QB365 Question Bank Software Study Materials

Combinatorics and Mathematical Induction Important 2 Marks Questions With Answers (Book Back and Creative)

11th Standard

Maths

Total Marks : 60

2 Marks

30 x 2 = 60

1) In how many ways 10 pigeons can be placed in 3 different pigeon holes?

Answer : Since each pigeon can occupy any of these 3 holes

Total number of ways of placing 10 pigeons = 3^{10}

2) Find the value of 4!+5!

Answer : = 4 × 3 × 2 × 1 + 5 × 4 × 3 × 2 × 1

= 4 × 3 × 2 × 1 (1+5)

= 24 × 6 = 144

3) Suppose 8 people enter an event in a swimming meet. In how many ways could the gold, silver and bronze prizes be awarded?

Answer : Gold medal can be awarded to anyone of the 8 candidates in 8 ways.

Silver medal can be awarded to anyone of the remaining 7 candidates in 7 ways.

Bronze medal can be awarded to anyone of the remaining 6 candidates in 6 ways.

∴ Total numbers of ways of awarding the prize

= 8 × 7 × 6 = 336

4) How many ways can the product $a^2b^3c^4$ be expressed without exponents?

Answer : Given factors are two a's, 3b's and 4c's

Total number of exponents = 9.

Hence, number of ways the product can be expressed without exponents

= $\frac{9!}{2!3!4!} = \frac{9\times8\times7\times6\times5\times4\times!}{2\times3\times2\times4!}$ = 1260

5) A Kabaddi coach has 14 players ready to play. How many different teams of 7 players could the coach put on the court?

Answer : Here 7 players must be selected from 14 players. This can be done in $14C_7$ ways.

Hence, number of different teams of players

= $14C_7 = \frac{14!}{7!7!}$

= $\frac{14\times13\times12\times11\times10\times9\times8\times7!}{7!7\times6\times5\times4\times3\times2\times1}$

= $13 \times 11 \times 2 \times 3 \times 4$

= 3432.

6) There are 15 persons in a party and if each 2 of them shakes hands with each other, how many handshakes happen in the party?

Answer : The total number of handshakes is same as the number of ways of selecting 2 persons among 15 persons.

This can be done in $15C_2$ ways

∴ Number of handshakes = $15C_2 = \frac{15!}{2!13!} = \frac{15\times14\times13!}{2!13!} = \frac{15\times14}{2} = 15 \times 7$

= 105.

7) How many chords can be drawn through 20 points on a circle?

Answer : A chord is obtained by joining any two points on a circle

Number of chords drawn though 20 points is same as the number of ways of selecting 2 points out of 20 points.

This can be done in $20C_2$ ways.

Hence, total number of chords is $20C_2$

= $\frac{20!}{2!18!} = \frac{20\times19\times18!}{2\times18!} = \frac{20\times19}{2}$

= 10×19 = 190.

8) In a parking lot one hundred, one year old cars, are parked. Out of them five are to be chosen at random for to check its pollution devices. How many different set of five cars can be chosen?

Answer : 5 cars can be chosen out of 100 cars in $^{100}C_5$ ways

$= \frac{100!}{5!(100-5)!} = \frac{100 \times 99 \times 98 \times 97 \times 96 \times 95!}{5 \times 4 \times 3 \times 2 \times 95!}$

= 451725120

9) If $^nC_{12} = {}^nC_9$ find $^{21}C_n$.

Answer : We have $nCx = {}^nCy \Rightarrow x = y$ or $x + y = n$

$\Rightarrow 12 + 9 = n$

$\Rightarrow n = 21$

$\Rightarrow 21C_n = {}^{21}C_{21}$

$= 1 \quad [\because {}^nC_n = 1]$

10) A Woman wants to select one silk saree and one sungudi saree from a textile shop located at Kancheepuram. In that shop, there are 20 different varieties of silk sarees and 8 different varieties of sungudi sarees. In how many ways she can select her sarees?

Answer : The work is done when she selects one silk saree and one sungudi saree. The Woman can select a silk saree in 20 ways and sungudi saree in 8 ways. By the rule of product, the total number of ways of selecting these 2 sarees is $20 \times 8 = 160$.

11) Find the total number of outcomes when 5 coins are tossed once.

Answer : When a coin is tossed, the outcomes are in two ways which are {Head, Tail}.

By the rule of product rule, the number of outcomes when 5 coins are tossed is $2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

12) If $\frac{6!}{n!} = 6$, then find the value of n.

Answer : $\frac{6!}{n!} = \frac{1.2.3.4.5.6}{1.2.3...n} = 6$. As $n < 6$ we get, $n = 5$

13) Evaluate: $^5P_3$.

Answer : $^5P_3 = 5 \times 4 \times 3 = 60$

14) Evaluate: $^8P_4$.

Answer : $^8P_4 = 8 \times 7 \times 6 \times 5 = 1680$

15) If $^{10}P_r = {}^7P_{r+2}$ find r.

Answer : $^{10}P_r = {}^7P_{r+2}$

$\frac{10!}{(10-r)!} = \frac{7!}{(5-r)!}$

i.e., $\frac{10 \times 9 \times 8 \times 7!}{(10-r) \times (9-r) \times (8-r) \times (7-r) \times (6-r) \times (5-r)} = \frac{7!}{(5-r)!}$

$(10-r) \times (9-r) \times (8-r) \times (7-r) \times (6-r) = 10 \times 9 \times 8 = 6 \times 5 \times 4 \times 3 \times 2$

Therefore, $10-r = 6 \Rightarrow r = 4$

16) Evaluate the following: $^{100}C_{99}$

Answer : $^{100}C_{99} = \frac{100 \times 99!}{99!} = 100$

17) If $^nC_4 = 495$. find the value of n.

Answer : We know that, $^nC_4 = 495$

Therefore, $\frac{n \times (n-1) \times (n-2) \times (n-3)}{4 \times 3 \times 2 \times 1} = 495$

$\Longrightarrow n \times (n-1) \times (n-2) \times (n-3) = 495 \times 4 \times 3 \times 2 \times 1$

Factoring $495 = 3 \times 3 \times 5 \times 1$, and writing this product as a product of 4 consecutive numbers in the descending order we get, $n \times (n-1) \times (n-2) \times (n-3) = 12 \times 11 \times 10 \times 9$. Equating n with the maximum number, we obtain n = 12.

18) A salad at a certain restaurant consists of 4 of the following fruits: apple, banana, guava, pomegranate, grapes, papaya and pineapple. Find the total possible number of fruit salads.

Answer : There are seven fruits and we have to select four fruits for the fruit salad.

Hence, the total number of possible ways of making a fruit salad is $^7C_4 = {}^7C_3 = 35$.

19) A Mathematics club has 15 members. In that 8 are girls. 6 of the members are to be selected for a competition and half of them should be girls. How many ways of these selections are possible?

Answer : There are 8 girls and 7 boys in the mathematics club.
The number of ways of selecting 6 members in that half of them girls (3 girls and 3 others) is $^8C_3 \times {}^7C_3 = 56 \times 35 = 1960$.

20) Suppose one girl or one boy has to be selected for a competition from a class comprising 17 boys and 29 girls. In how many different ways can this selection be made?

Answer : The first task of selecting a girl can be done in 29 ways. The second task of selecting a boy can be done in 17 ways. It follows from the sum rule, that there are 17 + 29 = 46 ways of making this selection.

21) Find n if $(n+1)! = 12\times(n-1)!$

Answer : Given $(n+10! = 12\times(n-1)!$
$\Rightarrow (n+1)(n)(n-1)! = 12(n-1)!$
$\Rightarrow (n+1)n = 12$
$\Rightarrow n^2+n-12 = 0$
$\Rightarrow (n+4)(n-3) = 0$
$\Rightarrow n = -4$ or 3
$\Rightarrow n = 3$

22) A room has 6 doors. In how many ways can a man enter the room through one door and come out through a different door?

Answer : Clearly a person can enter the room through any one of the six doors. So, there are 6 ways of entering into the room. After entering into this room, the man can come out through any one of the remaining 5 doors. So, he can come out through a different door in 5 ways.
Hence, the number of ways in which a man can enter a room through one door and come out through a different door = 6×5 = 30.

23) If $nP_4 = 20 \times 3\ nP_2$, then find n.

Answer : Given $nP_4 = 20 \times nP_2$.

$$\Rightarrow \quad \frac{n!}{(n-4)!} = 20\times\frac{n!}{(n-2)!}$$

$$\Rightarrow \quad \frac{1}{(n-4)!} = \frac{20}{(n-2)(n-3)(n-4)!}$$

$\Rightarrow (n-2)(n-3) = 20 \Rightarrow n^2-5n+6 = 20$
$\Rightarrow n^2-5n-14 = 0$
$\Rightarrow (n-7)(n+2) = 0$
$\Rightarrow n = 7$ or $-2 \Rightarrow n = 7$

24) In how many ways can the letters of the word PENCIL be arranged so that N is always next to E.

Answer : Let us keep EN together and consider it as one letter.
Now, we have 5 letters which can be arranged in a row in $5P_5 = 5! = 120$ ways.
Hence, the total number of ways in which N is always next to E is 120.

25) How many words can be formed using the letter A thrice, the letter B twice and the letter C thrice?

Answer : We are given 8 letters namely AAABBCCC.
Clearly three are of one kind, 2 are of second kind and C are of third kind.
$\therefore$ Total number of permutations = $\frac{8!}{3!2!3!}$

$$= \frac{8^4\times7\times6\times5\times4\times3!}{3!\times2\times3\times2}$$

= 560.

26) If the ratio $2nC_3 : nC_3 = 11 : 1$, find n.

Answer : Given $\frac{2nC_3}{nC_3} = \frac{11}{1}$

$\Rightarrow$ $2nC_3 = 11.nC_3$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} = 11 \times \frac{n!}{(n-3)!3!}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} = 11 \times \frac{n(n-1)(n-2)(n-3)!}{(n-3)!}$$

$$\Rightarrow 2n(2n-1)(2n-2) = 11 \times n(n-1)(n-2)$$

$$\Rightarrow 4(2n-1)(n-1) = 11(n-1)(n-2)$$

$\Rightarrow$ 8n - 4 = 11n - 22 $\Rightarrow$ 4n

$\Rightarrow$ 22-4 = 3n

$\Rightarrow$ 3n = 18 $\Rightarrow$ n = 6.

27) How many chord can be drawn through 21 points on a circle?

Answer : A chord is obtained by joining any two points on a circle

$\therefore$ Total number of chords drawn through 21 points = $21C_2$

$= \frac{21!}{2!19!} = \frac{21 \times 20 \times 19!}{2 \times 19!} = 210$

28) Let p(n) be the statement "10n + 3" is prime. Show that p(2) is true but p(3) is not true.

Answer : Given p(n) : " 10n + 3" is prime

$\therefore$ p(2) = 10(2) + 3 = 20 + 3 = 23 which is a prime number.

$\Rightarrow$ p(2) is true.

Now, p(3) = 10(3) + 3 = 30 + 3 = 33 which is not a prime number.

$\Rightarrow$ p(3) is not true.

29) Let p(n) be the statement "7 divides 2^{3n}-1" What is p(n+1) =?

Answer : Given p(n) :"7 divides 2^{3n}- 1"

$\Rightarrow$ 2^{3n}- 1 = 7k [where k is a constant]

$\Rightarrow$ 2^{3n} = 7k+1 ...(1)

Now, p(n+1) is 7 divides $2^{3(n+1)}$-1

$2^{3(n+1)}-1 = 7k_1$

$2^{3n}.2^3-1 = 7k_1$

$8(2^{3n}) = 7k_1+1$ which is the required statement.

30) If the letters of the word BLEAT are permuted in all possible ways and the strings thus formed are arranged in the dictionary order, find the rank of the word BLEAT.

Answer : The rank of the word BLEAT

A - - - - = 4! = 24 ways

BA --- = 3! = 6 ways

BE - - - = 3! = 6 ways

BLA ---- = 2! = 2 ways

BLEAT = 1 way

The rank of the word BLEAT is 24 + 6 + 6 + 2 + 1 = 39.