QB365 Question Bank Software Study Materials

Random Variable and Mathematical Expectation Important 2,3 & 5 Marks Questions With Answers (Book Back and Creative)

12th Standard

Business Maths and Statistics

Total Marks : 50

2 Marks

10 x 2 = 20

1) Construct cumulative distribution function for the given probability distribution.

X	0	1	2	3
P(X = x)	0.3	0.2	0.4	0.1

Answer : We know Fx (x) = P(X ≤ x) for all x ∈ R

∴ F(0) = P(X ≤ 0) = P(0) = 0.3

F(1) = P(X ≤ 1) = P(0)+P(l)

= 0.3 + 0.2 = 0.5

F(2) = P(X ≤ 2) = P(0) + P(1) + P(2)

= 0.3 + 0.2 + 0.4 = 0.9

F(3) = P(X ≤ 3) = P(0) + P(1) + P(2) + P(3)

= 0.3 + 0.2 + 0.4 + 0.1 = 1

∴ Cumulative distribution function for the given probability distribution is 1

2) Define discrete random variable.

Answer : **Discrete random variable :**

A variable which can assume finite number of possible values or an infinite sequence of countable real numbers is called a discrete random variable.

3) Six men and five women apply for an executive position in a small company. Two of the applicants are selected for an interview. Let X denote the number of women in the interview pool. We have found the probability mass function of X.

X = x	0	1	2
P(x)	$\frac{2}{11}$	$\frac{5}{11}$	$\frac{4}{11}$

How many women do you expect in the interview pool?

Answer : Expected number of women in the interview pool is

$E(X)=\sum_x xP_X(x)$

$=\left[\left(0\times\frac{2}{11}\right)+\left(1\times\frac{5}{11}\right)+\left(2\times\frac{4}{11}\right)\right]$

$=\frac{13}{11}$

4) Let X be a random variable defining number of students getting A grade. Find the expected value of X from the given table

X = x	0	1	2	3
P(X=x)	0.2	0.1	0.4	0.3

Answer : Given probability mass function is

X = x	0	1	2	3
P(X=x)	0.2	0.1	0.4	0.3

$E(X)=\sum_{i=0}^{3} xp(x)$

= 0(0.2) + 1(0.1) + 2(0.4) + 3(0.3)

= 0.1+0.8+0.9

E(X) = 1.8

5) Let X be a continuous random variable with probability density function

$f_x(x)=\begin{cases}2x, & 0\le x\le 1\\ 0, & otherwise\end{cases}$

Find the expected value of X.

Answer : Given probability density function is

$$f_x(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$
$$E(X) = \sum_{x=3}^{4,5} xp(x)$$
$$E(X) = \int_0^1 x.f(x)dx = \int_0^1 x.2(xdx = 2\int_0^1 x^2 dx$$
$$= 2\left(\frac{x^3}{3}\right)_0^1 = \frac{2}{3}(1^3 - 0^3) = \frac{2}{3}$$
$$E(X) = \frac{2}{3}$$

6) In an investment, a man can make a profit of Rs. 5,000 with a probability of 0.62 or a loss of Rs. 8,000 with a probability of 0.38. Find the expected gain.

Answer : Given that in an investment profit is Rs. 5000 with probability of 0.62 or a loss of Rs.8000 with a probability of 0.38.

Hence, the probability mass function is

X = x	5000	-8000
P(X = x)	0.61	0.38

∴ Expected gain E(X) = 5000(0.62) - 8000 (0.32)

= 3100-3040

= Rs. 60

Hence, the expected gain is = Rs. 60

7) What do you understand by Mathematical expectation?

Answer : Mathematical expectation E(X) is an average of the values, that the random variable takes on, where each value is weighted by the probability that the random variable is equal to that value. Values that are most probable receive more weight. Each value x is multiplied by the approximate probability that X equals the valuex.

8) How do you define variance in terms of Mathematical expectation?

Answer : Var (X) = $E(X^2)$ - $[E(X)]^2$ where

$$E(X^2) = \begin{cases} \sum x^2 p(x) \text{for discrete random variable} \\ \int_{-\infty}^{\infty} x^2 P(x) \text{dx for continuous random variable} \end{cases}$$

9) Define Mathematical expectation in terms of discrete random variable.

Answer : Let X be a discrete random variable with probability mass function p(x), then its expected value is defined by E(X) =$\sum_x x.p(x)$

10) The time to failure in thousands of hours of an important piece of electronic equipment used in a manufactured DVD player has the density function

$$f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the expected life of this piece of equipment.

Answer : Given p.d.f. is f(x) = $\begin{cases} 2e^{-2x}, > 0 \\ 0, \quad otherwise \end{cases}$

Expected life of this piece of equipment is

$$E(X) = \int_{-\infty}^{\infty} x.f(x)dx = \int_0^{\infty} x.2e^{-2x}$$
$$= 2\int_0^{\infty} xe^{-2x}dx = 2\left[\frac{1!}{2^2}\right]$$
$[\because \int_0^{\infty} x^n e^{-ax}dx = \frac{n!}{a^{n+1}}$ gamma integral here n = 1, a = 2]
$$E(X) = \frac{2}{4} = \frac{1}{2}$$

3 Marks 5 x 3 = 15

11) What are the properties of

(i) discrete random variable and

(ii) continuous random variable?

Answer : **For discrete random variable:**

$p(x_i) \ge 0 \forall i$ and $\sum_{i=1}^{n} p(x_i) = 1$ and X takes only infinite number of values

For Continuous random variable:

$f(X) \ge 0 \forall x$ and $\int_{-\infty}^{\infty} f(x)dx = 1$ and X takes infinite number of values in the interval.

12) State the properties of distribution function.

Answer : 1) 0≤F(x)≤1, -∞

2) F(-∞) = 0 and F(∞) = 1

3) is a non-decreasing function, F(a) ≤F(b) for a

4) $\lim_{h\to 0}$ F(x+h) = F(x), since F(x) is continuous from the right

5) F'(x) = f(x)≥0

6) P(a≤x≤b) = F(b)-F(a)

13) Suppose the probability mass function of the discrete random variable is

X=x	0	1	2	3
p(x)	0.2	0.1	0.4	0.3

What is the value of E(3X + 2X^2) ?

Answer : $E(X)=\sum_x xP_x(x)$

= (0 × 0.2) + (1 × 0.1) + (2 × 0.4) + (3 × 0.3)

= 1.8

$E\left(x^2\right)=\sum_x x^2P_x(x)$

= (0^2 × 0.2) + (1^2 × 0.1) + (2^2 × 0.4) + (3^2 × 0.3)

= 4.4

E(3X + 2X$^{2)}$ = 3E(X) + 2E(X^2)

= (3 × 1.8) + (2 × 4.4)

= 14.2

14) The probability distribution function of a discrete random variable X is

$$F(x)=\begin{cases}2k, x=1\\3k, x=3\\4k, x=5\\0, \text{otherwise}\end{cases}$$

where k is some constant. Find (a) k and (b) P(X > 2).

Answer : Given probability distribution function is

X=x	1	3	5
P(X=x)	2k	3k	4k

a) Since the given function is a p.d.f. each p_i≥0 and Σp_i=1

⇒2k + 3k+4k = 1 ⇒ 9k = 1 ⇒ k = $\frac{1}{9}$

b) P(X > 2) = P(X = 3) + P(X = 5)

= 3k + 4k = 7k = 7($\frac{1}{9}$)

$\left[\because k=\frac{1}{9}\right]$

$\therefore P(X\geq 2)=\frac{7}{9}$

15) Consider a random variable X with p.d.f

$$f(x)=\begin{cases}3x^2, \text{ if } 0<x<1\\0, \text{ otherwise}\end{cases}$$

Find E(X) and V(3X-2).

Answer : Given p.d.f. is $f(x)=\begin{cases}3x^2, \text{ if } 0<x<1\\0, \text{ otherwise}\end{cases}$

$E(X)=\int_{-\infty}^{\infty}x.f(x)dx$

$=\int_0^1 x.(3x^2)dx$

$=3\int_0^1 x^3dx=3.\left[\frac{x^4}{4}\right]_0^1$

$=\frac{3}{4}(1-0)=\frac{3}{4}$

$E(X^2)=\int_0^1 x^2.3x^2dx=3\int_0^1 x^4dx=3.\left[\frac{x^5}{5}\right]_0^1$

$=\frac{3}{5}(1-0)=\frac{3}{5}$

$Var(X)=E(X^2)-[E(X)]^2=\frac{3}{5}-\left(\frac{3}{4}\right)^2$

$=\frac{3}{5}-\frac{9}{16}=\frac{48-45}{80}=\frac{3}{80}$

$V(3X-2)=3^2V(X)[\because V(aX+b)=a^2V(X)]$

$=9\times\frac{3}{80}=\frac{27}{80}$

$\therefore V(3X-2)=\frac{27}{80}$

5 Marks 3 x 5 = 15

16) The distribution of a continuous random variable X in range (–3, 3) is given by p.d.f.

$$f(x)=\begin{cases}\frac{1}{16}(3+x)^2, -3\le x\le -1\\ \frac{1}{16}\left(6-2x^2\right), -1\le x\le 1\\ \frac{1}{16}(3-x)^2, 1\le x\le 3\end{cases}$$

Verify that the area under the curve is unity.

Answer : Given p.d.f is

$$f(x)=\begin{cases}\frac{1}{16}(3+x)^2, & -3\le x\le -1\\ \frac{1}{16}(6-2x^2), & -1\le x\le 1\\ \frac{1}{16}(3-x)^2, & 1\le x\le 3\end{cases}$$

Area under the given curve

$$\int_{-3}^{3} f(c)dx=\int_{-3}^{-1}\frac{1}{16}(3+x)^2+\int_{-1}^{1}\frac{1}{16}(6-2x^2)dx+\int_{1}^{3}\frac{1}{16}(3-x)^2dx$$

$$=\frac{1}{16}\left[\frac{(3+x)^3}{3}\right]_{-3}^{-1}+\frac{1}{16}\left(6x-\frac{2x^3}{3}\right)_{-1}^{1}+\frac{1}{16}\left(\frac{(3-x)^3}{-3}\right)_{1}^{3}$$

$$=\frac{1}{16}\left[\left(\frac{2^3}{3}-0\right)+\left(6-\frac{2}{3}\right)-\left(-6+\frac{2}{3}\right)-\frac{1}{3}\left(0-\left(2^3\right)\right)\right]$$

$$=\frac{1}{6}\left[\frac{8}{3}+\frac{16}{3}-\left(\frac{-16}{3}\right)+\frac{8}{3}\right]$$

$$=\frac{1}{16}\left[\frac{8}{3}+\frac{16}{3}+\frac{16}{3}+\frac{8}{3}\right]$$

$$=\frac{1}{16}\left[\frac{8+16+16+8}{3}\right]$$

$$=\frac{1}{16}\times\frac{48}{3}=\frac{1}{16}\times 16=1$$

Hence, area under the given curve is unity.

17) Suppose that the time in minutes that a person has to wait at a certain station for a train is found to be a random phenomenon with a probability function specified by the distribution function $F(x)\begin{cases}0, & \text{for} \quad x<0\\ \frac{1}{2}, & \text{for} \quad 0\le x<1\\ 0, & \text{for} \quad 1\le x<2\\ \frac{1}{4}, & \text{for} \quad 2\le x<4\\ 0, & \text{for} \quad x\ge 4\end{cases}$

(a) Is the distribution function continuous? If so, give its probability density function?

(b) What is the probability that a person will have to wait

(i) more than 3 minutes,

(ii) less than 3 minutes and

(iii) between 1 and 3 minutes?

Answer : Given probability distribution function

$$F(x)\begin{cases}0, & \text{for} \quad x<0\\ \frac{1}{2}, & \text{for} \quad 0\le x<1\\ 0, & \text{for} \quad 1\le x<2\\ \frac{1}{4}, & \text{for} \quad 2\le x<4\\ 0, & \text{for} \quad x\ge 4\end{cases}$$

a) The distribution function F(x) is continuous since it is a step function

We know f'(x) = f(x)

∴ Probability density function

$$F(x)\begin{cases}0, & \text{for} \quad x\le 0\\ \frac{1}{2}, & \text{for} \quad 0\le x<1\\ 0, & \text{for} \quad 1\le x<2 \qquad \ldots(1)\\ \frac{1}{4}, & \text{for} \quad 2\le x<4\\ 0, & \text{for} \quad x\ge 4\end{cases}$$

(b) (i) Probability that a person will have to wait more than 3 minutes is P(X > 3)

∴ P(X > 3) = P(3≤X< 4)+P(X≥4)

$\frac{1}{4}+0=\frac{1}{4}$ [from (1)]

ii) Probability that a person will have to wait less than 3 minutes is P(X < 3)

∴ P(X< 3) = P(X≤0)+P(0≤x≤1)+P(1≤x≤2)+P(2≤x≤3)

0 + $\frac{1}{2}+0+\frac{1}{4}$[from (1)]

∴ P(X< 3) = $\frac{3}{4}$

= P(1≤X< 2)+P(2≤X< 3)

= 0 +$\frac{1}{4}=\frac{1}{4}$ [from (1)]

18) The probability density function of a random variable X is f(x) = $ke^{-|x|}$, -∞ < x < ∞ Find the value of k and also find mean and variance for the random variable.

Answer : We know that,

$\int_{-\infty}^{\infty} f(x)dx = 1$

$\int_{-\infty}^{\infty} ke^{-|x|}dx = 1$

$k\int_{-\infty}^{\infty} ke^{-|x|}dx = 1$

$2k\int_{-\infty}^{\infty} e^{-|x|}dx = 1$ ($\because x^2e^{-|x|}$ is an function)

$2k\int_{0}^{\infty} \left[\frac{e^{-x}}{-1}\right]_0^{\infty} = 1$

$k = \frac{1}{2}$

Mean of the random variable is

$E(X) = \int_{-\infty}^{\infty} xf(x)dx$

$E(X) = \int_{-\infty}^{\infty} xke^{-|x|}dx$ ($\because xe^{-|x|}$ is an odd function of x)

$= \frac{1}{2}\int_{-\infty}^{\infty} xe^{-|x|}$

= 0

$E\left(x^2\right) = \int_{-\infty}^{\infty} x^2f(x)dx$

$= \int_{-\infty}^{\infty} x^2ke^{-|x|}dx$

$= \frac{1}{2}\int_{-\infty}^{\infty} x^2e^{-|x|}dx$

$= \int_{0}^{\infty} x^2e^{-x}$ ($\because x^2e^{-|x|}$ is an even function)

$= \Gamma 3\left(\because \Gamma\left(\alpha\right) = \int_0^{\infty} x^{\alpha-1}e^{-x}dx, \alpha > 0; \Gamma n = (n-1)!\right)$

= 2

$V(X) = E\left(x^2\right) - [E(X)]^2$

$= 2 - [0]^2$

= 2