QB365 Question Bank Software Study Materials

Applications of Differential Calculus Important 2 Marks Questions With Answers (Book Back and Creative)

12th Standard

Maths

Total Marks : 40

2 Marks

20 x 2 = 40

1) A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water?

Answer : Let r be the radius of the ripple and A be the area of the ripple.

GIven $\frac{dr}{dt}$ = 2 cm/sec and r = 5 cm ... (1)

We know A = πr^2

Differentiating with respect to 't' we get,

$\frac{dA}{dt}$ = $\pi(2r).\frac{dr}{dt}$

= π(2) (5) (2) [using (1)]

$\frac{dA}{dt}$ = 20 πsq.cm/sec.

2) Find the points of x the curve y = $x^3 - 3x^2 + x - 2$ at which the tangent is parallel to the line y = x

Answer : The slope of the line y = x is 1. The tangent to the given curve will be parallel to the line, if the slope of the tangent to the curve at a point is also 1. Hence,

$\frac{dy}{dx} = 3x^2 - 6x + 1 = 1$

which gives $3x^2 - 6x = 0$

Hence, x = 0 and x = 2.

Therefore, at (0, –2) and (2, –4) the tangent is parallel to the line y = x.

3) Find the angle of intersection of the curve y = sin x with the positive x -axis.

Answer : The curve y = sin x intersects the positive x -axis. When y = 0 which gives, x =

$x = n\pi, n = 1, 2, 3, \dots$

Now, $\frac{dy}{dx} = cosx$. The slpoe $x = n\pi$ are $cos(n\pi) = (-1)^n$.

Hence, the required angle of intersection is m_2 = 0

$tan\theta = \frac{(-1)^n - 0}{1+((-1)^n(0)} = 1 \forall\ n$

4) Find the absolute extrema of the following functions on the given closed interval.

$f(x) = 6x^{\frac{3}{4}} - 3x^{\frac{1}{3}}; [-1, 1]$

Answer : $f'(x) = 6 \times \frac{4}{3}x^{\frac{4}{3}-1} - 3 \times \frac{1}{3}x^{\frac{1}{3}-1}$

= $8x^{\frac{1}{3}} - x^{\frac{-2}{3}}$

f'(x) = 0

$\Rightarrow 8x^{\frac{1}{3}} - \frac{1}{x^{\frac{2}{3}}} = 0$

$\Rightarrow \frac{8x-1}{x^{\frac{2}{3}}} = 0$

$\Rightarrow x = \frac{1}{8}$

Thus, the critical number is $x = \frac{1}{8}$

Evaluating f(x) at the end points = -1

x = 1 and at the critical number x = $\frac{1}{8}$

we get

$f(-1) = 6(-1)^{\frac{4}{3}} - 3(-1)^{\frac{1}{3}}$

= 6(1) - 3 (-1) = 6 + 3 = 9

$f(1) = 6(1)^{\frac{4}{3}} - 3(1)^{\frac{1}{3}} = 6 - 3 = 3$

$f\left(\frac{1}{8}\right) = 6\left(\frac{1}{8}\right)^{\frac{4}{3}} - 3\left(\frac{1}{8}\right)^{\frac{1}{3}}$

=$6\left(2^{-3}\right)^{\frac{4}{3}} - 3\left(2^{-3}\right)^{\frac{1}{3}}$

$= \frac{}{16} - \frac{}{2} = \frac{}{8} - \frac{}{2}$

$= \frac{3-12}{8} = \frac{-9}{8}$

From these values, the absolute maximum is 9 which occurs at x = -1 and the absolute minimum is $-\frac{9}{8}$ which occurs at x = $\frac{1}{8}$

5) Find the values in the interval (1, 2) of the mean value theorem satisfied by the function f (x) = x − x^2 for 1 ≤ x ≤ 2

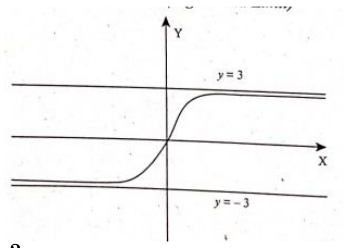
Answer : f (1) = 0 and f(2) = -2. Clearly f (x) is defined and differentiable in 1< x < 2.

Therefore, by the Mean Value Theorem, there exists a c∈(1, 2) such that by

$f'(c) = \frac{f(2)-f(1)}{2-1} = 1 - 2c$

That is $1 - 2c = -2 \Rightarrow c = \frac{3}{2}$

6) Find the asymptotes of the following curves $f(x) = \frac{3x}{\sqrt{x^2+2}}$

Answer : 

$f(x) = \frac{3x}{\sqrt{x^2+2}}$

$\lim\limits_{x \to \infty} \frac{3x}{\sqrt{x^2+2}} = \lim\limits_{\frac{1}{x} \to 0} \frac{3}{\sqrt{1+\frac{2}{x^2}}}$

$= \frac{3}{\sqrt{1+0}} = 3$

∴y = 3 is the horizontal asymptote.

Also $\lim\limits_{x \to -\infty} \frac{3x}{\sqrt{x^2+2}} = \lim\limits_{\frac{1}{x} \to 0^-} \frac{3}{\sqrt{1+\frac{2}{x^2}}} = -3$

∴ y = -3 is the horizontal asymptote

7) Compute the limit $\lim\limits_{x \to a}(\frac{x^n-a^n}{x-a})$

Answer : If we put directly x = a we observe that the given function is in an indeterminate form $\frac{0}{0}$.

As the numerator and the denominator functions are polynomials they both are differentiable.

Hence by an application of the l'Hôpital Rule we get,

$\lim\limits_{x \to a}(\frac{x^n-a^n}{x-a})$ = $\lim\limits_{x \to a}(\frac{n \times x^{n-1}}{1})$

= $n \times a^{n-1}$.

8) $\lim\limits_{\theta \to 0}(\frac{1-\cos m\theta}{1-\cos n\theta})$ =1, then prove that, $m = \pm n$

Answer : As this is an indeterminate form $\left(\frac{0}{0}\right)$ using the l'Hôpital's Rule

$\lim\limits_{\theta \to 0}(\frac{1-cosm\theta}{1-cosn\theta})$=$\lim\limits_{\theta \to 0}(\frac{m-sinm\theta}{n-sinn\theta})$

$\lim\limits_{\theta \times 0} \frac{m}{n}(\frac{\frac{\sin m\theta}{\theta}}{\frac{\sin n\theta}{\theta}}) = \frac{m^2}{n^2}$

Therefore, m^2 = n^2

That is $m = \pm n$.

9) Evaluate: $\lim\limits_{x \to \infty}(\frac{x^2+17x+29}{x^4})$.

Answer : This is an indeterminate of the form $(\frac{\infty}{\infty})$.

To evaluate this limit, we apply l 'Hôpital Rule.

$\lim\limits_{x \to \infty}(\frac{x^2+17x+29}{x^4}) = \lim\limits_{x \to \infty}(\frac{2x+17}{4x^3})$

= $\lim\limits_{x \to \infty}(\frac{2}{12x^2}) = 0$

10) Prove that the function f (x) = x^2 + 2 is strictly increasing in the interval (2,7) and strictly decreasing in the interval (−2, 0)

Answer : We have,

$f'(x) = 2x > 0, \forall x \in (2, 7)$ and

$f'(x) = 2x > 0, \forall x \in (-2, 0)$

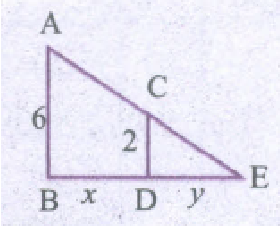
and hence the proof is completed.

11) A man 2 m high walks at a uniform speed of 5 km/ hr away from a lamp post 6 m high. Find the rate at which the length of his shadow increases?

Answer : Let AB be the lamp post. Let the man CD be at distance x m from lamp post and y m be the length of his shadow at any time t.

Given $\frac{dx}{dt}$ = 5 km / hr = 5000 m/hr

ΔABE and CDE are similar



$\therefore \frac{DE}{CD} = \frac{BE}{AB}$

$\Rightarrow \frac{y}{2} = \frac{x+y}{6}$

⇒ 6y = 2x+2y

⇒ 4y = 2x

⇒ y = $\frac{x}{2}$

$\Rightarrow \frac{dy}{dt} = \frac{1}{2}\frac{dx}{dt} = \frac{1}{2}(5000)$

= 2500 m/hr = 2.5 km/hr.

12) Find the point on the parabola y^2=18x at which the ordinate increases at twice the rate of the abscissa.

Answer : $\left(\frac{9}{8}, \frac{9}{2}\right)$

13) Verify Rolle 's Theorem for $f(x) = |x-1|, O \leq x \leq 2$

Answer : Rolle's theorem is not valid.

14) Prove that the function f(x)=$2x^2$+3x is strictly increasing on $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Answer : f(x) is strictly increasing $\left[-\frac{1}{2}, \frac{1}{2}\right]$

15) Determine the domain of concavity of the curve y=2-x^2

Answer : concave downward everywhere

16) show that the distance from the origin to the plane 3x + 6y + 2z + 7 = 0 is 1

Answer : Distance from the orgin (0, 0, 0) to the plane is

$= \left|\frac{3(0)-6(0)+2(0)+7}{\sqrt{9+36+4}}\right|$

$= \frac{7}{\sqrt{49}}$

$= \frac{7}{7} = 1$

17) Find the local maxima and local minima (if any) for the function cos x, $0 < x < \pi$

Answer : Let(x) = cos x, $0 < x < \pi$

f'(x) = -sinx

For maximum or minimum, $f'(x) = 0 \Rightarrow \sin x = 0$ But $\sin x \neq 0$ for $0 < x < \pi$

f(x) has neither local maxima nor local minima for 0 < x < π

18) Find absolute maxima and absolute minima for the function $f(x) = x^3, x \in [-2, 2]$

Answer : $f(x) = x^3$

$f'(x) = 3x^2$

$f'(x) = 0 \Rightarrow 3x^2 = 0$

$x = 0 \in [-2, 2]$

Now, $f(-2) = (-2)^3 = -8$

$f(0) = 0$

$f(2) = (2)^3 = 8$

∴ Absolute maximum is 8 at x = 2

Absolute minimum is - 8 at x = - 2

19) Evaluate: $\lim_{x \to 0^+} x \log x$

Answer : $\lim_{x \to 0^+} x \log x \quad [0 \times \infty$ Indeterminate form]

$= \lim_{x \to 0^+} \frac{\log x}{\frac{1}{x}} \quad [\frac{\infty}{\infty}$ Indeterminate form]

Applying L' Hopital's rule

$= \lim_{x \to 0^+} \frac{\left(\frac{1}{x}\right)}{\frac{-1}{x^2}}$

$= \lim_{x \to 0^+} (-x) = 0$

20) Evaluate: $\lim_{x \to 2} \frac{\sin \pi x}{2-x}$

Answer : $\lim_{x \to 2} \frac{\sin \pi x}{2-x} \quad (\frac{0}{0} form$

Applying L' Hopital's rule

$= \lim_{x \to 2} \frac{\pi \cos \pi x}{-1}$

$= -\pi \cos 2\pi = -\pi$