# QB365 Question Bank Software Study Materials

## Probability Distributions Important 2 Marks Questions With Answers (Book Back and Creative)

12th Standard

**Maths**

Total Marks : 30

2 Marks

15 x 2 = 30

1) Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.

**Answer :** Sample space s = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

n(S) = 8

Let X be the random variable denoting the number of heads

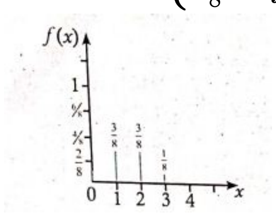
X = {0, 1, 2, 3}

| Values of randomn variabl | 0 | 1 | 2 | 3 | Total |
|---|---|---|---|---|---|
| Number of elements in inverse image | 1 | 3 | 3 | 1 | 8 |

Probability mass function

| x | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| f(x) | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

$$\therefore f(x)=\begin{cases}\frac{1}{8} & for \quad x=0,3\\ \frac{3}{8} & for \quad x=1,2\end{cases}$$

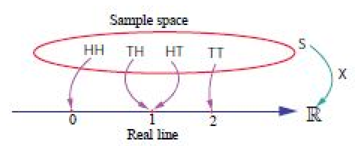


2) Suppose two coins are tossed once. If X denotes the number of tails,

(i) write down the sample space

(ii) find the inverse image of 1

(iii) the values of the random variable and number of elements in its inverse images




A mapping $X(.)$ from $S$ to $\mathbb{R}$

**Answer :** (i) The sample space S = {H,T}×{H,T}

That is S = {TT,TH,HT,HH}

(ii) Let X : S ⟶R be the number of tails

Then X (TT) = 2 (2 Tails)

X (TH ) = 1 (1 Tail)

X (HT) = 1 (1 Tail)

and X (HH) = 0 (0 Tails).

Then X is a random variable that takes on the values 0, 1 and 2.

Let X (ω) denotes the number of tails, this gives

$$X(\omega)=\begin{cases}2 & if \omega=TT\\ 1 & if \omega=HT,TH\\ 0 & if \omega=HH\end{cases}$$

The inverse images of 1 {TH, HT} . That is $X^{-1}\{1\}$ = {TH, HT}.

(iii) Number of elements in inverse images are shown in the table.

| Values of the Random Variable | 0 | 1 | 2 | Total |
|---|---|---|---|---|
| Number of elements in inverse image | 1 | 2 | 1 | 4 |

3) Compute P(X = k) for the binomial distribution, B(n, p) where

$n=10, p=\frac{1}{5}, k=4$

**Answer :** $\therefore q=1-p=1-\frac{1}{5}=\frac{4}{5}$

$\mathrm{P}(\mathrm{X}=x)=n\mathrm{C}_x p^x q^{n-x}, x=0,1,2,\ldots\ldots n$

$\mathrm{P}(\mathrm{X}=k)=\mathrm{P}(\mathrm{X}=4)$

$=10\mathrm{C}_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^{10-4}=210\left(\frac{1}{5^4}\right)\left(\frac{4^6}{5^6}\right)=210\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^6$

4) If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights

(i) exactly 10 will have a useful life of at least 600 hours;

(ii) at least 11 will have a useful life of at I least 600 hours;

(iii) at least 2 will not have a useful life of at : least 600 hours.

**Answer :** Given n = 12

P = 0.9

(i) P(X = 10) = $12C_{10}(0.9)^{10}(1 - 0.9)^2$

= $12C_{10}(0.9)^{10}$ $(0.1)^2$

(ii) P(X 2: 11) = R(X = 11) + P(X = 12)

= $12C_{11}(0.9)^{11}$ (0.1) + $12C_{12}(0.9)^{12}(0.1)^6$

= 12C $(0.9)^{11}$ (0.1) + $(0.9)^{12}$

= $12(0.9)^{11}$ (0.1) + $(0.9)^{12}$

= $(0.9)^{11}$ ((12)(0.1) + 0.9)

= (0.9)11 (1.2 + 0.9) = (0.9)11 (2.1)

(iii) P( at least 2 will not have a useful life of atleast 600 hours) = 1 - P( atleast 11 will have a useful life of atleast 600 hours)

= 1 - P(X ≥11)

= 1-2.1 $(0.9)^{11}$

5) If the sum of mean and variance of a binomial distribution is 4.8 for 5 trails, find the distribution.

**Answer :** Given, np + npq = 4.8

np(1 + q) = 4.8

5p(1 +(1-P)) = 4.8

$5p^2-10p+4.8=0$

$\left(p-\frac{6}{5}\right)\left(p-\frac{4}{5}\right)=0$

$p=\frac{6}{5}, \frac{4}{5}$

$p=\frac{4}{5}$

[∴ p cannot greater than 1]

$q=\frac{1}{5}$

Binomial distribution is P(X = x)

$={}^5C_x\left(\frac{4}{5}\right)^x\left(\frac{1}{5}\right)^{5-x}, \mathrm{x}=0,1,2,3,4,5$

6) The difference between the mean and the variance of a binomial distribution is 1 and the difference between their squares is 11. Find n.

**Answer :** Let m + 1 and m be the mean and variance

Given, $(m+1)^2-m^2=11$

$m^2+2m+1-m^2=11$

2 m = 10

m = 5

Mean = m + 1 = 6

np = 6, npq = 5, $q=\frac{5}{6}, p=\frac{1}{6}$

n = 36

7) The probability of success of an event is p and that of failure is q. Find the expected number of trails to get a first success.

**Answer :** The probability distribution

| **x** | 0 | 1 | 2 | 3 | 4 | ...... | n | ....... |
|---|---|---|---|---|---|---|---|---|
| **P(X = x)** | p | k | qp | $q^2p$ | $q^3p$ | ........ | $q^{n-1}p$ | ....... |

$E(X) = \sum px$
$= 1 \cdot p + 2 \cdot qp + 3 \cdot q^2p + \ldots. + nq^{n-1}p + \ldots$
$= p\left(1 + 2q + 3q^2 + \ldots.. + nq^{n-1} + \ldots\ldots\right)$
$= p(1-q)^{-2}$
$= p(p)^{-2} = \frac{p}{p^2} = \frac{1}{p}$

8) If $F(x) = \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1} x\right), -\infty < x < \infty$ is a distribution function of a continuos variable x, find $P(0 \le x \le 1)$

**Answer :** $F(x) = \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1} x\right)$
$P(0 \le x \le 1) = F(1) - F(0)$
$= \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1} 1\right) - \frac{1}{\pi}\left(\frac{\pi}{2} + \tan^{-1} 0\right)$
$= \frac{1}{\pi}\left(\frac{\pi}{2} + \frac{\pi}{4}\right) - \frac{1}{\pi}\left(\frac{\pi}{2} + 0\right)$
$= \frac{1}{\pi}\left(\frac{\pi}{2} + \frac{\pi}{4} - \frac{\pi}{2}\right) = \frac{1}{4}$

9) If X ~ B(n, p) such that F(X = 3) = 2 P(X= 2) and n = 5, find p.

**Answer :** Given $P(X = x) = {}^nC_x p^x q^{n-x}$
${}^5C_3 p^3 q^2 = 2\left({}^5C_2 p^2 q^3\right)$
p = 2q
p = 2(1-p)
3p = 2
$p = \frac{2}{3}$

10) For the probability density function $f(x) = \begin{cases} 2e^{-2x} & , x > 0 \\ 0 & , x \le 0 \end{cases}$, find f(2).

**Answer :** $F(2) = P(X \le 2)$
$= \int_{-\infty}^{2} f(x)dx$
$= \int_0^2 2e^{-2x}dx \ = 2\left[\frac{e^{-2x}}{-2}\right]_0^2 = -\left[e^{-4} - 1\right] = 1 - e^{-4}$

11) If the mean of the binomial distribution with 9 trial is 6, find the variance?

**Answer :** Given n = 9 and
mean = 6
np = 6
$\Rightarrow \ p = \frac{6}{9} = \frac{2}{3}$
$\therefore q = 1 - p$
$= 1 - \frac{2}{3} = \frac{1}{3}$
Variance = npg = $9 \times \frac{2}{3} \times \frac{1}{3} = 2$

12) If 10 coins are tossed, find the probability that exactly 5 heads appears.

**Answer :** Given = 10
$P(H) = \frac{1}{2} \Rightarrow p = \frac{1}{2}$
$\therefore q - 1 - p$
$= 1 - \frac{1}{2} = \frac{1}{2}$
$P(X = x) = nC_x p^x (1-p)^{n-x}$
$x = 0, 1, 2, \ldots n$
$\therefore P(X = 5) = 10C_5 p^5 q^5$
$= 10C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^5$
$= 10C_5 \left(\frac{1}{2}\right)^{10}$

13) If the p.d.f of a random variable X is given by
$f(x) = \frac{2x}{9}, 0 < x < 3$, then find E(3X + 8).

**Answer :** $\mathrm{E(X)} = \int_0^3 x \cdot f(x) dx$

$= \int_0^3 x \cdot \frac{2x}{9} dx = \frac{2}{9} \int_0^3 x^2 dx$

$= \frac{2}{9}\left[\frac{x^3}{3}\right]_0^3 = \frac{2}{27} \times 27 = 2$

$\therefore \mathrm{E(3X+8)} = 3\mathrm{E(X)} + 8 = 3(2) + 8 = 6 + 8 = 14$

14) A random variable X has the following p.d.f.

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| P(X-x) | a | 2a | 3a | 4a | 5a | 6a |

then find F(4).

**Answer :** Since X is the probability distribution function, then

$\Sigma p_i = 1 \Rightarrow a + 2a + 3a + 4a + 5a + 6a = 1$

$\Rightarrow\ 21a = 1$

$\Rightarrow\ a = \frac{1}{21}$

$\mathrm{F}(4) = \mathrm{P}(\mathrm{X} \leq 4)$

$= \mathrm{P}(\mathrm{X}=0) + \mathrm{P}(\mathrm{X}=1) + \mathrm{P}(\mathrm{X}=2)$

$= a + 2a + 3a + 4a + 5a$

$= 15a = \frac{15}{21}$

$\mathrm{F}(4) = \frac{5}{7}$

15) Suppose X is a binomial variate X ~ B(5, p) and P(X = 2) = P(X = 3), then find p.

**Answer :** Given n = 5 and

P(X = 2) = P(X = 3)

P(X = x) = $n\mathrm{C}_x p^x q^{n-x}$

x = 0, 1, 2,...n

Therefore (1) becomes,

$5\mathrm{C}_2 p^2 q^3 = 5\mathrm{C}_3 p^3 q^2$

$\Rightarrow \frac{5\times4}{1\times2} q = \frac{5\times4\times3}{3\times2\times1} p$

$\Rightarrow$ Since $p + q = p$

$\Rightarrow p + p = 1$

$\Rightarrow 2p = 1$

$\Rightarrow p = \frac{1}{2}$