QB365 Question Bank Software Study Materials

Applications of Vector Algebra Important 2 Marks Questions With Answers (Book Back and Creative)

12th Standard

Maths

Total Marks : 40

20 x 2 = 40

1) A particle is acted upon by the forces $(\hat{3i} - \hat{2j} + \hat{2k})$ and $(\hat{2i} + \hat{j} - \hat{k})$ is displaced from the point (1, 3, -1) to the point (4, -1, λ). If the work done by the forces is 16 units, find the value of λ.

Answer : Resultant of the given forces is $\vec{F}$ = $(\hat{3i} - \hat{2j} + \hat{2k})$ + $(\hat{2i} + \hat{j} - \hat{k})$ = $\hat{5i} - \hat{j} + \hat{k}$

The displacement of the particle is given by

$\vec{d}$ = $(\hat{4i} - \hat{j} + \lambda\hat{k}) - (\hat{i} + 3\hat{j} - \hat{k})$ = $(3\hat{i} - \hat{4j} + (\lambda+1)\hat{k})$

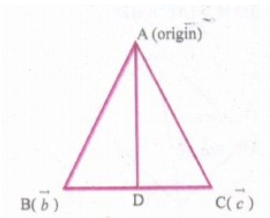
As the work done by the forces is 16 units, we have

$\vec{F}.\vec{d}$ = 16

That is $(\hat{5i} - \hat{j} + \hat{k}).(3\hat{i} - \hat{4j} + (\lambda+1))\hat{k}$ = 16 ⇒ λ + 20 = 16

So, λ = - 4

2) Prove by vector method that the median to the base of an isosceles triangle is perpendicular to the base.

Answer : 

Let ABC be an isosceles triangle with AB = AC and let AD is the median

D is mid-point of BC.

$\overrightarrow{AD} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$\overline{DA} \cdot \overrightarrow{DB} = -\overrightarrow{AD} \cdot \left(\frac{-1}{2}\overrightarrow{CB}\right)$

$= -\overrightarrow{AD} \cdot \left(\frac{1}{2}\overrightarrow{BC}\right)$

$= \frac{1}{2}\overrightarrow{AD} \cdot \overrightarrow{BC}$

$= \frac{1}{4}(\overrightarrow{AB} + \overrightarrow{AC}) \cdot (\overrightarrow{BA} + \overrightarrow{AC})$

$= \frac{1}{4}(\overrightarrow{AB} + \overrightarrow{AC}) \cdot (\overrightarrow{AC} - \overrightarrow{AB})$

$= \frac{1}{4}[(\overrightarrow{AC} \cdot \overrightarrow{AC}) - (\overrightarrow{AB} \cdot \overrightarrow{AB})]$

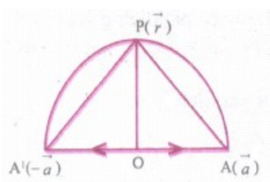
$= \frac{1}{4}(AC^2 - AB^2)$

$= \frac{1}{4}(0) = 0$

$\overrightarrow{DA} \cdot \overrightarrow{DB} = 0$

$\overrightarrow{DA} \perp \overrightarrow{DB}$

3) Prove by vector method that an angle in a semi-circle is a right angle.

Answer : 

Let O be the centre of the semi-circle and AA^1 be the diameter.

Let P be any point on the circumference of the semi circle.

Taking O as the origin, let the position vectors of A and P be a and $\vec{r}$ respectively.

Let us prove that $\angle APB = 90°$

W.K.T OA = OB = OP (because of radius)

$\overrightarrow{PA} = \overrightarrow{PO} + \overrightarrow{OA}$

$\overrightarrow{PB} = \overrightarrow{PO} + \overrightarrow{OB}$

$= P\acute{O} - O\acute{A}$

$\overrightarrow{PA} \cdot \overrightarrow{PB} = (\overrightarrow{PO} + \overrightarrow{OA})(\overrightarrow{PO} - \overrightarrow{OA})$

$= \overrightarrow{PO}^2 - \overrightarrow{OA}^2 = 0$

$\overrightarrow{PA} \perp \overrightarrow{PB}$

$\Rightarrow \angle APB = 90^\circ$. Hence proved.

4) Find the volume of the parallelepiped whose coterminous edges are represented by the vectors $-6\hat{i} + 14\hat{j} + 10\hat{k}, 14\hat{i} - 10\hat{j} - 6\hat{k}$ and $2\hat{i} + 4\hat{j} - 2\hat{k}$

Answer : Let $\vec{a} = -6\hat{i} + 14\hat{j} + 10\hat{k}$, $\vec{b} = 14\hat{i} - 10\hat{j} - 6\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} - 2\hat{k}$

Volume of the parallelepiped having $\vec{a}, \vec{b}$ and $\vec{c}$ as its co-terminus edges is $\vec{a}.(\vec{b} \times \vec{c})$.

$\therefore \vec{a}.(\vec{b} \times \vec{c}) = \begin{vmatrix} -6 & 14 & 10 \\ 14 & -10 & -6 \\ 2 & 4 & -2 \end{vmatrix}$

$= -6\begin{vmatrix} -10 & -6 \\ 4 & -2 \end{vmatrix} - 14\begin{vmatrix} 14 & -6 \\ 2 & -2 \end{vmatrix} + 10\begin{vmatrix} 14 & -10 \\ 2 & 4 \end{vmatrix}$

= -6(20 + 24) - 14(-28 + 12) + 10(56 + 20)

= -6(44) -14(-16) + 10(76)

= -264 + 224 + 760 = 720.

$\therefore$ Volume of the required parallelepiped = 720 cubic units.

5) If $\vec{a} = \hat{i} - \hat{k}, \vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}, \vec{c} = y\hat{i} + x\hat{j} + (1+x+y)\hat{k}$ show that $[\vec{a}, \vec{b}, \vec{c}]$ depends on neither x nor y.

Answer : Given $\vec{a} = \hat{i} - \hat{k}, \vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}, \vec{c} = y\hat{i} + x\hat{j} + (1+x+y)\hat{k}$

$[\vec{a}, \vec{b}\vec{c}] = \vec{a}.(\vec{b} \times \vec{c}) = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$

$= 1\begin{vmatrix} 1 & 1-x \\ x & 1+x-y \end{vmatrix} + 0 - y\begin{vmatrix} x & y \\ y & x \end{vmatrix}$

= $[(1+x-y)-x(1-x)]-[x^2-y]$

$= 1 + \cancel{x} - \cancel{y} - \cancel{x} + \cancel{x}^2 - \cancel{x}^x + \cancel{y}$

= 1

$\therefore [\vec{a}\vec{b}\vec{c}]$ = 1 for all values of x and y

$\therefore [\vec{a}\vec{b}\vec{c}]$ depends on neither x nor y.

6) Find the vector and Cartesian form of the equations of a plane which is at a distance of 12 units from the origin and perpendicular to $6\hat{i} + 2\hat{j} - 3\hat{k}$

Answer : Let $\hat{d} = 6\hat{i} + 2\hat{j} - 3\hat{k}$ and p = 12

If $\hat{d}$ is the unit normal vector in the direction of the vector $6\hat{i} + 2\hat{j} - 3\hat{k}$

then $\hat{d} = \frac{d}{|d|} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$

If $\hat{r}$ is the position vector of an arbitrary point (x, y, z) on the plane, then using $\vec{r}.\hat{d} = p$, the vector equation of the plane in normal form is $\vec{r}.\frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k}) = 12$

Substituting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in the above equation, we get $(x\hat{i} + y\hat{j} + z\hat{k}).\frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k}) = 12$

Applying dot product in the above equation and simplifying, we get 6x + 2y - 3z = 84, which is the Cartesian equation of the required plane.

7) If the Cartesian equation of a plane is 3x - 4y + 3z = -8, find the vector equation of the plane in the standard form.

Answer : If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector of an arbitrary point (x, y, z) on the plane, then the given equation can be written as $(x\hat{i} + y\hat{j} + z\hat{k}).(3\hat{i} - 4\hat{j} + 3\hat{k}) = -8$ or $(x\hat{i} + y\hat{j} + z\hat{k}).(-3\hat{i} + 4\hat{j} - 3\hat{k}) = 8$.

That is, $\hat{r}.(-3\hat{i} + 4\hat{j} - 3\hat{k}) = 8$ which is the vector equation of the given plane in standard form.

8) Find the vector and Cartesian equations of the plane passing through the point with position vector $4\hat{i} + 2\hat{j} - 3\hat{k}$ and normal to vector $2\hat{i} - \hat{j} + \hat{k}$

Answer : If the position vector of the given point is $\vec{a}=4\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{n}=2\hat{i}-\hat{j}+\hat{k}$, then the equation of the plane passing through a point and normal to a vector is given by $(\vec{r}-\vec{a}).\vec{n}=0$ or $\vec{r}.\vec{n}=\vec{a}.\vec{n}$

Substituting $\vec{a}=4\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{n}=2\hat{i}-\hat{j}+\hat{k}$ in the above equation, we get

$\vec{r}.(2\hat{i}-\hat{j}+\hat{k})=(4\hat{i}+2\hat{j}-3\hat{k}).(2\hat{i}-\hat{j}+\hat{k})$

Thus, the required vector equation of the plane is $\vec{r}.(2\hat{i}-\hat{j}+\hat{k})$= 3. If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ then we get the Cartesian equation of the plane 2x − y + z = 3.

9) Verify whether the line $\frac{x-3}{-4}=\frac{y-4}{-7}=\frac{z+3}{12}$ lies in the plane 5x-y+z = 8.

Answer : Here (x_1, y_1, z_1) = (3, -4, -3) and direction ratios of the given straight line are (a, b, c) = (-4, -7, 12).

Direction ratios of the normal to the given plane are (A, B, C) = (5, -1, 1).

We observe that, the given point (x_1, y_1, z_1) = (3, 4, -3) satisfies the given plane 5x-y+z = 8

Next, aA+bB+cC = (-4)(5)+(-7)(-1)+(12)(1) = -1 $\neq$ 0.

So, the normal to the plane is not perpendicular to the line.

Hence, the given line does not lie in the plane.

10) Find the angle between the planes $\vec{r}.(\hat{i}+\hat{j}-2\hat{k})$ = 3 and 2x - 2y + z =2

Answer : Given planes are $\vec{r}.(\hat{i}+\hat{j}-2\hat{k})$ and

$2x-2y+z=2\Rightarrow\vec{r}.\left(2\hat{i}-2\hat{j}+\hat{k}\right)=3$

$\therefore\vec{n}_1=\hat{i}+\hat{j}-2\hat{k}$ and $\vec{n}_2=2\hat{i}-2\hat{j}+\hat{k}$

Angle between the plane is'

$cos\theta=\frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}=\frac{|1(2)+1(-2)-2(1)|}{\sqrt{1^2+1^2+(-2)^2}.\sqrt{2^2+(-2)^2+1^2}}$

= $\frac{|-2|}{\sqrt{6}.\sqrt{9}}=\frac{2}{\sqrt{6}(3)}=\frac{2}{3\sqrt{6}}$

$\theta=cos^{-1}\left(\frac{2}{3\sqrt{6}}\right)$

11) A force of magnitude 6 units acting parallel to $2\hat{i}-2\hat{j}+\hat{k}$ displaces the point of application from (1, 2, 3) to (5, 3, 7). Find the work done.

b

12) Find the parametric form of vector equation of a line passing through a point (2, -1, 3) and parallel to line $\vec{r}=\left(\hat{i}+\hat{j}\right)+t\left(2\hat{i}+\hat{j}-2\hat{k}\right)$

p=-1

13) If the planes $\vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)=7$ and $\vec{r}.\left(\lambda\hat{i}+2\hat{j}-7\hat{k}\right)=26$ are perpendicular. Find the value of λ.

Δ=4

14) Let $\vec{a},\vec{b},\vec{c}$ be unit vectors such $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$. Prove that $\vec{a}=\pm2\left(\vec{b}\times\vec{c}\right)$

Type I even degree reciprocal equation

15) Prove that for any two vectors $\vec{a}$ and $\vec{b}|\vec{a}+\vec{b}|\leq|\vec{a}|+|\vec{b}|$ (Triangle inequality)

Answer : We have

$|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2+2(\vec{a}\cdot\vec{b})$

$\Rightarrow|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta$

$\leq|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\ [\because\cos\theta\leq1]$

$\Rightarrow|\vec{a}+\vec{b}|^2\leq(|\vec{a}|+|\vec{b}|)^2$

$\Rightarrow|\vec{a}+\vec{b}|\leq|\vec{a}|+|\vec{b}|$

16) For what value of m the vectors $\vec{a}$ and $\vec{b}$ perpendicular to each other.

$(i)\ \vec{a}=m\hat{i}+2\hat{j}+\hat{k}\ and\ \vec{b}=4\hat{i}-9\hat{j}+2\hat{k}$

$(ii)\ \vec{a}=5\hat{i}-9\hat{j}+2\hat{k}\ and\ \vec{b}=m\hat{i}+2\hat{j}+\hat{k}$

Answer : (i) Given $\vec{a} \perp \vec{b}$

$\therefore \vec{a} \cdot \vec{b} = 0 \Rightarrow (m\hat{i} + 2\hat{j} + \hat{k}) \cdot (4\hat{i} - 9\hat{j} + 2\hat{k}) = 0$

$\Rightarrow 4m - 18 + 2 = 0 \Rightarrow m = 4$

(ii) $(5\hat{i} - 9\hat{j} + 2\hat{k}) \cdot (m\hat{i} + 2\hat{j} + \hat{k}) = 0$

$\Rightarrow 5m - 18 + 2 = 0 \Rightarrow m = \frac{16}{5}$

17) Prove that $\left|\begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}\right| = abc$ if and only if $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular.

Answer : $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular $\Leftrightarrow \left|\left[\vec{a}, \vec{b}, \vec{c}\right]\right|$

$\Leftrightarrow |[\vec{a}, \vec{b},\ \vec{c}]| = |\vec{a}||\vec{b}||\vec{c}|$

$\Leftrightarrow |[\vec{a}, \vec{b},\ \vec{c}]| = abc$

18) If the edges $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}, \vec{b} = -5\hat{i} + 7\hat{j} - 3\hat{k},\ \vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$ meet at a vertex, find the volume of the parallelepiped.

Answer : Volume of the parallelepiped $= \begin{bmatrix} \vec{a}, & \vec{b}, & \vec{c} \end{bmatrix}$

$$= \begin{vmatrix} -3 & 7 & 5 \\ -5 & 7 & -3 \\ 7 & -5 & -3 \end{vmatrix}$$

= -264

The volume cannot be negative

Volume of parallelepiped = 264 cu. units.

19) If $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$ prove that $(\vec{c} \times \vec{a}) \times \vec{b} = \vec{0}$

Answer : Given $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$

$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$

$\Rightarrow (\vec{a} \cdot \vec{b})\vec{c} = (\vec{b} \cdot \vec{c})\vec{a}$

$\Rightarrow (\vec{c} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{b})\vec{c} = \vec{0}$

$\Rightarrow (\vec{c} \times \vec{a}) \times \vec{b} = \vec{0}$

20) Find the shortest distance between the parallel lines $\vec{r} = (\hat{i} - \hat{j}) + t(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} + \hat{k}) + s(2\hat{i} - \hat{j} + \hat{k})$

Answer : Comparing the given equations with

$\vec{r} = \vec{a} + s\vec{b}$ and $\vec{r} = \vec{c} + t\vec{b}$

$\vec{a} = \hat{i} - \hat{j} \quad \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{c} = 2\hat{i} + \hat{j} + \hat{k}$

$\vec{c} - \vec{a} = \hat{i} + 2\hat{j} + \hat{k}$

$$|(\vec{c} - \vec{a}) \times \vec{b}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{vmatrix}$$

$= \hat{i}(2 + 1) - \hat{j}(1 - 2) + \hat{k}(-1 - 4)$

$= 3\hat{i} + \hat{j} - 5\hat{k}$

$|(\vec{c} - \vec{a}) \times \vec{b}| = \sqrt{9 + 1 + 25} = \sqrt{35}$

$|\vec{b}| = \sqrt{4 + 1 + 1} = \sqrt{6}$

distance = $\frac{|(\vec{c} - \vec{a}) \times \vec{b}|}{|\vec{b}|}$

$= \frac{\sqrt{35}}{\sqrt{6}} = \sqrt{\frac{35}{6}}$ units.