QB365 Question Bank Software Study Materials

Complex Numbers Important 2 Marks Questions With Answers (Book Back and Creative)

12th Standard

Maths

Total Marks : 40

20 x 2 = 40

1) Evaluate the following if z = 5−2i and w = −1+3i

z + w

Answer : (z+w)

= (5-2i) + (-1+3i)

= (5-1) + i(-2+3)

= 4+i(1)

= 4+i

2) Find the following $\left|\frac{2+i}{-1+2i}\right|$

Answer : $\left|\frac{2+i}{-1+2i}\right| = \frac{|2+i|}{|-1+2i|} = \frac{\sqrt{2^2+1^2}}{\sqrt{(-1)^2+2^2}} = 1 \left(\because \left|\frac{z_1}{z_2}\right| = \left|\frac{z_1}{z_2}\right|, z_2 \neq 0\right)$

3) Show that the equation z^2 = $\bar{z}$ has four solutions.

Answer : We have, $z^2 = \bar{z}$

$\Rightarrow |z|^2 = |z|$

$|z|(|z|-1) = 0$

$\Rightarrow |z| = 0, or\, |z| = 1$

$|z| = 0 \Rightarrow z = 0$ is a solution $|z| = 1 \Rightarrow z\bar{z} = 1 \Rightarrow \bar{z} = \frac{1}{z}$

Given $z^2 = \bar{z} \Rightarrow z^2 = \frac{1}{z} \Rightarrow z^3 = 1$

It has 3 non-zero solutions. Hence including zero solution, there are four solutions.

4) Find the modulus of the following complex numbers

$\frac{2i}{3+4i}$

Answer : $\frac{2i}{3+4i}$

Let z = $\frac{2i}{3+4i}$

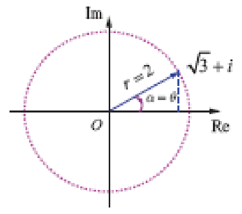
|z| = $\left|\frac{2i}{3+4i}\right| = \frac{|2i|}{|3+4i|} = \frac{\sqrt{2^2}}{\sqrt{3^2+4^2}} = \frac{2}{\sqrt{9+16}}$

= $\frac{2}{\sqrt{25}} = \frac{2}{5}$.

5) Find the modulus and principal argument of the following complex numbers.

$\sqrt{3}+i$.

Answer : $\sqrt{3}+i$



Modulus = $\sqrt{x^2+y^2} = \sqrt{\left(\sqrt{3}\right)^2+1^2} = \sqrt{3+1} = 2$

$\alpha = tan^{-1}\left|\frac{y}{x}\right| = tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$

Since the complex number $\sqrt{3+i}$ lies in the first quadrant, has the principal value

$\theta = \alpha = \frac{\pi}{6}$

Therefore, the modulus and principal argument of $\sqrt{3+i}$ are 2 and $\frac{\pi}{6}$ respectively.

6) Evaluate the following if z = 5−2i and w = −1+3i

z − iw

Answer : z-iw

= (5-2i) - i(-1+3i)

= (5-2i) + (+1-3i^2)

5 - 2i + i - 3(-1) = 5 - i + 3 = 8 - i

7) Evaluate the following if z = 5−2i and w = −1+3i

2z + 3w

Answer : 2z+3w

= 2(5-2i)+3(-1+3i)

= 10-4i-3+9i

= (10-3)+ i(-4+9)

= 7+5i

8) Simplify the following:

$i^{-1924} + i^{2018}$

Answer : $(i)^{-1924} + (i)^{2018} = (i)^{-1924+0} + (i)^{2016+2} = (i)^0 + (i)^2 = 1 - 1 = 0$

9) Find the following $\left|\overline{(1+i)}(2+3i)(4i-3)\right|$

Answer : $\left|\left(\overline{1+i}\right)(2+3i)(4i-3)\right| = \left|\left(\overline{1+i}\right)\right||2+3i||4i-3|$ $(\because |z_1 z_2 z_3| = |z_1||z_2||z_3|)$

= |1+i| |2+3i| |-3+4i| $(\because |z| = |\bar{z}|)$

= $\left(\sqrt{1^2+1^2}\right)\left(\sqrt{2^2+3^2}\right)\left(\sqrt{(3)^2+4^2}\right)$.

$= (\sqrt{2})(\sqrt{13})(\sqrt{25}) = 5\sqrt{26}$

10) Find the following $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$

Answer : $\left|\frac{i(2+i)^2}{(1+i)^2}\right| = \frac{|i|\left|(2+i)^3\right|}{\left|(1+i)^2\right|} = \frac{(\sqrt{4+1})^3}{(\sqrt{2})^2}$ $\left(\because \left|\frac{z_1}{z_2}\right| = \left|\frac{z_1}{z_2}\right|, z_2 \neq 0\right)$

= $\frac{(\sqrt{5})^3}{2} = \frac{5\sqrt{5}}{2}$.

11) If $(\cos\theta + i \sin\theta)^2 = x + iy$, then show that $x^2+y^2 = 1$

Answer : $(\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$

[By De moivre's theorem]

⇒ cos 2θ + isin 2θ = x + iy

Equating the real and imaginary parts we get,

x = cos 2θ, y = sin 2θ

∴ $x^2 + y^2 = \cos^2 2\theta + \sin^2 2\theta = 1$

Hence proved

12) Find the value of the complex number $(i^{25})^3$.

Answer : $i^{25} = (i^4)^6 \times i^1 = i^6 \times i = i$

∴ $|i^{25}| = |i| = 1$

13) Find the lcast positive integer n such that $\left(\frac{1+i}{1-i}\right)^n$

Answer : $\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$

$= \frac{1+i+i+i^2}{1-i^2} = \frac{1+2i-2i}{1+1} = \frac{2i}{2} = i$

Given $\left(\frac{1+i}{1-i}\right)^n = 1 \Rightarrow i^n = 1$

since $i^4 = i^8 = i^12..... = 1$

Hence the least positive integer n =4

14) If z = - 2 + 4i, then find $\text{Im}\left(\frac{z}{\bar{z}}\right)$.

Answer : $\frac{z}{\bar{z}}=\frac{-2+4i}{-2-4i}\times\frac{-2+4i}{-2+4i}$

$=\frac{(-2+4i)^2}{(-2)^2-(4i)^2}$

$=\frac{(-2)^2+(4i)^2+2(-2)(4i)}{4+16}$

$=\frac{4-16-16i}{20}$

$=\frac{-12-16i}{20}=\frac{4(-3-4i)}{20}$

$=\frac{-3}{5}-\frac{4i}{5}$

$\operatorname{Im}\left(\frac{z}{\bar{z}}\right)=\frac{-4}{5}$

15) Find the values of x and y for which the numbers - 3 + ix^2y and $x^2 + y^2$ + 4i are complex conjugates of each other.

Answer : - 3 + ix^2y = $x^2 + y^2$ + 4i

Equating real and imaginary parts

$x^2+y=-3$ ....(1)

$x^2y=-4$ ....(2)

$x^2=\frac{-4}{y}(\text{ or })\mathrm{y}=\frac{-4}{x^2}$

Substituting in (1)

$\frac{-4}{x^2}+x^2=-3$

$x^4+3x^2-4=0$

$\left(x^2+4\right)\left(x^2-1\right)=0$ [x^2+ 4≠0 for any real 'x']

$\Rightarrow x^2-1=0\Rightarrow x=\pm1$

From(2), y = -4 when x = ±1

Hence x = 1, y = ±2i (or)

x = -1, y = -4

16) If (a + ib) (c + id) (e + if) (g + ih) = A + iB, then prove that $\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right)=A^2+B^2$

Answer : (a + ib) (c + id) (e + if) (g + ih) = A + iB

Taking Modulus

|(a + ib)(c + id)(e + if)(g + ih)| = |A + iB|

|a+ib| |c+id| |e+if| |g+ih| = |A+iB|

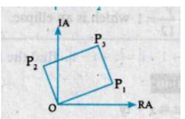
$\sqrt{a^2+b^2}\sqrt{c^2+d^2}\sqrt{e^2+f^2}\sqrt{g^2+h^2}=\sqrt{A^2+B^2}$

Squariag on both sides

$\left(a^2+b^2\right)\left(c^2+d^2\right)\left(e^2+f^2\right)\left(g^2+h^2\right)=A^2+B^2$

Hence proved.

17) From the figure, what does the point P_3 represent if the points P_1, and P_2 represent two complex numbers z_1 and z_2.

Answer : 

This is a Parallelogram $OP_1P_2P_3$ then the mid point of P_1P_2 and OP_3 are the same.

But the mid point of P_1P_2 is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

So, the co-ordinates of P_3 are ($x_1 + x_2$, $y_1 + y_2$) Thus the point P_3 corresponds to sum of the complex numbers z_1 and z_2

$\Rightarrow z_1 + z_2$

18) If |z + 1- i| = |z + i -1|, find the locus of z.

Answer : Let z = x + iy

$|x+iy+1-i|=|x+iy+i-1|$

$|(x+1)+i(y-1)|=|(x-1)+i(y+1)|$

$\sqrt{(x+1)^2+(y-1)^2}=\sqrt{(x-1)^2+(y+1)^2}$

Squaring on both sides

$(x+1)^2+(y-1)^2=(x-1)^2+(y+1)^2$

4x = 4y

x - y = 0 which is a straight line.

19) Simplify: $\frac{(\cos\theta+i\sin\theta)^4}{(\sin\theta+i\cos\theta)^5}$

Answer : $\frac{(\cos\theta+i\sin\theta)^4}{(\sin\theta+i\cos\theta)^5}=\frac{(\cos\theta+i\sin\theta)^4}{i^5\left(\frac{1}{i}\sin\theta+\cos\theta\right)^5}$

$=\frac{(\cos\theta+i\sin\theta)^4}{i(\cos\theta-i\sin\theta)^3}=\frac{(\cos\theta+i\sin\theta)^4}{i(\cos\theta+i\sin\theta)^{-5}}$

$=\frac{1}{i}[\cos\theta+i\sin\theta]^9$

$=\sin 9\theta-i\cos 9\theta$

20) Express $-1+i\sqrt{3}$ in polar form

Answer : $-1+i\sqrt{3}=r\cdot(\cos\theta+i\sin\theta)$

$r=\sqrt{(-1)^2+(\sqrt{3})^2}=\sqrt{1+3}=2$

$-1+i\sqrt{3}$ lies on second quadrant

$\theta=\pi-\alpha$

$\alpha=\tan^{-1}\frac{|y|}{|x|}$

$=\tan^{-1}\frac{\sqrt{3}}{1}$

$\alpha=\frac{\pi}{3}$

$\theta=\pi-\frac{\pi}{3}$

$\theta=\frac{2\pi}{3}$

$-1+\mathrm{i}\sqrt{3}=2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

$=2\left(\cos\left(2k\pi+\frac{2\pi}{3}\right)+i\sin\left(2k\pi+\frac{2\pi}{3}\right)\right),k\in Z$