QB365 Question Bank Software Study Materials

Ordinary Differential Equations Important 2 Marks Questions With Answers (Book Back and Creative)

12th Standard

Maths

Total Marks : 40

2 Marks

20 x 2 = 40

1) For each of the following differential equations, determine its order, degree (if exists)

$y\left(\frac{dy}{dx}\right)=\frac{x}{\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^3}$

Answer : $y\left(\frac{dy}{dx}\right)=\frac{x}{\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^3}$

is the given differential equation.

$\Rightarrow y\left(\frac{dy}{dx}\right)^2+\left(\frac{dy}{dx}\right)^4=x$

The highest derivative is 1 and its maximum power is 4.

∴ Order 1, degree 4.

2) Find the differential equation of the family of all non-vertical lines in a plane.

Answer : General equation of a straight line is

ax + by + c = 0 ....(1)

where a, b, c ∈ R.

Since, the lines are non - vertical,we have b $\neq$ 0

Dividing b' by equation (1),

$(\frac{a}{b})x+y+(\frac{c}{b})=0$

$Ax+y=C=0, where A=\frac{a}{b}, C=\frac{c}{b}$ .....(2)

Thus, eventhough 3 arbitrary constants (a, b, c) are present in (1), they can be considered as 2 constants only, as above (2).

Differentiating (1) with respect to x

a + b $\frac{dy}{dx}=0$

Differentiating again with respect to 'x' we get,

(b) $\frac{d^2y}{dx^2}=0\Rightarrow\frac{d^2y}{dx^2}=0\quad[\because b\neq 0]$ .....(3)

This is the differential equation of family of all non - vertical lines in a plane.

3) Find the differential equations of the family of all the ellipses having foci on the y-axis and centre at the origin.

Answer : The equation of the family of ellipses having centre at the origin & foci on the y-axis, is given

$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ ...(1)

where b > a & a, b are the parameters or a,b are arbitrary constant.

Differentiating equation (1) twice successively, because we have two arbitrary constant) we get

$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$

$2\left(\frac{x}{a^2}+\frac{y}{b^2}\frac{dy}{dx}\right)=0$

$\frac{x}{a^2}+\frac{y}{b^2}\frac{dy}{dx}=0$ ....(2)

Again differentiating equation (2)

$\frac{1}{a^2}+\frac{y}{b^2}\frac{d^2y}{dx^2}+\frac{dy}{dx}\frac{dy}{dxb^2}=0$

$\frac{1}{a^2}+\frac{y}{b^2}\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\frac{1}{b^2}=0$

multiply by x

$\frac{x}{a^2}+\frac{xy}{b^2}\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2\frac{x}{b^2}=0$ ....(3)

Equation (3)-(2)

$\frac{x}{a^2}+\frac{xy}{b^2}\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2\left(\frac{x}{b^2}\right)-\left(\frac{x}{a^2}+\frac{y}{b^2}\frac{dy}{dx}\right)=0$

$\frac{xy}{b^2}\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2\frac{x}{b^2}-\frac{y}{b^2}\frac{dy}{dx}=0$

Taking $\frac{1}{b^2}$ outside, we get

$\frac{1}{b^2}\left[xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}\right]=0$

$xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$

is the required differential equation.

4) Find the differential equation of the curve represented by $xy = ae^x + be^{-x} + x^2$.

Answer : Given equation of curve is

$xy = ae^x + be^{-x} + x^2$ ...(1)

where a &b are aribitrary constant. differentiate equation (1) twice successively, because we have two arbitray constant.

$x\frac{dy}{dx}+y(1)=ae^x-be^{-x}+2x$ ...(2)

$x\frac{d^2y}{dx^2}+\frac{dy}{dx}(1)+\frac{dy}{dx}=ae^x+be^{-x}+2$

$x\frac{d^2y}{dx^2}+\frac{2dy}{dx}=ae^x+be^{-x}+2$ ...(3)

From (1), we get $xy-x^2=ae^x+be^{-x}$ ...(4)

Substituting equation (a) in (3), we get

$\therefore x\frac{d^2y}{dx^2}+\frac{2dy}{dx}-xy+x^2-2=0$ which is the required differential equaiton.

5) Find value of m so that the function $y = e^{mx}$ is a solution of the given differential equation.

y '+ 2y = 0

Answer : Given = e^{mx} is the solution of

y' + 2y = 0 ...(1)

$y = e^{mx}$ ...... (2)

$\frac{dy}{dx}=e^{mx}.m$

$\frac{dy}{dx}=ym$

$\frac{dy}{dx}-my=0$

⇒ y' - my = 0 ...(3)

Comparing equation (1) & (3),

we get m = -2

6) Determine the order and degree (if exists) of the following differential equations:

$\frac{dy}{dx}=x+y+5$

Answer : In this equation, the highest order derivative is $\frac{dy}{dx}$ whose power is 1.

Therefore, the given differential equation is of order 1 and degree 1.

7) Determine the order and degree (if exists) of the following differential equations:

$\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2=x^2log\left(\frac{d^2y}{dx^2}\right)$

Answer : In the given differential equation, the highest order derivative is $\frac{d^2y}{dx^2}$ whose power is 1.

Therefore, the given differential equation is of order 2.

The given differential equation is not a polynomial equation in its derivatives and so its degree is not defined.

8) Form the differential equation by eliminating the arbitrary constants A and B from y = A cos x + B sin x.

Answer : y = Acos x + Bsin x ... (1)

Differentiating (1) twice successively, we get

$\frac{dy}{dx}$ = −Asin x + Bcos x. ...(2)

$\frac{d^2y}{dx^2}$ = -Acos x − Bsin x = −(A cos x + B sin x). ...(3)

Substituting (1) in (3), we get $\frac{d^2y}{dx^2}$ + = 0 as the required differential equation

9) Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant.

Answer : The equation of the family of parabolas is given by y^2 ax = 4, a is an arbitrary constant. ... (1)

Differentiating both sides of (1) with respect to x , we get $2y\frac{dy}{dx}=4a\Rightarrow a=\frac{y}{2}\frac{dy}{dx}$

Substituting the value of a in (1) and simplifying, we get $\frac{dy}{dx}=\frac{y}{2x}$ as the required differential equation.

10) Solve: $\frac{dy}{dx}$ = $(3x+y+4)^2$.

Answer : To solve the given differential equation, we make the substitution 3x + y + 4 = z.

Differentiating with respect to x, we get $\frac{dy}{dx} = \frac{dz}{dx}$-3.

So the given differential equation becomes $\frac{dz}{dx}$ = z^2+ 3.

In this equation variables are separable. So, separating the variables and integrating, we get the general solution of the given differential equation as $\frac{1}{\sqrt{3}} tan^{-1}\left(\frac{3x+y+4}{\sqrt{3}}\right) = x + C$

11) Express each of the following physical statements in the form of differential equation.

For a certain substance, the rate of change of vapor pressure P with respect to temperature T is proportional to the vapor pressure and inversely proportional to the square of the temperature.

Answer : Let P represent the vapour pressure and T represent the vapour temperature.

Given $\frac{dp}{dt} \alpha \quad p.\frac{1}{T^2}$

[∴ Inversely proportional to the square of the temperature]

$\Rightarrow \frac{dp}{dt} = \frac{kP}{T^2}$ where k is a constant.

12) Express each of the following physical statements in the form of differential equation.

A saving amount pays 8% interest per year, compounded continuously. In addition, the income from another investment is credited to the amount continuously at the rate of Rs. 400 per year.

Answer : Let x represent the principal in the saving amount.

R = 8% and N = 1.

∴ Interest = $\frac{PNR}{100} = \frac{x \times 1 \times 8}{100} = \frac{2x}{25}$

∴ Given $\frac{dx}{dt}$ = interest + Rs. 400.

∴ $\frac{dx}{dt} = \frac{2x}{25} + 400$

13) Form the differential equation satisfied by are the straight lines in my-plane.

Answer : Equation of family of straight lines in my plane is y = mx - c where m and c are arbitrary constraints.

Differentiating, y' = m

Differentiating again, y" = 0, is the required differential equation.

14) A curve passing through the origin has its slope e^x, Find the equation of the curve.

Answer : Given slope = $\frac{dy}{dx} = e^x$

$\Rightarrow dy = e^x dx$

$\int dy = \int e^x dx$

$\Rightarrow y = e^x + c$

Since the curve passes through (0, 0),

0 = e^0+c

⇒ 0 = 1+c

⇒c = -1

y = ex^{-1} is the required equation of the curve

15) Solve: $\frac{dy}{dx} + y = e^{-x}$

Answer : This is a linear differential equation

Here P = 1, Q = e^{-x}

$\therefore \int p\, dx = \int 1.dx = x$

$I.F = e^{\int pdx} = e^x$

The solution is

$ye^{\int pdx} \int Qe^{\int p\, dx} dx + c$

$\Rightarrow ye^x = \int e^{-x}.e^x dx + c = \int dx + c$

$\int ye^x = x + c$

16) Find the order and degree of $y + \frac{dy}{dx} = \frac{1}{4}\int ydx$

Answer : order 2 ; degree 1

17) Form the Differential Equation representing the family of curves y = A cos(x + B) where A and B are parameters.

Answer : $\frac{d^2y}{dx^2} + y = 0$

18) Form the D.E of family of parabolas having vertex at the origin and axis along positive y-axis.

Answer : $x\frac{dy}{dx} = 2y$

19) Solve: $\frac{dy}{dx} + y = 1$

Answer : x + log (1 - y) = c

20) solve: x dy + y dx = xy dx

Answer : $|xy| = ce^x$