QB365 Question Bank Software Study Materials

Applications of Matrices and Determinants Important 2,3 & 5 Marks Questions With Answers (Book Back and Creative)

12th Standard

Business Maths and Statistics

Total Marks : 75

2 Marks

10 x 2 = 20

1) Find the rank of the matrix $\begin{pmatrix} 1 & 5 \\ 3 & 9 \end{pmatrix}$

Answer : Let A = $\begin{pmatrix} 1 & 5 \\ 3 & 9 \end{pmatrix}$

Order of A is 2×2

$\therefore \rho$ (A) ≤ 2

Consider the second order minor

$\begin{vmatrix} 1 & 5 \\ 3 & 9 \end{vmatrix} = -6 \neq 0$

There is a minor of order 2, which is not zero.

$\therefore \rho$ (A) ≤ 2

2) Find the rank of the matrix $\begin{pmatrix} -5 & -7 \\ 5 & 7 \end{pmatrix}$

Answer : Let A = $\begin{pmatrix} -5 & -7 \\ 5 & 7 \end{pmatrix}$

Order of A is 2×2

$\therefore \rho(A) \leq 2$

Consider the second order minor $\begin{vmatrix} -5 & -7 \\ 5 & 7 \end{vmatrix}$=0

Since the second order minor vanishes,$\rho(A) \neq 2$

Consider a first order minor $|-5| \neq 0$

There is a minor of order 1, which is not zero

$\therefore \rho(A) = 1$

3) Find the rank of each of the following matrices.

$\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

Answer : Let $A = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$

Order of A is 2×2

$\therefore \rho(A) \leq 2$ [Since minimum of (2, 2) is 2]

Consider the second order minor

$\begin{vmatrix} 5 & 6 \\ 7 & 8 \end{vmatrix} = 40 - 42$

= $-2 \neq 0$

There is a minor of order 2, which is not zero

$\therefore \rho(A) = 2$

4) Find the rank of the matrix A = $\begin{pmatrix} 1 & -3 & 4 & 7 \\ 9 & 1 & 2 & 0 \end{pmatrix}$

Answer : Given A = $\begin{pmatrix} 1 & -3 & 4 & 7 \\ 9 & 1 & 2 & 0 \end{pmatrix}$

$\begin{pmatrix} 1 & -3 & 4 & 0 \\ 0 & 28 & -34 & -63 \end{pmatrix} R_2 \rightarrow R_2 - 9R_1$

$\sim \begin{pmatrix} 1 & -3 & 4 & 0 \\ 0 & 0 & \frac{10}{3} & -63 \end{pmatrix} R_2 \rightarrow R_2 + \frac{28}{3}.R_1$

The last equivalent matrix is in echelon form and there are 2 non-zero rows

$\therefore \rho(A) = 2$

5) Find the rank of each of the following matrices.

$\begin{pmatrix} 1 & -1 \\ 3 & -6 \end{pmatrix}$

Answer : Let $A = \begin{pmatrix} i & -1 \\ 3 & -6 \end{pmatrix}$

Order of A is 2 × 2

$\therefore \rho(A) \leq 2$ [Since minimum of (2, 2) is 2]

Consider the second order minor

$\begin{vmatrix} 1 & -1 \\ 3 & -6 \end{vmatrix} = -6 - (-3)$

= -6 + 3 = -3

$\neq 0$

There is a minor of order 2, which is not zero

$\therefore \rho(A) = 2$

6) Find the rank of each of the following matrices.

$\begin{pmatrix} 1 & 4 \\ 2 & 8 \end{pmatrix}$

Answer : Let A = $\begin{pmatrix} 1 & 4 \\ 2 & 8 \end{pmatrix}$

Order of A is 2 × 2 [Since minimum of (2,2) is 2]

Consider the second order minor $\begin{vmatrix} 1 & 4 \\ 2 & 8 \end{vmatrix}$

= 8-8

= 0

Since the second order minor vanishes $\rho(A) \neq 2$

Consider a first order minor $[1] \neq 0$

There is a minor of order 1, which is not zero

$\therefore \rho\left(A\right) = 1$

7) Find the rank of the matrix $\begin{bmatrix} 7 & -1 \\ 2 & 1 \end{bmatrix}$

Answer : Let $A = \begin{bmatrix} 7 & -1 \\ 2 & 1 \end{bmatrix}$

The order of A is 2 x 2

$\rho(A) \leq min(2, 2)$

$\begin{bmatrix} 7 & -1 \\ 2 & 1 \end{bmatrix} = 7 - (-2) = 7 + 29 \neq 0$

The highest order of non-vanishing minor of A is 2

$\therefore \rho(A) = 2$

8) Solve: 2x + 3y = 4 and 4x + 6y = 8 using Cramer's rule.

Answer : $\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 12 - 12 = 0$

$\Delta x = \begin{vmatrix} 4 & 3 \\ 8 & 6 \end{vmatrix} = 24 - 24 = 0$

$\Delta x = \begin{vmatrix} 4 & 3 \\ 8 & 6 \end{vmatrix} = 24 - 24 = 0$

$\therefore \Delta = \Delta x = \Delta y = 0$

$\therefore$ The system is consistent with infinite number of solutions

let y = k, $k \epsilon R$

$\therefore 2x + 3k = 4 \Rightarrow 2x = 4 - 3k$

$\Rightarrow x = \frac{1}{2}(4 - 3k), k \epsilon R$

$\therefore$ Solution set is $\left\{ \frac{4 - 3k}{2}, k \right\}, k \epsilon R$

9) Find the rank of $\begin{pmatrix} 7 & -1 \\ 2 & 1 \end{pmatrix}$

Answer : $A=\begin{pmatrix}7 & -1\\ 2 & 1\end{pmatrix} R_2\to 7R_2-2R_1$
$\sim\begin{pmatrix}7 & -1\\ 0 & 9\end{pmatrix}$
$\rho(A)=2$

10) Solve by determinant method:

(i) 2x - y = 3, 5x + y = 4

(ii) 2x + 3y = 7, 2x + y = 5

(iii) 6x - 7y = 16, 9x - 5y = 35

(iv) 3x + 2y = 5, x + 3y = 4

(v) x + 2y = 3, x + y = 2

Answer : (i) $\Delta=\begin{vmatrix}2 & -1\\ 5 & 1\end{vmatrix}=2+5=7\neq 0$

$\Delta_x=\begin{vmatrix}3 & -1\\ 4 & 1\end{vmatrix}=3+4=7$

$\Delta_y=\begin{vmatrix}2 & 3\\ 5 & 4\end{vmatrix}=8-15=-7$

$x=\frac{\Delta_x}{\Delta}=\frac{7}{7}=1$

$y=\frac{\Delta_y}{\Delta}=\frac{-7}{7}=-1$

Solutionis x = 1, y = -1

(ii) 2x + 3y = 7, 2x + y = 5

$\Delta=\begin{vmatrix}2 & 3\\ 2 & 1\end{vmatrix}=2-6=-4\neq 0$

$\Delta_x=\begin{vmatrix}7 & 3\\ 5 & 1\end{vmatrix}=7-15=-8$

$\Delta_y=\begin{vmatrix}2 & 7\\ 2 & 5\end{vmatrix}=10-14=-4$

$x=\frac{\Delta_x}{\Delta}=\frac{-8}{-4}=2$

$y=\frac{\Delta_y}{\Delta}=\frac{-4}{-4}=1$

(iii) 6x - 7y = 16, 9x - 5y = 35

$\Delta=\begin{vmatrix}6 & -7\\ 9 & -5\end{vmatrix}=-30+63=33\neq 0$

$\Delta_x=\begin{vmatrix}16 & -7\\ 35 & -5\end{vmatrix}=-80+245=165$

$\Delta_y=\begin{vmatrix}6 & 16\\ 9 & 35\end{vmatrix}=210-144=66$

$x=\frac{\Delta_x}{\Delta}=\frac{165}{33}=5$

$y=\frac{\Delta_y}{\Delta}=\frac{66}{33}=2$

Solution is x = 5, y = 2

(iv) 3x + 2y = 5, x + 3y = 4

$\Delta=\begin{vmatrix}3 & 2\\ 1 & 3\end{vmatrix}=9-2=7\neq 0$

$\Delta_x=\begin{vmatrix}5 & 2\\ 4 & 3\end{vmatrix}=15-8=7$

$\Delta_y=\begin{vmatrix}3 & 5\\ 1 & 4\end{vmatrix}=12-5=7$

$x=\frac{\Delta_x}{\Delta}=\frac{7}{7}=1$

$y=\frac{\Delta_y}{\Delta}=\frac{7}{7}=1$

Solution is x = 1, y = 1

(v) x + 2y = 3, x + y = 2

$\Delta=\begin{vmatrix}1 & 2\\ 1 & 1\end{vmatrix}=1-2=-1\neq 0$

$\Delta_x=\begin{vmatrix}3 & 2\\ 2 & 1\end{vmatrix}=3-4=-1$

$\Delta_y=\begin{vmatrix}1 & 3\\ 1 & 2\end{vmatrix}=2-3=-1$

$x=\frac{\Delta_x}{\Delta}=\frac{-1}{-1}=1$

y $=\frac{\Delta_y}{\Delta}=\frac{-1}{-1}=1$

Solution is x = 1, y = 1

3 Marks

10 x 3 = 30

11) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$

Answer : Let A = $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{pmatrix}$

Order of A is 3×4

$\therefore \rho(A) \leq 3$

Consider the third order minors

$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 4 & 1 \\ 3 & 6 & 3 \end{vmatrix} = 0, \begin{vmatrix} 1 & -1 & 3 \\ 2 & 1 & -2 \\ 3 & 3 & -7 \end{vmatrix} = 0$

$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & -2 \\ 3 & 6 & -7 \end{vmatrix} = 0, \begin{vmatrix} 2 & -1 & 3 \\ 4 & 1 & -2 \\ 6 & 3 & -7 \end{vmatrix} = 0$

Since all third order minors vanishes, $\rho(A) \neq 3$

Now, let us consider the second order minors,

Consider one of the second order minors $\begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} = 6 \neq 0$

There is a minor of order 2 which is not zero.

$\therefore \rho(A) = 2$

12) If A= $\begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and B= $\begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$, then find the rank of AB and the rank of BA.

Answer : Given

A = $\begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$ and B = $\begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$

$AB = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix}$

= $\begin{pmatrix} 1-2+5 & -2+4-1 & 3-6+1 \\ 2+6+20 & -4-12+45 & 6+18-4 \\ 3+4+15 & -6-8+3 & 9+12-3 \end{pmatrix}$

= $\begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix} = \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix}$

MATRIX (AB)	**ELEMENTARY TRANSFORMATION**
$AB = \begin{pmatrix} -6 & 1 & -2 \\ 28 & -12 & 20 \\ 22 & -11 & 18 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & -6 & -2 \\ -12 & 28 & 20 \\ -11 & 22 & 18 \end{pmatrix}$	$C_1 \leftrightarrow C_2$
$\sim \begin{pmatrix} 1 & -6 & -2 \\ -12 & 28 & 20 \\ -11 & 22 & 18 \end{pmatrix}$	$R_2 \to R_2 + 12R_1$
$\sim \begin{pmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ 0 & -44 & -4 \end{pmatrix}$	$R_3 \to R_3 + 11R_1$
$\sim \begin{pmatrix} 1 & -6 & -2 \\ 0 & -44 & -4 \\ 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - R_2$

The matrix is in echelon form and the number of non-zero rows is 2.

$\therefore \rho(AB) = 2$

Now $BA = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -6 \\ 5 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$

= $\begin{pmatrix} 1-4+9 & 1+6-6 & -1-8+9 \\ -2+8-18 & -2-12+12 & 2+16-18 \\ 5+2-3 & 5-3+2 & -5+4-3 \end{pmatrix}$

= $\begin{pmatrix} 6 & 1 & 0 \\ -12 & -2 & 0 \\ 4 & 4 & -4 \end{pmatrix}$

MATRIX (BA)	**ELEMENTARY TRANSFORMATION**
$BA = \begin{pmatrix} 6 & 1 & 0 \\ -12 & -2 & 0 \\ 4 & 4 & -4 \end{pmatrix}$	
$\sim \begin{pmatrix} 1 & 6 & 0 \\ -2 & -12 & 0 \\ 4 & 4 & -4 \end{pmatrix}$	$C_1 \leftrightarrow C_2$
$\sim \begin{pmatrix} 1 & 6 & 0 \\ 0 & 0 & 0 \\ 4 & 4 & -4 \end{pmatrix}$	$R_2 \to R_2 + 2R_1$
$\sim \begin{pmatrix} 1 & 6 & 0 \\ 0 & 0 & 0 \\ 0 & -20 & -4 \end{pmatrix}$	$R_3 \to R_3 - 4R_1$

The number of non-zero rows is 2.

$\therefore \rho(BA) = 2$

13) Show that the following system of equations have unique solution:

x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6 by rank method.

Answer : Given non-homogeneous equations are

x + y + z = 3

x + 2y + 3z = 4

x + 4y + 9z = 6

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 11 & 4 & 9 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

AX = B

AUGMENTED MATRIX	**ELEMENTARY TRANSFORMATION**
$\begin{pmatrix} 1 & 1 & 1\,3 \\ 1 & 2 & 3\,4 \\ 1 & 4 & 9\,6 \end{pmatrix}$	
$\begin{pmatrix} 1 & 1 & 1\,3 \\ 0 & 1 & 2\,1 \\ 0 & 3 & 8\,R \end{pmatrix}$	$R_2 \to R_2 - R_1$ $R_3 \to R_3 - R_1$
$\begin{pmatrix} 1 & 1 & 1\,3 \\ 0 & 1 & 2\,1 \\ 0 & 0 & 2\,0 \end{pmatrix}$	$R_3 \to R_3 - R_2$

Clearly the last equivalent matrix is in echelon form and it has three non-zero rows

$\therefore \rho(A) = 3 \quad \rho([A, B]) = 3$

$\rho(A) = \rho([A, B]) = 3$

∴ The given system is consistent and has unique solution.

To find the solution, let us rewrite the above echelon form into the matrix form

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

⇒ x + y + z = 3

y + 2z = 1

$(3) \Rightarrow 2x = 0 \Rightarrow z = \frac{0}{2} = 0$

$(2) \Rightarrow y + 2(0) = 1 \Rightarrow y + 0 = 1 \Rightarrow y = 1 - 0 = 1$

$(1) \Rightarrow x + 1 + 0 = 3$

$\Rightarrow x + 1 = 3$

$\Rightarrow x = 3 - 1$

$\Rightarrow x = 2$

∴ Solution set [2, 1, 0]

14) The following table represents the number of shares of two companies A and B during the month of January and February and it also gives the amount in rupees invested by Ravi during these two months for the purchase of shares of two companies. Find the the price per share of A and B purchased during both the months

Months	**Number of Shares of the company**		**Amount invested by Ravi (in Rs)**
	A	**B**	
January	10	5	125
February	9	12	150

Answer : Let the price of one share of A be x

Let the price of one share of B be y

$\therefore$ By given data, we get the following equations

10x + 5y = 125

9x + 12y = 150

$\triangle = \begin{vmatrix} 10 & 5 \\ 9 & 12 \end{vmatrix} = 75 \neq 0$

$\triangle_x = \begin{vmatrix} 125 & 5 \\ 150 & 12 \end{vmatrix} = 750$

$\triangle_y = \begin{vmatrix} 10 & 125 \\ 9 & 150 \end{vmatrix} = 375$

$\therefore$ Cramer’s rule

$x = \frac{\triangle x}{\triangle} = \frac{750}{75} = 10$

$y = \frac{\triangle y}{\triangle} = \frac{375}{75} = 5$

The price of the share A is Rs10 and the price of the share B is Rs. 5.

15) A commodity was produced by using 3 units of labour and 2 units of capital, the total cost is Rs 62. If the commodity had been produced by using 4 units of labour and one unit of capital, the cost is Rs 56. What is the cost per unit of labour and capital? (Use determinant method).

Answer : Let Rs. x represents the cost per unit of labour and Rs. y represents the cost per unit of capital

Given

3x + 2y = 62

4x +y = 56

$\Delta = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} = 3(1) - 4(2) = 3 - 8 = -5$

Since $\Delta \neq 0$ the system is consistent with unique solution and Cramer's rule can be applied.

$\Delta x = \begin{vmatrix} 62 & 2 \\ 56 & 1 \end{vmatrix} = 62(1) - 56(2) = 62 - 112 = -50$ $\Delta_y = \begin{vmatrix} 3 & 62 \\ 4 & 56 \end{vmatrix} = 3(56) - 4(62)$

= 168 - 248 = -80

$x = \frac{\Delta x}{\Delta} = \frac{-50}{-5} = 10$

$y = \frac{\Delta y}{\Delta} = \frac{-80}{-5} = 16.$

$\therefore$ Cost per unit oflabour is Rs. 10 and the cost per unit of capital is Rs. 16.

16) Find the rank of the matrix

$A = \begin{pmatrix} 2 & 4 & 5 \\ 4 & 8 & 10 \\ -6 & -12 & -15 \end{pmatrix}$

Answer : The order of A is 3 x 3

$\therefore \rho(A) \leq min(3,3)$

$\Rightarrow \rho(A) \leq 3$

MATRIX	ELEMENTARY TRANSFORMATION
$\begin{pmatrix} 2 & 4 & 5 \\ 4 & 8 & 10 \\ -6 & -12 & -15 \end{pmatrix}$	
$- \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ -3 & -3 & -3 \end{pmatrix}$	$C_1 \to C_1 \div 2$ $C_2 \to C_2 \div 4$ $C_3 \to C_3 \div 5$
$- \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$R_2 \to R_1 - 2R_1$ $R_3 \to R_3 + 3R_1$

The last equivalent matrix is in echelon form and it has one non-zero row

$\therefore \rho(A) = 1$

17) Show that the equations 2x - y + z = 7, 3x + y - 5z = 13, x + y + z = 5 are consistent and have a unique solution.

Answer : The non-homogeneous equation are

2x - y + z = 7, 3x + y - 5z = 13, x + y + z = 5

AUGMENTED MATRIX [A,B]	**ELEMENTARY TRANSFORMATION**
$\begin{pmatrix} 2 & -1 & 1 & 7 \\ 3 & 1 & -5 & 13 \\ 1 & 1 & 1 & 5 \end{pmatrix}$	
$-\begin{pmatrix} 1 & 1 & 1 & 5 \\ 3 & 1 & -5 & 13 \\ 2 & -1 & 1 & 7 \end{pmatrix}$	$R_1 \leftrightarrow R_3$
$\sim\begin{pmatrix} 1 & 1 & 1 & 5 \\ 0 & -2 & -8 & -2 \\ 0 & -3 & -1 & -3 \end{pmatrix}$	$R_2 \to R_2 - 3R_1$ $R_3 \to R_3 - 2R_1$
$\sim\begin{pmatrix} 1 & 1 & 1 & 5 \\ 0 & -2 & -8 & -2 \\ 0 & 0 & 11 & 0 \end{pmatrix}$	$R_3 \to R_3 - \frac{3}{2}R_2$

Clearly $\rho(A) = 3$ and $\rho(A, B)$ = 3 = Number of unknowns

∴ The given system is consistent and has unique solution.

18) Two products A and B currently share the market with shares 60% and 40% each respectively. Each week some brand switching latees place. Of those who bought A the previous week 70% buy it again whereas 30% switch over to B. Of those who bought B the previous week, 80% buy it again whereas 20% switch over to A. Find their shares after one week and after two weeks.

Answer : Transition probability matrix

$$T = \begin{matrix} & A & B \\ A & 0\cdot7 & 0\cdot3 \\ B & 0\cdot2 & 0\cdot8 \end{matrix}$$

Shares after one week

$(\cdot 6 \cdot 4)\begin{pmatrix} \cdot 7 & \cdot 3 \\ \cdot 6 & \cdot 8 \end{pmatrix}$

= (-6×·7+-4x·2 ·6×·3+·4×·8)

= z:(-42+·08 ·18+·32)= (·50 ·50)

⇒ A = 50% and B = 50%

Shares after two weeks $(\cdot 5 \cdot 5)\begin{pmatrix} \cdot 7 & \cdot 3 \\ \cdot 2 & \cdot 8 \end{pmatrix}$

= (-5×·7+·5×.2 ·5×·3 +·5×·8)

= (-35+·10 ·15 + 40) = (-45 ·55)

A = 45% and B = 55%

19) Show that the equations $x + y + z = -3, 3x + y - 2z = -2, 2x + 4y + 7z = 7$ are not consistent.

Answer : In matrix form

$$\begin{pmatrix} 1 & 1 & 1 \\ 3 & 1 & -2 \\ 2 & 4 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \\ 7 \end{pmatrix}$$

Augmented matrix

$$(A, B) = \begin{pmatrix} 1 & 1 & 1 & -3 \\ 3 & 1 & -2 & -2 \\ 2 & 4 & 7 & 7 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 2 & 5 & 13 \end{pmatrix} R_2 \to R_3 - 2R_1$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 0 & 0 & 20 \end{pmatrix} \quad R_3 \to R_3 + R_2$$

$\rho(A, B) = 3, \rho(A) = 2$

$\rho(A, B) \neq \rho(A)$

The system is inconsistent.

20) Solve, by Cramer's rule x + y = 2, y + z = 6, z + x = 4.

Answer : $\Delta = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1-0) - 1(0-1)$

Cramer's rule is applicable

$\Delta_x = \begin{vmatrix} 2 & 1 & 0 \\ 6 & 1 & 1 \\ 4 & 0 & 1 \end{vmatrix} = 2(1-0) - 1(6-4)$

= 2 - 2 = 0

$\Delta_y = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 6 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 1(6-4) - 2(0-1)$

= 2 + 2 = 4

$\Delta_z = \begin{vmatrix} 1 & 1 & 2 \\ 0 & 1 & 6 \\ 1 & 0 & 4 \end{vmatrix} = 1(4-0) - 1(0-6) + 2(0-1)$

= 4 + 6 - 2 = 8

$x = \frac{\Delta_x}{\Delta} = \frac{0}{2} = 0$

y $= \frac{\Delta_y}{\Delta} = \frac{4}{2} = 2$

$z = \frac{\Delta_z}{\Delta} = \frac{8}{2} = 4$

Solution is x = 0, y = 2, z = 4

5 Marks 5 x 5 = 25

21) Show that the equations 5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5 are consistent and solve them by rank method.

Answer : Given non-homogeneous equations are

5x+ 3y + 7z = 4

3x + 26y + 2z = 9

7x + 2y + 10z = 5

The matrix equation corresponding to the given system is

$$\begin{pmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 5 \end{pmatrix}$$

AUGMENTED MATRIX [A, B]	**ELEMENTARY TRANSFORMATION**
$\begin{pmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	
$\sim \begin{pmatrix} 3 & 26 & 2 & 9 \\ 5 & 3 & 7 & 4 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	$R_1 \leftrightarrow R_2$
$\begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 5 & 3 & 7 & 4 \\ 7 & 2 & 10 & 5 \end{pmatrix}$	$R_1 \to R_1 \div 3$
$\begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-121}{3} & \frac{11}{3} & -11 \\ 7 & 2 & 5 & 5 \end{pmatrix}$	$R_2 \to R_2 - 5R_1$
$\begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-121}{3} & \frac{11}{3} & -11 \\ 0 & \frac{-176}{3} & \frac{16}{3} & -16 \end{pmatrix}$	$R_3 \to R_3 - 7R_1$
$\sim \begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-11}{3} & \frac{1}{3} & -1 \\ 0 & \frac{-11}{3} & \frac{1}{3} & -1 \end{pmatrix}$	$R_2 \to R_2 \div 11$ $R_3 \to R_3 \div 16$
$\begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} & 3 \\ 0 & \frac{-11}{3} & \frac{1}{3} & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$R_3 \to R_3 - R_2$

$\therefore$ The system is consistent with infinitely many solutions let us rewrite the above echelon form into matrix form

$$\begin{pmatrix} 1 & \frac{26}{3} & \frac{2}{3} \\ 0 & \frac{-11}{3} & \frac{1}{3} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$$

$$x + \frac{26}{3}y + \frac{2}{3}z = 3$$

$$x + \frac{26}{3}y + \frac{2}{3}z = 3$$

let z = k where k∈ R

$$(2) \Rightarrow \frac{-11}{3}y + \frac{k}{3} = -1$$

$$\Rightarrow \frac{-11}{3}y = -1 - \frac{k}{3} = \frac{-3-k}{3}$$

$$\Rightarrow \ -11y = -3 - k$$

11y = 3 + k

$$\Rightarrow \quad y = \frac{1}{11}(3+k)$$

Substituting $y = \frac{1}{11}(3+k)$ and z = k in (1) we get,

$$x + \frac{26}{3}\left(\frac{3+k}{11}\right) + \frac{2}{3}k = 3$$

$$= \frac{26}{3}\left(\frac{3+k}{11}\right) - \frac{2k}{3} + 3$$

$$\frac{78-26k}{33} - \frac{2k}{3} + 3$$

$$\frac{78-26k-22k+99}{33}$$

$$78 - 26k - 22k + 99$$

$\frac{[illegible]}{33}$

$\frac{21-48k}{33}=\frac{3(7-16k)}{33}$

= $\frac{1}{11}(7-6k)$

$\therefore$ Solution set is $\left\{\frac{1}{11}(7-16k)\,,\frac{1}{11}(3+k),k\right\}$K $\in$ R

Hence, for different values of k, we get infinitely many solutions.

22) In a market survey three commodities A, B and C were considered. In finding out the index number some fixed weights were assigned to the three varieties in each of the commodities. The table below provides the information regarding the consumption of three commodities according to the three varieties and also the total weight received by the commodity

Commodity Variety	**Variety**			**Total weight**
	I	**II**	**III**	
A	1	2	3	11
B	2	4	5	21
C	3	5	6	27

Find the weights assigned to the three varieties by using Cramer’s Rule.

Answer : Let the weight assigned to the three varieties be Rs. x, Rs. y and Rs. z respectively By the given data,

x + 2y + 3z = 11

2x + 4y + 5z = 21

3x + 5y + 6z = 27

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix} = 1\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 2\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} + 3\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$$

= 1(24 - 25) -2(12 - 15) + 3(10 - 12)

= 1(-1) -2 (-3) + 3(-2)

= -1+6 - 6 = -1$\neq$ 0.

Since $\Delta \neq 0$ the system is consistent with unique solution and Cramer's rule can be applied.

$$\Delta x = \begin{vmatrix} 11 & 2 & 3 \\ 21 & 4 & 5 \\ 27 & 5 & 6 \end{vmatrix}$$

$$= 11\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 2\begin{vmatrix} 21 & 5 \\ 27 & 6 \end{vmatrix} + 3\begin{vmatrix} 21 & 4 \\ 27 & 5 \end{vmatrix}$$

= 11(24 - 25) - 2(126 - 135) + 3(105 - 108)

= 11(-1) - 2(-9) + 3 (-3)

= 11+18-9

= -2

$$\Delta y = \begin{vmatrix} 1 & 11 & 3 \\ 2 & 21 & 5 \\ 3 & 27 & 6 \end{vmatrix}$$

= $\begin{vmatrix} 21 & 5 \\ 27 & 6 \end{vmatrix} - 11\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} + 3\begin{vmatrix} 2 & 21 \\ 3 & 27 \end{vmatrix}$

= 1(126 - 135) - 11(12 -15) + 3(54 - 63)

= - 9 - 11(-3) + 3(-9)

= - 9 + 33 - 27

= 3

$$\Delta z = \begin{vmatrix} 1 & 2 & 11 \\ 2 & 4 & 21 \\ 3 & 5 & 27 \end{vmatrix} = 1\begin{vmatrix} 4 & 21 \\ 5 & 27 \end{vmatrix} - 2\begin{vmatrix} 2 & 21 \\ 3 & 27 \end{vmatrix} + 11\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$$

= 1(108 - 105) - 2(54 - 63) + 11(10 - 12)

= 1(3) - 2(-9) + 11(-2)

= 3 + 18 - 22

= -1

$$x = \frac{\Delta x}{\Delta} = \frac{-2}{-1} = 2$$

$$y = \frac{\Delta y}{\Delta} = \frac{-3}{1} = 3$$

and $z = \frac{\Delta z}{\Delta} = \frac{-1}{-1} = 1$

Hence, the weights assigned to the three varieties are 2, 3 and 1 respectively

23) Two products A and B currently share the market with shares 50% and 50% each respectively. Each week some brand switching takes place. Of those who bought A the previous week, 60% buy it again whereas 40% switch over to B. Of those who bought B the previous week, 80% buy it again where as 20% switch over to A. Find their shares after one week and after two weeks. If the price war continues, when is the equilibrium reached?

Answer : Transition probability matrix

$$T = \begin{array}{c} \\ A \\ B \end{array}\begin{array}{c} \begin{array}{cc} A & B \end{array} \\ \begin{pmatrix} \cdot 6 & \cdot 4 \\ \cdot 2 & \cdot 8 \end{pmatrix} \end{array}$$

By the given data

A = 50% = ·5

B = 50% = ·5

Shares after one week

$$\left(\cdot 5 \quad \cdot 5\right)\begin{pmatrix} \cdot 6 & \cdot 4 \\ \cdot 2 & \cdot 8 \end{pmatrix}$$

= $\left(\cdot 5\right)\left(\cdot 5\right)\left(\cdot 2\right)\cdot 5\left(4\right)+\cdot 5\left(\cdot 8\right)$

= $\left(\cdot 30+\cdot 10\cdot 20+40\right)$

= $\left(\cdot 40 \quad \cdot 60\right)$

∴ Shares after one week for products A and Bare 40% and 60% respectively.

Shares after two weeks

$$\left(\cdot 4 \quad \cdot 6\right)\begin{pmatrix} \cdot 6 & \cdot 4 \\ \cdot 2 & \cdot 8 \end{pmatrix}$$

$\left(\left(\cdot 4\right)\left(\cdot 6\right)+\left(\cdot 6\right)\left(\cdot 2\right)\cdot\left(\cdot 4\right)\left(\cdot 4\right)+\cdot 6\left(0.8\right)\right)$

= $\left(\cdot 24+\cdot 12 \quad \cdot 16+\cdot 48\right)$

= $\left(\cdot 36 \quad \cdot 64\right)$

∴ Shares after two week for products A and Bare 36% and 64% respectively.

At equilibrium, we must have (A B) T = (A B)

where A+B = 1

$$\left(A \quad B\right)\begin{pmatrix} \cdot 6 & \cdot 4 \\ \cdot 2 & \cdot 8 \end{pmatrix}=\left(A \quad B\right)$$

⇒ = (-6A+·2B -4A+·8B) = (A B)

Equating the corresponding entries on both sides

we get,

·6A+ ·2B = A

⇒ ·6A + ·2(1 - A) = A

⇒ ·6A + ·2 - ·2A = A

⇒ ·2 = A - ·6A + ·2A

⇒ ·2 = A (1 - ·6 + ·2)

⇒ ·2 = A (·4 + ·2)

⇒ ·2 = A (·6)

$$\Rightarrow A=\frac{\cdot 2}{\cdot 6}=\cdot 33\Rightarrow A=33$$

and B = 1 - A = 1 - ·33 = ·67⇒ B = 67%

∴ Equilibrium is reached when A = 33% and B = 67%

24) Solve the equations x + 2y + z = 7, 2x − y + 2z = 4, x + y − 2z = −1 by using Cramer’s rule

Answer : $\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 2 & -1 & 2 \\ 1 & 1 & -2 \end{vmatrix}$

= $1\begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} - 2\begin{vmatrix} 2 & 2 \\ 1 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$

= 1(2 -2) - 2(-4 -2) + 1(2 + 1)

= 1(0)-2(-6)+1(3)

= 12 + 3 = 15$\neq$0.

Since $\Delta \neq 0$ Cramer's rule can be applied and thesystem is consistent with unique solution.

$\Delta x = \begin{vmatrix} 7 & 2 & 1 \\ 4 & -1 & 2 \\ -1 & 1 & -2 \end{vmatrix}$

= $7\begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} - 2\begin{vmatrix} 4 & 2 \\ -1 & -2 \end{vmatrix} + 1\begin{vmatrix} 4 & -1 \\ -1 & 1 \end{vmatrix}$

= 7 (2 -2) -2 (-8 + 2) + 1 (4 - 1)

= 7 (0) - 2(-6) + 1(3)

= 12 + 3 = 15

$\Delta y = \begin{vmatrix} 1 & 7 & 1 \\ 2 & 4 & 2 \\ 1 & -1 & -2 \end{vmatrix}$

= $1\begin{vmatrix} 4 & 2 \\ -1 & -2 \end{vmatrix} - 7\begin{vmatrix} 2 & 2 \\ 1 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix}$

= 1 (- 8 + 2) -7(-4 -2) + 1(-2 -4)

= 1 (-6) -7 (-6) + 1 (-6)

= - 6 + 42 - 6 = 30

$\Delta z = \begin{vmatrix} 1 & 2 & 7 \\ 2 & -1 & 4 \\ 1 & 1 & -1 \end{vmatrix}$

= $1\begin{vmatrix} -1 & 4 \\ 1 & -1 \end{vmatrix} - 2\begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix} + 7\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$

= 1 (1 - 4) - 2(- 2 - 4) + 7(2 + 1)

= 1(-3)-2(-6)+7(3)

= - 3 + 12 + 21 = 30

$x = \frac{\Delta x}{\Delta} = \frac{15}{15} = 1$

$y = \frac{\Delta y}{\Delta} = \frac{\overset{2}{30}}{15} = 2$

$z = \frac{\Delta z}{\Delta} = \frac{\overset{2}{30}}{15} = 2$

$\therefore$ Solution set is {1, 2, 2}

25) Solve the following equation by using Cramer's rule

x + y + z = 6, 2x + 3y− z =5, 6x−2y− 3z = −7

Answer : $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ 6 & -2 & -3 \end{vmatrix}$

= $1\begin{vmatrix} 3 & -1 \\ -2 & -3 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 6 & 3 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 6 & -2 \end{vmatrix}$

= 1(-9 -2) -1(-6 +6) + 1(-4 -18)

= 1(-11) -1(0) +1(-22)

= -11 -22 = -33 $\neq$0

Since $\Delta \neq 0$

Cramer's rule can be applied and the system is consistent with unique solution

$\Delta x = \begin{vmatrix} 6 & 1 & 1 \\ 5 & 3 & -1 \\ -7 & -2 & -3 \end{vmatrix}$

= $6\begin{vmatrix} 3 & -1 \\ -2 & -3 \end{vmatrix} - 1\begin{vmatrix} 5 & -1 \\ -7 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 3 \\ -7 & -2 \end{vmatrix}$

= 6 (-9 -2) -1(-15 -7) + 1(-10 +21)

= 6 (-11) -1 (-22) + 1 (11)

= -66 + 22 + 11= - 33

$\Delta y = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 5 & -1 \\ 6 & -7 & -3 \end{vmatrix}$

= $1\begin{vmatrix} 5 & -1 \\ -7 & -3 \end{vmatrix} - 6\begin{vmatrix} 2 & -1 \\ 6 & -3 \end{vmatrix} + 1\begin{vmatrix} 2 & 5 \\ 6 & -7 \end{vmatrix}$

= 1(-15 -7) -6(-6 +6) + 1(-14 -30)

= 1(-22) -6(0) + 1 (-44)

= -22 - 44 = - 66

$\Delta z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 3 & 5 \\ 6 & -2 & -7 \end{vmatrix}$

= $= 1\begin{vmatrix} 3 & 5 \\ -2 & -7 \end{vmatrix} - 1\begin{vmatrix} 2 & 5 \\ 6 & -7 \end{vmatrix} + 6\begin{vmatrix} 2 & 3 \\ 6 & -2 \end{vmatrix}$ = 1(-21 +10) -1(-14 -30) +6 (-4 -18)

=1(-11) -1(-44) +6(-22)

= -11 + 44 - 132 = - 99

$$x = \frac{\Delta x}{\Delta} = \frac{-33}{-33} = 1$$

$$y = \frac{\Delta y}{\Delta} = \frac{-66}{-33} = 2$$

$$z = \frac{\Delta z}{\Delta} = \frac{-99}{-33} = 3$$

$\therefore$ Solution set is {1, 2, 3}