QB365 Question Bank Software Study Materials

Differentials and Partial Derivatives Important 2 Marks Questions With Answers (Book Back and Creative)

12th Standard

Maths

Total Marks : 40

2 Marks

20 x 2 = 40

1) Use the linear approximation to find approximate values of $\sqrt[4]{15}$

Answer : Let f(x) = $x^{\frac{1}{4}}$, x_0 16, Δx = -1

∴ $\sqrt[4]{15}$ = f(16) +f'(16) (-1) ... (1)

$f(16) = (16)^{\frac{1}{4}} = (2^4)^{\frac{1}{4}} = 2^1 = 2$

$f'(16) = \frac{1}{4}x^{\frac{1}{4}-1} = \frac{1}{4}x^{-\frac{3}{4}} = \frac{2}{4x^{\frac{3}{4}}}$

$f'(125) = \frac{1}{6(2^4)x^{\frac{3}{4}}} = \frac{1}{4(2^3)} = \frac{1}{4(8)} = \frac{1}{32}$

∴ becomes

$\sqrt[4]{15} = 2 + \frac{1}{32}(-1)$

$= 2 - \frac{1}{32} = 2 - 0.0312$

$\sqrt[4]{15} = 1.968$

2) Let f (x, y) = 0 if xy ≠ 0 and f (x, y) = 1 if xy = 0.

Calculate: $\frac{\partial f}{\partial x}(0,0), \frac{\partial f}{\partial y}(0,0).$

Answer : Note that the function f takes value 1 on the x, y-axes and 0 everywhere else on R^2. So let us calculate

$\frac{\partial f}{\partial x}(0,0)$ = $\lim_{h\longrightarrow 0} \frac{f(0+h,0)-f(0,0)}{h} = \lim_{h\longrightarrow 0} \frac{1-1}{h} = 0;$

$\frac{\partial f}{\partial y}(0,0)$ = $\lim_{k\longrightarrow 0} \frac{f(0,0+k)-f(0,0)}{k} = \lim_{k\longrightarrow 0} \frac{1-1}{k} = 0$

This completes (i).

3) Evaluate $\lim_{(x,y)\to(0,0)} cos = \left(\frac{e^x siny}{y}\right)$, if the limit exists.

Answer : $\lim_{(x,y)\to(0,0)} cos = \left(\frac{e^x siny}{y}\right)$ = $cos\left(e^0 \frac{siny}{y}\right)$

= cos[(1)(1)] = cos (1) $\left[\because \lim_{y\to 0} \frac{siny}{y} = 1\right]$

4) If v(x, y, z) = $x^3 + y^3 + z^3$ + 3xyz, show that $\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial z \partial y}$

Answer : Given v(x, y, z) = $x^3 + y^3 + z^3 + xyz^3$

$\frac{\partial v}{\partial z}$ = 0 + 0 +$3z^2$ + 3xy = $3z^2$ + 3xy

$\frac{\partial v}{\partial y}$ = 0 + $3y^2$ + 0 + 3xz = $3y^2$ + 3xz

Now, $\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial z}\right)$ = 0 + 3x = 3x ...(1)

$\frac{\partial^2 v}{\partial z \partial y} = \frac{\partial}{\partial z}\left(\frac{\partial v}{\partial y}\right)$ = 0 + 3x = 3x ..... (2)

From (1) and (2),

$\frac{\partial^2 v}{\partial y \partial z} = \frac{\partial^2 v}{\partial z \partial y}$

5) If u (x, y) = $x^2y + 3xy^4$, x = e^t and y = sin t, find $\frac{du}{dt}$ and evaluate If at t = 0.

Answer : Given u (x, y) = $x^2y + 3xy^4$, x = e^t, y = sin t

$\frac{du}{dt} = \frac{\partial u}{\partial x}.\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}.\frac{\partial y}{\partial t}$ ...(1)

x = e^t

⇒ $\frac{dx}{dt} = e^t$

y = sin t

⇒ $\frac{dy}{dt} = cos\ t$

$\frac{\partial u}{\partial x} = 2xy + 3y^4$;
$\frac{\partial u}{\partial y} = x^2 + 12xy^3$
$\Rightarrow \frac{\partial u}{\partial x} = 2e^t sint + 3sin^4 t$
$\frac{\partial u}{\partial y} = e^{2t} + 12e^t sin^3 t$
Substituting in (1) we get,
$\frac{du}{dt}$ = ($2e^t$ sin t + 3 $\sin^4$t)e^t + (e^2t + 12e^t $\sin^3$ t) cos t
$\frac{du}{dt}$ = 2e^{2t}sin t + 3 e^t $\sin^4$ t + e^{2t} cos t + 12e^t $\sin^4$ t cos t
= e^t[2e^t sin t + 3 $\sin^4$ t + cos t + 12 $\sin^3$ t cos t]
$\left(\frac{du}{dt}\right)_{t=0}$ = e^0[2(1) (0) +3(0) + 1+ 12 (0)]
= 1 [1] = 1
∴ $\left(\frac{du}{dt}\right)$ = 1

6) If w (x, y, z) = x^2 + y^2 + y^2, x = e^t, y = e^t sin t, z = e^t cos t, find $\frac{dw}{dt}$

Answer : Given w (x, y, z) = x^2 +y^2 +y^2,
x = e^t, y = e^t sin t, z = e^t cos t
$\frac{\partial u}{\partial x}$ = 2x; $\frac{\partial u}{\partial y}$ = 2y; $\frac{\partial u}{\partial z}$ = 2z
$\frac{\partial u}{\partial x}$ = 2e^t
$\frac{\partial u}{\partial y}$ = 2e^t sin t
$\frac{\partial u}{\partial z}$ = 2e^t cos t
$\frac{dx}{dt}$ = e^t
$\frac{dy}{dt}$ = e^t cas t + sin t e^t
⇒ $\frac{dz}{dt}$ = et (- sin t) + cos t et
By chain rule
$\frac{dw}{dt} = \frac{\partial u}{\partial x}.\frac{dx}{dt} + \frac{dw}{\partial y}.\frac{dy}{dt} + \frac{\partial w}{\partial z}.\frac{dz}{dt}$
∴ $\frac{dw}{dt}$ = 2e^t(e^t) + 2e^t sin t (e^t cos t +sin t e^t) - e^t sin t + 2e^t cas t (e^t cos t - e^t sin t)
$\therefore \frac{dw}{dt} = e^{2t}\left[2 + 2\sin t\cos t + 2\sin^2 t - 2\sin t\cos t + 2\cos^2 t\right]$
= e^{2t} [2 + 2] = 4e^{2t}

7) In each of the following cases, determine whether the following function is homogeneous or not. If it is so, find the degree.
f(x, y) = x^2y + 6x^3 + 7

Answer : Given f(x, y) = x^2y + 6x^3 + 7
f(λx, λy) = $\lambda^2 x^2 \lambda y + 6\lambda^3 x^3 + 7$
= λ^3 x^2 y + 6λ^3 x^3 + 7
≠ λf(x, y)
There is no common λ in this equation.
It is not homogeneous

8) In each of the following cases, determine whether the following function is homogeneous or not. If it is so, find the degree.
$U(x, y, z) = xy + sin\left(\frac{y^2 - 2x^2}{xy}\right)$

Answer : Given $U(x, y, z) = xy + sin\left(\frac{y^2 - 2x^2}{xy}\right)$
$u(\lambda x, \lambda y, \lambda z) = \lambda x \lambda y + sin\left(\frac{\lambda^2 y^2 - 2\lambda^2 z^2}{\lambda x \lambda y}\right)$
$= \lambda^2 xy + sin\left(\frac{y^2 - 2x^2}{xy}\right)$
≠ λp. u (x, y, z)
There is no common λ
∴. It is not homogeneous.

9) If u(x, y) = $\frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$

Answer : Given u (x, y) = $\frac{x^2+y^2}{\sqrt{x+y}}$

$u(\lambda x, \lambda y) = \frac{\lambda^2 x^2+\lambda^2 y^2}{\sqrt{\lambda x+\lambda y}}$

= $\frac{\lambda^2(x^2+y^2)}{\sqrt{\lambda}(\sqrt{x+y})}$

= $\lambda^{2-\frac{1}{2}}u(x,y)$

= $\lambda^{\frac{3}{2}}u(x,y)$

∴ u (x, y) is a homogeneous function of degree $\frac{3}{2}$

∴ By Euler's theorem,

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ = n.u = $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{3}{2}u$

Hence, proved.

10) Show that F(x,y) = $\frac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1.

Answer : We compute

F(λx, λy) = $\frac{(\lambda x)^2+5(\lambda x)(\lambda y)-10(\lambda y)^2}{3\lambda x+7\lambda y}=\frac{\lambda^2}{\lambda}\left(\frac{x^2+5xy-10y^2}{3x+7y}\right)=\lambda F(x,y)$

for all λ ∈ R. So F is a homogeneous function of degree 1.

11) If of f(x, y) = $x^2 + y^3 + 2xy^2$ find f_{xx}, f_{yy}, f_{xy} and f_{yx}.

Answer : Given f(x,y) = $x^2 + y^3 + 2xy^2$

$f_x = 3x^2 + 2y^2$

$f_{xx} = 6x$

$f_y = 0 + 3y^2 + 4xy$

$= 3y^2 + 4xy$

$f_{yy} = 6y + 4x$

$f_{xy} = 4y$

$f_{yx} = 4y$

12) IF u(x, y) = $x^2 + 3xy + y^2$, x, y, ∈ R, find tha linear appraoximation for u at (2, 1)

Answer : Given u(x, y) = $x^2 + 3xy + y^2$

$u(x_o, y_o)$ = u(2,1)

= $2^2 + 3(2)(1) + 1^2$

= 4 + 6 + 1 = 11

$\frac{\partial u}{\partial x}$ = 2x+ 3y

$\left(\frac{\partial u}{\partial x}\right)_{(2,1)}$ = 2 + 3 = 5

$\frac{\partial u}{\partial y}$ = 3x+ 2y

$\left(\frac{\partial u}{\partial y}\right)_{(2,1)}$ = 6 + 2 = 8

Linear approximation

L(x,y) = $U(x_o, y_o)$ + $\left(\frac{\partial u}{\partial x}\right)_{(x_0,y_0)}$ $(x - x_o)$ + $\left(\frac{\partial u}{\partial y}\right)_{(x_0,y_0)}$ $(y - y_o)$

L (x,y) = 11 + 5 (x - 2) + 8 (y - 1)

= 11 + 5x - 10 + 8y - 8

L(x,y) = 5x + 8y - 7

13) If w=log(x^2+y^2),x=cosθ,y=sinθ, find $\frac{dw}{d\theta}$

Answer : $\frac{\vartheta^2 u}{\vartheta x \vartheta y}$

14) If $w = e^{x^2+y^2}$,x=cosθ,y=sinθ, find $\frac{dw}{d\theta}$

Answer : $\frac{dw}{d\theta}=0$

15) If y = $x^3 + 2x^2$ find dy when x = 2 and dx =0.1

Answer : $f(x) = x^3 + 2x^2$
$f'(x) = 3x^2 + 4x$
$dy = \left(3x^2 + 4x\right) dx$
$dy = (3(4) + 4(2))0.1$
= (12+8) 0.1
$= 20 \times 0.1 = 2$

16) The radius of a sphere was measured and found to be 21 cm with a possible error in measurement of atmost 0.05 cm. What is the maximum error in using this value of the radius to compute the volume of the sphere?

Answer : Volume of the sphere V $= \frac{4}{3}\pi r^3$
$dV = \frac{4}{3}\pi 3r^2 dr$
$dV = 4\pi(21)^2(0.05)$
= 277
The maximum error in the calculated volume is about 277 cm^3.

17) Find the differential of the function y = x tan x, x = $\pi/4$, dx = 1

Answer : y = x tan x
$dy = \left(x\sec^2 x + \tan x\right) dx$
$= \left(\pi/4 \sec^2 \pi/4 + \tan \pi/4\right) \times 1$
$= (\pi/4 \times 2 + 1) \times 1$
$= \left(\frac{\pi}{2} + 1\right) \times 1 = \left(\frac{3.14}{2} + 1\right)$
dy = 2.57

18) If $u = \sin^{-1}\left(\frac{x^4+y^4}{x^2+y^2}\right)$then check whether it is a homogeneous function or not.

Answer : $u = \sin^{-1}\left(\frac{x^4+y^4}{x^2+y^2}\right)$
u is not a homogeneous function
Let $\sin u = \left(\frac{x^4+y^4}{x^2+y^2}\right) = f(x, y)$
$\mathrm{f}(\lambda \mathrm{x}, \lambda \mathrm{y}) = \frac{\lambda^4 x^4+\lambda^4 y^4}{\lambda^2 x^2+\lambda^2 y^2}$
$= \frac{\lambda^4(x^4+y^4)}{\lambda^2(x^2+y^2)} = \lambda^2 + (\mathrm{x}, \mathrm{y})$
f is a homogeneous function of degree 2.

19) If U is a homogeneous function of x and y of degree n, Prove that $\mathrm{x}\frac{\partial^2 u}{\partial x \partial y} + \mathrm{y}\frac{\partial^2 U}{\partial y^2} = (\mathrm{n} - 1)\frac{\partial U}{\partial y}$

Answer : Since U is a homogeneous function in x and y of degree n, U_y is homogeneous function in x and y of degree n - 1.
Applying Euler's theorem for U_y we have
$x(U_y)_x + y(U_y)_y = (n-1)U_y$
$xU_{yx} + yU_{yy} = (n-1)U_y$
$x\frac{\partial^2 U}{\partial x \partial y} + y\frac{\partial^2 U}{\partial y} = (n-1)\frac{\partial U}{\partial y}$

20) If f(x, y) = $x^3 - 3x + y^2 + 5x + 6$ then find f_x at (1, -2)

Answer : f(x, y) = $x^3 - 3x + y^2 + 5x + 6$
$f_x = 3x^2 - 6x + 5$
at (1, -2)
$f_x = 3(1)-6(1)+5$
$f_x = 2$