QB365 Question Bank Software Study Materials

Applications of Integration Important 2 Marks Questions With Answers (Book Back and Creative)

12th Standard

Maths

Total Marks : 40

2 Marks

20 x 2 = 40

1) Evaluate $\int_0^1 xdx$, as the limit of a sum.

Answer : Here f (x) = x, a = 0 and b = 1. Hence, we get

$\int_a^b f(x)dx = \lim\limits_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right) \Rightarrow \int_0^1 xdx = \lim\limits_{n\to\infty} \frac{1}{n}\sum_{r=1}^{n} \frac{r}{n}$

$= \lim\limits_{n\to\infty} \frac{1}{n^2}[1+2+\ldots+n]$

$= \lim\limits_{n\to\infty} \frac{1}{n^2}\frac{n(n+1)}{2} = \lim\limits_{n\to\infty} \frac{1}{2}\left(1+\frac{1}{n}\right) = \frac{1}{2}$

2) Evaluate: $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}}$ x cos x dx.

Answer : Let f (x) = x cos x

Then f (−x) = (−x) cos(−x) = −x cos x = − f (x).

So f (x) = x cos x is an odd function.

Hence, applying the property, for odd function f(x), $\int_{-a}^{a} f(x)dx = 0$

$\therefore$ we get $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} xcosxdx = 0$

3) Evaluate the following definite integrals:

$\int_3^4 \frac{dx}{x^2-4}$

Answer : $\int_3^4 \frac{dx}{x^2-4}$

$\int_3^4 \frac{dx}{x^2-4} = \int_3^4 \frac{dx}{x^2-2^2}$

$\left[\because \int \frac{dx}{x^2-a^2} = \frac{1}{2a}log\left|\frac{x-a}{x+a}\right| + c\right]$

$= \frac{1}{4}\left[log\left(\frac{4-2}{4+2}\right) - log\left(\frac{3-2}{3+2}\right)\right]$

$= \frac{1}{4}log\left[\left(\frac{2}{6}\right) - log\ \frac{1}{5}\right]$

$= \frac{1}{4}log\left(\frac{1}{3}\times 5\right)$

$= \frac{1}{4}log\left(\frac{5}{3}\right)$

4) Evaluate $\int_b^\infty \frac{1}{a^2+x^2}dx, a > 0, b \in R$

Answer : $\int_b^\infty \frac{1}{a^2+x^2}dx = \left[\frac{1}{a}tan^{-1}\frac{x}{a}\right]_b^\infty = \frac{1}{a}tan^{-1}\infty - \frac{1}{a}tan^{-1}\frac{b}{a} = \frac{1}{a}\left[\frac{\pi}{2} - tan^{-1}\frac{b}{a}\right]$

5) Evaluate $\int_0^{\frac{\pi}{2}}$ ($\sin^2$ x + $\cos^4$ x) dx

Answer : Given that I = $\int_0^{\frac{\pi}{2}}$ ($\sin^2$x + $\cos^4$x)dx = $\int_0^{\frac{\pi}{2}}$ $\sin^2$x dx+ $\int_0^{\frac{\pi}{2}}$ $\cos^4$x dx $\frac{1}{2}\times\frac{\pi}{2}+\frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2} = \frac{7\pi}{16}$

6) Evaluate $\int_0^{\frac{\pi}{2}} \begin{vmatrix} cos^4x & 7 \\ sin^5x & 3 \end{vmatrix}$ dx

Answer : I = $\int_0^{\frac{\pi}{2}}$ (3cos ^{4}x-7sin^{5}x)dx = 3 $\int_0^{\frac{\pi}{2}}$ $\cos^4$ x dx-7 $\int_0^{\frac{\pi}{2}}$ $\sin^5$ x dx

= 3 × $\frac{3}{4}$ × $\frac{1}{2}$ × $\frac{\pi}{2}$ - 7 × $\frac{4}{3}$ × $\frac{2}{3}$ = $\frac{9\pi}{16}$ - $\frac{56}{15}$.

By applying the reduction formula III iteratively, we get the following results (stated without proof):

(i) If n is even and m is even,

$\int_0^{\frac{\pi}{2}}$ sin ^{m} x cos ^{n} x dx = $\frac{(n-1)}{m+n}$ $\frac{(n-3)}{m+n-2}$ $\frac{(n-5)}{m+n-4}$ ...... $\frac{1}{m+2}$ $\frac{m-1}{m}$ $\frac{m-3}{m-2}$ $\frac{m-5}{m-4}$ ... $\frac{1}{2}$ $\frac{\pi}{2}$

(ii) If n is odd and m is any positive integer (even or odd), then

$\int_0^{\frac{\pi}{2}}$ sin ^{m} x cos^{n}x dx = $\frac{(n-1)}{m+n}$ $\frac{(n-3)}{m+n-2}$ $\frac{(n-5)}{m+n-4}$ ... $\frac{2}{m+3}$ $\frac{1}{m+1}$

7) Find the values of the following:

$\int_0^{\frac{\pi}{2}}$ sin ^{5}x $\cos^4$xdx

Answer : $\int_0^{\frac{\pi}{2}} sin^4 xcos^6 xdx = \frac{(6-1)}{(6+4)} \cdot \frac{(6-3)}{(6+4-2)} \cdot \frac{(6-5)}{(6+4-4)} \cdot \frac{(4-1)}{(4)} \cdot \frac{(4-3)}{(4-2)} \cdot \frac{\pi}{2}$
$= \frac{(5)}{(10)} \frac{(3)}{(8)} \frac{(1)}{(6)} \frac{(3)}{(4)} \frac{(1)}{(2)} \frac{\pi}{2} = \frac{3\pi}{512}$
Also, $\int_0^{\frac{\pi}{2}} sin^4 xcos^6 xdx = \int_0^{\frac{\pi}{2}} sin^6 xcos^4 xdx = \frac{(3)}{(10)} \frac{(1)}{(8)} \frac{(5)}{(6)} \frac{(3)}{(4)} \frac{(1)}{(2)} \frac{\pi}{2} = \frac{3\pi}{512}$

8) Evaluate $\int_0^1 x^3(1-x)^4 dx$

Answer : $\int_0^1 x^m(1-x)^n dx = \frac{m!\times n!}{(m+n+1)!}$
$\therefore \int_0^1 x^3(1-x)^4 dx = \frac{3!\times 4!}{(3+4+1)!} = \frac{3!\times 4!}{8!} = \frac{3\times2\times1\times4\times3\times2\times1}{8\times7\times6\times5\times4\times3\times2\times1} = \frac{1}{280}$

9) Evaluate the following
$\int_0^{\frac{\pi}{2}} sin^3\theta cos^5\theta d\theta$

Answer : $I = \int_0^{\frac{\pi}{2}} sin^3\theta cos^5\theta d\theta$
$\mathrm{m} = 3, \mathrm{n} = 5$
$\int_0^{\pi/2}(cos^5\theta - cos^7\theta)sin\theta d\theta$
$t = cos\theta$
$dt = -sin\theta d\theta$
$= \int_1^0 (t^2 - t^7)(-dt)$
$= \int_1^0 (t^5 - t^7)(dt) = [\frac{t^6}{6} - \frac{t^8}{8}]$
$\frac{1}{6} - \frac{1}{8} = \frac{8-6}{48}$
$= \frac{1}{24}$
Aliter method:
$I = \int_0^{\frac{\pi}{2}} sin^3\theta cos^5\theta d\theta$
Here m = 3, which is odd and n = 5, which is odd
$I = \int_0^{\frac{\pi}{2}} sin^m\theta cos^n xdx$
$\frac{n-1}{m+n} \cdot \frac{n-3}{m+n-2} \cdot \frac{n-5}{m+n-4} \cdots \frac{2}{m+3} \cdot \frac{1}{m+1}$
$\int_0^{\pi/2} \sin^3\theta \cos^5\theta d\theta = \frac{A}{8} \times \frac{2}{6} \times \frac{1}{A}$
$= \frac{1}{24}$

10) Evaluate $\int_0^\infty e^{-ax}x^n dx$, where a > 0 .

Answer : Making the substitution t = ax, we get dt = adx and x = 0 ⇒ t = 0 and x = ∞ ⇒ t = ∞
Hence, we get
$\int_0^\infty e^{-ax}x^n dx = \int_0^\infty e^{-t}\left(\frac{t}{a}\right)^n \frac{dt}{a} = \int_0^\infty e^{-t}t^n dt$
$= \frac{1}{a^{n+1}} \int_0^\infty e^{-x}x^n dx = \frac{n!}{a^{n+1}}$
Thus
$\int_0^\infty e^{-ax}x^n dx = \frac{n!}{a^{n+1}}$

11) Evaluate $\int_0^{\frac{\pi}{2}} e^{3x}cosxdx$

Answer : $\frac{1}{10}\left[e^{\frac{3\pi}{2}} - 1\right]$

12) Evaluate $\int_0^1 \frac{e^x}{1+e^{2x}}dx$

Answer : $tan^{-1}e - \frac{\pi}{4}$

13) Find the area of the region bounded by the curves $y=2^x.y=-2x-x^2$ and the lines x=0 and x=2

Answer : $\frac{a}{log2} - \frac{20}{2}$

14) Find the area of the region bounded by the curve y = sin x and the ordinate x=0 $x = \frac{\pi}{3}$

Answer : $\frac{1}{2}$

15) Find the area of the region bounded by the curve y=sin x and the ordinate x=0.$x = \frac{\pi}{3}$

Answer : $\frac{1}{2}$

16) Write the Reduction Formulae III and IV

Answer : G

17) Evaluate $\int_0^2 \sqrt{\frac{2+x}{2-x}} dx$

Answer : $I = \int_0^2 \sqrt{\frac{2+x}{2-x} \times \frac{2+x}{2+x}} dx$

$= \int_0^2 \sqrt{\frac{(2+x)^2}{4-x^2}} dx$

$= \int_0^2 \frac{2+x}{\sqrt{4-x^2}} dx$

$= \int_0^2 \frac{2}{\sqrt{4-x^2}} dx + \int_0^2 \frac{x}{\sqrt{4-x^2}} dx$

$= \left[2\sin^{-1}\frac{x}{2}\right]_0^2 - \left[\sqrt{4-x^2}\right]_0^2 = 2 \times \frac{\pi}{2} + 2$

$I = \pi + 2$

18) Evaluate $\int_0^\pi \theta \sec^2 \theta d\theta$

Answer : $u = \theta \;\; dv = \sec^2 \theta d\theta$

$u' = 1 \;\; v = \tan\theta$

$v_1 = \log\sec\theta$

$\int_0^{\pi/4} \theta \sec^2 \theta d\theta = [\theta\tan\theta - \log\sec\theta]_0^{\pi/4}$

$= \frac{\pi}{4} - \log\sqrt{2}$

$= \frac{\pi}{4} - \frac{1}{2}\log 2$

$\int_0^{\pi/4} \theta \sec^2 \theta d\theta = \frac{1}{4}(\pi - 2\log 2)$

19) Prove that $\int_0^{\frac{\pi}{2}} \frac{f(\sin x)}{f(\sin x)+f(\cos x)} dx = \frac{\pi}{4}$

Answer : Let I $= \int_0^{\frac{\pi}{2}} \frac{f(\sin x)}{f(\sin x)+f(\cos x)} dx$ ....(1)

$I = \int_0^{\frac{\pi}{2}} \frac{f\left(\sin\left(\frac{\pi}{2}-x\right)\right)}{f\left(\sin\left(\frac{\pi}{2}-x\right)\right)+f\left(\cos\left(\frac{\pi}{2}-x\right)\right)} dx$

$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$I = \int_0^{\frac{\pi}{2}} \frac{f(\cos x)}{f(\cos x)+f(\sin x)} dx$ ....(2)

From (1) and (2) we get,

$2I = \int_0^{\frac{\pi}{2}} \frac{f(\sin x)+f(\cos x)}{f(\sin x)+f(\cos x)} dx$

$= \int_0^{\pi/2} dx = [x]_0^{\pi/2}$

$2I = \frac{\pi}{2}$

$\therefore I = \frac{\pi}{4}$

Hence proved.

20) Evaluate $\int_0^1 \frac{|x|}{x} dx$

Answer : $\mathrm{I} = \mathrm{I} = \int_0^1 \frac{|x|}{x} \mathrm{~d}x$

$= \int_0^1 1 \mathrm{~d}x$

$= [x]_0^1$

$\mathrm{I} = 1$

$f(x) = \begin{cases} \frac{x}{x}, x > 0 \\ \frac{-x}{x}, x \le 0 \end{cases}$

$= \begin{cases} +1, x > 0 \\ -1, x \le 0 \end{cases}$