QB365 Question Bank School

HALF YEARLY EXAMINATION

10th Standard

Maths

Total Marks : 100

SECTION A 14X1=14

CHOOSE THE BEST ANSWER

1) If there are 1024 relations from a set A = {1, 2, 3, 4, 5} to a set B, then the number of elements in B is

(a) 3 **(b) 2** (c) 4 (d) 8

2) If f(x) = $2x^2$ and g(x) = $\frac{1}{3x}$, then f o g is

(a) $\frac{3}{2x^2}$ (b) $\frac{2}{3x^2}$ **(c)** $\frac{2}{9x^2}$ (d) $\frac{1}{6x^2}$

3) $7^{4k} \equiv$ ________ (mod 100)

(a) 1 (b) 2 (c) 3 (d) 4

4) If the sequence t_1, t_2, t_3... are in A.P. then the sequence t_6, t_{12}, t_{18},.... is

(a) a Geometric Progression **(b) an Arithmetic Progression**

(c) neither an Arithmetic Progression nor a Geometric Progression (d) a constant sequence

5) The square root of $\frac{256x^8y^4z^{10}}{25x^6y^6z^6}$ is equal to

(a) $\frac{16}{5}\left|\frac{x^2z^4}{y^2}\right|$ (b) $16\left|\frac{y^2}{x^2z^4}\right|$ (c) $\frac{16}{5}\left|\frac{y}{xz^2}\right|$ **(d)** $\frac{16}{5}\left|\frac{xz^2}{y}\right|$

6) Graph of a linear equation is a ____________

(a) straight line (b) circle (c) parabola (d) hyperbola

7) If number of columns and rows are not equal in a matrix then it is said to be a

(a) diagonal matrix **(b) rectangular matrix** (c) square matrix (d) identity matrix

8) If in $\triangle$ABC, DE || BC, AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm then the length of AE is

(a) 1.4 cm (b) 1.8 cm (c) 1.2 cm (d) 1.05 cm

9) The slope of the line joining (12, 3), (4, a) is $\frac{1}{8}$. The value of 'a' is

(a) 1 (b) 4 (c) -5 **(d) 2**

10) The angle of elevation of a cloud from a point h metres above a lake is β. The angle of depression of its reflection in the lake is 45°. The height of location of the cloud from the lake is

(a) $\frac{h(1+tan\beta)}{1-tan\beta}$ (b) $\frac{h(1-tan\beta)}{1+tan\beta}$ (c) h tan(45°-β) (d) none of these

11) The total surface area of a cylinder whose radius is $\frac{1}{3}$ of its height is

(a) $\frac{9\pi h^2}{8}$ sq.units (b) $24\pi h^2$ sq.units **(c)** $\frac{8\pi h^2}{9}$ **sq.units** (d) $\frac{56\pi h^2}{9}$ sq.units

12) The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is

(a) $\frac{4}{3}\pi$ (b) $\frac{10}{3}\pi$ (c) 5π (d) $\frac{20}{3}\pi$

13) Which of the following is incorrect?

(a) P(A) > 1 (b) $0 \leq P(A) \leq 1$ (c) P(ϕ) = 0 (d) $P(A) + P(\bar{A}) = 1$

14) The equation of a line passing through the origin and perpendicular to the line 7x - 3y + 4 = 0 is

(a) 7x - 3y + 4 = 0 (b) 3x - 7y + 4 = 0 **(c) 3x + 7y = 0** (d) 7x - 3y = 0

SECTION B 10X2=20

ANSWER ANY 10 QUESTIONS
Q.NO. 28 IS COMPULSORY

15) If X = {–5, 1, 3, 4} and Y = {a, b, c}, then which of the following relations are functions from X to Y ?
R_1= {(–5, a), (1, a), (3, b)}

Answer : R_1 = {(–5, a), (1, a), (3, b)}
We may represent the relation R_1 in an arrow diagram
R_1 is not a function as 4 ∈ X does not have an image in y.

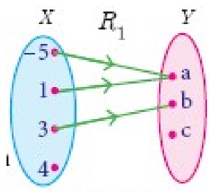


Fig. 1.15(a)

16) Find k if f o f(k) = 5 where f(k) = 2k - 1.

Answer : f o f(k) = f(f(k))
= 2(2k - 1) -1 = 4k - 3
Thus, f o f(k) = 4k - 3
But, it is given that f o f(k) = 5
Therefore 4k - 3 = 5 ⇒ k = 2

17) A positive integer when divided by 88 gives the remainder 61. What will be the remainder when the same number is divided by 11?

Answer : Let the positive integer be 'n'
So n = 88 (p) + 61, where p be an integer
n = 88 (p) + (5 x 11 + 6)
n = 8 x 11 x p + 5 x 11 + 6
n = 11 (8p + 5) + 6
Dividing both the sides by 11, we get
$\frac{n}{11} = (8p + 5) + \frac{6}{11}$
When the same number n is divided by 11 the remainder will be 6.

18) Find the 8^{th} term of the G.P 9,3,1,....

Answer : The find the 8th term we have use the n^{th} term formula $tn = ar^{n-1}$
First term a = 9, common radio r = $\frac{t_2}{t_1} = \frac{3}{9} = \frac{1}{3}$
$t_8 = 9 \times \left(\frac{1}{3}\right)^{8-1} = 9 \times \left(\frac{1}{3}\right)^{7} = \frac{1}{243}$
Therefore the 8th term of the G.P is $\frac{1}{243}$

19) Determine the nature of the roots for the following quadratic equations
$9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0, a \neq 0, b \neq 0$

Answer : $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0$

a b c

$\Delta = b^2 - 4ac$

$= (-24abcd)^2 - 4 \times 9a^2b^2 \times 16c^2d^2$

$= 576a^2b^2c^2d^2 - 576a^2b^2c^2d^2$

$= 0$

∴ The roots are real and equal.

20) Solve the following quadratic equations by formula method

$2x^2 - 5x + 2 = 0$

Answer : $2x^2-5x+2 = 0$

The formula for finding roots of a quadratic equation $ax^2 + bx + c = 0$ is

$2x^2-5x+2 = 0$

a b c

$$\therefore = x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times 2}}{2 \times 2}$$

$$= \frac{5 \pm \sqrt{25-16}}{4}$$

$$= \frac{5 \pm \sqrt{9}}{4} = \frac{5 \pm 3}{4} = \frac{8}{4}, \frac{2}{4}$$

∴ Solutions is 2, $\frac{1}{2}$

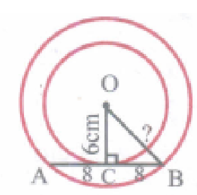
21) If $A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$ then verify $(A^T)^T = A$

Answer : If $A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$, $A^T = \begin{bmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{bmatrix}$

$(A^T)^T = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix} = A$

∴ verified

22) In two concentric circles, a chord of length 16 cm of larger circle becomes a tangent to the smaller circle whose radius is 6 cm. Find the radius of the larger circle.

Answer : 

Let o be the center of concentric circies and APB be the chord of length 16 cm of the larger circle touching the smaller circle at p.

Then OP ⊥ AB and p is the midpoint of AB.

AP = PB = 8 cm

In LOPA, we have

$OA^2 = OP^2 + AP^2$ [By pythagoras Theorem]

$OA^2 = 6^2 + 8^2$

$OA^2 = 36 + 64$

$OA^2 = 100$

OA = 10cm

Radius of the larger circle in 10 cm

23) Show that the given points are collinear: (-3, -4) , (7, 2) and (12, 5)

Answer : Given points (- 3, - 4), (7, 2) and (12, 5)
Let the points be A (- 3, - 4),8 (2, 2) and C (12, 5)
Slope of a line = $\frac{y_1-y_2}{x_1-x_2}$
Slope of AB = $\frac{-4-2}{-3-7} = \frac{-6}{-10} = \frac{3}{5}$
Slope of BC = $\frac{2-5}{7-12} = \frac{-3}{-5} = \frac{3}{5}$
Slope of AB = Slope of BC
The points A, B and C are collinear

24) Find the equation of a straight line passing through the point P(-5, 2) and parallel to the line joining the points Q(3, -2) and R(-5, 4).

Answer : The vertices Q(3, - 2) and R(- 5, 4)
slope of the line QR $= \frac{y_1-y_2}{x_1-x_2}$
$= \frac{-2-4}{3+5} = \frac{-6}{8} = \frac{-3}{4}$
Slope of the line parallel to QR is $-\frac{3}{4}$
Equation of the line passing through
P(- 5, 2) and having slope $-\frac{3}{4}$ is
$y - y_1 = m(x - x_1)$
y - 2 = $-\frac{3}{4}(x+5)$
4y - 8 = -3x - 15
3x + 4y + 7 = 0

25) prove that $\sqrt{\frac{1+cos\theta}{1-cos\theta}}$ = cosec θ + cotθ

Answer : $\sqrt{\frac{1+cos\theta}{1-cos\theta}}$=$\sqrt{\frac{1+cos\theta}{1-cos\theta} \times \frac{1+cos\theta}{1+cos\theta}}$ [multiply numerator and denominator by the conjugate of 1 - cosθ]
=$\sqrt{\frac{(1+cos\theta)^2}{(1-cos\theta)^2}}$ =$\frac{1+cos\theta}{\sqrt{sin^2\theta}}$ [since sin$^2\theta$ + cos$^2\theta$ = 1]
=$\frac{1+cos\theta}{sin\theta} = cosec\theta + cot\theta$

26) A garden roller whose length is 3 m long and whose diameter is 2.8 m is rolled to level a garden. How much area will it cover in 8 revolutions?

Answer : Given that, diameter d = 2.8 m and height = 3 m
radius r = 1.4 m
Area covered in one revolution = curved surface area of the cylinder
= 2πrh sq. units
$2 \times \frac{22}{7} \times 1.4 \times 3 = 26.4$
Area covered in 1 revolution = 26.4 m^2
Area covered in 8 revolutions = 8 x 26.4 = 211.2
Therefore, area covered is 211.2 m^2

27) The volume of a solid right circular cone is 11088 cm^3. If its height is 24 cm then find the radius of the cone.

Answer : Let r and h be the radius and height of the cone respectively.
Given that, volume of the cone = 11088 cm^3
$\frac{1}{3}\pi r^2 h = 11088$
$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$
$r^2 = 441$
Therefore, radius of the cone r = 21 cm.

28) Find the standard deviation of first 50 natural numbers.

Answer : Standard deviation of first n natural numbers

$= \sqrt{\frac{n^2-1}{12}}$

S.D. of first 50 natural numbers $= \sqrt{\frac{50^2-1}{12}}$

$= \sqrt{\frac{2500-1}{12}} = \sqrt{\frac{2499}{12}}$

$= \sqrt{208.25} = 14.43$

Standard deviation of first 50 natural numbers

= 14.43

SECTION C 10X5=50

ANSWER ANY 10 QUESTIONS

Q.NO.42 IS COMPULSORY

29) Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that

A x (B - C) = (A x B) - (A x C)

Answer : Given

A = {1,2,3,4,5,6,7}

B = {2,3,5,7}

C = {2}

A x (B - C) = (A x B) - (A x C)

B - C = {2,3,5,7} - {2}

= {3,5,7}

A x (B-C) = {1,2,3,4,5,6,7} x {3.5.7}

={(1,3),(1,5),(1,7),(2,3),(2,5),(2,7) (3,3),(3,5),(3,7),(4,3),(4,5),(4,7) (5,7),(5,3),(5,5) (6,3),(6,5),(6,7),(7,3),(7,5),(7,7)} ....(1)

A x B = {1,2,3,4,5,6,7} x {2,3,5,7}

= {(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7) (3,2),(3,3),(3,5),(3,7),(4,2),(4,3),(4,5),(4,7) (5,2),(5,3),(5,5),(5,7),(6,2),(6,3),(6,5),(6,7) (7,2),(7,3),(7,5),(7,7)}

A x C = {1,2,3,4,5,6,7} x {2}

= {(1,2),(2,2),(3,2),(4,2),(5,2)(6,2),(7,2)}

(A x B) - (A x C) = {(1,3),(1,5),(1,7),(2,3) (2,5),(2,7),(3,3),(3,5) (3,7),(4,3),(4,5),(4,7), (5,3),(5,5),(5,7),(6,3) (6,5),(6,7),(7,3),(7,5),(7,7)} ..(2)

From (1) and (2), it is clear that

A x (B - C) = (A x B) - (A x C)

Hence verified

30) A function f: [-5,9] ⟶ R is defined as follows:

$$f(x) = \begin{cases} 6x+1 & \text{if } -5 \leq x < 2 \\ 5x^2-1 & \text{if } 2 \leq x < 6 \\ 3x-4 & \text{if } 6 \leq x \leq 9 \end{cases}$$

Find

i) f(-3) + f(2)

ii) f(7) - f(1)

iii) 2f(4) + f(8)

iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

Answer : f: [-5,9] $\longrightarrow$ R

(i) f(-3) + f(2)

= [6(-3) + 1] + [$5(2)^2$ - 1]

= (-18 + 1) + (20 - 1)

= -17 + 19 = 2.

(ii) f(7) - f(1)

= [3(7) - 4] - [6(1) + 1]

= (21 - 4) - (6 + 1)

=17 - 7 = 10

(iii) 2 f(4) + f(8)

= 2 [$5(4)^2$ - 1] + [3(8) - 4]

= 2[80 - 1] + [24 - 4]

= 158 + 20 = 178

(iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

f(-2) = 6x + 1 = 6(-2) + 1 = -11

f(6) = 3x - 4 = 3(6) - 4 = 14

f(4) = $5x^2$ - 1 = $5(4^2)$ - 1 = 79

f(-2) = 6x + 1 = 6(-2) + 1 =-11

$\frac{2f(-2)-f(6)}{f(4)+f(-2)} = \frac{2(-11)-14}{79+(-11)} = \frac{-22-14}{68}$

= $\frac{-36}{68} = \frac{-9}{17}$

31) Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

Answer : The natural numbers between 300 and 600 which are divisible by 7 are 301, 308, 315, …, 595.

The sum of all natural numbers between 300 and 600 is 301 + 308 + 315 +...+ 595

The terms of the above series are in A.P.

First term a = 301; common difference d = 7; Last term l = 595.

$n = \left(\frac{l-a}{d}\right) + 1 = \left(\frac{595-301}{7}\right) + 1 = 43$

Since, $S_n = \frac{n}{2}[a + l]$, we have $s_{43} = \frac{43}{2}[301 + 595]$ = 19264

32) If A, B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if P(A ∩ B) = $\frac{1}{6}$, P(B ∩ C) = $\frac{1}{4}$, P(A ∩ C), $\frac{1}{8}$, P(A P(A U B U C) = $\frac{9}{10}$, P(A ∩ B ∩ C) = $\frac{1}{15}$, then find P(A), P(B) and P(C)?

Answer : Given $P(A \cap B) = \frac{1}{6}$

$P(B \cap C) = \frac{1}{4}$

$P(A \cap C) = \frac{1}{8}$

$P(A \cup B \cup C) = \frac{9}{10}$

$P(A \cap B \cap C) = \frac{1}{15}$

Also given that P(B) = 2 P(A)

P(C) = 3 P(A)

Now

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

$$\frac{9}{10} = \mathrm{P(A)} + 2\mathrm{P(A)} + 3(\mathrm{P(A)}) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$$

$$\frac{9}{10} = 6\mathrm{P(A)} - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$$

$$6\mathrm{P(A)} = \frac{9}{10} + \frac{1}{6} + \frac{1}{4} + \frac{1}{8} - \frac{1}{15}$$

$$6\mathrm{P(A)} = \frac{108+20+30+15-8}{120}$$

$$6\mathrm{P}(A) = \frac{165}{120}$$

$$\mathrm{P(A)} = \frac{165}{120 \times 6} = \frac{11}{48}$$

$$\mathrm{P(B)} = 2 \times \frac{11}{48} = \frac{11}{24}$$

$$\mathrm{P(C)} = 3 \times \frac{11}{48} = \frac{11}{16}$$

$$\mathrm{P(A)} = \frac{11}{48}; \mathrm{P(B)} = \frac{11}{24}; \mathrm{P(C)} = \frac{11}{16}$$

33) In an interschool atheletic meet, with 24 individual events, securing a total of 56 points, a first place secures 5 points, a second place secures 3 points, and a third place secures 1 point. Having as many third place finishers as first and second place finishers, find how many athletes finished in each place.

Answer : Let the number of I, II and III place finishers be x , y and z respectively.

Total number of events = 24; Total number of points = 56.

Hence, the linear equations in three variables are

x + y + z = 24 …(1) 5x + 3y + z = 56 …(2) x + y = z …(3)

Substituting (3) in (1) we get, z + z = 24 gives, z = 12

Therefore, (3) equation will be, x + y =12

(3) will be, x + y = 12

(2) is $5x + 3y = 44$

$3\times$(3) is $3x + 3y = 36$ (–)

$2x = 8$ we get, $x = 4$

Substituting x = 4, z = 12 in (3) we get, y = 12 - 4 = 8

Therefore, Number of first place finishers is 4

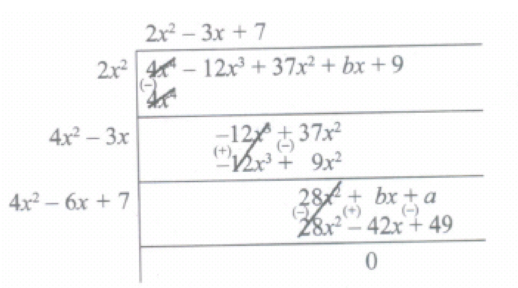
Number of second place finishers is 8

Number of third place finishers is 12.

34) Find the values of a and b if the following polynomials are perfect squares

$4x^4 - 12x^3 + 37x^2 + bx + a$

Answer :



b=-42

a=49

35) If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ show that A^2 - (a + d) A = (bc - ad)I_2

Answer : $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^2 = A \cdot A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$

$(a + d)A = (a + d) \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$= \begin{bmatrix} a(a+d) & b(a+d) \\ c(a+d) & d(a+d) \end{bmatrix}$

$= \begin{bmatrix} a^2 + ad & ab + bd \\ ac + cd & ad + d^2 \end{bmatrix}$

A^2 - (a + d) A

$= \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} - \begin{bmatrix} a^2 + ad & ab + bd \\ ac + cd & ad + d^2 \end{bmatrix}$

$= \begin{bmatrix} bc - ad & 0 \\ 0 & bc - ad \end{bmatrix}$

Now (bc - ad) $I_2 = (bc - ad) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

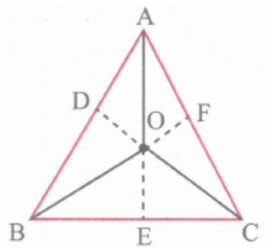
$= \begin{bmatrix} bc - ad & 0 \\ 0 & bc - ad \end{bmatrix}$

From (1) and (2)

A^2 - (a + d)A = (bc - ad) I_2

Hence proved.

36) Show that the angle bisectors of a triangle are concurrent.

Answer : 

Let $\triangle$ABC be B triangle points D, E, F are angular bisectors of $\angle A, \angle B$ and $\angle C$ respectively. By angular bisector theorem we have

$\frac{BD}{DC} = \frac{AB}{AC} \Rightarrow AB = \frac{BD \times AC}{DC}$

$\frac{AC}{BC} = \frac{AF}{FB} \Rightarrow AC = \frac{AF \times BC}{FB}$

$\frac{AE}{EC} = \frac{AB}{BC} \Rightarrow AB = \frac{AE \times BC}{EC}$

From (1) and (3), we have

$\frac{BD \times AC}{DC} = \frac{AE \times BC}{EC}$

Now substituting (2) in (4) we have

$\frac{BD \times \left(\frac{AF \times BC}{FB}\right)}{DC} = \frac{AE \times BC}{EC}$

$\frac{BD \times AF \times BC}{DC \times FB} = \frac{AE \times BC}{EC}$

$BD \times AF \times EC = \frac{AE \times BC \times DC \times FB}{BC}$

$BD \times AF \times CE = EA \times FB \times DC$

$\therefore \frac{BD \times AF \times CE}{EA \times FB \times DC} = 1$

Hence by Ceva's theorem we conclude that the angle bisectors of a triangle are concurrent.

37) Find the area of the quadrilateral formed by the points (8, 6), (5, 11), (-5, 12) and (-4, 3).

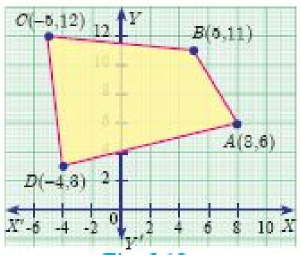
Answer : Before determining the area of quadrilateral, plot the vertices in a graph.
Let the vertices be A(8, 6), B(5, 11), C(-5, 12) and D(-4, 3).
Therefore, area of the quadrilateral ABCD

$=\frac{1}{2}$ { $(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)$ }

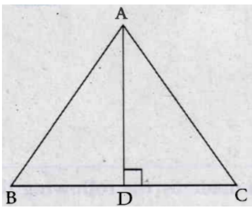
$=\frac{1}{2}$ { (80 + 60 - 15 - 24) - (30 - 55 - 48 + 24)}

$=\frac{1}{2}$ {109 + 49 }

$=\frac{1}{2}$ { 158 } = 79 sq.units



38) Find the equation of the median and altitude of Δ ABC through A where the vertices are A(6, 2), B(-5,-1) and C(1, 9)

Answer : 

Given vertices are A (6, 2), B (- 5, - 1) and C (1, 9)
Median through A :
Let D be the mid point of BC

Mid point of BC $= \text{D}\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \text{D}\left(\frac{-5+1}{2}, \frac{-1+9}{2}\right)$

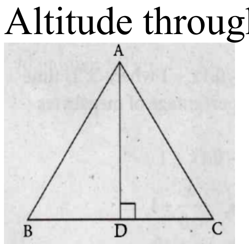
= D (-2 , 4)
Now AD is the median.

Equation of AD $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$\frac{y-2}{4-2} = \frac{x-6}{-2-6}$

$\frac{y-2}{2} = \frac{x-6}{-8}$

- 4y + 8 = x - 6
x + 4y - 14 = 0
Altitude through A



Altitude is passing through 'A' and perpendicular to BC.
Now,

Slope of BC = $\frac{y_1-y_2}{x_1-x_2} = \frac{-1-9}{-5-1} = \frac{-10}{-6} = \frac{5}{3}$

Slope of Altitude $= -\frac{3}{5}$

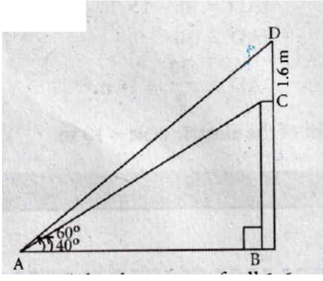
Equation of the altitude which is passing through A (6,2)and having slope $-\frac{3}{5}$ is

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{3}{5}(x - 6)$

5y- 10 = -3x + 18
3x + 5y - 28 = 0

39) A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 40°. Find the height of the pedestal. (tan 40° = 0.8391, $\sqrt{3}$ = 1.732)

Answer : 

Let CD be the statue of tall 1.6 m.
BC be the pedestal.
From the right triangle $\triangle$ABC

$$\tan 40^\circ = \frac{BC}{AB}$$

$$0.8391 = \frac{BC}{AB}$$

$$AB = \frac{BC}{0.8391} \qquad ....(1)$$

From the right triangle $\triangle$ABD

$$\tan 60^\circ = \frac{BD}{AB}$$

$$\sqrt{3} = \frac{BC+CD}{AB}$$

$$1.732 = \frac{BC+1.6}{AB}$$

$$AB = \frac{BC+1.6}{1.732} \qquad ....(2)$$

From (1) and (2)

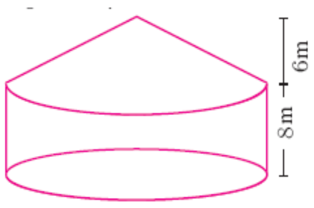
$$\frac{BC}{0.8391} = \frac{BC+1.6}{1.732}$$

1.732 BC = 0.8391 (BC + 1.6)
1.732 BC = 0.8391 BC + (0.8391) (1.6)
1.732 BC - 0.8391 BC = 1.34256
0.8929 BC = 1.34256

$$\mathrm{BC} = \frac{1.34256}{0.8929} = \frac{13425.6}{8929} = 1.5 \text{ m}$$

Height of the pedestal = 1.5 m

40) Arul has to make arrangements for the accommodation of 150 persons for his family function. For this purpose, he plans to build a tent which is in the shape of cylinder surmounted by a cone. Each person occupies 4 sq. m of the space on ground and 40 cu. meter of air to breathe. What should be the height of the conical part of the tent if the height of cylindrical part is 8 m?

Answer : 

Let h_1 and h_2 be the height of cylinder and cone respectively
Area for one person = 4 sq.m
Total number of persons = 150
Therefore total base area = 150 x 4
$\pi r^2 = 600$ ......(1)
Volume of air required for 1 person = $40 m^3$
Total Volume of air required for 150 persons = 150 x 40 = 6000 m^3

$$\pi r^2 h_1 + \frac{1}{3}\pi r^2 = 6000$$

$$\pi r^2 \left(h_1 + \frac{1}{3}h_2\right) = 6000$$

$$600 \left(8 + \frac{1}{3}h_2\right) = 6000 \quad [using\ (1)]$$

$$8 + \frac{1}{3}h_2 = \frac{6000}{600}$$

$$\frac{1}{3}h_2 = 10 - 8 = 2$$

h_2 = 6 m
Therefore, the height of the conical tent h_2 is 6 m

41) The internal and external diameter of a hollow hemispherical shell are 6 cm and 10 cm respectively. If it is melted and recast into a solid cylinder of diameter 14 cm, then find the height of the cylinder.

Answer : Hollow Hemisphere
Internal diameter = 6 cm
Internal radius 'r' = 3 cm
External diameter = 10 cm
External radius 'R' = 5 cm

$$\left.\begin{array}{l}\text{Volume of hemisphere (or)}\\ \text{Volume of material used}\end{array}\right\} = \frac{2}{3}\pi\left(\mathrm{R}^3 - \mathrm{r}^3\right) \text{ cu. units}$$

$$= \frac{2}{3}\pi\left(5^3 - 3^3\right)$$

$$= \frac{2}{3}\pi(125 - 27) = \frac{196\pi}{3} \text{ cm}^3$$

Cylinder
Diameter = 14 cm
radius = 7 cm
height = h
Volume of cylinder $= \pi r^2 h\ cu.units$

$$= \pi(7)^2 h$$

$$= 49\pi h \text{ cm}^3$$

Given that hollow hemisphere is melted and cast into a solid cylinder
Volume of cylinder = volume of hollow hemisPhere

$$49\pi h = \frac{196\pi}{3}$$

$$h = \frac{196}{3\times 49} = \frac{4}{3} = 1.33$$

Height of the cylinder = 1.33 cm.

42) If m times m^{th} term of an A.P is equal to n times its n^{th} term then show that the (m+n)th term of the A.P is Zero.

Answer : On subtracting equation (2) from equation (1), equation (3) from equation (2)and equation (1) from equation (3), we get
x - y = (l - m)d
y - z = (m - n)d
z - x = (n -l)d
(x - y)n + (y - z)l + (z - x)m = [(l -m)n + (m -n)l + (n -l)m]d
= [ln - mn + lm - nl + nm - lm]d = 0.

SECTION D 2X8=16

ANSWER BOTH THE QUESTIONS

43) a) Graph the following quadratic equations and state their nature of solutions.
$x^2 - 4x + 4 = 0$

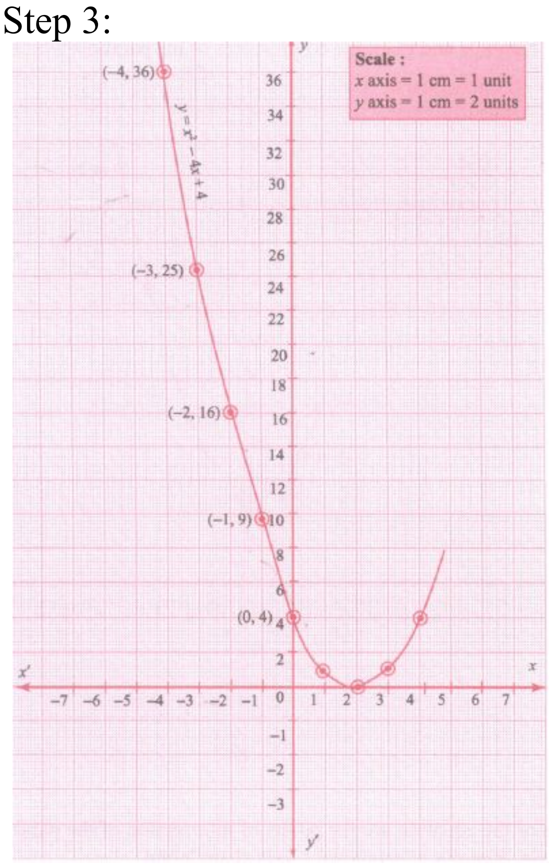
Answer : $x^2 - 4x + 4 = 0$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-4x	16	12	8	4	0	-4	-8	-12	-16
4	4	4	4	4	4	4	4	4	4
$y=x^2-4x+4$	36	25	16	9	4	1	0	1	4

Step 1: Points to be plotted: (-4,36), (-3, 25), (-2, 16), (-1, 9), (0, 4), (1, 1), (2, 0), (3, 1), (4,4)

Step 2: The point of intersection of the curve with x axis is (2, 0)

Step 3:



Since there is only one point of intersection with x axis, the quadratic equation $X^2 - 4x + 4 = 0$ has real and equal roots.

∴ Solution{2,2}

(OR)

b) A garment shop announces a flat 50% discount on every purchase of items for their customers. Draw the graph for the relation between the Marked Price and the Discount. Hence find

i. the marked price when a customer gets a discount of Rs. 3250 (from graph)

ii. the discount when the marked price is Rs. 2500.

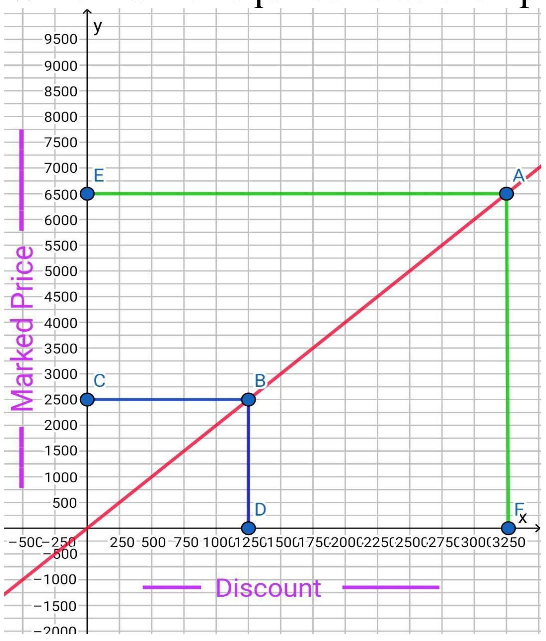
Answer : Let y be the marked price of an item and x be the discount on that item
Now it is given that the garment shop announces a flat 50% discount on every purchase of items for their customers

$\Longrightarrow x = y \times \frac{50}{100}$

$\Longrightarrow \ x = y \times \frac{1}{2}$

$\Longrightarrow y = 2x$

Which is the required relationship between the Marked Price and the Discount.



(1) We have to find the marked price when a customer gets a discount of 3250

As discount is 3250

In the graph is represent the point F (3250, 0)

If we draw a line parallel to Y axis and passing through the point F(3250, 0) then it intersects the line y = 2x at B(3250, 6500)

Accordingly we get the point A (3250, 6500)

Hence the Marked Price = 6500

Manually it can be checked

As discount is 3250

x = 3250

So y = 2 x 3250 = 6500

(2) We have to find the discount when the marked price is 2500

As marked price is 2500

In graph it represent the point C(0, 2500)

If we draw a line parallel to X axis and passing through the point C(0, 2500) then it intersects the line y = 2x at B(1250, 2500)

Accordingly we get the point B(1250, 2500)

Hence the discount = 1250

Manually it can be checked

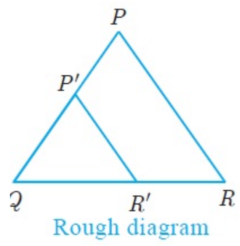
As marked price is 2500

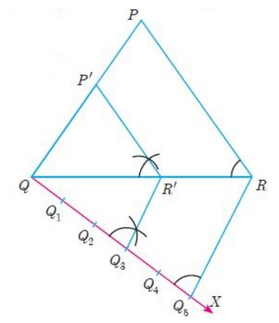
y = 2500

So x = 2500 ÷ 2 = 1250

44) a) Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{3}{5}$ of the corresponding sides of the triangle PQR (scale factor $\frac{3}{5} < 1$)

Answer : Given a triangle PQR we are required to construct another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the triangle PQR.





Steps of construction

1. Construct a △ PQR with any measurement
2. Draw a ray QX making an acute angle with QR on the side opposite to vertex P.
3. Locate 5 (the greater of 3 and 5 in $\frac{3}{5}$) points.

Q_1Q_2, Q_3, Q_4 and Q_5 on QX so that $QQ_1 = Q_1Q_2 = Q_2Q_3 = Q_4Q_5$

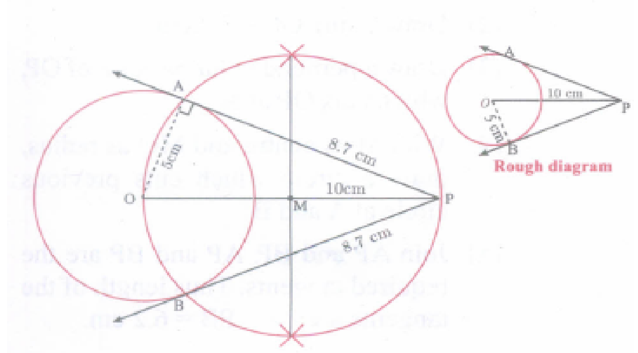
4. Join Q_5R and draw a line through Q_3 (the third point, 3 being smaller of 3 and 5 in $\frac{3}{5}$) parallael to Q_5R to intersect QR at R'.
5. Draw line through R' parallel to the line RP to intersect QP at P'.

Then, △P'QR' is the required triangle each of whose sides is three-fifths of the corresponding sides of △ PQR.

(OR)

b) Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents.

Answer : The distance between the point from the centre is 10 cm.



Length of the tangents PA - PB = 8.7 cm

Construction:

Steps:

(1) With O as centre, draw a circle of radius 5cm.

(2) Draw a line OP = 10 cm.

(3) Draw a perpendicular bisector of OP which cuts OP at M.

(4) With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

(5) Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA and PB = 8.7 cm