QB365 Question Bank Software Study Materials

Applications of Matrices and Determinants Important 2 Marks Questions With Answers (Book Back and Creative)

12th Standard

Maths

Total Marks : 40

20 x 2 = 40

1) If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is non-singular, find A^{-1}.

Answer : We first find adj A. By definition, we get adj A = $\begin{bmatrix} +M_{11} & -M_{12} \\ -M_{21} & +M_{22} \end{bmatrix}^T = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T = \begin{bmatrix} d & -c \\ -c & a \end{bmatrix}$.

Since A is non-singular, |A| = ad - bc ≠ 0.

As $A^{-1} = \frac{1}{|A|}$ adj A, we get A^{-1} = $\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

2) Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

Answer : Let A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$. Then, A^T = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

So, we get

AA^T = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

= $\begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = I_2

Similarly, we get $A^TA = I_2$. Hence $AA^T = A^TA = I_2$ ⇒ A is orthogonal.

3) Find the adjoint of the following:

$\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

Answer : $\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

Let A = $\begin{pmatrix} -3 & 4 \\ 6 & 2 \end{pmatrix}$

adj A = $\begin{pmatrix} 2 & -4 \\ -6 & -3 \end{pmatrix}$

[Interchange the elements in the leading diagonal and change the sign of the elements in off diagonal]

4) If adj(A) = $\begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A.

Answer : Given adj A = $\begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$

We know that A = $\pm\frac{1}{\sqrt{|adj A|}}$ adj (adj A)..(1)

|adj A| = $2\begin{vmatrix} 12 & -7 \\ 0 & 2 \end{vmatrix} + 4\begin{vmatrix} -3 & -7 \\ -2 & 2 \end{vmatrix} + 2\begin{vmatrix} -3 & 12 \\ -2 & 0 \end{vmatrix}$

[Expanded along R_1]

= 2(24-0)+4(-6-14)+2(0+24)

= 2(24)+4(-20)+2(24) = 48-80+48

= 96-80 = 16

Now, adj (adj A)

= $\begin{bmatrix} +\begin{vmatrix} 12 & -7 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} -3 & -7 \\ -2 & 2 \end{vmatrix} & +\begin{vmatrix} -3 & 12 \\ -2 & 0 \end{vmatrix} \\ -\begin{vmatrix} -4 & 2 \\ 0 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & 2 \\ -2 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & -4 \\ -2 & 0 \end{vmatrix} \end{bmatrix}^T$

$$\begin{bmatrix} +\begin{vmatrix} -4 & 2 \\ 12 & -7 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ -3 & -7 \end{vmatrix} & +\begin{vmatrix} 2 & -4 \\ -3 & 12 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} +(24-0)-(6-14)+(0+24) \\ -(-8-0)+(4+4)-(0-8) \\ +(28-24)-(-14+6)+(24-12) \end{bmatrix}^T$$

$$= \begin{bmatrix} 24 & 20 & 24 \\ 8 & 8 & 8 \\ 4 & 8 & 12 \end{bmatrix}^T = \begin{bmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{bmatrix}$$

$$= 4 \begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

Substituting (2) and (3) in (1) we get,

$$A = \frac{1}{\sqrt{16}}.4 \begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

$$A = \pm\frac{4}{4} \begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix} = \pm \begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

5) If adj(A) = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

Answer : Given adj (A) = $\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

We know that $A^{-1} = \pm\frac{1}{\sqrt{|adj A|}}$ (adj A) ....(1)

$|adj\ A| = 0 + 2\begin{vmatrix} 6 & -6 \\ -3 & 6 \end{vmatrix} + 0$

[Expanded along R_1]

= 2(36-18) = 2(18) = 36

$$\therefore A^{-1} = \pm\frac{1}{\sqrt{36}} \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$$

$$= \pm\frac{1}{6} \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}.$$

6) Find adj(adj (A)) if adj A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

Answer : Given adj A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$$\text{Now adj(adj A)} = \begin{bmatrix} +\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 0 \\ -1 & 1 \end{vmatrix} & +\begin{vmatrix} 0 & 2 \\ -1 & 0 \end{vmatrix} \\ -\begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ -1 & 0 \end{vmatrix} \\ +\begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} & +\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} +(2-0) & -(0) & +(0+2) \\ -(0) & +(1+1) & -(0) \\ +(0+2) & -(0) & +(2-0) \end{bmatrix}^T = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix}^T$$

$$\text{adj(adj A)} = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

7) Find the rank of each of the following matrices:

$$\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$$

Answer : Let A = $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 1 \\ 3 & 3 & 6 \end{bmatrix}$. Then A is a matrix of order 3 × 3. So ρ(A) ≤ min {3, 3} = 3.

The highest order of minors of A is 3. There is only one third order minor of A.

It is $\begin{vmatrix} 3 & 2 & 5 \\ 1 & 1 & 1 \\ 3 & 3 & 6 \end{vmatrix}$ = 3(6 - 6) - 2(6 - 6) + 5(3 - 3) = 0. So, ρ(A) < 3.

Next consider the second - order minors of A.

We find that the second order minor $\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix}$ = 3 - 2 ≠ 0. So ρ(A) = 2.

8) Find the rank of the following matrices which are in row-echelon form :

$\begin{bmatrix} 2 & 0 & -7 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Answer : Let A = $\begin{bmatrix} 2 & 0 & -7 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Then A is a matrix of order 3 × 3 and ρ(A) ≤ 3

The third order minor |A| = $\begin{vmatrix} 2 & 0 & -7 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix}$ = (2)(3)(1) = 6 ≠ 0. So, ρ(A) = 3.

Note that there are three non-zero rows.

9) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

Answer : Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$. Applying elementary row operations, we get

A $\xrightarrow[R_3 \longrightarrow R_3 - 3R_1]{R_2 \longrightarrow R_2 - 2R_1}$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & -6 & -4 \end{bmatrix}$ $\xrightarrow{R_3 \longrightarrow R_3 - 2R_2}$ $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -2 \\ 0 & 0 & 0 \end{bmatrix}$.

The last equivalent matrix is in row-echelon form. It has two non-zero rows. So, ρ(A) = 2.

10) Find the inverse of each of the following by Gauss-Jordan method:

$\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

Answer : $\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

Let A = $\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$

Applying Gauss - Jordan method, we get

$[A|I_2] = \left[\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 5 & -2 & 0 & 1 \end{array}\right]$

$\xrightarrow{R_1 \to R_1 \div 2} \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 5 & -2 & 0 & 1 \end{array}\right]$

$\xrightarrow{R_2 \to R_2 - 5R_1} \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 5 & -2 & -\frac{5}{2} & 1 \end{array}\right]$

$\xrightarrow{R_2 \to R_2 \times 2} \left[\begin{array}{cc|cc} 1 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & -5 & 2 \end{array}\right]$

$\xrightarrow{R_1 \to R_1 + \frac{1}{2}R_2} \left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & -5 & 2 \end{array}\right]$

∴ We get $A^{-1} = \begin{bmatrix} -2 & 1 \\ -5 & 2 \end{bmatrix}$

11) Solve the following system of linear equations, using matrix inversion method:

5x + 2y = 3, 3x + 2y = 5.

Answer : The matrix form of the system is AX = B , where A = $\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \end{bmatrix}$, B = $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$

We find |A| = $\begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix}$ = 10 - 6 = 4 ≠ 0. So, A^{-1} exists and A^{-1} = $\frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

Then, applying the formula X = A^{-1}B, we get

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix} = \begin{bmatrix} \frac{-4}{4} \\ \frac{16}{4} \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}.$$

So the solution is (x = −1, y = 4).

12) Solve the following system of linear equations by matrix inversion method:

2x + 5y = −2, x + 2y = −3

Answer : 2x+5y = -2, x+2y = -3

The matrix form of the system is

$$\begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$

⇒ AX = B where

A = $\begin{pmatrix} 2 & 5 \\ 1 & 2 \end{pmatrix}, B = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

X = $\begin{pmatrix} x \\ y \end{pmatrix}$

⇒ = A^{-1}B

|A| = $\begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix}$= 4 - 5 = -1 ≠ 0

∴ A^{-1} = $\frac{1}{|A|} adj A = \frac{1}{-1}\begin{bmatrix} 2 & -5 \\ -1 & 2 \end{bmatrix}$

= $\begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}$

∴ X = A^{-1}B = $\begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} -2 \\ -3 \end{bmatrix}$

= $\begin{bmatrix} 4-15 \\ -2+6 \end{bmatrix} = \begin{bmatrix} -11 \\ 4 \end{bmatrix}$

∴ Solution set is x = -11, y = 4

13) Solve, by Cramer's rule, the system of equations

$x_1 - x_2 = 3$, $2x_1 + 3x_2 + 4x_3 = 17$, $x_2 + 2x_3 = 7$.

Answer : First we evaluate the determinants

Δ = $\begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix}$ = 6 ≠ 0, Δ_1 = $\begin{vmatrix} 3 & -1 & 0 \\ 17 & 3 & 4 \\ 7 & 1 & 2 \end{vmatrix}$ = 12, Δ_2 = $\begin{vmatrix} 1 & 3 & 0 \\ 2 & 17 & 4 \\ 0 & 7 & 2 \end{vmatrix}$ = -6, Δ_3 = $\begin{vmatrix} 1 & -1 & 3 \\ 2 & 3 & 17 \\ 0 & 1 & 7 \end{vmatrix}$ = 24

By Cramer's rule, we get $x_1 = \frac{\Delta_1}{\Delta} = \frac{12}{6}$ = 2, $x_2 = \frac{\Delta_2}{\Delta} = \frac{-6}{6}$ = -1, $x_3 = \frac{24}{6}$ = 4.

So, the solution is (x_1 = 2, x_2 = -1, x_3 = 4).

14) Test for consistency of the following system of linear equations and if possible solve:

x - y + z = -9, 2x - 2y + 2z = -18, 3x - 3y + 3z + 27 = 0.

Answer : Here the number of unknowns is 3.

The matrix form of the system is AX = B, where

A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & -3 & 3 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B = $\begin{bmatrix} -9 \\ -18 \\ -27 \end{bmatrix}$.

Applying elementary row operations on the augmented matrix[A | B], we get

[A | B] = $\begin{bmatrix} 1 & -1 & 1 & -9 \\ 2 & -2 & 2 & -18 \\ 3 & -3 & 3 & -27 \end{bmatrix} \xrightarrow[R_3 \longrightarrow R_3 - 3R_1]{R_2 \longrightarrow R_2 - 2R_1,} \begin{bmatrix} 1 & -1 & 1 & -9 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

So, ρ(A) = ρ ([A | B]) = 1 < 3.

From the echelon form, we get the equivalent equations x - y + z = -9, 0 = 0, 0 = 0.

The equivalent system has one non-trivial equation and three unknowns.

Taking y = s, z = t arbitrarily, we get x - s + t = -9; x = -9 + s - t.

So, the solution is (x = -9 + s - t, y = s, z = t), where s and t are parameters.

The above solution set is a two-parameter family of solutions.

Here, the given system of equations is consistent and has infinitely many solutions which form a two parameter family of solutions.

15) Find the adjoint of the following:

$\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

Answer : $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

Let A = $\begin{pmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix}$

adj A = $\begin{pmatrix} +\begin{vmatrix} 4 & 1 \\ 7 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} & +\begin{vmatrix} 3 & 4 \\ 3 & 7 \end{vmatrix} \\ -\begin{vmatrix} 3 & 1 \\ 7 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 3 \\ 3 & 7 \end{vmatrix} \\ +\begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} & +\begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} \end{pmatrix}$

= $\begin{bmatrix} +(8-7)-(6-3)+(21-12) \\ -(6-7)+(4-3)-(14-9) \\ +(3-4)-(2-3)+(8-9) \end{bmatrix}^T$

= $\begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}^T$

adj A = $\begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$

16) Find the rank of the following matrices by minor method:

$\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$

Answer : $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$

Let A = $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$

A is a matrix of order 3 × 2

∴ $\rho(A) \le \min(3, 2) = 2$

The highest order of minor of A is 2

It is $\begin{vmatrix} -1 & 3 \\ 4 & -7 \end{vmatrix}$ = 7-12 = 5 ≠ 0

∴ $\rho(A) = 2$

17) Find the rank of the following matrices by minor method:

$$\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$$

Answer : $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

Let A = $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

A is a matrix of order (2 × 4)

$\therefore \rho(A) \le \min(2, 4) = 2$

The highest order of minor of A is 2

It is $\begin{vmatrix} 1 & -2 \\ 3 & -6 \end{vmatrix}$ = -6 + 6 = 0

Also, $\begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix}$ = -1 + 0 = -1 ≠ 0

$\therefore \rho(A) = 2$

18) Solve the following system of linear equations by matrix inversion method :

2x − y = 8 , 3x + 2y = −2.

Answer : 2x-y = 8, 3x+2y+2 = -2

The matrix form of the system is

$$\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \end{bmatrix}$$

⇒ AX = B where A = $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

B = $\begin{bmatrix} 8 \\ -2 \end{bmatrix}$

⇒ $X = A^{-1}N$

Now, |A| = $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ = 4 + 3 = 7

$\therefore A^{-1} = \frac{1}{|A|}\text{adj } A$

$= \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

$\therefore X = A^{-1}B = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ -2 \end{bmatrix}$

$= \frac{1}{7}\begin{bmatrix} 16 - 2 \\ -24 - 4 \end{bmatrix}$

$= \frac{1}{7}\begin{bmatrix} 14 \\ -28 \end{bmatrix} = \begin{bmatrix} \frac{14}{7} \\ \frac{-28}{7} \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

∴ x = 2, y = -4

Hence, the solution set is {2, -4}

19) Find the rank of the following matrices which are in row-echelon form :

$$\begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Answer : Let A = $\begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. Then A is a matrix of order 3 × 3 and ρ(A) ≤ 3.

The only third order minor is |A| = $\begin{vmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{vmatrix}$ = (-2)(5)(0) = 0. So ρ(A) ≤ 2.

There are several second order minors. We find that there is a second order minor, for example, $\begin{vmatrix} -2 & 2 \\ 0 & 5 \end{vmatrix}$ = (-2)(5) = -10 ≠ 0.

So, ρ(A) = 2.

Note that there are two non-zero rows. The third row is a zero row.

20) Find the rank of the following matrices which are in row-echelon form :

$$\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Answer : Let A = $\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Then A is a matrix of order 4 × 3 and $\rho(A) \le 3$.

The last two rows are zero rows. There are several second order minors.

We find that there is a second order minor, for example, $\begin{vmatrix} 6 & 0 \\ 0 & 2 \end{vmatrix}$ = (6)(2) = 12 ≠ 0. So, $\rho(A) = 2$.

Note that there are two non-zero rows. The third and fourth rows are zero rows.