

12th Standard-Maths

ONE MARKS

	Type A	Type B
1)	4	4
2)	1	1
3)	4	4
4)	1	3
5)	4	3
6)	1	2
7)	2	4
8)	2	4
9)	4	2
10)	2	1
11)	1	4
12)	3	1
13)	3	4
14)	4	2
15)	1	3
16)	4	2
17)	2	3
18)	4	4
19)	1	3
20)	3	4

TWO MARKS

21) $x+y+z=100$, $1+2y+5z=30$

22) $3\vec{i} + 2\vec{j} + 9\vec{k} = a(\vec{i} + m\vec{j} + 3\vec{k})$

Equating component vectors $a = 3$, $am = 2$ then $m = 2/3$

23) $(\frac{1+i}{1-i} \times \frac{1+i}{1+i}) = 2i/2 = i$ then $i^4 = 1$. ie $n=4$

24) only comet diagram

25) $dy/dx = \cos x$

$dy/dx = 0$ then $\cos x = 0 \rightarrow x = (2n+1)\pi/2$. $n \in \mathbb{Z}$.

26) Domain : $(-\infty, \infty)$

Extent : vertical : $(-\infty, \infty)$

Horizontal : $(-\infty, \infty)$

27) $\frac{\frac{\pi}{3}}{\frac{\pi}{6} + \sqrt{\tan x}} = \frac{\frac{\pi}{3}}{\frac{\pi}{6} + \sqrt{\tan(\frac{\pi}{6} + \frac{\pi}{3} - x)}} = \frac{\frac{\pi}{3}}{\frac{\pi}{6} + \sqrt{\tan(\frac{\pi}{2} - x)}} = \frac{\frac{\pi}{3}}{\frac{\pi}{6} + \sqrt{\cot x}}$

28) closer axiom :

Let $A = a/b \in \mathbb{Q} - 0$, $B = -a/b \in \mathbb{Q} - 0$, then $A+B = a/b - a/b = 0 \notin \mathbb{Q} - 0$

So its not closed.

29) Limits of integral is 0 to 3.

$F(3) = \int_0^3 3e^{-3x} dx = 3 \left[\frac{e^{-3x}}{-3} \right]_0^3 = -(e^{-9} - 1) = -e^{-9} + 1$

30) $f(x)$ is not differentiable for $x=2$ and $x=5$. so rolls theorem is not exists.

THREE MARKS

31) simply we take $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A+B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ SO } \rho(A) = 3, \rho(B) = 3, \rho(A+B) = 0$$

Hence $\rho(A) + \rho(B) \neq \rho(A+B)$

32) $\vec{a} \times \vec{b} = -\vec{i} + 2\vec{j} + 2\vec{k}$ and $|\vec{a} \times \vec{b}| = 3$, $\mu = 6$

Required vector = $\mu \left(\pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} \right) = \pm (-2\vec{i} + 4\vec{j} + 4\vec{k})$

33) let $z = \sin\theta + i \cos\theta$ and $\bar{z} = 1/z = \sin\theta - i \cos\theta$

$$\frac{1 + \sin\theta - i \cos\theta}{1 + \sin\theta + i \cos\theta} = \frac{1 + \bar{z}}{1 + z} = \frac{1 + \frac{1}{z}}{1 + z} = 1/z = \sin\theta - i \cos\theta$$

Hence $\frac{1 + \sin\theta - i \cos\theta}{1 + \sin\theta + i \cos\theta}^n = (\sin\theta - i \cos\theta)^n = \cos^n\left(\frac{\pi}{2} - \theta\right) - i \sin\left(\frac{\pi}{2} - \theta\right)$

34) Equation of tangent $x + yt^2 = 2ct$

A, B coordinates are $(2ct, 0)$ and $(2c/t, 0)$ mid point of AB = $\left(\frac{2ct+0}{2}, \frac{0+0}{2}\right) = (ct, 0)$

35) $f(x) = \tan^{-1}(\sin x + \cos x)$, $dy/dx = \frac{\cos x - \sin x}{2 + \sin 2x} > 0$ in $x \in (0, \pi/4)$

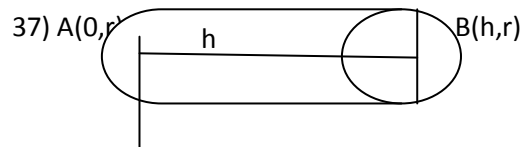
So $f(x)$ is st. increasing in $(0, \pi/4)$

36) $f(tx, ty) = \frac{1}{\sqrt{(tx)^2 + (ty)^2}} = \frac{1}{t\sqrt{x^2 + y^2}} = t^{-1} f(x, y)$

So f is Homogeneous fn. of deg -1

By eulers thm

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = -1$$



Equation of the line AB is $\frac{y-r}{r-r} = \frac{x-0}{h-0}$

$$\frac{y-r}{0} = \frac{x}{h} \rightarrow y-r=0 \text{ ie } y=r$$

Volume of the cylinder = $\pi \int_0^h y^2 dx$

$$= \pi \int_0^h r^2 dx$$

$$= \pi r^2 [x]_0^h = \pi r^2 h \text{ cu.unit}$$

38)

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

39) $n=120$

$$P=1/2, q= 2/3$$

$$\text{Mean} = np = 120 \times 1/3 = 40$$

$$\text{Variance} = npq = 40 \times 2/3 = 40/3$$

40) $yx^3 dx = -e^{-x} dy$

$$\frac{x^3 dx}{e^{-x}} = -\frac{dy}{y}$$

$\int e^x x^3 dx = -\int \frac{dy}{y}$ using bernoullies integral formula

$$e^x (x^3 - 3x^2 + 6x - 6) + \log y = c$$

FIVE MARKS

41) a) $[A,B] = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 3 & \mu & 0 \\ 2 & 1 & 2 & 0 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \mu - 12 & 0 \\ 0 & 0 & 8 - \mu & 0 \end{bmatrix}$$

Case (i) if $\mu \neq 8$

$$\rho(A) = \rho(A, B) = 3$$

So it has trivial solution, $(x, y, z) = (0, 0, 0)$

Case (ii) if $\mu = 8$

$$\rho(A) = \rho(A, B) = 2 < 3$$

So it has non trivial solution, $(x, y, z) = (c, -4c, c)$, where $c \neq 0$

b) diagram and correct proof

42) a) Cartesian equation

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l & m & n \end{vmatrix} = 0$$

ie $8x - 10y - 7z + 11 = 0$

$$\text{b) } x^6(x^5 - 1) + 1(x^5 - 1) = 0$$

$$(x^5 - 1)(x^6 + 1) = 0 \rightarrow x^6 + 1 = 0 \text{ and } x^5 - 1 = 0 \text{ ie } x^6 = -1 \text{ and } x^5 = 1$$

Case(i) $x^6 = -1$

$$x = (-1)^{\frac{1}{6}}$$

$$= (cis\pi)^{\frac{1}{6}}$$

$$= cis\pi\left(\frac{2k+1}{6}\right), k=0,1,2,3,4,5$$

Case(ii) $x^5 = 1$

$$x = 1^{\frac{1}{5}}$$

$$= (ciso)^{\frac{1}{5}}$$

$$= (cis2k\pi)^{\frac{1}{5}}$$

$$= cis\frac{2k\pi}{5}, k=0,1,2,3,4$$

43)a

w.k.t focal property of ellipse $F_1P + F_2P = 2a$

$$2a=9 \quad \text{and foci}(\pm ae, 0)=(\pm 3, 0) \quad \text{so } ae=3$$

$$a=9/2, b^2 = a^2 - (ae)^2, b^2 = 45/4$$

hence the required ellipse equation is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ ie, } \frac{x^2}{\frac{81}{4}} + \frac{y^2}{\frac{45}{4}} = 1$$

b) $x= 2r\cos\theta, y=2r \sin\theta$

area of tectangle = $2x.2y$

$$A(\theta) = 2r\cos\theta.2r \sin\theta$$

$$\frac{d}{d\theta} A(\theta) = 4r^2 \cos 2\theta$$

$$\frac{d}{d\theta} A(\theta) = 0 \text{ then } \theta = \pi/4$$

$$A''(\theta) = -8r^2 \sin 2\theta < 0 \text{ for } \theta = \pi/4$$

So A is maximum for $\theta = \pi/4$

$$x=y= \sqrt{2}r,$$

Required area= $x.y$

$$= \sqrt{2}r. \sqrt{2}r$$

$$= 2r^2$$

44) a

$$x= 100t - \frac{25}{2} t^2$$

(i) Initial velocity, so $t=0$

$$v= dx/dt=100 - (25/2) 2t$$

$$v= 100-0=100$$

ii)time taken. maximum height so $v=0$

$$\text{ie } 100-25t=0$$

$$t=4 \text{ sec}$$

iii) maximum height reached when $t=4 \text{ sec}$

$$x= 100(4)- (25/2)16$$

$$x = 200 \text{ mts}$$

iv) velocity

if $t = 4 + 4 = 8$ sec

$$v = 100 - 25(8) = -100 \text{ m/sec}$$

b) equation

$$\frac{(y+1)^2}{16} - \frac{(x-1)^2}{9} = 1, a^2=16, b^2 = 9$$

$$e=5/4$$

centre:(1,-1)

foci:(1,4), (1,-6)

vertices : (1,3), (1,-4)

45)a) $P(z_1 < z < z_2) = 70\%$

ie $P(z_1 < z < z_2) = .70$ ie $P(z_1 < z < 0) = .35$ and $P(0 < z < z_2) = .35$

z_1 and z_2 lies left and right side of the normal curve

So $z_1 = -1.04$ and $z_2 = 1.04$

$$Z = \frac{x-\mu}{\sigma} = \frac{x-34}{16} = -1.04$$

$$X = 17.36 \text{ and } z = \frac{x-34}{16} = 1.04 \rightarrow x = 50.64$$

70% student score b/w 17.36 to 50.64

b) $Y = \sin x, y = \cos x$

Then $\sin x = \cos x \rightarrow x = \frac{\pi}{4} \in (0, \pi)$

$$\text{Required area} = \int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi} (\sin x - \cos x) dx$$

$$= 2\sqrt{2} \text{ sq.unit}$$

46) a)

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt}$$

$$= -\sin t + 2\cos t + 2t \dots \dots \dots (1)$$

$$w = \cos t + 2\sin t + t^2, \frac{dw}{dt} = -\sin t + 2\cos t + 2t \dots \dots \dots (2)$$

hence proved

b) $dT/dt = k(T-s)$, $s=15$

$$T = s + ce^{kt} \dots\dots\dots(1)$$

$t=0$ then $T=100$

(1) $\Rightarrow c = 85$

$t = 5$, $T=60$ then $e^{5k} = \frac{45}{85}$

$t=10$, then $T = ?$

$$T = 15 + 85 e^{10k} = 38.82$$

47) a

- (i) Identity element of group is unique
- (ii) Inverse of each element of group is unique
- (iii) Reversal law
- (iv) Cancellation law
- (v) $a^{-1}a = 1 = a^{-1}a$

b) C.F = $Ae^{2x} + Be^{-\frac{2}{5}x}$

P.I1 = $(-5/12)xe^{-\frac{2}{5}x}$

P.I2 = $(-2/7)e^x$

P.I3 = $-3/4$

G.S $y = C.F + P.I1 + P.I2 + P.I3$

$$= Ae^{2x} + Be^{-\frac{2}{5}x} + (-5/12)xe^{-\frac{2}{5}x} + (-2/7)e^x - 3/4$$

