QB365 Question Bank Software Study Materials

Theory of Equations Important 2 Marks Questions With Answers (Book Back and Creative)

12th Standard

Maths

Total Marks : 40

20 x 2 = 40

1) If α, β and γ are the roots of the cubic equation $x^3+2x^2+3x+4 = 0$, form a cubic equation whose roots are, 2α, 2β, 2γ

Answer : The roots of $x^3+2x2+3x+4 = 0$ are ∝, β, ɤ

∴ ∝+β+ɤ = -co-efficient of x^2 = -2 ...(1)

∝β + βɤ + ɤ∝ = co-effficient of x = 3 ....(2)

-∝βɤ = +4 ⇒ ∝βɤ = -4 ...(3)

Form a cubic equation whose roots are 2∝, 2β, 2ɤ

2∝+2β+2ɤ = 2(∝+β+ɤ) = 2(-2) = -4 [from (1)]

4∝β+4βɤ+4ɤ∝ = 4(∝β+βɤ+ɤ∝) = 4(3) = 12 [from (2)]

(2∝)(2β)(2ɤ) = 8(∝βɤ) = 8(-4) = -32 [from (3)]

∴ The required cubic equation is

x^3-(2∝+2β+2ɤ)x^2 + (2∝β+2βɤ+2ɤ∝)x - (2∝)(2β)(2ɤ) = 0

⇒ x^3+(-4)x^2+12x+32 = 0

⇒ $x^3+4x^2+12x+32 = 0$

2) If p is real, discuss the nature of the roots of the equation $4x^2+ 4px + p + 2 = 0$ in terms of p.

Answer : The discriminant Δ =$((4p)^2 - 4(4)(p+2) = 16(p^2-p-2) = 16(p+1)(p-2)$. So we get

Δ < 0 if -1 < p < 2

Δ = 0 if p = -1 or p = 2

Δ > 0 if ∞. Thus the given polynomial has

imaginary roots if -1 < p < 2

equal real roots if p = −1 or p = 2;

distinct real roots if -∞ < p < -1 or 2 < p < ∞

3) Find a polynomial equation of minimum degree with rational coefficients, having 2-$\sqrt{3}$ as a root.

Answer : Since 2-$\sqrt{3}$i is a root and the coefficients are rational numbers, 2+$\sqrt{3}$i is also a root. A required polynomial equation is given by

x^2 −(Sum of the roots) x + Product of the roots = 0

and hence

x^2- 4x +1 = 0 is a required equation.

4) Show that the equation $2x^2- 6x +7 = 0$ cannot be satisfied by any real values of x.

Answer : Δ = b^2- 4ac = -20 < 0. The roots are imaginary numbers.

5) If $x^2+2(k+2)x+9k = 0$ has equal roots, find k.

Answer : Here Δ = b^2−4ac = 0 for equal roots. This implies $4(k + 2)^2 = 4(9)k$. This implies k = 4 or 1.

6) Obtain the condition that the roots of $x^3+ px^2+ qx + r = 0$ are in A.P.

Answer : Let the roots be in A.P. Then, we can assume them in the form α-d, α, α+d

Applying the Vieta's formula (α-d)+α+(α+d) = $\frac{p}{1}$ = p ⇒ 3α = -p ⇒ α = -$\frac{p}{3}$.

But, we note that αis a root of the given equation. Therefore, we get

$\left(-\frac{p}{3}\right)^3+p\left(-\frac{p}{3}\right)^3+q\left(-\frac{p}{3}\right)^3+r$ = 0 ⇒ 9 pq = $2p^3$+27r.

7) It is known that the roots of the equation $x^3- 6x^2- 4x + 24 = 0$ are in arithmetic progression. Find its roots.

Answer : Let the roots be a−d, a, a+d.

Then the sum of the roots is 3a which is equal to 6 from the given equation.

Thus 3a = 6 and hence a = 2.

The product of the roots is $a^3 - ad^2$ which is equal to −24 from the given equation.

Substituting the value of a, we get $8-2d^2 = -24$ and hence d = ±4.

If we take d = 4 we get −2, 2, 6 as roots and if we take d = −4, we get 6, 2, −2 as roots (same roots given in reverse order) of the equation.

8) If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, form a cubic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

Answer : The roots of $x^3+2x^2+3x+4 = 0$ are ∝, β, ɤ

∴ ∝ + β + ɤ = -co-efficient of x^2 = -2 ....(1)

∝β + βɤ + ɤ∝ = co-effficient of x = 3 ....(2)

-∝βɤ = +4 ⇒ ∝βɤ = -4 ....(3)

From the cubic equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

$$\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\gamma\alpha+\alpha\beta}{\alpha\beta\gamma}=\frac{3}{-4}=\frac{-3}{4}$$

$$\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}=\frac{\gamma+\alpha+\beta}{\alpha\beta\gamma}=\frac{-2}{-4}=\frac{1}{2}$$

$$\left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)\left(\frac{1}{\gamma}\right)=\frac{1}{\alpha\beta\gamma}=\frac{1}{-4}=-\frac{1}{4}$$

∴ The required cubic equation is

$$x^3-\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)x^2+\left(\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}\right)x-\left(\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma}\right)$$

$$\Rightarrow x^3+\frac{3}{4}x^2+\frac{1}{2}x+\frac{1}{4}=0$$

Multiplying by 4 we get,

$4x^3 + 3x^2 + 2x + 1 = 0$

9) If α, β and γ are the roots of the cubic equation $x^3 + 2x^2 + 3x + 4 = 0$, form a cubic equation whose roots are −α, -β, -γ

Answer : The roots of $x^3+2x^2+3x+4 = 0$ are ∝, β, ɤ

∴ ∝ + β + ɤ = -co-efficient of x^2 = -2 ...(1)

∝β + βɤ + ɤ∝ = co-effficient of x = 3 ...(2)

-∝βɤ = +4 ⇒ ∝βɤ = -4 ...(3)

Form the equation whose roots are ∝-β-ɤ

∴ -∝-β-ɤ = -(∝+β+ɤ)

= -(-2) = 2

∝β + βɤ + ɤ∝ = 3

(-∝)(-β)(-ɤ) = -(∝βɤ) = -(-4) = 4

∴ The required cubic equation is

x^3-(-∝-β-ɤ)x^2+(∝β+βɤ+ɤ∝)

x-[(-∝)(-β)(-ɤ)] = 0

⇒ $x^3-(2)x^2+3x-4 = 0$

⇒ $x^3-2x^2+3x-4 = 0$

10) Construct a cubic equation with roots $2, \frac{1}{2}$ and 1

Answer : The cubic equation is

$x^3 - x^2 (\alpha + \beta + \gamma) + x (\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma = 0$

$$x^3-x^2\left(2+\frac{1}{2}+1\right)+x\left(1+\frac{1}{2}+2\right)-(2)\left(\frac{1}{2}\right)(1)=0$$

$$x^3-x^2\left(\frac{4+1+2}{2}\right)+x\left(\frac{2+1+4}{2}\right)-1=0$$

$$x^3-x^2\left(\frac{7}{2}\right)+x\left(\frac{7}{2}\right)-1=0$$

$2x^3 - 7x^2 + 7x - 2 = 0$

11) Formulate into a mathematical problem to find a number such that when its cube root is added to it, the result is 6.

Answer : Let that number be x $\therefore \sqrt[3]{x}+x=6$

$\Rightarrow \sqrt[3]{x}=6-x$

Taking power 3 both sides we get.

$\left(x^{\frac{1}{3}}\right)^{3}=(6-x)^{3}$

$x=6^{3}-3(6^{2})x+3(6)(x^{2})-x^{3}$

$[\because (a-b)^{3}=a^{3}-3a^{2}b+3ab^{2}-b^{3}]$

$\Rightarrow x=216-108x+18x^{2}-x^{3}$

$\Rightarrow x^{3}+108x-18x-216+x=0$

⇒ x^3-18x^2+109x -216 = 0. Which is the requires mathematical problem.

12) Construct a cubic equation with roots 2, −2, and 4.

Answer : Here ∝ = 2, β = -2 and ɤ = 4

x^3-(2-2+4)x^2+(- 4 - 8 + 8x)x-(2)(-2)(4) = 0

⇒x^3- 4x^2- 4x+16 = 0

13) If sin ∝, cos ∝ are the roots of the equation ax^2 + bx + c-0 (c ≠ 0), then prove that $(n + c)^2$ - b^2 + c^2

Answer : Sum of the roots = sin ∝ + cos ∝ = $\frac{-b}{a}$

Product of the roots = sin ∝ cos ∝ = $\frac{c}{a}$

Now 1 = $\cos^2$∝ + $\sin^2$ ∝

= $(\sin ∝ + \cos ∝)^2$ - 2 sin ∝ cos ∝

$1=\frac{b^{2}}{a^{2}}-\frac{2c}{a} \Rightarrow 1=\frac{b^{2}-2ac}{a^{2}}$

⇒ a^2 = b^2 - 2ac ⇒ a^2 + 2ac = b^2

Adding c^2 both sides, a^2 +2ac+c^2 = b^2+c^2

⇒ $(a+c)^2$ = b^2 + c^2

14) Find value of a for which the sum of the squares of the equation x^2 - (a- 2) x - a -1 = 0 assumes the least value.

Answer : Let ∝, β are the roots of the equation

Sum of the roots $\alpha+\beta=\frac{-b}{a}$

$=\frac{[-(a-2)]}{1}=a-2$

Product of the roots $=\alpha\beta=\frac{c}{a}$

$=\frac{-(a+1)}{1}=-(a+1)$

we have $\alpha^{2}\beta^{2}=(\alpha+\beta)^{2}-2\alpha\beta$

$=(a-2)^{2}+2(a+1)$

$=a^{2}-4a+4+2a+2$

$=(a-1)^{2}+5$

Thus $\alpha^{2}+\beta^{2}$ is least if a = 1

15) Find the Interval for a for which 3x^2+2(a^2+1) x+(a^2-3a+2) possesses roots of opposite sign.

Answer : The quadratic equation 3x^2 + 2(a^2 + 1)x + (a^2 - 3a + 2)

Will have two roots of opposite sign if it has real roots and the product of the roots is negative.

⇒ 4$(a^2+1)^2$-12(a^2-3a+2)$\geq$ 0 and $\frac{a^{2}-3a+2}{3}<0$

Both of these conditions are true if

⇒ a^2-3a+2< 0

⇒ (a-1)(a-2)< 0

⇒ 1< a< 2

16) Find the number of positive and negative roots of the equation x^7 - 6x^6 + 7x^5 + 5x^2+2x+2

Answer : Let p(x) = x^7-6x^6+7x^5+5x^2+2x+2

It has only one change of sign. Now,

p(-x) = $(-x)^7$ -6$(-x)^6$ +7$(-x)^5$ +5$(-x)^2$ +2(-x)+2

= -x^7 -6x^6 -7x^5 + 5x^2 - 2x + 2

It has two, change of sign.

Hence, p(x) has one positive root and has at least two negative roots.

17) Find a polynomial equation of the lowest degree with rational co-efficients having $\sqrt{3}$ and 1 - 2i as two of its roots.

Answer : Since quadratic surds occur in pairs as roots, -$\sqrt{3}$ is also a root.

Since complex roots occur in conjugate pairs, 1+ 2i is also a root of the required polynomial equation. Therefore the desired equation is given by

$(x-\sqrt{3})(x+\sqrt{3})(x-(1-2i))(x-(1+2i))=0$

i.e., $x^4-2x^3+2x^2+6x-15=0$

18) If the roots of the equation $x^3+\mathrm{p}x^2+\mathrm{q}x+\mathrm{r}=0$ are in arithmetic progression, show that $2p^3 - 9pq + 27r = 0$

Answer : Let the roots of the given equation be a - d, a, a + d.

Then $\mathrm{S}_1=a-d+a+a+d=3a=-p \Rightarrow a=-\frac{p}{2}$

Since a is a root, it satisfies the given polynomial

$\Rightarrow \left(-\frac{p}{3}\right)^3+p\left(-\frac{p}{3}\right)^2+q\left(-\frac{p}{3}\right)+r=0$

On simplification, we obtain $2p^3-9pq+27r=0$

19) Form an equation whose roots are three times those of the equation $x^3-x^2+x+1=0$

Answer : To obtain the required equation, we have to multiply the co-efficients of x^3, x^2, x and 1 by 1, 3, 3^2 and 3^3 respectively.

Thus $x^3-3x^2+9x+27=0$ is the desired equation.

20) Form an equation whose roots are the reciprocals of the roots of $x^4-5x^3+7x^2-4x+5=0$

Answer : We obtain the required equation, by replacing the co-efficients in the reverse order, as

$5x^4-4x^3+7x^2-5x+1=0$