QB365 Question Bank Software Study Materials

Binomial Theorem, Sequences and Series Important 2 Marks Questions With Answers (Book Back and Creative)

11th Standard

Maths

Total Marks : 60

2 Marks

30 x 2 = 60

1) Show that the sum of $(m + n)^{th}$ and $(m - n)^{th}$ term of an A.P is equal to twice the m^{th} term.

Answer : $T_n = a + (n - 1)d$

$T_{m+n} = a + (m + n - 1)d$

& $T_{m-n} = a + (m - n - 1)d$

$T_{m+n} + T_{m-n} = a + (m + n - 1)d + a + (m - n - 1)d$

$= 2a + d(m + n - 1 + m - n - 1)$

$= 2a + d(2m - 2)$

$= 2[a + (m - 1)d]$

$T_{m+n} + T_{m-n} = 2.\ T_m$

2) Find $\sqrt[3]{1001}$ approximately. (two decimal places).

Answer : Given $\sqrt[3]{1001}=(1000+1)^{\frac{1}{3}}=(1000)^{\frac{1}{3}}\left(1+\frac{1}{1000}\right)^{\frac{1}{3}}$

$=10^{3\times\frac{1}{3}}\left[1+\frac{1}{1000}\right]^{\frac{1}{3}}$

$\sqrt[3]{1001}=10(1+.001)^{\frac{1}{3}}$

$=10\left[1+\frac{.001}{3}+\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(\frac{.000001}{2}\right)\right] app$

$=10\left[1+.00033-\frac{.000001}{9}\right] app$

= 10 [1.00033 - .00000011] app

= 10 [1.000329]

= 10 [1.00033]

$(1000)^{\frac{1}{3}}\cong 10.0033$

3) Write the first 6 terms of the sequences whose n^{th} terms are given below and classify them as arithmetic progression, geometric progression, arithmetic geometric progression, harmonic progression and none of them $\frac{(n+1)(n+2)}{(n+3)(n+4)}$

Answer : $a_1=\frac{(1+1)(1+2)}{(1+3)(1+4)}=\frac{2(3)}{4(5)}=\frac{6}{20}=\frac{3}{10}$

$a_2=\frac{(2+1)(2+2)}{(2+3)(2+4)}=\frac{3(4)}{5(6)}=\frac{12}{30}=\frac{2}{5}$

$a_3=\frac{(3+1)(3+2)}{(3+3)(3+4)}=\frac{4(5)}{6(7)}=\frac{10}{21}$

$a_4=\frac{5(6)}{7(8)}=\frac{15}{28}$

$a_5=\frac{6(7)}{8(9)}=\frac{7}{12}$

$a_6=\frac{7(8)}{9(10)}=\frac{28}{45}$

The sequence is $\frac{3}{10},\frac{2}{5},\frac{10}{21},\frac{15}{28},\frac{7}{12},\frac{28}{45}$

None of A.P, G.P or H.P

4) Write the first 6 terms of the sequences whose n^{th} terms are given below and classify them as arithmetic progression, geometric progression,arithmetic-geometric progression, harmonic progression and none of them $4\left(\frac{1}{2}\right)^n$

Answer : Let $a_n = 4\left(\frac{1}{2}\right)^n$

$a_1=4\left(\frac{1}{2}\right)^1=\frac{4}{2}=2$

$a_{21}=4\left(\frac{1}{2}\right)^2=\frac{4}{4}=1$

$a_3=4\left(\frac{1}{2}\right)^3=\frac{4}{8}=\frac{1}{2}$

$a_5=4\left(\frac{1}{2}\right)^4=\frac{4}{32}=\frac{1}{8}$

$a_6=4\left(\frac{1}{2}\right)^5=\frac{4}{64}=\frac{1}{16}$

the sequence is $2,1,\frac{1}{2},\frac{1}{4},\frac{1}{8},\frac{1}{16}$

a = 2; r = $\frac{?}{2}$

It is of the form. a, ar, ar^2

It is geometric progression.

5) Write the first 6 terms of the sequences whose nth terms are given below and classify them as arithmetic progression, geometric progression, arithmetic -geometric progression, harmonic progression and none of them $\frac{(-1)^n}{n}$

Answer : Let $a_n = \frac{(-1)^n}{1}$

$a_1 = \frac{(-1)^1}{1} = -1, a_2 = \frac{(-1)^2}{2} = \frac{1}{2}, a_3 = \frac{(-1)^3}{3} = \frac{-1}{3}$

$a_4 = \frac{(-1)^4}{4} = \frac{1}{4}, a_5 = \frac{(-1)^5}{5} = -\frac{1}{5}, a_6 = \frac{(-1)^6}{6} = \frac{1}{6}$

$\therefore$ The sequence is $-1, \frac{1}{2}, -\frac{1}{3}, \frac{12}{4}, -\frac{1}{5}, \frac{1}{6}, \ldots$

That is $-1, \frac{1}{2}, -\frac{1}{3}, \frac{12}{4}, -\frac{1}{5}, \frac{1}{6}, \ldots$

Consider 1, 2, 3, 4, ... which is an A.P.

Since d = 2 - 1 = 3 - 2 = 1

and -1, 1, -1,1, ... is a G.P. where

$r = \frac{1}{-1} = \frac{-1}{1} = -1$

Hence this is an arithmetico-geometric progression.

6) Write the first 6 terms of the sequences whose nth terms are given below and classify them as arithmetic progression, geometric progression, arithmetic -geometric progression, harmonic progression and none of them $\frac{2n+3}{3n+4}$

Answer : Let $a_n = \frac{2n+3}{3n+4}$

$a_1 = \frac{2+3}{3+4} = \frac{5}{9}$

$a_2 = \frac{4+3}{6+4} = \frac{7}{10}$

$a_3 = \frac{6+3}{9+4} = \frac{9}{13}$

$a4 = \frac{8+3}{12+4} = \frac{11}{16}$

$a_5 = \frac{10+3}{15+4} = \frac{13}{19}$

$a_6 = \frac{12+3}{18+4} = \frac{15}{22}$

$\frac{5}{9}, \frac{7}{10}, \frac{9}{13}, \frac{11}{16}, \frac{13}{19}, \frac{15}{22} \ldots$

this is neither A.P, G.P nor AGP

7) Write the first 6 terms of the sequences whose nth terms are given below and classify them as arithmetic progression, geometric progression, arithmetic -geometric progression, harmonic progression and none of them 2018

Answer : 2018

Let a_n = 2018

then the first 6 terms are 2018, 2018, 2018, 2018, 2018, 2018

It is not an AP, GP, AGP and HP.

8) Write the first 6 terms of the sequences whose nth terms are given below and classify them as arithmetic progression, geometric progression, arithmetic - geometric progression, harmonic progression and none of them $\frac{3n-2}{3^{n-1}}$

Answer : Let $a_n = \frac{3n-2}{3^{n-1}}$

$a_1 = \frac{1}{30} = 1$

$a_2 = \frac{3(2)-2}{3^1} = \frac{4}{3}$

$a_3 = \frac{3(3)-2}{3^2} = \frac{7}{9}$

$a_4 = \frac{3(4)-2}{3^3} = \frac{10}{27}$

$\therefore$ The sequence $\frac{1}{1}, \frac{4}{3}, \frac{7}{9}, \frac{10}{27}, \ldots$

$= 1, 4\left(\frac{1}{3}\right), 7\left(\frac{1}{3}\right)^2 + 10\left(\frac{1}{3}\right)^3 + \ldots$

$\therefore$ 1, 4, 7, 10 is an A.P and $\left(\frac{1}{3}\right)^0, \left(\frac{1}{3}\right)^1, \left(\frac{1}{3}\right)^2$ g.P

Hence the given sequence is an arithmetics - geometric progression.

9) Write the first 6 terms of the sequences whose n^{th} term a_n given below

$$a_n = \begin{cases} n+1 & if \quad n \quad is \quad odd \\ n & if \quad n \quad is \quad even \end{cases}$$

Answer : $a_n = \begin{cases} n+1 & if \quad n \quad is \quad odd \\ n & if \quad n \quad is \quad even \end{cases}$

a_1 = 1 + 1 = 2, a2 = 2, a_3 = 3 + 1 = 4

a_4 = 4, a_5 = 5 +1 = 6, a_6 = 6

hence the first 6 terms are 4, 2, 2, 4, 6, 6...

10) Write the first 6 terms of the sequences whose n^{th} term a_n given below $a_n = \begin{cases} n & \text{if } n \text{ is } 1, 2 \text{ or } 3 \\ a_{n-1} + a_{n-2} + a_{n-3} & \text{if } n > 3 \end{cases}$

Answer : $z_n = \begin{cases} n \\ a_{n-1} + a_{n-2} + a_{n-3} \end{cases}$

a_1 = 1, a_2 = 2, a_3 = 3

a_4 = a_3 + a_2 + a_1 = 3 + 2 + 1 = 6

a_5 = a_4 + a_3 + a_2 = 6 + 3 + 2 = 11

a_6 = a_5 +a_4 +a_3 = 11 + 6 + 3 = 20

the first 6 terms are 1, 2, 3, 6, 11, 20

11) Write the n^{th} term of the following sequences

2,2,4,4,6,6

Answer : 2,2,4,4,6,6

Given sequences is 2, 2, 4, 4, 6, 6,

the odd term are 2, 4, 6 .. and even terms are also 2, 4, 6

$\therefore \; a_{n=} \begin{cases} n+1 \\ 1 \end{cases}$

if n is odd

if n is even

12) Write the n^{th} term of the following sequences

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$

Answer : Consider the terms in the numerator 1, 2, 3....

a = 1, d = 2 -1 = 1 a_n = a + (n-1) d

a_n = 1 + (n-1) (1) = 1 + n - 1 = n

The terms in the denominator are 2, 3, 4, 5, 6...

here a = 2, d = 1

a_n = 2 + (n-1) 1 = 2 + n -1 = n + 1

Hence n^{th} term of the given sequence is $\frac{n}{n+1}$

13) Write the n^{th} term of the following sequences

$\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{9}{10}$

Answer : Numerators are 1, 3, 4, 7, 9

a = 1 d = 2 -1

a_n = 1 + (n -1) 2 = 1 + 2n - 2 = 2n -1

denominator 2, 4, 6, 8, 10

a = 2, d = 2

a_n = 1 +(n- 1) 2 = 1 + 2n - 2 = 2n

Hence n^{th} term of the given sequence is $\frac{2n-1}{2n} = 1 - \frac{1}{2n}$

14) Write the n^{th} term of the following sequences

6,10, 4, 12, 2, 14, 0, 16, -2...

Answer : odd terms are 6, 4, 2, 0...

t_n = 6 +(n -1) (-2) = 6 -2n + 2

= 8 -2n

Even terms are 10, 12, 14 , 16

Here a = 1 , d = 2

t_n = 10 + (n - 1) (2) = 10 + 2n -2

= 8 + 2n

n^{th} term of the given sequence is $\begin{cases} 8-2n \\ 8+2n \end{cases}$

15) Find the expansion of $(2x + 3)^5$.

Answer : By taking a = 2x, b = 3 and n = 5 in the binomial expansion of $(a + b)^n$ we get

$(2x + 3)^5 = (2x)^5 + 5(2x)^4 3 + 10(2x)^3 3^2 + 10(2x)^2 3^3 + 5(2x)3^4 + 3^5$

$= 32x^5 + 240x^4 + 720x^3 + 1080x^2 + 810x + 243$.

16) Evaluate 98^4 .

Answer : By taking a = 100, b = 2 and n = 4 in the binomial expansion of $(a - b)^n$ we get

$98^4 = (100-2)^4$

$= {}^4C_0 100^4 - {}^4C_1\ 100^3 2 + {}^4C_2\ 100^2 2^2 - {}^4C3\ 100^1 2^3 + {}^4C_4\ 100^0 2^4$

= 100000000 - 8000000 + 240000 - 3200 + 16

= 92236816.

17) Find the middle term in the expansion of $(x +y)^6$.

Answer : Here n = 6, which is even.

Thus the middle term in the expansion of $(x +y)^6$ is the term containing $x^{\frac{6}{2}} y^{\frac{6}{2}}$, that is the term ${}^6C_3\ x^3y^3$ which is equal to $20x^3y^3$.

18) Find the middle terms in the expansion of $(x + y)^7$.

Answer : As n = 7 which is odd, the terms containing x^4y^3 and x^3y^4 are the two middle terns.

They are ${}^7C_3\ x^4y^3$ and ${}^7C_4 x^3y^4$ which are equal $35x^4y^3$ and $35x^3y^4$.

19) Find the sum up to n terms of the series : $1 + \frac{6}{7} + \frac{11}{49} + \frac{16}{343} + \ldots$

Answer : Here a = 1 d = 5 and $r = \frac{1}{7}$ $S_n = \frac{a-(a+(n-1)d)r^n}{1-r} + dr\left(\frac{1-r^{n-1}}{(1-r)^2}\right)$

$$= \frac{1-(1+5(n-1))\left(\frac{1}{7}\right)^n}{1-\frac{1}{7}} + 5 \times \frac{1}{7}\left(\frac{1-\left(\frac{1}{7}\right)^{n-1}}{\left(1-\frac{1}{7}\right)^2}\right)$$

$$= \frac{1-\frac{5n-4}{7^n}}{\frac{6}{7}} + \frac{\frac{5}{7}\left(7^{n-1}-1\right)}{7^{n-1}\left(\frac{6}{7}\right)^2} \Rightarrow \frac{7^n-5n+4}{7^{n-1}6} + \frac{5\left(7^{n-1}-1\right)}{7n^{-1}6}$$

20) Write the first 6 terms of the sequences whose n^{th} term an is given below:

$$a_n = \begin{cases} 1 \\ 2 & if\ n=1 \\ a_{n-1} + a_{n-2} & if\ n=2, \\ & if\ n>3 \end{cases}$$

Answer : $a_n = \begin{cases} 1 \\ 2 \\ a_{n-1} + a_{n-2} if n > 2 \end{cases}$

$a_1 = 1$, $a_2 = 2$

$a_3 = a_2 + a_1$ =2 +1 = 3

$a_4 = a_3 + a_2$ = 3+2 = 5

$a_5 = a_4 + a_3$ = 5 +3 = 8

$a_6 = a_5 + a_4$ = 8 +5 = 13

hence the first 6 terms are 1, 2, 3, 5, 8, 13

21) Find the sum of first n terms of the series $1^2 + 3^2 + 5^2 + \ldots$

Answer : Given series is $1^2 + 3^2 + 5^2 + ...$

Let Tn be the nth term

T_n = (nth term of 1, 3, 5,...)^2

= $[1+(n-1)2]^2 = (1 + 2n - 2)^2 = (2n-1)^2$

= $4n^2 + 1 - 4n$

∴ Sum of n terms = $\sum 4n^2 - 4n + 1 = 4\sum n^2 - 4\sum n + n$

= $4\frac{(n)(n+1)(2n+1)}{6} - \frac{4n(n+1)+n}{2}$

= $\frac{n}{2}$ [2(n + 1)(n + 1) - 6(n + 1) + 3]

= $\frac{n(4n^2-1)}{3}$

22) Find the general term in the expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$

Answer : Given $\left(\frac{4x}{5} - \frac{5}{2x}\right)^9$

Here n = 9, x = $\frac{4x}{5}$ and a = $\left(\frac{-5}{2x}\right)$

$\therefore T_{r+1} = 9C_r\left(\frac{4x}{5}\right)^{9-r}\left(\frac{-5}{2x}\right)^r$

$= 9C_r.\frac{4^{9-r}}{5^{9-r}}.x^{9-r}(-1)^r.\frac{5^r}{2^r.x^r}$

$= (-1)^r 9Cr\frac{12^{18-3r}}{5^{9-2r}}.\frac{5^r}{2^r}.x^{9-2r}$

$T_{r+1} = (-1)^r 9Cr\frac{12^{18-3r}}{5^{9-2r}}.x^{9-2r}, 0 \le r \le 9.$

23) Find the middle term in $\left(x - \frac{1}{2y}\right)^{10}$

Answer : Given $\left(x - \frac{1}{2y}\right)^{10}$

Here n = 10, x = x and $a = \left(\frac{-1}{2y}\right)$

Middle term = $T_{\frac{10+2}{2}} = T_6$

General term is $T_{r+1} = nCrx^{n-r}a^r$

Putting r = 5 we get,

$T_6 = 10C_5x^{10-5}\left[-\frac{1}{2y}\right]^5 = \frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1}.x^5\left(\frac{-1}{32.y^5}\right)$

$= -225.x^5.\frac{1}{32y^5}T_6 = \frac{-63x^5}{8y^5}$

24) Find the 5th term in the sequence whose first three terms are 3, 3, 6 and each term after the second is the sum of the two terms preceding it.

Answer : Let T_n be the nth term of the sequence

Then, given $T_1 = 3$, $T_2 = 3$, $T_3 = 6$ and

$T_n = T_{n-1} + T_{n-2}$, n > 2.

$T_3 = T_2 + T_1 = 3 + 3 = 6$

$T_4 = T_3 + T_2 = 6 + 3 = 9$

$T_5 = T_4 + T_3 = 9 + 6 = 15$.

25) If $x = a + \frac{a}{r} + \frac{a}{r^2} + ... + \infty, y = b - \frac{b}{r} + \frac{b}{r^2} + ... + \infty$ $z = c + \frac{c}{r^2} + \frac{c}{r^4} + ... + \infty$ then show that $\frac{xy}{z} = \frac{ab}{c}$

Answer : Given x = a + $\frac{a}{r} + \frac{a}{r^2} + ...\infty$

$= a(1 + \frac{1}{r} + \frac{1}{r^2} + ... + \infty)$ $[\because S = \frac{a}{1-r}]$

= $a(\frac{1}{1-\frac{1}{r}}) = a(\frac{r}{r-1})$

x = $\frac{ar}{r-1}$ --- (1)

y = $b - \frac{b}{r} + \frac{b}{r^2}...b(\frac{1}{1-(\frac{-1}{r})}) = \frac{b}{1+\frac{1}{r}}$

y = $\frac{br}{r+1}$ --- (2)

z = c + $\frac{c}{r^2} + \frac{c}{r^4} + ..$ = $\frac{c}{1-\frac{1}{r^2}} = \frac{cr^2}{r^2-1}$ ---- (3)

∴ $\frac{xy}{z} = (\frac{ar}{r-1}.\frac{br}{r+1})/\frac{cr^2}{r^2-1} = \frac{abr^2}{r^2-1} \times \frac{r^2-1}{cr^2} = \frac{ab}{c}$ [∵ using (1),(2) and (3)]

⇒ $\frac{xy}{z} = \frac{ab}{c}$

26) If a, b, c are in A.P., show that $(a-c)^2 = 4(b^2 - ac)$.

Answer : Given a, b, c are in A.P

$\Rightarrow b=\frac{a+c}{2}$

RHS = $4[b^2 - ac]$

$=4\left[\left(\frac{a+c}{2}\right)^2-ac\right]=4\left[\left(\frac{a+c}{4}\right)^2-ac\right]$

$=4\left[\frac{(a+c)^2-4ac}{4}\right]=a^2+c^2+2ac-4ac$

= $a^2 + c^2 - 2ac$

= $(a - c)^2$ = LHS

Hence proved.

27) Find the term independent of x in the expansion of $(x^2+\frac{3}{x})^{15}$.

Answer : $T_{11} = {}^{15}C_{10}3^{10}$

28) With usual notation find the sum $C_0 + 3C_1 + 5C_2 + ... + (2n + 1)C_n$ where C_r is representing ${}^nC_r$.

Answer : $n.2^n$

29) Find the $\sqrt[3]{126}$ approximately to two decimal places.

Answer : $\sqrt[3]{126}=(125)^{1/3}=(125+1)^{1/3}=\left\{125\left(1+\frac{1}{125}\right)\right\}^{1/3}=(125)^{1/3}\left[1+\frac{1}{125}\right]^{1/3}$
$=5\left[1+\frac{1}{3}\times\frac{1}{125}+\ldots\right]\left(\therefore\frac{1}{125}<1\right)=5\left[1+\frac{1}{3}(0.008)\right]=5(1+0.002666)=5.01$

30) Fidd the coefficient of x^5 in the expression of $\left(x+\frac{1}{x^3}\right)^{17}$

Answer : Suppose (r + 1)th term contains x^5 in the binomial expansion of $\left(x+\frac{1}{x^3}\right)^{17}$

Now, $T_{r+1}={}^{17}C_r(x)^{17-r}\left(\frac{1}{x^3}\right)^r$

$={}^{17}C_r x^{17-r}x^{-3r}$

$={}^{17}C_T x^{17-4r}$

This term will contain x^5, if

$17-4r=5$

$4r=12$

$r=3$

So $(\mathrm{r}+1)^{\mathrm{th}}$. ie) 4^{th} term contains x^5

$\mathrm{T}_4={}^{17}\mathrm{C}_3x^5$

The co - efficient of x^5 is ${}^{17}\mathrm{C}_3$.