QB365 Question Bank Software Study Materials

Probability Distributions Important 2,3 & 5 Marks Questions With Answers (Book Back and Creative)

12th Standard

Business Maths and Statistics

Total Marks : 50

2 Marks

10 x 2 = 20

1) A fair coin is tossed 6 times. Find the probability that exactly 2 heads occurs.

Answer : Let X be a random variable follows binomial distribution with probability value p = 1/2 and q = 1/2

probability that exactcy 2 heads occur are as follows

P(X = 2) = $\binom{6}{x} p^x q^{n-x}$

$= \binom{6}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{6-2}$

$= \frac{15}{64}$

2) Verfy the following statement:

The mean of a Binomial distribution is 12 and its standard deviation is 4.

Answer : Mean: np = 12

$SD = \sqrt{npq} = 4$

$npq = 4^2 = 16, \frac{np}{npq} = \frac{12}{16} = \frac{3}{4}$

$q = \frac{4}{3} > 1$

Since p + q cannot be greater than unity, the Statement is wrong

3) In tossing of a five fair coin, find the chance of getting exactly 3 heads.

Answer : Let X be a random variable follows binomial distribution with p = q = 1/2

P (3 heads) = $5C_x\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{5-x}$

$= 5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{5-3}$

$= 5C_3\left(\frac{1}{2}\right)^5$

$= \frac{5}{16}$

4) In a Poisson distribution the first probability term is 0.2725. Find the next Probability term

Answer : Given that p(0) = 0.2725

$\frac{e^{-\lambda}\lambda^0}{0!} = 0.2725$

⇒ $e^{-\lambda}$ = 0.2725 (by using exponent table)

λ = 1.3

∴ p(X=1) = $e^{-1.3}$(1.3)/1!

= $e^{-1.3}$(1.3)

= 0.2725 x 1.3

= 0.3543

5) Define Binomial distribution.

Answer : A random variable X is said to follow binomial distribution with parameter n and p, if it assumes only non-negative value and its probability mass function is given by

P(X = x) = P(x) = q = 1-p $= \begin{cases} n_{C_x} P^x q^{n-x}, x = 0, 1, 2, \ldots n; \\ 0, \quad otherwise \end{cases}$

6) Define Bernoulli trials.

Answer : A random experiment whose outcomes are of two types namely success S and failure P, occurring with probabilities p and q is called a Bernoulli trial.

7) Write down the conditions for which the binomial distribution can be used.

Answer : The binomial distribution can be used under the following conditions.

(i) The number of trials en' is finite,

(ii) The trials are independent of each other.

(iii) The probability of success 'p' is constant for each trial.

(iv) In every trial there are only two possible outcomes namely success or failure.

8) Write any 2 examples for Poisson distribution.

Answer : (i) Number of lightnings per second.

(ii) Number of printing mistakes per page in a textbook.

9) Define Normal distribution.

Answer : A random variable X is said to follow a normal distribution with parameters mean μ and variance σ^2, if its probability density function is given by

$$f(x:\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{\frac{-1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} \quad \begin{matrix}-\infty<x<\infty\\-\infty<\mu<\infty\\\sigma>0\end{matrix}$$

10) Write down the conditions in which the Normal distribution is a limiting case of binomial distribution.

Answer : Normal distribution is a limiting case of Binomial distribution under the following conditions.

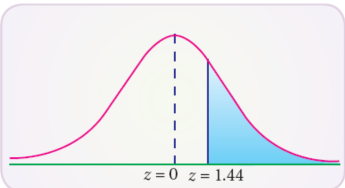
(i) n, the number of trials is infinitely large i.e. $n \longrightarrow \infty$

(ii) Neither p(or q) is very small.

3 Marks 5 x 3 = 15

11) Assume the mean height of children to be 69.25 cm with a variance of 10.8 cm. How many children in a school of 1,200 would you expect to be over 74 cm tall?

Answer :



Let the distribution of heights be normally distributed with mean mean 68.22 and standard deviation = 3.286

$$Z=\frac{X-\mu}{\sigma}=\frac{X-69.25}{3.286}$$

When X = 74

$$Z=\frac{X-\mu}{\sigma}=\frac{74-69.25}{3.286}=1.4455$$

Now P(Z > 74) = P(Z > 1.44)

= 0.5 – 0.4251

= 0.0749

Expected number of children to be over 74 cm out of 1200 children

= 1200 × 0.0749 ≈ 90 children

12) Mention the properties of binomial distribution.

Answer : (i) Binomial distribution is symmetrical if p = q = 0.5. It is skew symmetric if p≠q. It is positively skewed if p < 0.5 and it is negatively skewed if p > 0.5.

(ii) For binomial distribution, variance is less than mean.

Variance = npq = (np)q < np < mean.

13) The mortality rate for a certain disease is 7 in 1000. What is the probability for just 2 deaths on account of this disease in a group of 400? [Given $e^{(-2.8)}$ = 0.06]

Answer : Let X be probability of mortality rate for a certain disease

∴ p = $\frac{7}{1000}$ and n = 400

$$\therefore \lambda = np = \frac{7}{1000}\times 400 = \frac{28}{10} = 2.8$$

Hence X follows poisson distribution with p(x, λ)

= $\frac{e^{-\lambda}.\lambda^x}{x!}$

∴ P (just 2 deaths) = P(X = 2)

= $\frac{e^{-2.8}(2.8)^2}{2!}$ [∵ p(x,λ) = $\frac{e^{-\lambda}.\lambda^x}{x!}$, λ = 2.8, x = 2]

= $\frac{(0.06)(2.8)^2}{2}$ = $(0.03)(2.8)^2$ = 0.2352

∴ Probability for just 2 deaths on account of this disease = 0.2352.

14) If electricity power failures occur according to a Poisson distribution with an average of 3 failures every twenty weeks, calculate the probability that there will not be more than one failure during a particular week.

Answer : Given average = λ = $\frac{3}{20}$ = 0.15

P(will not be more than one failure)

= P(X ≤ 1)

= P(X = 0) + P(X = 1)

= $\frac{e^{-\lambda}.\lambda^0}{0!} + \frac{e^{-\lambda}.\lambda^1}{1!}$

= $e^{-\lambda}$ (1+λ) =$e^{-0.15}$ (1+0.15)

= (0.8607) (1.15)

= (0.98981 [$e^{-0.15}$ =0.8607]

∴ Probability that there will not be more than one failure = 0.98981

15) Entry to a certain University is determined by a national test. The scores on this test are normally distributed with a mean of 500 and a standard deviation of 100. Raghul wants to be admitted to this university and he knows that he must score better than at least 70% of the students who took the test. Raghul takes the test and scores 585. Will he be admitted to this university?

Answer : Given μ = 500, σ = 100

P(X< 585)

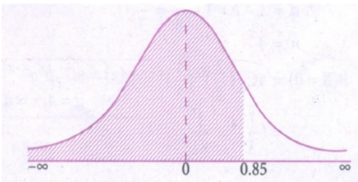
When X = 585, Z = $\frac{X-\mu}{\sigma}$

= $\frac{585-500}{100} = \frac{85}{100}$ = 0.85

P(X < 858) = P(Z< 0.85)

P(-∞ = 0.5 + 0.3023

= 0.8023



When his score is 585,

80.23% of people have scored less than Rahul

∴ Rahul will be admitted to the university.

5 Marks 3 x 5 = 15

16) An insurance company has discovered that only about 0.1 per cent of the population is involved in a certain type of accident each year. If its 10,000 policy holders were randomly selected from the population, what is the probability that not more than 5 of its clients are involved in such an accident next year? (e^{-10}=.000045)

Answer : p = probability that a person will involve in an accident in a year

= 0.1/100 = 1/1000

given n = 10,000

so, λ = np = 10000$\left(\frac{1}{10000}\right)$ = 10

Probability that not more than 5 will involve in such an accident in a year

P(X $\leq$ 5) = P(X = 0)+P(X = 1)+P(X = 2)+P(X = 3)+P(X = 4)+P(X = 5)

$= e^{-10}[1 + \frac{10}{1!} + \frac{10^2}{2!} + \frac{10^3}{3!} + \frac{10^4}{4!} + \frac{10^5}{5!}]$

= 0.06651

17) If the probability that an individual suffers a bad reaction from injection of a given serum is 0.001, determines the probability that out of 2,000 individuals

(a) exactly 3, and

(b) more than 2 individuals will suffer a bad reaction.

Answer : Consider a 2,000 individuals getting injection of a given serum , n = 2000

Let X be the number of individuals suffering a bad reaction.

Let p be the probability that an individual suffers a bad reaction = 0.001

and q = 1– p = 1– 0.001 = 0.999

Since n is large and p is small, Binomial Distribtuion approximated to poisson distribution

So, λ = np = 2000 × 0.001 = 2

(i) Probability out of 2000, exactly 3 will suffer a bad reaction is

$P(X=3)=\frac{e^{-\lambda}\lambda^{x}}{x!}=\frac{e^{-2}2^{3}}{3!}=0.1804$

(ii) Probability out of 2000, more than 2 individuals will suffer a bad reaction

= P(X > 2)

1-[P(X≤2)]

= 1 – [P(x = 0) + P(x = 1) + P(x = 2)]

$=1-\left[\frac{e^{-2}2^{0}}{0!}+\frac{e^{-2}2^{1}}{1!}+\frac{e^{-2}2^{2}}{2!}\right]$

$=1-e^{2}\left(\frac{2^{0}}{0!}+\frac{2^{1}}{1!}+\frac{2^{2}}{2!}\right)$

= 0.323

18) In a distribution 30% of the items are under 50 and 10% are over 86. Find the mean and standard deviation of the distribution.

Answer : Plot the variable X = 50 on the left side and

X = 86 on the right side of the curve.

Given P(-∞< Z_1 < -Z_1) = 0.3

⇒ P(-Z_1 < Z < Z_1) = 0.2 [∵ 0.5 - 0.3 = 0.2]

⇒ P(0 < Z < Z_1) = 0.2 [By symmetry]

⇒ Z_1 = -0.52 [From the normal distribution table 'and it lies on the negative side]

⇒ -0.52 = $\frac{X-\mu}{\sigma}$ ⇒ -0.52 σ = 50-μ

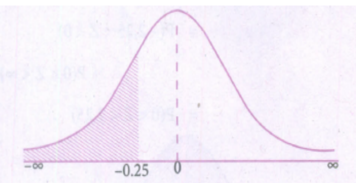
⇒ 50-μ = -0.52 σ ....(1)

Also given P(Z_2 ∴ P($Z_2$2) = 0.4 [∵ 0.5-0.1 = 0.4]

⇒ Z_2 = 1.28 (from the table)

⇒ 1.28 = $\frac{86-\mu}{\sigma}$

⇒ 86-μ = 1.28 σ ...(2)



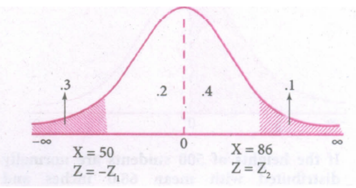
Substituting σ = 20 in (2) we get,

86-μ = (1.28)(20)

86-μ = 25.6

μ = 86-25.6

μ = 60.4



Hence, the mean is 60.4 and standard deviation is 20.