QB365 Question Bank Software Study Materials

Inverse Trigonometric Functions Important 2 Marks Questions With Answers (Book Back and Creative)

12th Standard

Maths

Total Marks : 40

20 x 2 = 40

1) Find the principal value of

$Sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Answer : We know that $\sin^{-1}$: [-1, 1] $\rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$is given by

$\sin^{-1}$x = y if and only if x = sin y for $-1 \leq x \leq$ and -$\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$. Thus,

$Sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$= $\frac{\pi}{4}$, Since $\frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$and sin $\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

2) Find all values of x such that -6$\pi \leq x \leq 6\pi$ and cos x = 0

Answer : cos x = 0

$\Rightarrow x = (2n+1)\frac{\pi}{2}, n\varepsilon Z$

But $-6\pi \leq x \leq 6\pi$

$\therefore$ n can take values from

$x = (2n+1)\frac{\pi}{2}, n = 0 \pm 1, \pm 2, \ldots \pm 5$, -6

3) Find the value of

$2cos^{-1}\left(\frac{1}{2}\right) + sin^{-1}\left(\frac{1}{2}\right)$

Answer : $2cos^{-1}\left(\frac{1}{2}\right) + sin^{-1}\left(\frac{1}{2}\right)$

Let $cos^{-1}\left(\frac{1}{2}\right) = x$ and$sin^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow cosx = \frac{1}{2}$

$\Rightarrow cosx = cos\frac{\pi}{3}\ \left[\therefore \frac{\pi}{3}\varepsilon\,[0,\pi]\right]$

$\Rightarrow x = \frac{\pi}{3}$

[$\therefore$ Principal domain of sin is $\therefore \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and principal domain of cos is$[0, \pi]$

$siny = \frac{1}{2}$

$siny = sin\frac{\pi}{6}\ \left[\because \frac{\pi}{6}\varepsilon\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]\right]$

$\Rightarrow x = \frac{\pi}{3}$ and $y = \frac{\pi}{6}$

$\therefore\quad 2cos^{-1}\left(\frac{1}{2}\right) + sin^{-1}\left(\frac{1}{2}\right)$

= $2\left(\frac{\pi}{3}\right) + \frac{\pi}{6} = \frac{2\pi}{3} + \frac{\pi}{6}$

= $\frac{4\pi+\pi}{6} = \frac{5\pi}{6}$

4) Find the principal value of $\tan^{-1}(\sqrt{3})$

Answer : Let $\tan^{-1}(\sqrt{3})$ = y

Then, tan y =$\sqrt{3}$

Thus, y = $\frac{\pi}{3}$

Since $\in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Thus the principal value of $\tan^{-1}(\sqrt{3})$ = $\frac{\pi}{3}$

5) Find the value of

$sin\left[\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right]$

Answer : $sin\left[\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right] = sin\left[\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right] = sin\left(\frac{\pi}{2}\right) = 1$

6) Find the value of $sin^{-1}\left(sin\left(\frac{5\pi}{4}\right)\right)$

Answer : $sin^{-1}\left(sin\left(\frac{5\pi}{4}\right)\right)$

= $sin^{-1}\left(sin\left(\pi + \frac{\pi}{4}\right)\right) \because \frac{5\pi}{4} \notin \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

= $sin^{-1}\left(sin\left(-\frac{\pi}{4}\right)\right)$

= $\frac{-\pi}{4}\epsilon\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

7) Find the value of

$cos^{-1}\left(\frac{1}{2}\right)+sin^{-1}(-1)$

Answer : $cos^{-1}\left(\frac{1}{2}\right)+sin^{-1}(-1)$

Let $cos^{-1}\left(\frac{1}{2}\right)=x$ and -1 = sin y

$\Rightarrow cosx=\frac{1}{2}=cos\frac{\pi}{3}$

$x=\frac{\pi}{3}$

$siny=-1=-sin\left(\frac{-\pi}{2}\right)$

= $-sin\left(\frac{-\pi}{2}\right)$

$\left[\because sin\left(-\theta\right)=-sin\ \theta\ and\ \frac{\pi}{2}\varepsilon\left[\frac{-\pi}{2},\frac{\pi}{2}\right]\right]$

$\Rightarrow y=\frac{-\pi}{2}$

$\therefore cos^{-1}\left(\frac{1}{2}\right)+sin^{-1}\left(-1\right)=x+y$

= $\frac{\pi}{3}-\frac{\pi}{2}=\frac{2\pi-3\pi}{6}=-\frac{\pi}{6}$

$\therefore cos^{-1}\left(\frac{1}{2}\right)+sin^{-1}(-1)=-\frac{\pi}{6}$

8) Find the domain of

g(x) = $\sin^{-1}$x + $\cos^{-1}$x

Answer : Given g(x) = $\sin^{-1}$ x + $\cos^{-1}$x

From the definition of $\sin^{-1}$x.

$-1\leq x\leq 1$ ...(1)

Also from the definition of $\cos^{-1}$x

$-1\leq x\leq 1$ ....(2)

$\therefore$ From (1) & (2),

Domain ofg(x) = [-1, 1] U [-1, 1]

= [-1, 1]

Hence the domain of g(x) is [-1, 1].

9) Find the value of

tan($\tan^{-1}$(-0.2021)).

Answer : tan($\tan^{-1}$(0.2021))

= -0.2021[$\because$ tan ($\tan^{-1}$x) = x for any real number]

10) Find the principal value of $sin^{-1}\left(sin\left(-\frac{\pi}{3}\right)\right)$

Answer : We know that $\sin^{-1}$: [-1, 1] $\rightarrow\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$is given by

$\sin^{-1}$x = y if and only if x = sin y for $-1\leq x\leq$ and -$\frac{\pi}{2}\leq y\leq\frac{\pi}{2}$. Thus,

$sin^{-1}\left(sin\left(-\frac{\pi}{3}\right)\right)$ = -$\frac{\pi}{3}$, since -$\frac{\pi}{3}\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

11) Find all the values of x such that

-3$\pi\leq x\leq$-3π and sin x = -1

Answer : sin x = -1

$\Rightarrow sinx=sin\left(\frac{-\pi}{2}\right)\Rightarrow x=\frac{-\pi}{2},\frac{3\pi}{2},\frac{7\pi}{2}...$

$\Rightarrow x=(4n-1)\frac{\pi}{2},n\varepsilon Z.$

$\Rightarrow x=(4n-1)\frac{\pi}{2}$

n takes the values $0,\pm1,\pm2,\pm3and\pm4$

since when n = -4, $x=\frac{-17\pi}{2}<-8\pi$

12) Find the principal value of $tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

Answer : Let $tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)=y$ where $\frac{-\pi}{2}<y<\frac{\pi}{2}$

$\Rightarrow tan\ y=\frac{-1}{3}=-tan\frac{\pi}{6}=tan\left(\frac{-\pi}{6}\right)$

$y=\frac{-\pi}{6}\left[\because\frac{-\pi}{6}\epsilon\left(\frac{-\pi}{2},\frac{\pi}{2}\right)\right]$

The principal value of $tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)=6$

13) Prove that $tan^{-1}\left(\frac{1}{7}\right)+tan^{-1}\left(\frac{1}{13}\right)=tan^{-1}\left(\frac{2}{9}\right)$

Answer : L.H.S = $tan^{-1}\left(\frac{1}{7}\right)+tan^{-1}\left(\frac{1}{13}\right)$

= $tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{3}}{1-\left(\frac{1}{7}\right)\left(\frac{1}{13}\right)}\right)=tan^{-1}\left(\frac{\frac{13+7}{91}}{\frac{91-1}{91}}\right)=\tan^{-1}\left(\frac{\frac{20}{91}}{\frac{90}{91}}\right)=\tan^{-1}\left(\frac{20}{91}\times\frac{91}{90}\right)$

$=\tan^{-1}\left(\frac{20^2}{90^9}\right)=\tan^{-1}\left(\frac{2}{9}\right)=\text{RHS}$

Hence proved.

14) Prove that $2tan^{-1}\left(\frac{2}{3}\right)=tan^{-1}\left(\frac{12}{5}\right)$

Answer : LHS = $2tan^{-1}\left(\frac{2}{3}\right)$

= $tan^{-1}\left(\frac{2}{3}\right)+tan^{-1}\left(\frac{2}{3}\right)$

= $tan^{-1}\left(\frac{\frac{2}{3}+\frac{2}{3}}{1-\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)}\right)=tan^{-1}\left(\frac{\frac{4}{3}}{\frac{9-2}{9}}\right)$

= $tan^{-1}\left(\frac{4}{3}\times\frac{9}{7}\right)=tan^{-1}\left(\frac{12}{7}\right)$

= RHS

Hence proved

15) Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Answer : Let $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=y$, where $\frac{-\pi}{2}<y<\frac{\pi}{2}$

$\Rightarrow \sin y=\frac{1}{\sqrt{2}}\Rightarrow \sin y=\sin\frac{\pi}{4}\Rightarrow y=\frac{\pi}{4}$

The principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi}{4}$

16) Find the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$

Answer : Let $\operatorname{cosec}^{-1}(-\sqrt{2})$ = y, where $y\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]\{-0\}$

$\Rightarrow \operatorname{cosec} y=-\sqrt{2}$

$\Rightarrow \operatorname{cosec} y=\operatorname{cosec}\left(\frac{-\pi}{4}\right)\ [\because \operatorname{cosec}(-\theta)=-\operatorname{cosec}\theta]$

$y=\frac{-\pi}{4}$

The principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$ is $-\left(\frac{\pi}{4}\right)$

$\Rightarrow y=\frac{-\pi}{4}$

17) Prove that $3\cos^{-1}x=\cos^{-1}\left(4x^3-3x\right), x\in\left[\frac{1}{2},1\right]$

Answer : Let $\cos^{-1}x=\theta\Rightarrow x=\cos\theta$

Consider $\cos 3\theta=4\cos^3\theta-3\cos\theta$

$\Rightarrow 3\theta=\cos^{-1}\left(4\cos^3\theta-3\cos\theta\right)$

$\Rightarrow 3\cos^{-1}x=\cos^{-1}\left(4x^3-3x\right)$

18) Evaluate $\tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right)$

Answer : $\tan^{-}\left(\sin\left(\frac{-\pi}{2}\right)\right)=\tan^{-1}\left(-\sin\left(\frac{\pi}{2}\right)\right)$

$=\tan^{-1}(-1)=-\frac{\pi}{4}$

19) Find the value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$

Answer : $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)=\tan^{-1}\left(\tan\left(\pi+\frac{\pi}{8}\right)\right)$

$=\tan^{-1}\left(\tan\left(\frac{\pi}{8}\right)\right)=\frac{\pi}{8}$

20) Evaluate $\tan\left(\tan^{-1}(-4)\right)$

Answer : Since, $\tan\left(\tan^{-1}x\right)=x,\forall x\in\mathbb{R}$

$\tan\left(\tan^{-1}(-4)\right)=-4$