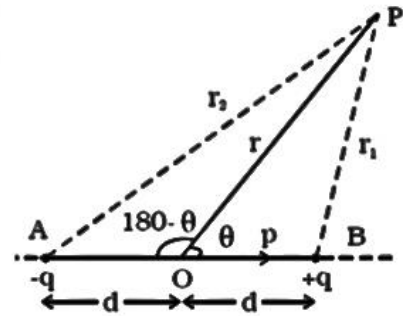
$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad (1.33)$$

+2 PHYSICS**STUDY MATERIAL****4. Derive an expression for electrostatic potential due to an electric dipole**

1. Consider an electric dipole AB. Let P be the point at a distance r from the midpoint of the dipole and θ be the angle between PO and the axis of the dipole OB.

2. Potential at P due to charge (+q) = $\frac{1}{4\pi\epsilon_0} \frac{q}{r_1}$

Potential at P due to charge (-q) = $\frac{1}{4\pi\epsilon_0} \left(-\frac{q}{r_2} \right)$



Total potential at P due to dipole is, $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r_1} - \frac{1}{4\pi\epsilon_0} \frac{q}{r_2}$

$$V = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad \dots(1)$$

3. Applying cosine law, $r_1^2 = r^2 + d^2 - 2rd \cos \theta$

Using the Binomial theorem and neglecting higher powers,

$$\frac{1}{r_1} = \frac{1}{r} \left(1 + \frac{d}{r} \cos \theta \right) \quad \dots(2)$$

4. Similarly, $r_2^2 = r^2 + d^2 - 2rd \cos (180 - \theta) = r^2 + d^2 + 2rd \cos \theta$.

5. $\frac{1}{r_2} = \frac{1}{r} \left(1 - \frac{d}{r} \cos \theta \right) \quad \dots(3)$

6. Substituting equation (2) and (3) in equation (1) and simplifying

$$V = \frac{q}{4\pi\epsilon_0} \frac{1}{r} \left(1 + \frac{d}{r} \cos \theta - 1 + \frac{d}{r} \cos \theta \right)$$

$$\therefore V = \frac{q \cdot 2d \cos \theta}{4\pi\epsilon_0 \cdot r^2} = \frac{1}{4\pi\epsilon_0} \frac{p \cdot \cos \theta}{r^2} \quad \dots(4)$$

7. Special cases:

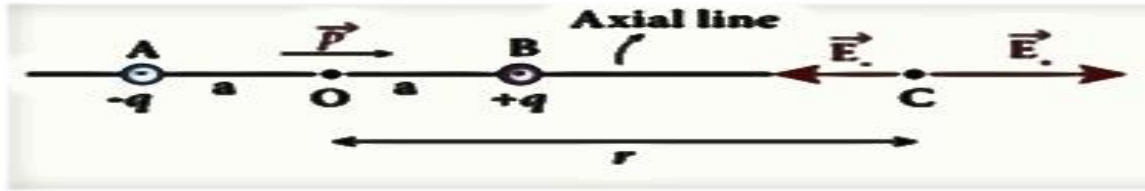
(i) If $\theta = 0^\circ$; $V = \frac{p}{4\pi\epsilon_0 r^2}$

(ii) If $\theta = 180^\circ$; $V = -\frac{p}{4\pi\epsilon_0 r^2}$

(iii) If $\theta = 90^\circ$; $V = 0$

+2 PHYSICS**STUDY MATERIAL****5. Calculate the electric field due to a dipole on axial and equatorial plane****Electric field due to an electric dipole on the axial line**

- ❖ Consider an electric dipole placed on x axis.
- ❖ A point C is located at a distance of r from the midpoint O of the dipole along the axial line.



The electric field at a point C due to +q

$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-a)^2} \text{ along } BC$$

\hat{p} – Direction is -q to +q and along BC

$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0} \frac{q}{(r-a)^2} \hat{p} \dots\dots\dots(1)$$

The electric field at a point C due to -q

$$\vec{E}_- = -\frac{1}{4\pi\epsilon_0} \frac{q}{(r+a)^2} \hat{p} \dots\dots\dots(2)$$

Total electric field at C calculated using super position principle

$$\begin{aligned} \vec{E}_{\text{tot}} &= \vec{E}_+ + \vec{E}_- \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{(r-a)^2} \hat{p} - \frac{1}{4\pi\epsilon_0} \frac{q}{(r+a)^2} \hat{p} \\ &= \frac{q}{4\pi\epsilon_0} \left(\frac{4ra}{(r^2-a^2)^2} \right) \hat{p} \end{aligned}$$

$r \gg a$

$$(r^2 - a^2)^2 = r^4$$

$$\begin{aligned} \vec{E}_{\text{tot}} &= \frac{1}{4\pi\epsilon_0} \left(\frac{4aq}{r^3} \right) \hat{p} \quad \text{since } 2aq\hat{p} = \vec{p} \\ &= \frac{1}{4\pi\epsilon_0} \left(\frac{2\vec{p}}{r^3} \right) \end{aligned}$$

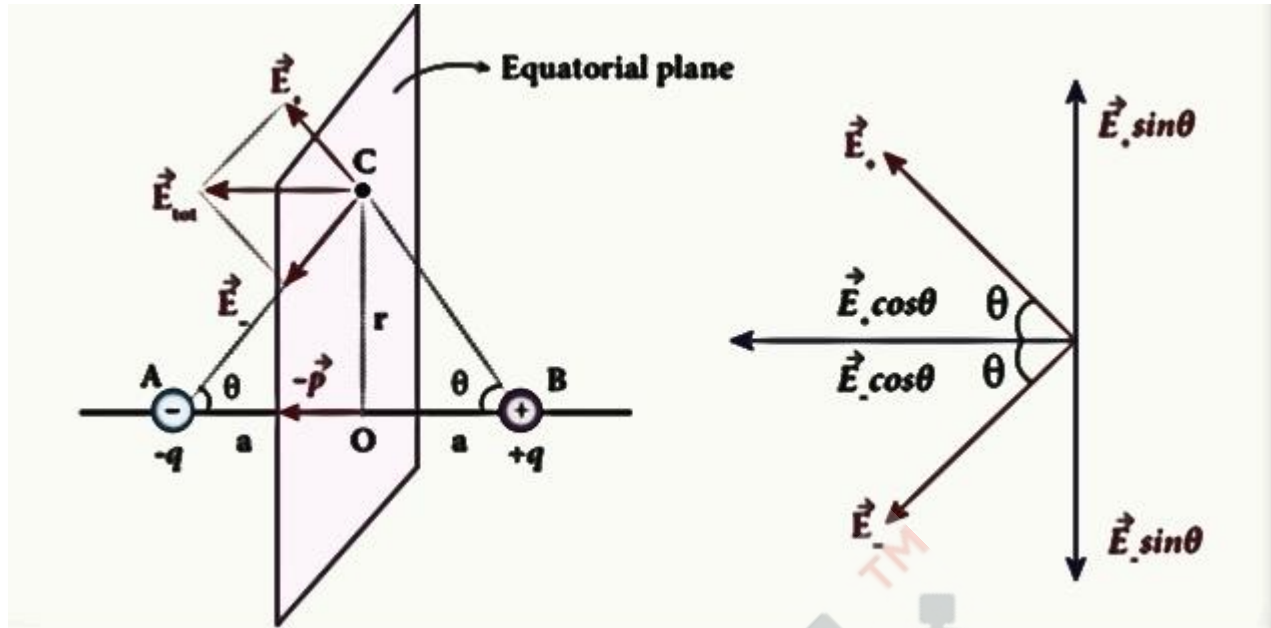
Electric field due to an electric dipole on the equatorial plane

- ❖ Consider point C is located at a distance of r from the midpoint O of the dipole on the equatorial plane.
- ❖ C is equidistant from +q & -q, the magnitude of electric field of +q & -q are the same
- ❖ Direction of \vec{E}_+ along BC
- ❖ Direction of \vec{E}_- along CA
- ❖ \vec{E}_+ & \vec{E}_- resolved into two components : One component parallel to dipole and perpendicular to it
- ❖ Perpendicular components $E_+ \sin\theta$ & $E_- \sin\theta$ are oppositely directed so cancel each other.

$$\vec{E}_{\text{tot}} = -|\vec{E}_+| \cos\theta \hat{p} - |\vec{E}_-| \cos\theta \hat{p}$$

+2 PHYSICS

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$$|\vec{E}_+| = |\vec{E}_-| = \frac{1}{4\pi\epsilon_0} \frac{q}{(r+a)^2}$$

$$\vec{E}_{\text{tot}} = -\frac{1}{4\pi\epsilon_0} \frac{2q \cos \theta \hat{p}}{(r+a)^2}$$

$$\vec{E}_{\text{tot}} = -\frac{1}{4\pi\epsilon_0} \frac{2q \cos \theta \hat{p}}{(r+a)^{\frac{3}{2}}}$$

$$\vec{E}_{\text{tot}} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{(r+a)^{\frac{3}{2}}}$$

$$r \gg a \quad (r^2 + a^2)^{3/2} = r^3$$

$$\vec{E}_{\text{tot}} = \frac{1}{4\pi\epsilon_0} \left(\frac{\vec{p}}{r^3} \right)$$

$$\text{since } \cos \theta = \frac{a}{\sqrt{r^2 + a^2}}$$

$$\text{since } 2aq\hat{p} = \vec{p}$$

6. Discuss the various properties of conductors in electrostatic equilibrium

- ❖ The electric field is zero everywhere **inside the conductor**. This is true regardless of **whether the conductor is solid or hollow**.
- ❖ There is no net charge inside the **conductors**. The charges must reside only on the **surface of the conductors**.
- ❖ The electric field outside the conductor **is perpendicular to the surface of the conductor and has a magnitude of $\frac{\sigma}{\epsilon_0}$** where σ is the surface charge density at that point.
- ❖ The electrostatic potential has the **same value on the surface and inside of the conductor**.
- ❖ Since the electric field is zero inside the conductor, the potential is the same as the surface of the conductor. Thus at electrostatic equilibrium, the conductor is always at equipotential

+2 PHYSICS**STUDY MATERIAL****7. Obtain the expression for capacitance for a parallel plate capacitor**

Consider a capacitor with two parallel plates each of cross-sectional area A and separated by a distance d

The electric field between two infinite parallel plates is uniform and is given by

$$\mathbf{E} = \frac{\sigma}{\epsilon_0} \quad \sigma - \text{Surface charge density on the plates}$$

$$(\sigma = Q/A)$$

The electric field between the plates is

$$\mathbf{E} = \frac{Q}{A\epsilon_0}$$

Since the electric field is uniform, the electric potential between the plates having separation d is

$$\mathbf{V} = \mathbf{E}d = \frac{Qd}{A\epsilon_0}$$

capacitance of the capacitor is given by

$$C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{A\epsilon_0}} = \frac{\epsilon_0 A}{d}$$

$$C \propto A \quad C \propto \frac{1}{d}$$

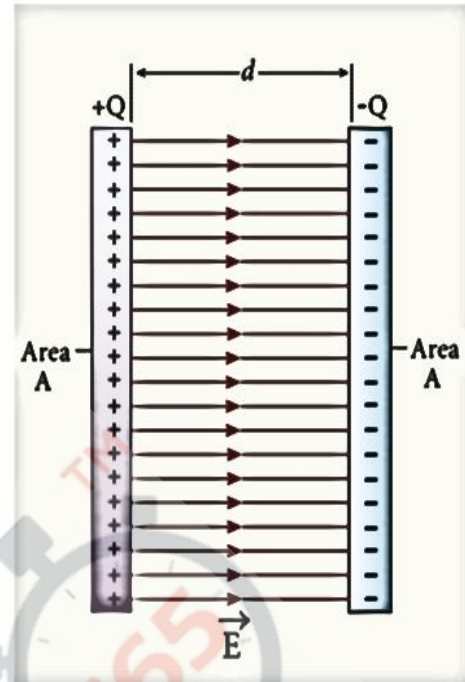


Figure 1.56 Capacitance of a parallel plate capacitor

8. Obtain the expression for energy stored in parallel plate capacitor

- ❖ Capacitor not only stores the charge but also it stores energy.
- ❖ When a battery is connected to the capacitor, electrons of total charge $-Q$ are transferred from one plate to the other plate.
- ❖ To transfer the charge, work is done by the battery. This work done is stored as electrostatic potential energy in the capacitor.

To transfer an infinitesimal charge dQ for a potential difference V , the work done is given by

$$dW = V dQ$$

$$V = \frac{Q}{C}$$

The total work done to charge a capacitor is

$$W = \int_0^Q \frac{Q}{C} dQ = \frac{Q^2}{2C}$$

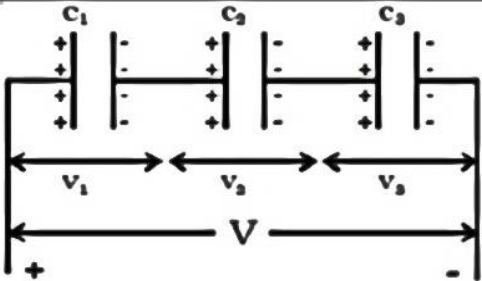
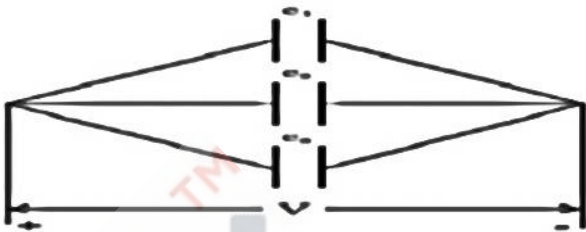
This work done is stored as electrostatic potential energy (U_E) in the capacitor

$$U_E = \frac{Q^2}{2C} = CV^2$$

$$U_E \propto C \quad U_E \propto V^2$$

+2 PHYSICS**STUDY MATERIAL**

9. Derive the expression for resultant capacitance when capacitors are connected in series and in parallel

Capacitors in series	Capacitors in parallel
1. C_1, C_2, C_3 , capacitors are connected in series. C_s is the effective capacitance.	1. C_1, C_2, C_3 , capacitors are connected in parallel. C_p is the effective capacitance.
	
3. Charge in each capacitor is same.	3. Potential in each capacitor is same.
4. $V = V_1 + V_2 + V_3$	4. $q = q_1 + q_2 + q_3$
5. $V_1 = \frac{q}{C_1}; V_2 = \frac{q}{C_2}; V_3 = \frac{q}{C_3}$ $V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} = q \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right]$	5. $q_1 = C_1 V, q_2 = C_2 V, q_3 = C_3 V$ $q = C_1 V + C_2 V + C_3 V$
6. $V = \frac{q}{C_s}$ $\frac{q}{C_s} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$	6. $q = C_p V$ $C_p V = V (C_1 + C_2 + C_3)$
7. $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$	7. $C_p = C_1 + C_2 + C_3$
8. The reciprocal of the effective capacitance is equal to the sum of reciprocal of the capacitance of the individual capacitors.	8. The effective capacitance of the capacitors connected in parallel is the sum of the capacitances of the individual capacitors.

+2 PHYSICS**STUDY MATERIAL****10. Explain in detail the construction and working of a van de Graff generator**

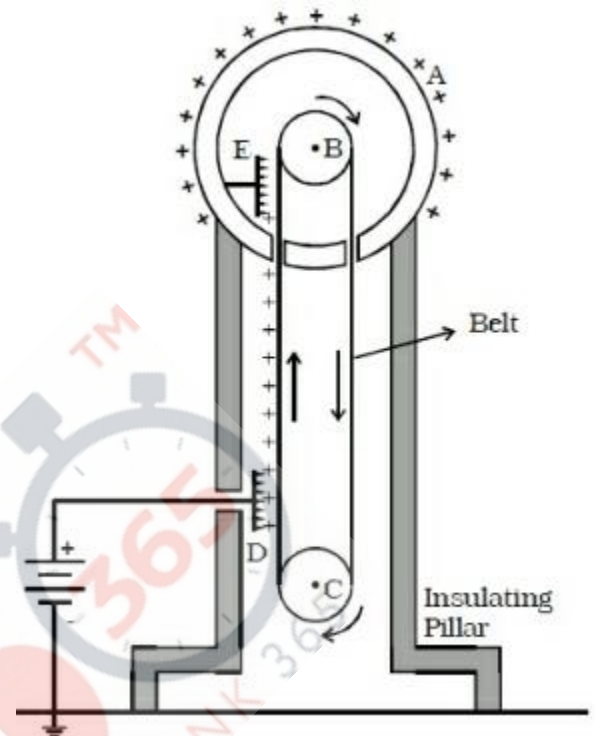
It is a machine which produces a large amount of electrostatic potential difference, up to several million volts (10^7 V).

Principle

Electrostatic induction and action at points

Construction

- ❖ A large hollow spherical conductor is fixed on the insulating stand .
- ❖ A pulley B is mounted at the center of the hollow sphere and another pulley C is fixed at the bottom.
- ❖ A belt made up of insulating materials like silk or rubber runs over both pulleys.
- ❖ The pulley C is driven continuously by the electric motor.
- ❖ Two comb shaped metallic conductors E and D are fixed near the pulleys.
- ❖ The comb D is maintained at a positive potential of 10^4 V by a power supply.
- ❖ The upper comb E is connected to the inner side of the hollow metal sphere.

**Working**

- ❖ Due to the high electric field near comb D, air between the belt and comb D gets ionized. The positive charges are pushed towards the belt and negative charges are attracted towards the comb D.
- ❖ The positive charges stick to the belt and move up.
- ❖ When the positive charges reach the comb E, a large amount of negative and positive charges are induced on either side of comb E due to electrostatic induction.
- ❖ As a result, the positive charges are pushed away from the comb E and they reach the outer surface of the sphere.
- ❖ Since the sphere is a conductor, the positive charges are distributed uniformly on the outer surface of the hollow sphere.
- ❖ At the same time, the negative charges nullify the positive charges in the belt due to corona discharge before it passes over the pulley.
- ❖ When the belt descends, it has almost no net charge.
- ❖ At the bottom, it again gains a large positive charge. The belt goes up and delivers the positive charges to the outer surface of the sphere.
- ❖ This process continues until the outer surface produces the potential difference of the order of 10^7 which is the limiting value.
- ❖ The leakage of charges can be reduced by enclosing the machine in a gas filled steel chamber at very high pressure.

+2 PHYSICS**STUDY MATERIAL****Unit – 2 Current Electricity Important 3m & 5m****1. State and explain Kirchoff's rules****Kirchoff's first rule (Current rule or Junction rule)**

- ❖ It states that the algebraic sum of the currents at any junction of a circuit is zero. It is a statement of conservation of electric charge.
- ❖ Current entering the junction is taken as positive and current leaving the junction is taken as negative.

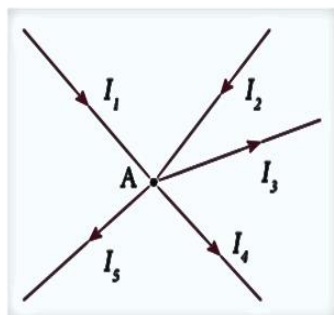


Figure 2.23 Kirchoff's current rule

Applying this law to the junction A

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

Kirchoff's Second rule (Voltage rule or Loop rule)

- ❖ It states that in a closed circuit the algebraic sum of the products of the current and resistance of each part of the circuit is equal to the total emf included in the circuit.
- ❖ This rule follows from the law of conservation of energy for an isolated system.

2. Explain the determination of the internal resistance of a cell using voltmeter

- ❖ The emf of cell ξ is measured by connecting a high resistance voltmeter across it without connecting the external resistance R .
- ❖ Since the voltmeter draws very little current for deflection, the circuit may be considered as open. Hence the voltmeter reading gives the emf of the cell.
- ❖ Then, external resistance R is included in the circuit and current I is established in the circuit..

The potential drop across the resistor R is

$$V = IR \dots\dots\dots(1)$$

Due to internal resistance r of the cell, the voltmeter reads a value V , which is less than the emf of cell ξ .

$$V = \xi - Ir$$

$$Ir = \xi - V \dots\dots\dots(2)$$

Dividing (2) by (1) we get

$$r = \left(\frac{\xi - V}{V}\right) R \quad \text{Since } \xi, V \text{ and } R \text{ are known, internal resistance } r \text{ can be determined.}$$

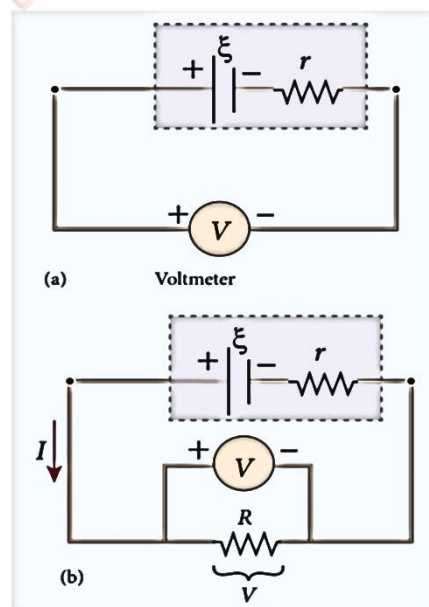
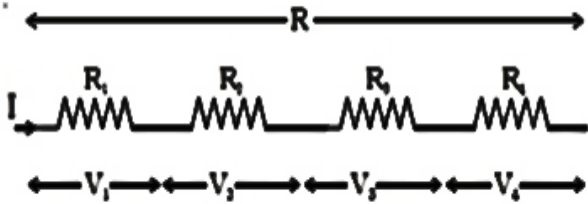
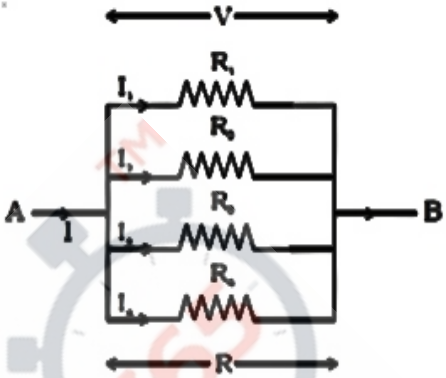


Figure 2.20 Internal resistance of the cell

+2 PHYSICS**STUDY MATERIAL****3. Explain the equivalent resistance of a series and parallel resistor network**

Resistors in series	Resistors in parallel
1. R_1, R_2, R_3, R_4 Resistors are connected in series. R_s is the effective resistance.	1. R_1, R_2, R_3, R_4 are Resistors connected in parallel. R_p is the effective resistance.
2. 	2. 
3. Current flowing through each resistor is the same.	3. Potential difference (V) across each resistor is same.
4. $V = V_1 + V_2 + V_3 + V_4$	4. $I = I_1 + I_2 + I_3 + I_4.$
5. $V_1 = IR_1, V_2 = IR_2, V_3 = IR_3, V_4 = IR_4$ and $V = IR_s$ $IR_s = IR_1 + IR_2 + IR_3 + IR_4$ (Or) $R_s = R_1 + R_2 + R_3 + R_4$	5. $I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}, I_4 = \frac{V}{R_4}$ $I = \frac{V}{R_p} \quad \frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \frac{V}{R_4}$ $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$
6. The equivalent resistance of a number of resistors in series connection is equal to the sum of the resistance of individual resistors.	6. The sum of the reciprocal of the resistance of the individual resistors is equal to the reciprocal of the effective resistance of the combination.

4. Obtain the condition for bridge balance in Wheatstone's bridge.

+2 PHYSICS**STUDY MATERIAL**

- ❖ An important application of Kirchhoff's rules is the Wheatstone's bridge. It is used to compare resistances and also helps in determining the unknown resistance in electrical network.
- ❖ The bridge consists of four resistances P, Q, R and S connected .
- ❖ A galvanometer G is connected between the points B and D.
- ❖ The battery is connected between the points A and C. The current through the galvanometer is I_G and its resistance is G.

Applying Kirchhoff's current rule to junction B

$$I_1 - I_G - I_3 = 0 \dots\dots\dots(1)$$

Applying Kirchhoff's current rule to junction D,

$$I_2 - I_G - I_4 = 0 \dots\dots\dots(2)$$

Applying Kirchhoff's voltage rule to loop ABDA,

$$I_1P + I_GG - I_2R = 0 \dots\dots\dots(3)$$

Applying Kirchhoff's voltage rule to loop ABCDA,

$$I_1P + I_3Q - I_2R - I_4S = 0 \dots\dots\dots(4)$$

When the points B and D are at the same potential, the bridge is said to be balanced , no current flows through galvanometer ($I_G = 0$).

Substituting $I_G = 0$ in (1) , (2) , (3) , (4)

$$I_1 = I_3 \dots\dots\dots(5) \quad I_2 = I_4$$

.....(6)

$$I_1P = I_2R \dots\dots\dots(7)$$

Substituting (5) & (6) in (4)

$$I_1P + I_1Q - I_2R - I_2S = 0$$

$$I_1(P + Q) = I_2(R + S) \dots\dots\dots(8)$$

Dividing (8) by (7)

$$\frac{P+Q}{P} = \frac{R+S}{R}$$

$$1 + \frac{Q}{P} = 1 + \frac{S}{R}$$

$$\frac{P}{Q} = \frac{R}{S}$$

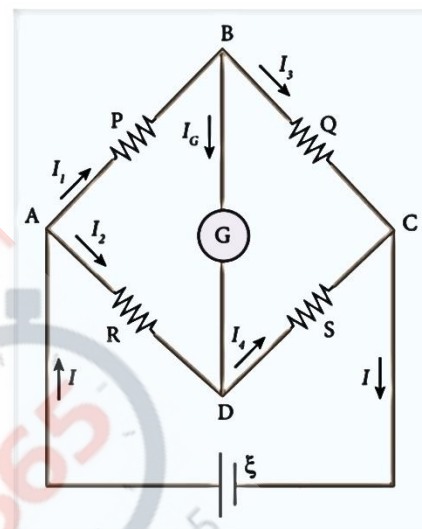


Figure 2.25 Wheatstone's bridge

5. Explain determination of internal resistance of a cell by potentiometer

- ❖ To measure the internal resistance of a cell, the circuit connections are made
- ❖ The end C of the potentiometer wire is connected to the positive terminal of the battery Bt and the negative terminal of the battery is connected to the end D through a key K_1 . This forms the primary circuit.
- ❖ The positive terminal of the cell ξ whose internal resistance is to be determined is also connected to the end C of the wire.
- ❖ The negative terminal of the cell ξ is connected to a jockey through a galvanometer and a high resistance.

+2 PHYSICS

STUDY MATERIAL

- ❖ A resistance box R and key K₂ are connected across the cell ξ. With K₂ open, the balancing point J is obtained and the balancing length CJ = l₂ is measured. Since the cell is in open circuit, its emf is

$$\xi \propto l_1$$

- ❖ A suitable resistance (say, 10 Ω) is included in the resistance box and key K₂ is closed. Let r be the internal resistance of the cell. The current passing through the cell and the resistance R is given by

$$I = \frac{\xi}{R + r} \dots\dots\dots(1)$$

- ❖ The potential difference across R is

$$V = \frac{\xi R}{R + r}$$

- ❖ When this potential difference is balanced on the potentiometer wire, let l₂ be the balancing length. $\frac{\xi R}{R + r} \propto l_2 \dots\dots\dots(2)$

❖ From (1) & (2)

$$\frac{R+r}{R} = \frac{l_1}{l_2}$$

$$\text{❖ } r = R \left(\frac{l_1 - l_2}{l_2} \right)$$

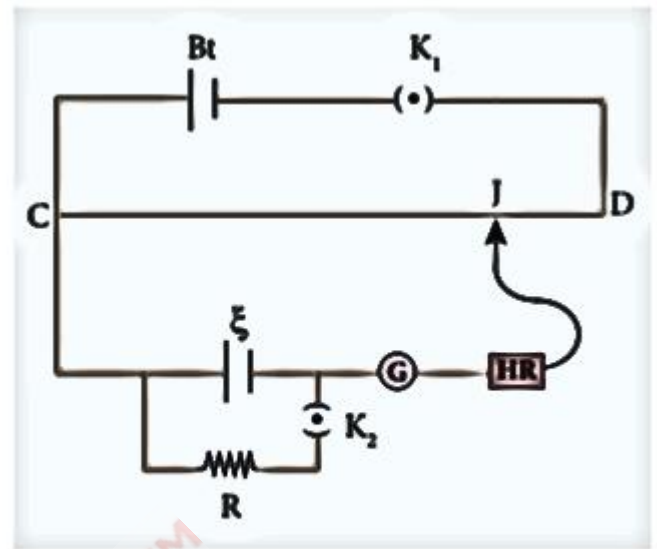
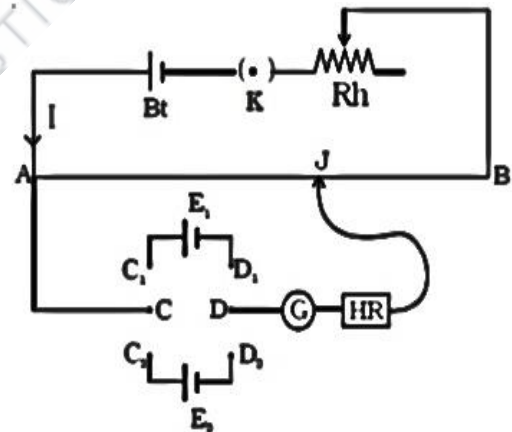


Figure 2.29 measurement of internal resistance

6. How the emf of two cells are compared using potentiometer?

- The end A of potentiometer is connected to the terminal C of a DPDT switch.
- Battery, key and rheostat are connected in series with B. terminal D is connected to the jockey (J) through a galvanometer and high resistance.
- Let I be the current flowing through the primary circuit and r be the resistance of the potentiometer wire per metre length.



- The jockey is moved on the wire and adjusted for zero deflection in galvanometer.

• E₁ = Irl₁ ----->(1)

• E₂ = Irl₂ ----->(2)

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$E_2 = E_1 \frac{l_2}{l_1}$$