QB365 Question Bank Software Study Materials

Two Dimensional Analytical Geometry-II Important 2 Marks Questions With Answers (Book Back and Creative)

12th Standard

Maths

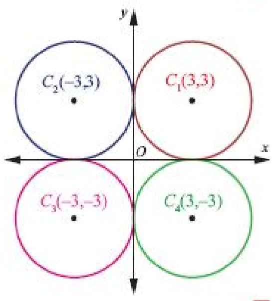
Total Marks : 40

20 x 2 = 40

1) A circle of radius 3 units touches both the axes. Find the equations of all possible circles formed in the general form.

Answer : As the circle touches both the axes, the distance of the centre from both the axes is 3 units, centre can be (±3, ±3) and hence there are four circles with radius 3, and the required equations of the four circles are

$x^2 + y^2 \pm 6x \pm 6y + 9 = 0$.



2) Find the centre and radius of the circle $3x^2 + (a + 1)y^2 + 6x - 9y + a + 4 = 0$.

Answer : Coefficient of x^2 = Coefficient of y^2 (characteristic (ii) for a second degree equation to represent a circle).

That is, 3 = a + 1 and a = 2 .

Therefore the equation of the circle is

$3x^2 + 3y^2 + 6x - 9y + 6 = 0$

$x^2 + y^2 + 2x - 3y + 2 = 0$

So, centre is $\left(-1, \frac{3}{2}\right)$ and radius $r = \sqrt{1 + \frac{9}{4} - 2}$

$= \sqrt{\frac{5}{2}}$

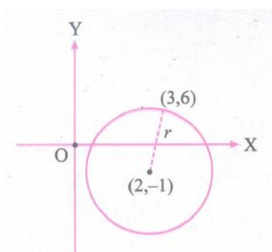
3) If y = 4x + c is a tangent to the circle $x^2 + y^2 = 9$, find c

Answer : The condition for the line y = mx + c to be a tangent to the circle $x^2 + y^2 = a^2$ is $c^2 = a^2(1 + m^2)$ from

Then $c = \pm\sqrt{9(1+16)}$

$c = \pm 3\sqrt{17}$

4) Find the equation of the circle with centre (2, -1) and passing through the point (3, 6) in standard form.

Answer : 

Given centre is (2, -1) and passing through the point (3, 6)

∴ r = distance between (2, -1) and (3, 6)

$= \sqrt{(2-3)^2 + (-1-6)^2}$

$= \sqrt{(-1)^2 + (-7)^2}$

$= \sqrt{1 + 49} = \sqrt{50}$

∴ Equation of the circle is

$(x - h)^2 + (y - k)^2 = r^2$

$(x-2)^2+(y+1)^2 = (\sqrt{50})^2$

$(x-2)^2+(y+1)^2 = 50$

5) Find the equation of circles that touch both the axes and pass through (-4, -2) in general form.

Answer : Since the circle touch both the axis. Its equation will be

$(x + a)^2 + (y + a)^2 = a^2$ ....(1)

It passes through (-4, -2)

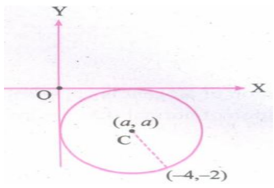
$\therefore (-4 + a)^2 + (-2 + a)^2 = a^2$

$$16 + \cancel{a^2} + 8a + 4 + a^2 + 4a = \cancel{a^2}$$

$\Rightarrow a^2 + 12a + 20 = 0$

$\Rightarrow (a + 10)(a + 2) = 0$

a = -10 or -2



Case (i):

When a = -10, (1) becomes

$(x + 10)^2 + (y + 10)2 = 10^2$

$$\Rightarrow x^2 + \cancel{100} + 20x + y^2 + \cancel{100} + 20y = 160$$

$\Rightarrow\ x^2 + y^2 + 20x + 20y + 100 = 0$

Case (ii):

When a = -2, (1) becomes

$\Rightarrow (x + 2)^2 + (y + 2)^2 = 2^2$

$$x^2 + 4x + 4 + y^2 + 4y + \cancel{4} = \cancel{4}$$

$x^2 + y^2 + 4x + 4y + 4 = 0$

Hence, equation of the circles are

$x^2 + y^2 + 4x + 4y + 4 = 0$

or $x^2 + y^2 + 20x + 20y + 100 = 0$

6) Identify the type of conic section for each of the equations.

$3x^2+3y^2-4x+3y+10 = 0$

Answer : Here A = 3, B = 0, C = 3, D = -4, E = 3 and F = 10

A = C and B = 0 (No xy term)

Hence, the given equations represents a circle.

7) Identify the type of the conic for the following equations :

$11x^2-25y^2-44x+50y-256 = 0$

Answer : A = 11, C = -25, D = -44, E = - 50, and F = -256

Here A ≠ C and A and C are of opposite signs. Hence, the given equation represents a hyperbola.

8) Find centre and radius of the following circles.

$x^2 + y^2 + 6x - 4y + 4 = 0$

Answer : Equation of the circle is

$x^2 + y^2 + 6x - 4y + 4 = 0$.

Here $2g = 6 \Rightarrow g = 3$

$2f = -4 \Rightarrow f = -2$ and $c = 4$

Centre is $(-g, -f) \Rightarrow (-3, 2)$

$r = \sqrt{g^2 + f^2 - c} = \sqrt{3^2 + (-2)^2 - 4}$

$= \sqrt{9 + 4 - 4}$

$= \sqrt{9}$

= 3 unit

9) Find centre and radius of the following circles.

$x^2+y^2-x+2y-3 = 0$

Answer : Equation of the circle is $x^2 + y^2 - x + 2y - 3 = 0$

Here $2g = -1 \Rightarrow g = \frac{-1}{2}$

$2f = 2 \Rightarrow f = 1$ and $c = -3$

Centre is $(-g, -f) = \left(\frac{1}{2}, -1\right)$

and $r = \sqrt{g^2 + f^2 - c}$

$= \sqrt{\frac{1}{4} + 1 + 3}$

$= \sqrt{\frac{1}{4} + 4} = \sqrt{\frac{1+16}{2}}$

$r = \sqrt{\frac{17}{2}}$ units.

10) Find the equation of the hyperbola in each of the cases given below:

Centre (2, 1) one of the foci (8, 1) and corresponding directrix x = 4.

Answer : ae = distance between centre and focus

$ae = \sqrt{(8-2)^2 - (1-1)^2} = \sqrt{6^2} = 6$ ...(1)

Also $\frac{a}{e} = \sqrt{(4-2)^2 + (1-1)^2} = \sqrt{2^2} = 2$

[∴ (4, 1) is a point on the directrix]

$(1) \times (2) \rightarrow a\not{e} \times \frac{a}{\not{e}} = 6 \times 2$

$\Rightarrow a^2 = 12$

$(1) \rightarrow a^2e^2 = 36$

$12(e^2) = 36$

$\Rightarrow e^2 = 3$

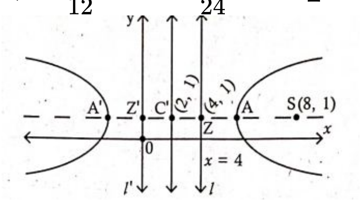
$\Rightarrow e = \sqrt{3}$

Also, $b^2 = a^2(e^2 - 1) = 12(3 - 1) = 12(2) = 24$

∴ Equation of the hyperbola is

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$\Rightarrow \frac{(x-2)^2}{12} - \frac{(y-1)^2}{24} = 1$



11) Find the equation of the parabola with vertex at the origin, passing through (2, -3) and symmetric about x-axis

Answer : Since the parabola is symmetric about x-axis, it is either open upward or downward.

Let the equation be $x^2 = 4ay$ ...(1)

Since (2, -3) lies on the parabola,

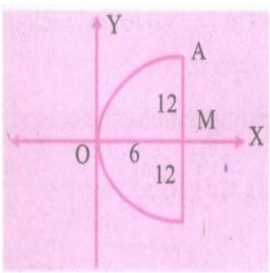
$2^2 = 4a(-3) \Rightarrow a = \frac{-1}{3}$

Substituting $a = \frac{-1}{3}$ in (1) we get,

$x^2 = 4\left(\frac{-1}{3}\right)y \Rightarrow 3x^2 = -4y$. Which is the required equation of the parabola.

12) If a parabolic reflector is 24 cm in diameter and 6 cm deep, find its locus.

Answer : Let AOB be the vertical section of the reflector and m is the mid-point of AB. Let the equation of the parabola be $y^2 = 4ax$ A(6, 12) lies on (1)



∴ $12^2 = 4a(6) \Rightarrow a = 6$

∴ Focus is (a, 0) = (b, 0)

Hence focus coincides with m, the mid-point of AB.

13) The equation $y = \frac{1}{32}x^2$ models cross sections of parabolic mirrors that are used for solar energy. There is a heating tube located at the focus of each parabola; how high is this tube located above the vertex of the parabola?

Answer : a = 8

So the heating tube needs to be placed at focus (0,a) . Hence the heating tube needs to be placed 8 units above the vertex of the parabola.

14) Find the length of the tangent from (2, 3) to the circle $x^2+y^2-4x-3y+12=0$

Answer : The length of the tangent to the circle $x^2+y^2+2gx+2fy+c=0$ from the point (x_1, y_1) is $\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$

Length of the tangent to the given circle

$=\sqrt{x_1^2+y_1^2-4x_1-3y_1+12}$

$=\sqrt{2^2+3^2-4.2-3.3+12}$

$=\sqrt{4+9-8-9+12}$

$=\sqrt{8}$

$=2\sqrt{2}$ units

15) Find the equations and lengths of major and minor axes of $\frac{(x-1)^2}{9^2}+\frac{(y+1)^2}{16}=1$

Answer : Let x - 1 = X and y + 1 = Y

$\therefore$ The given equation becomes $\frac{X^2}{9}+\frac{Y^2}{16}=1$

Clearly the major axis is along Y-axis and the minor axis is along X-axis.

The equation of major axis is X = 0 and the equation of minor axis is Y = 0.

i.e., the equation of major axis is x - 1 = 0 and the equation of minor axis is y + 1 = 0

Here $a^2=16,\ b^2=9$

$\therefore$ Length of major axis (2a) = 3

$\therefore$ Length of minor axis (2b) = 6

16) Find the equations of directrices, latus rectum and length of latus rectums of the following ellipse $25x^2+9y^2=225$

Answer : $\therefore \frac{x^2}{9}+\frac{y^2}{25}=1$

Here $a^2=25,\ b^2=9$

$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$

The equations of the directrices are y = $\pm\frac{a}{e}$

$y=\frac{\pm 25}{4}$

Equations of the latus rectum are y = $\pm$ ae

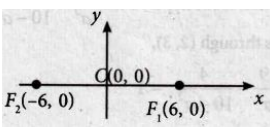
y = $\pm$4

Length of the latus rectum is $\frac{2b^2}{a}=\frac{2\times 9}{5}$

$=\frac{18}{5}$

17) Find the equation of the hyperbola whose foci are ($\pm$ 6, 0) and length of the transverse axis is 8.

Answer : From the given data the transverse a:ris is along x-axis



The equation is of the form

$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

The centre is the midpoint of F_1 and F_2

i.e., C is $\left(\frac{-6+6}{2},\frac{0+0}{2}\right)=(0,0)$

The length of the transverse axis 2a = 8

$\Rightarrow a=4$

$F_1F_2=2ae=12\Rightarrow ae=6$

$\therefore 4e=6$

$e=\frac{6}{4}=\frac{3}{2}$

$b^2=a^2\left(e^2-1\right)=16\left(\frac{9}{4}-1\right)$

$=\frac{16\times 5}{4}=20$

$\therefore$ The required equation is $\frac{x^2}{16}-\frac{y^2}{20}=1$

18) Find the equations and length of transverse and conjugate axes of the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$

Answer : The centre is at the origin, the transverse axis is along x -axis and the conjugate axis is along the y -axis i.e., transverse axis is x axis

i.e., y = 0 and the conjugate axis y- axis i.e., x = 0.

Hence $a^2 = 9, b^2 = 4 \Rightarrow a = 3, b = 2$

Length of transverse axis = 2a = 6

Length of conjugate axis = 2b = 4

19) Find the equations of directrices, latus rectum and length of latus rectrum of the hyperbola $9x^2 - 36x - 4y^2 - 16y + 56 = 0$

Answer : $\frac{(y+2)^2}{9} - \frac{(x-2)^2}{2} = 1$

The transverse axis is parallel to y axis.

$a^2 = 9 \quad b^2 = 4$

$a = 3 \quad b = 2$

$c^2 = a^2 + b^2$

$= 9 + 4c^2 = 13$

$c = \sqrt{13}$

$ae = \sqrt{13}$

$3e = \sqrt{13} \Rightarrow e = \frac{\sqrt{13}}{3}$

$\text{Centre } (h, k) = (2, -2)$

$\text{directrix } y = \pm\frac{a}{e} + k$

$= \pm\frac{3}{\sqrt{13}} - 2$

$y = \pm\frac{9}{\sqrt{13}} - 2$

Length of latus rectum $= \frac{2b^2}{a}$

$= \frac{2(4)}{3} = \frac{8}{3}$

20) Find the equation of the parabola. if the curve ie open leftward, vertex is (2,1) and passing through the point (1, 3)

Answer : Since the curve is open leftward, the required equation of the parabola is

$(y-k)^2 = -4a(x-h)$

Given vertex (h, k) = (2, 1)

$\therefore (y-1)^2 = -4a(x-2)$ ....(2)

Since this pass through (1, 3) we get

$(3-1)^2 = -4a(1-2)$

$4 = -4a(-1)$

$a = 1$

$\therefore (1) \Rightarrow (y-1)^2 = -4(x-2)$

which is required equation of the parabola