QB365 Question Bank Software Study Materials

Discrete Mathematics Important 2 Marks Questions With Answers (Book Back and Creative)

12th Standard

Maths

Total Marks : 40

2 Marks

20 x 2 = 40

1) Identify the valid statements from the following sentences.

Answer : (1) Mount Everest is the highest mountain of the world.

(2) 3+ 4 = 8 .

(3) 7 + 5 > 10 .

(4) Give me that book.

(5) (10 − x) = 7.

(6) How beautiful this flower is!

(7) Where are you going?

(8) Wish you all success.

(9) This is the beginning of the end.

The truth value of the sentences (1) and (3) are T, while that of (2) is F. Hence they are statements.

The sentence (5) is true for x = 3 and false for x ≠ 3 and hence it may be true or false but not both. So it is also a statement.

The sentences (4), (6), (7), (8) are not statements, because (4) is a command, (6) is an exclamatory, (7) is a question while (8) is a sentence expressing one's wishes and (9) is a paradox.

2) Write the statements in words corresponding to ¬p, p ∧ q , p ∨ q and q ∨ ¬p, where p is 'It is cold' and q is 'It is raining'.

Answer : (1) ¬p: It is not cold.

(2) p ∧ q: It is cold and raining.

(3) p ∨ q: It is cold or raining.

(4) q ∨ ¬p: It is raining or it is not cold

Observe that the statement formula ¬ p has only 1 variable p and its truth table has 2 = (2^1) rows. Each of the statement formulae p ∧ q and p ∨ q has two variables p and q. The truth table corresponding to each of them has 4 = (2^2) rows. In general, it follows that if a statement formula involves n variables, then its truth table will contain 2^n rows.

3) How many rows are needed for following statement formulae?

((p ∧ q) ∨ (¬r ∨¬s)) ∧ (¬ t ∧ v))

Answer : ((p ∧ q) ∨ (¬r ∨¬s)) ∧ (¬ t ∧ v)) contains 6 variables p, q, r, s, t, and v. Hence the corresponding truth table will contain 2^6 = 64 rows.

4) Consider p→q : If today is Monday, then 4 + 4 = 8.

Answer : Here the component statements p and q are given by,

p: Today is Monday; q: 4 + 4 = 8.

The truth value of p→q is T because the conclusion q is T.

An important point is that p→q should not be treated by actually considering the meanings of p and q in English. Also it is not necessary that p should be related to q at all.

5) Determine whether ∗ is a binary operation on the sets given below.

a*b = b = a.|b| on R

Answer : Given a*b = a.|b| on R

a,b ∈ R ⇒ a.|b| ∈R as a ∈ R and |b|∈R.

Hence * is a binary operation on R

6) Determine whether ∗ is a binary operation on the sets given below.

(a*b) = a√b is binary on R

Answer : √b is not defined for negative values, b which also ∈ R.

Hence, a√b is not defined for all a, b ∈R

* is not a binary operation on R

7) Fill in the following table so that the binary operation ∗ on A = {a, b, c} is commutative.

*	a	b	c
a	b		
b	c	b	a
c	a		c

Answer : Given * on A is commutative

Given b * a = c ⇒ a * b = c

Given c * a = a ⇒ a * c = a

Given b * c = a ⇒ c * b = a

Hence

*	a	b	c
a	b	**c**	**a**
b	c	b	a
c	a	**a**	c

8) Write the converse, inverse, and contrapositive of each of the following implication.

If x and y are numbers such that x = y, then $x^2 = y^2$

Answer : If x and y are numbers such that x = y, then $x^2 = y^2$

Converse statement :

If x and y are numbers such that $x^2 = y^2$ then x = y

Inverse statement :

If x and y are numbers such that $x \neq y$ then $x^2 \neq y^2$

Contrapositive statement :

If x and y are numbers such that $x^2 \neq y^2$ then $x \neq y$

9) Construct the truth table for the following statements.

¬(p ∧ ¬q)

Answer : Truth Table for ~(p ∧ ~q)

p	**q**	**~q**	**p ∧ ~q**	**~(p ∧ ~q)**
T	T	F	F	T
T	F	T	T	F
F	T	T	F	T
F	F	T	F	T

10) Construct the truth table for the following statements.

(¬p ⟶ r) ∧ (p ↔ q)

Answer : Truth Table for (~p ⟶ r) ∧ (p ↔ q)

p	**q**	**r**	**~ p**	**~ p ⟶ r**	**p ↔ q**	**(~p ⟶ r) ∧ (p ↔ q)**
T	T	T	F	T	T	T
T	T	F	F	T	T	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	F	F	F
F	F	T	T	T	T	T
F	F	F	T	F	F	F

11) Show that p v (~p) is a tautology.

Answer :

p	~p	p v (~q)
T	F	T
F	T	T

The last column contains only T

The given statement is a tautology

12) Show that p v (q ∧ r) is a contingency.

Answer :

p	r	q	q ∧ r	p v (q ∧ r)
T	T	T	T	T
T	F	F	F	T
T	T	F	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	F
F	T	F	F	F
F	F	F	F	F

∴p v (q Λ r) is a contingency

13) In the set of integers under the operation * defined by a * b = a + b - 1. Find the identity element.

Answer : Let a be any element and e be the identity element.

The a * e = e * a = a

a * e = a ⇒ a + e -1 = a ⇒ e-1 = 0 ⇒ e = 1

∴ The identity element is 1

14) Let S be the set of positive rational numbers and is defined by a * b = $\frac{ab}{2}$. Then find the identity element and the inverse of 2.

Answer : Let a ∈ S and e be the identity element.

Then a * e = a ⇒ $\frac{ae}{2}$ = a

⇒ ae = 2a ⇒ e = 2

Let a^{-1} be the inverse of $\frac{1}{2}$

Then $\frac{1}{2} * a^{-1} = e = \frac{1}{2} * a^{-1} = e$

⇒ $\frac{a^{-1}}{2}\frac{a^{-1}}{2} = 2 \Rightarrow a^{-1} = 8.$

15) Let G = {1, w, w^2) where w is a complex cube root of unity. Then find the universe of w^2. Under usual multiplication.

Answer : Clearly 1 is the identity element of G

$w^2 . a^{-1} = e \Rightarrow w^2 . a^{-1} = a^{-1} = w$

Since $w^2 . w = w^3 = 1$

Inverse of w^2 is w.

16) In an algebraic structure the identity element (if exists) must be unique.

Answer : Let (S, ∗) be an algebraic structure. Assume that the identity element of S exists in S .

It is to be proved that the identity element is unique. Suppose that e_1 and e_2 be any two identity elements of S.

First treat e_1 as the identity and e_2 as an arbitrary element of S

Then by the existence of identity property $e_2 * e_1 = e_1 * e_2 = e_2$ ... (1)

Interchanging the role of e1 and e2 $e_1 * e_2 = e_2 * e_1 = e_1$ ... (2)

From (1) and (2), $e_1 = e_2$. Hence the identity element is unique which completes the proof

17) Let * be a binary operation on the set of all nonzero real numbers, given by $a * b = \frac{ab}{5} \forall a, b \in R$. Find the value of x, given that $2 * (x * 5) = 10$

Answer : $a * b = \frac{ab}{5}$

$2 * (x * 5) = 10$

$2 * \left(\frac{5x}{5}\right) = 10$

$2 * x = 10$

$\frac{2x}{5} = 10 \Rightarrow x = \frac{50}{2} = 25$

18) Show that subtraction and division are not binary operations on N.

Answer : $N \times N \to N$ given by (a, b) $\Rightarrow$ a - b is not binary operation, because as the image of (2, 5) under '-' is 2 - 5

$= -3 \notin N$

Similarly, $\div$ is not abinary operation on N.

Forexample $1 * 2 = 1 \div 2 = \frac{1}{2} \notin N$

19) Show that $p \wedge q \to p$ is a tautology.

Answer :

p	**q**	**p** $\wedge$ **q**	$p \wedge q \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

The last columns contains only 'T'

$\therefore$ The given statement is a tautology.

20) Let * be a binary operation on set Q of rational numbers defind as $a * b = \frac{ab}{8}$. Write the identity for *, if any.

Answer : Let e be the identity element in Q.

Then for every a € Q such that

A*e = e * a = a

Given a^* b $= \frac{ab}{8}$

Now, a^*e $= a$

$\frac{ae}{8} = a$

e $= 8 \in$ Q

Hence 8 is the identity element for given operation.