QB365 Question Bank Software Study Materials

Differential Equations Important 2,3 & 5 Marks Questions With Answers (Book Back and Creative)

12th Standard

Business Maths and Statistics

Total Marks : 50

2 Marks

10 x 2 = 20

1) Find the order and degree of the following differential equations

$\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2+4y=0$

Answer : $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2+4y=0$

Highest order derivative is $\frac{d^2y}{dx^2}$

∴ order = 2

Power of the highest order derivative $\frac{d^2y}{dx^2}$ is 1.

∴ Degree = 1

2) Solve the following differential equations

$\frac{d^2y}{dx^2}-6\frac{dy}{dx}+8y=0$

Answer : The auxiliary equation is $m^2 - 6m + 8 = 0$

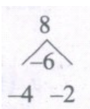
⇒ (m-4)(m-2) = 0

⇒ m = 2, 4

The roots are real and different

∴ Complementary function CF is $Ae^{2x} + Be^{4x}$

∴ The general solution is $y = Ae^{2x} + Be^{4x}$



3) Solve the following differential equations

$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=0$

Answer : The auxiliary equation is $m^2 - 4m + 4 = 0$

⇒ $(m - 2)^2 = 0$

⇒ m 2,2

The roots are real and equal

∴ Complementary function CF is $(Ax + B)e^{2x}$

∴ The general solution is $y = (Ax + B)e^{2x}$

4) Solve the following differential equations: $\frac{d^2y}{dx^2}+16y=0$

Answer : The auxiliary equation is $m^2 + 16 = 0$

$m_2 = -16$

⇒ $m_2 = \pm\sqrt{-16} = \pm 4i$

Hence α = 0 and β = 4

∴ Complementary function CF is

$e^{ax} = [A \cos \beta x + B \sin \beta x]$

$CF = e^{0x}[A \cos 4x + B \sin 4x]$

= A cos 4x + B sin 4x

[∵ e^o= 1]

∴ The general solution is y = A cos 4x + B sin 4x

5) Find the order and degree of the following differential equations.

$\frac{d^3y}{dx^3}+3\left(\frac{dy}{dx}\right)^3+2\frac{dy}{dx}=0$

Answer : The highest derivative is third order and its power is one

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∴ order : 3,

degree : 1

6) Find the order and degree of the following differential equations.

$\frac{d^3y}{dx^3} = 0$

Answer : The highest derivative is of third order and its power is 1.

Order is 3 and degree is 1.

7) Find the differential equation of the following

$xy = c^2$

Answer : Differentiating w.r.t 'x' we get,

$x.\frac{dy}{dx} + y(1) = 0$ [Product rule]

$\Rightarrow x\frac{dy}{dx} + y = 0$ which is the required differentiated equation.

8) Find the order and degree of the following differential equation

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = 0$

Answer : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = 0$

Highest order derivative is $\frac{d^2y}{dx^2}$

∴ order = 2

Power of the highest order derivative $\frac{d^2y}{dx^2}$ is 1

∴ Degree = 1

9) Find the order and degree of the following differential equation

$\left[1 + \frac{d^2y}{dx^2}\right]^{\frac{3}{2}} = a\frac{d^2y}{dx^2}$

Answer : $\left[1 + \frac{d^2y}{dx^2}\right]^{\frac{3}{2}} = a\frac{d^2y}{dx^2}$

Here we eliminate the radical sign.

Squaring both sides, we get

$\left[1 + \frac{d^2y}{dx^2}\right]^{\frac{3}{2}} = a^2(\frac{d^2y}{dx^2})^2$

∴ order = 2, ∴ Degree = 3

10) Find the order and degree of the following differential equation

$y = 2\left(\frac{dy}{dx}\right)^2 + 4x\frac{dx}{dy}$

Answer : $y = 2\left(\frac{dy}{dx}\right)^2 + 4x\frac{dy}{dx}$

$y = 2\left(\frac{dy}{dx}\right)^2 + 4x\frac{1}{\left(\frac{dy}{dx}\right)}$

$y\left(\frac{dy}{dx}\right) = \left(\frac{dy}{dx}\right)^3 + 4x$

∴ order = 1, ∴ Degree = 3

3 Marks 5 x 3 = 15

11) Form the differential equation by eliminating α and β from $(x - \alpha)^2 + (y - \beta)^2 = r^2$

Answer : Given equation is $(x - \alpha)^2 + (y - \beta)^2 = r^2$

Differentiating w.r.t. 'x' we get,

$2(x - \alpha)^2 + (y - \beta)\frac{dy}{dx} = r^2$

$\Rightarrow (x - \alpha) + (y - \beta)\frac{dy}{dx} = 0$....(2)

Differentiating again w.r.t. 'x' we get

$1+(y-\beta)\ \frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{dy}{dx} = 0$

$\Rightarrow 1+(y-\beta)\ \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

$\Rightarrow y-\beta = \frac{-1\left(1+\left(\frac{dy}{dx}\right)^2\right)}{\frac{d^2y}{dx^2}}$ .....(3)

Substituting this value in (2) we get

$(x-\alpha) = \left(\frac{1+\left(\frac{dy}{dx}\right)^2\frac{dy}{dx}}{\frac{d^2y}{dx^2}}\right)$ ......(4)

Substituting (3) and (4) in (1) we get

$\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\left(\frac{dy}{dx}\right)^2\right\}}{\left(\frac{d^2y}{dx^2}\right)^2} + \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^2}{\left(\frac{d^2y}{dx^2}\right)^2}$

$= \left(\frac{d^2y}{dx^2}\right)^2$

$\Rightarrow \left[1 + \left(\frac{dy}{dx}\right)^2\right]\left[\left(\frac{dy}{dx}\right)^2 + 1\right] = r^2\left(\frac{d^2y}{dx^2}\right)$

$\Rightarrow \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3$

$= r^2\left(\frac{d^2y}{dx^2}\right)^2$

12) Find the differential equation of the family of all straight lines passing through the origin.

Answer : Let the equation of straight lines passing through the origin be

$y = mx$ ..(1)

where m is the arbitrary constant

Differentiating w.r.t 'x' we get,

$\frac{dy}{dx} = m(1) \Rightarrow \frac{dy}{dx} = m$ .....(2)

Substituting (2) in (1) we get,

$y = x\frac{dy}{dx}.$

13) Solve $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$

Answer : Given $\frac{dy}{dx} = e^{x-y} + x^2e^{-y} = e^{-y}e^x + e^{-y}x^2$

$= e^{-y}(e^x + x^2)$

Separating the variables, we get $e^y dy=(e^x + x^2)dx$

Integrating, we get $\int e^y dy = \int (e^x+x^2)dx$

$e^y = e^x + \frac{x^3}{3} + c$

14) Solve: $\frac{dy}{dx} = y \sin 2x$

Answer : Separating the variables, we get,

$\frac{dy}{x} = \sin 2x\ dx$

Integrating both sides we get,

$\int \frac{dy}{y} = \int \sin 2x$

$\Rightarrow \log y = \frac{-\cos 2x}{1} + c$

15) Find the differential equation of the following

$y = c\,(x - c)^2$

Answer : Given equation is y = $c(x-c)^2$

⇒ y = $c(x^2-2xc+c^2)$....(1)

Differentiating w.r.t 'x' we get

$\frac{dy}{dx}$-2c(x-c) ...(2)

(1) ÷ (2) gives

$\frac{y}{\frac{dy}{dx}} = \frac{c(x-c)^2}{2c(x-c)} = \frac{y}{\frac{dy}{dx}} = \frac{x-c}{2}$

x-c = $\frac{2y}{\frac{dy}{dx}} \Rightarrow c = x - \frac{2y}{\frac{dy}{dx}}$

Substituting the value of c in (1) we get

y=$\left(x - \frac{2y}{\frac{dy}{dx}}\right)\left(x - x + \frac{2y}{\frac{dy}{dx}}\right)^2$

=$\left(x - \frac{2y}{\frac{dy}{dx}}\right)\left(\frac{4y^2}{\left(\frac{dy}{dx}\right)^2}\right)$

y =$\left(\frac{x\frac{dy}{dx}-2y}{\frac{dy}{dx}}\right)\left(\frac{4y^2}{\left(\frac{dy}{dx}\right)^2}\right)$

$\Rightarrow y\left(\frac{dy}{dx}\right)^3 = 4y^2\left(x\frac{dy}{dx} - 2y\right)$

$\Rightarrow \left(\frac{dy}{dx}\right)^3 = 4y\left(x\frac{dy}{dx} - 2y\right)$

$\Rightarrow \left(\frac{dy}{dx}\right)^3 = 4xy\left(\frac{dy}{dx}\right) - 8y^2$

$\Rightarrow \left(\frac{dy}{dx}\right)^3 - 4xy\left(\frac{dy}{dx}\right) + 8y^2 = 0$

5 Marks 3 x 5 = 15

16) If $\frac{dy}{dx}$ + 2y tan x = sin x and if y = 0 when x = $\frac{\pi}{3}$ express y in terms of x.

Answer : Given differential equation is of the form

$\frac{dy}{dx}$+Py = Q where

P = 2 tanx; Q = sinx

$\int pdx = \int 2\tan x dx$ = 2(log secx) = log $\sec^2$x

∴ Integrating factor (I. F) =$e^{\int pdx} = e^{log \quad sec^{2x}}$

= $\sec^2$x

Hence the solution is

$ye^{\int pdx} = \int Q.e^{\int pdx}dx + c$

⇒ y $\sec^2$x =$\int \sin x \sec^2 x \, dx$+c

⇒ y $\sec^2$x =$\int \sin x. \frac{1}{\cos^2 x}$ dx + c

⇒ y $\sec^2$x =$\int \frac{\sin x}{\cos x}. \frac{1}{\cos x}$ dx + c

⇒ y $\sec^2$x =$\int \tan x$ sec x dx + c

⇒ y $\sec^2$x = sec x + c ...(1)

Also its given when x = $\frac{\pi}{3}$, y = 0

∴ $0(sec^2\frac{\pi}{3}) = sec\frac{\pi}{3} + c$

⇒ 0 = 2+c ⇒ c = -2

∴ (1) becomes

y sec 2x = sec x-2

17) Suppose that the quantity demanded $Q_d = 29 - 2p - 5\frac{dp}{dt} + \frac{d^2p}{dt^2}$ and quantity supplied Q_s = 5 + 4p where p is the price. Find the equilibrium price for market clearance.

Answer : For market clearance, the required condition is $Q_d = Q_s$

$\Rightarrow 29 - 2p - 5\frac{dp}{dt} + \frac{d^2p}{dt} = 5 + 4p$

$\Rightarrow 24 - 6p - 5\frac{dp}{dt} + \frac{d^2p}{dt^2} = 0$

$\Rightarrow \frac{d^2p}{dt^2} + 5\frac{dp}{dt} - 6p = -24$

$(D^2-5D-6)p = -24$

The auxiliary equation is

$m^2 - 5m - 6 = 0$

$(m-6)(m+1) = 0$

$\Rightarrow m = 6, -1$

$C.F = Ae^{6t} + Be^{-t}$

$P.I = \frac{1}{\phi(D)} f(x)$

$= \frac{1}{D^2-5D-6}(-24)e^{0t}$

$= \frac{-24}{-6}$ (Replace D by 0)

= 4

The general solution is p = C.F + P.I

$= Ae^{6t} + Be^{-t} + 4$

18) A manufacturing company has found that the cost C of operating and maintaining the equipment is related to the length 'm' of intervals between overhauls by the equation $m^2\frac{dC}{dm} + 2mC = 2$ and c = 4 and when m = 2. Find the relationship between C and m.

Answer : Given $m^2\frac{dc}{dm} + 2mc = 2$

Dividing by m^2, we get,

$\frac{dC}{dm} + \frac{2c}{m} = \frac{2}{m^2}$

This is of form $\frac{dy}{dx} + Py = Q$

where $P = \frac{2}{m}$ and $Q = \frac{2}{m}$

$\int pdm = \int \frac{2}{m}dm = 2\log m = \log m^2$

∴ Integrating Factor (LF.) = $e^{\int Pdm} = e^{logm^2}$

$= m^2$

∴ The solution is

$ce^{\int Pdm} = \int Qe^{\int Pdm} m + c.K$

$cm^2 = \int \frac{2}{m^2}m^2dm + K = \int 2dm + cK$

$cm^2 = 2m + K$

Given that c = 4, when m = 2

$4(2^2) = 2(2) + K \Rightarrow 16 - 4 = K$

$\Rightarrow K = 12$

∴ (1) becomes

$cm^2 = 2m + 12$

$\Rightarrow cm^2 = 2(m + 6)$