QB365 Question Bank Software Study Materials

Numerical Methods Important 2,3 & 5 Marks Questions With Answers (Book Back and Creative)

12th Standard

Business Maths and Statistics

Total Marks : 50

2 Marks

10 x 2 = 20

1) Find (i) Δe^{ax}

(ii) $\Delta^2 e^x$

(iii) Δ log x

Answer : (i) $\Delta e^{ax} = e^{a(x+h)} - e^x$

$= e^{aX}.e^h - e^{ax}$ [$\because a^{m+n} = a^m.a^n$]

$= e^{ax}[e^h - 1]$

(ii) $\Delta^2 e^x = \Delta.[\Delta e^x]$

$= \Delta[e^{x+h} - e^x]$

$= \Delta[e^x e^h - e^x]$

$= \Delta e^x [e^h - 1]$

$= (e^h - 1)\Delta e^x$

$= (e^h - 1).(e^h - 1).e^x$

$= (e^h - 1)^2.e^x$

(iii) Δ log x = log(x+h) − log x

= log $\frac{x+h}{x}$

= log $\left(\frac{x}{x} + \frac{h}{x}\right)$

= log $\left(1 + \frac{h}{x}\right)$

2) Evaluate $\Delta^2\left(\frac{1}{x}\right)$ by taking '1' as the interval of differencing.

Answer : $\Delta^2\left(\frac{1}{x}\right) = \Delta\left(\Delta\left(\frac{1}{x}\right)\right)$

Now $\Delta\left[\frac{1}{x}\right] = \frac{1}{1+x} - \frac{1}{x}$

$\Delta^2\left(\frac{1}{x}\right) = \Delta\left(\frac{1}{1+x} - \frac{1}{x}\right)$

$= \Delta\left(\frac{1}{1+x}\right) - \Delta\left(\frac{1}{x}\right)$

Similarly $\Delta^2\left(\frac{1}{x}\right) = \frac{2}{x(x+1)(x+2)}$

3) If f (x) = e^{ax} then show that f(0), Δf(0), Δ^2f(0) are in G.P

Answer : Given f(x) = e^{ax}

$\therefore$ f(0) = e^{0x} = 1 (1)

Δf(x) = f(x + h) - f(x) (2)

Δf(0) = f(1) - f(0) = $e^{a(1)} - e^0 = e^{a-1}$ (2)

Δ^2f(0) = Δ [Δf(0)]

= Δ[f(1) + f(0)] = Δf(1) - f(1) - Δ(0)

= f(2) - f(1) - (e^a - 1)

= $e^{2a} - e^a - e^a + 1$

Δ^2f(0) = $e^{2a} - 2e^a + 1$

From (1), (2) & (3), the three terms are 1, e^a - 1, $e^{2a} - 2e^a + 1$

$\Rightarrow$ 1, e^a -1, $(e^a - 1)^2$ [$\because (a - b)^2 = a^2 - 2ab + b^2$]

Common ratio r = $\frac{e^a - 1}{1} = \frac{(e^a - a)^2}{e^a - 1}$ = e^a - 1

Since the common ratio is same throughout Δf(0) and Δ^2f(0) forms a G.P.

4) Prove that

$(1 + \Delta)(1 - \nabla) = 1$

Answer : LHS = $(1 + \Delta)(1 - \nabla)$

[$\because \Delta$ = E - 1 &
E 1

$\nabla = \frac{E-1}{E}$]

$= (1 + \Delta)(1 - \nabla)$

$= (1 + E - 1)\left(1 - \frac{E-1}{E}\right)$

$= E\left(1 - \frac{E-1}{E}\right)$

$= E - E\left(\frac{E-1}{E}\right)$

$= E - (E - 1)$

$= E - E + 1 = 1$

= RHS.

5) Prove that

$\nabla\Delta = \Delta - \nabla$

Answer : LHS = $\Delta\nabla = (E-1)\left(\frac{E-1}{E}\right)$

$= E\left(\frac{E-1}{E}\right) - 1\left(\frac{E-1}{E}\right)$

$= \Delta - \nabla$ [$\because \Delta = E - 1$ & $\nabla = \frac{E-1}{E}$]

= RHS

Hence Proved.

6) Prove that

$E\nabla = \Delta = \nabla E$

Answer : LHS = $E\nabla$

$= E\left(\frac{E-1}{E}\right)$

[$\because \Delta = \frac{E-1}{E}$]

[$\because \Delta = E - 1$]

$= E - 1 = \Delta$

RHS

Also $\nabla E = \left(\frac{E-1}{E}\right) \cdot E$

$= E - 1 = \Delta$ = RHS

$\therefore E\nabla = \Delta = \nabla E$

Hence proved.

7) Find the missing term from the following data.

x	20	30	40
y	51	-	34

Answer : Since only two values of yare given, the polynomial which fits the data is of degree 1.

Hence 2nd differences are zeros

$\therefore \Delta^2(y_0) = 0$

$\Rightarrow (E-1)^2 y_o = 0$

$\Rightarrow (E^2 - 2E + 1)\, y_o = 0$

$\Rightarrow y_2 - 2y_1 + y_o = 0$

$\Rightarrow 34 - 2y_1 + 51 = 0$

$\Rightarrow 85 - 2y_1 = 0$

$\Rightarrow 2y_1 + 85 \Rightarrow y_1 = \frac{85}{2}$

$\Rightarrow y_1 = 42.5$

8) Evaluate $\Delta\left(\frac{1}{x}\right)$ y taking 1 as the interval of differencing.

Answer : $\Delta\left(\frac{1}{x}\right) = \left(\frac{1}{x+1} - \frac{1}{x}\right)$

$= \frac{x-x-1}{(x+1)(x)} = \frac{-1}{(x)(x+1)}$

9) Prove that $f(3) = f(2) + \Delta I(1) + \Delta^2 f(1)$ by taking 1 will givea- as the interval of differencing.

Answer : We know that $f(3) - f(2) = \Delta f(2)$

$\because f(2) - f(1) = \Delta f(1)$

$\Delta f(2) = \Delta[f(1) + \Delta f(1)]$

$= \Delta f(1) + \Delta^2 f(1)$

$f(3) = f(2) + \Delta f(1) + \Delta^2 f(1)$

10) Calculate a forward difference table for the following data

x	20	30	40	50
y	51	43	34	24

Answer :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	51			
		-8		
30	43		-1	
		-9		0
40	34		-1	
		-10		
50	42			

3 Marks 5 x 3 = 15

11) Construct a forward difference table for the following data

x	0	10	20	30
y	0	0.174	0.347	0.518

Answer : The Forward difference table is given below

x	**y**	**Δy**	**Δ^2y**	**Δ^3y**
0	0			
		0.174		
10	0.174		-0.001	
		0.173		-0.001
20	0.347		-0.002	
		0.171		
30	0.518			

12) From the following table find the missing value

x	2	3	4	5	6
f(x)	45.0	49.2	54.1	-	67.4

Answer : Since only four values of f(x) are given, the polynomial which fits the data is of degree three. Hence fourth differences are zeros.

(ie) $\Delta^4 y_0 = 0$, $\therefore (E-1)^4 y = 0$

$(E^4 - 4E^3 + 6E^2 - 4E + 1)y_0 = 0$

$E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$67.4 - 4y_3 + 6(54.1) - 4(4.2) + 45 = 0$

$240.2 = 4y_3 \therefore y_3 = 60.05$

13) If $y = x^3 - x^2 + x - 1$ calculate the values of y for x = 0,1,2,3,4,5 and form the forward differences table.

Answer : when x = 0, y = 0 + 0 + 0 - 1 ⇒ y = -1

when x = 1, y = 1^3-1^2+ 1-1 ⇒ y=0.

when x = 2, y = 2^3 - 2^2 + 2 - 1 ⇒ y = 8 - 4 + 1 ⇒ y = 5

when x = 3, y = 3^3 - 3^2+ 3 - 1 ⇒ y = 27 - 9 + 2 ⇒ y = 20

when x = 4, y = 4^3 - 4^2+ 4 -1 ⇒ y= 64 -16 + 3 ⇒ y = 51

when x = 5, y = 5^3 - 5^2+ 5 - 1 ⇒ y = 125 - 25 + 4 ⇒ y = 104.

Hence, the forward difference table is

x	**y**	**Δy**	**Δ^2y**	**Δ^3y**	**Δ^4y**
0	-1				
		1			
1	0		4		
		5		6	
2	5		10		0
		15		6	
3	20		16		0
		31		6	
4	51		22		
		53			
5	104				

14) If h = 1 then prove that $(E^{-1}\Delta)x^3 = 3x^2 - 3x + 1$.

Answer : Given h = 1

LHS = $(E^{-1}\Delta)\ x^3$

= $\Delta(E^{-1}(x^3))$

= $\Delta(x - h)^3$ [∵ $E^{-1}f(x) = f(x - nh)$]

= $\Delta(1 - h)^3$ [∵ h = 1]

= $(x - 1+ 1)^3 - (x - 1)^3$ [∵ Δf(x) =f(x + h) - f(x)]

= $x^3 - (x - 1)^3$

= $x^3 - (x^3 - 3x^2 + 3x - 1)$

[∵ $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$]

= $x^3 - x^3 + 3x^2 - 3x + 1$

= $3x^2 - 3x + 1$

= RHS

Hence proved

15) The following data relates to indirect labour expenses and the level of output

Months	Jan	Feb	Mar	Apr	May	June
Units of output	200	300	400	640	540	580
Indirect labour expenses (Rs)	2500	2800	3100	3820	3220	3640

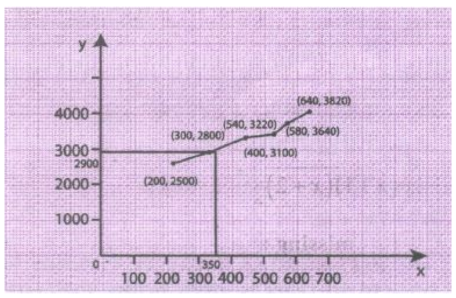
Estimate the expenses at a level of output of 350 units, by using graphic method

Answer : Scale:

In x-axis 1 cm = 100 units

In y-axis 1 cm = 1000 units

Plot the points (200, 2500), (300, 2800) (400, 3100) (640; 3820), (540,3220) and (580, 3640).



At x = 350, draw a vertical line and from the intersecting point on the curve, draw a horizontal line.

From the graph, we find that when x = 350, y = 2900.

Hence, the expense at a level of 350 units is Rs. 2900

5 Marks 3 x 5 = 15

16) The values of y = f(x) for x = 0,1,2, ...,6 are given by

x	0	1	2	3	4	5	6
y	2	4	10	16	20	24	38

Estimate the value of y (3.2) using forward interpolation formula by choosing the four values that will give the best approximation.

Answer : Since we apply the forward interpolation formula,last four values of f(x) are taken into consideration (Take the values from x = 3).

The forward interpolation formula is

$y_{(x=x_0+nh)} = y_0 + \frac{n}{1!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \ldots$

$x_0 + nh = 3.2$, $x_0 = 3$,y = 1

$\therefore n = \frac{1}{5}$

The difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^2 y$
3	16			
		4		
4	20		0	
		4		10
5	24		10	
6	38			

$y_{(x=3.2)} = 16 + \frac{1}{5}(4) + \frac{\frac{1}{5}\left(\frac{-4}{5}\right)}{2}(0) + \frac{\frac{1}{5}\left(\frac{-4}{5}\right)\left(\frac{-9}{5}\right)}{6} \times 10$

= 16+0.8+0+0.48

= 17.28

17) The following data are taken from the steam table

Temperature C^0	140	150	160	170	180
Pressure kg f / cm^2	3.685	4.854	6.302	8.076	10.225

Find the pressure at temperature t = 175^0

Answer : Since the pressure required is at the end of the table, we apply Backward interpolation formula. Let temperature be x and the pressure be y.

$y_{(x=x_0+nh)} = y_0 + \frac{n}{n!}\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-)(n-2)}{3!}\Delta^3 y_0 + \ldots$

To find y at x = 175

$\therefore x_n + nh = 175$, $x_n = 180$, h = 10 ⇒ n = −0.5

x	y	Δy	$\Delta^2 y$	$\Delta^2 y$	$\Delta^4 y$
140	3.685				
		1.169			
150	4.854		0.279		
		1.448		02.047	
160	6.032		0.326		0.002
		1.774		0.049	
170	8.076		0.375		
		2.149			
180	10.225				

$y_{(x=1750} = 10.225 + (-0.5)(2.149) + \frac{(-0.5)(0-5)}{2!}(0.375) + \frac{(-0.5)(0-5)(1.5)}{3!}(0.049) + \frac{(-0-5)(0.5)(1.5)(2.5)}{4!}(0.002)$ =

10.225−1.0745−0.046875−0.0030625 − 0.000078125

= 9.10048438

= 9.1

18) Using interpolation, find the value of f(x) when x = 15

x	3	7	11	19
f(x)	42	43	47	60

Answer : Using interpolation, find the value of f(x) when x =1

Here the intervals are unequal. By Lagrange's interpolation formula, we have

$x_0=3$, $x_1 = 7$, $x_2 = 11$, $x_3 = 19$

$y_0 = 42$, $y_1 = 43$, $y_2 = 47$, $y_3 = 60$ and x = 15

$\therefore$ Y =f(x) = $\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} \times y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} \times y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} \times y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} \times y_3$

= $\frac{(15-7)(15-11)(15-19)}{(3-7)(3-11)(3-19)} \times 42 + \frac{(15-3)(15-7)(15-19)}{(11-3)(11-7)(11-19)} \times 43 + \frac{(15-3)(15-7)(15-19)}{(11-3)(11-7)(11-19)} \times 47 + \frac{(15-3)(15-7)(15-11)}{(19-3)(19-7)(19-11)} \times 60$

$= \frac{(8)(4)(-4)}{(-4)(-8)(-16)} \times 42 + \frac{(12)(4)(-4)}{(4)(-4)(-12)} \times 43 + \frac{(12)(8)(-4)}{(8)(4)(-8)} \times 47 + \frac{(12)(8)(4)}{(16)(12)(8)} \times 60$

$= \frac{42}{4} - 43 + \frac{12 \times 47}{8} + \frac{60 \times 4}{16}$

= $\frac{21}{2} - 43 + 70.5 + 15$

= 10.5 - 43 + 70.5 + 15

y= 53

Hence when x = 15, f(x) = 53.