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QB365 Question Bank Software Study Materials

Differential Calculus - Differentiability and Methods of Differentiation Important 2 Marks Questions With Answers (Book Back and Creative)

11th Standard

Maths

Total Marks : 52

2 Marks

26 x 2 = 52

1) Find the derivatives of the following functions using first principle. f(x) = - x^2 + 2

Answer : $f(x)=-x^2+2$

$f(x+h)=-(x+h)^2+2=-x^2-h^2-2xh+2$

$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to0}\frac{-x^2-h^2-2xh+2+x^2-2}{h}$

$=\lim_{h\to0}\frac{+h(-h-2x)}{h}$

= -0 - 2x

$f'(x)=-2x$

2) Determine whether the following function is differentiable at the indicated values. f(x) = |x^2 - 1| at x = 1

Answer : $f(x)=\begin{cases}-\left(x^2-1\right) & \text{if } x<1\\ \left(x^2-1\right) & \text{if } x>1\end{cases}$

$f'\left(1^-\right)=\lim_{x\to1^-}\frac{f(x)-f(1)}{x-1}=\lim_{x\to1^-}\frac{-(x^2-1)-(0)}{x-1}$

$=\lim_{x\to1^-}\frac{-(x-1)(x+1)}{(x-1)}$

$=\lim_{x\to1^-}-(x+1)=-(1+1)$

= -2

$f'\left(1^+\right)=\lim_{x\to1^+}\frac{f(x)-f(1)}{x-1}=\lim_{x\to1^+}\frac{(x^2-1)-0}{x-1}$

$=\lim_{x\to1^+}\frac{(x-1)(x+1)}{(x-1)}=\lim_{x\to1^+}(x+1)$

= 1 + 1 = 2

$f'\left(1^-\right)\neq f'\left(1^+\right)$

∴ It is not differentiable, at x = 1

3) Differentiate the following with respect to x : y = x^3 + 5x^2 + 3x + 7

Answer : $\frac{dy}{dx}=3x^2+10x+3.$

4) Find the derivatives of the following functions with respect to corresponding independent variables: f(x) = x - 3 sinx

Answer : $f(x)=x-3\sin x$

$f'(x)=1-3\cos x$

5) Find the derivatives of the following functions with respect to corresponding independent variables: y = cos x - 2 tan x

Answer : y = cos x - 2tan x

$\frac{dy}{dx}=\frac{d}{dx}(\cos x)-2\frac{d}{dx}(\tan x)$

= -sinx - 2$\sec^2$ x

6) Differentiate : $y=(x^3-1)^{100}$

Answer : Take u = x^3-1 so that

y = u^{100}

and $\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$

= $100u^{100-1}$ × ($3x^2$-0)

= $100(x^3-1)^{99}$ × 3 x^2

$= 300\ x^2(x^3-1)^{99}$.

7) Find f '(x) if f(x) = $\frac{1}{3\sqrt{x^2+x+1}}$

Answer : First we write : f(x) = $(x^2 + x + 1)^{\frac{-1}{3}}$

Then, f '(x) = $-\frac{1}{3}(x^2+x+1)^{\frac{-1}{3}-1}\frac{d}{dx}(x^2+x+1)$

$= -\frac{1}{3}(x^2+x+1)^{\frac{-4}{3}} \times (2x+1)$

$= -\frac{1}{3}(2x+1)(x^2+x+1)^{\frac{-4}{3}}$.

8) Differentiate 2^x.

Answer : Let y = 2^x = $e^{x\log 2}$.

Take u = (log 2)x so that

y = e^u

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^u \times log2 = e^{xlog2}$

= $(\log 2)2^x$.

9) Differentiate the following: y = $(x^2 + 4x + 6)^5$

Answer : Given y = $(x^2 + 4x + 6)^5$

Let u = $x^2 + 4x + 6$

y = u^5

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= 5u^4 \cdot (2x+4)$

$= 5\left(x^2+4x+6\right)^4(2x+4)$

10) Differentiate the following: y = tan 3x

Answer : y = tan 3x

Take u = 3x ⇒ $\frac{du}{dx} = 3$

$y = \tan u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \sec^2 u(3) = \sec^2(3x) \cdot 3$

$= 3\sec^2(3x)$

11) Differentiate the following: y = cos (tan x)

Answer : y = cos(tan x)

Take $u = \tan x \Rightarrow \frac{du}{dx} = \sec^2 x$

$y = \cos u$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = -\sin u \cdot \left(\sec^2 x\right)$

$= -\sin(\tan x)\sec^2 x$

12) Differentiate the following: y = $(1 + \cos^2 x)^6$

Answer : y = $(1 + \cos^2 x)^6$

u = $1 + \cos^2 x$

∴ $\frac{du}{dx}$ = 2 cosx(-sin x) = -2 sinx cosx = -sin 2x

y = u^6

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ = $6u^5$ (-2sin x cos x)

= -6 $(1 + \cos^2 x)^5$ sin 2x $[\because \sin 2A = 2\sin A\cos A]$

13) Differentiate the following: $y = \frac{e^{3x}}{1+e^x}$

Answer : $y = \frac{e^{3x}}{1+e^x}$

$\frac{dy}{dx} = \frac{(1+e^x).\frac{d}{dx}(e^{3x}) - e^{3x}.\frac{d}{dx}(1+e^x)}{(1+e^x)^2} = \frac{(1+e^x).e^{3x}.3 - e^{3x}(0+e^x)}{(1+e^x)^2}$

$= \frac{3e^{3x}+3e^{4x}-e^{4x}}{(1+e^x)^2} = \frac{3e^{3x}+2e^{4x}}{(1+e^x)2}$.

14) Differentiate: $y = \frac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^5}$

Answer : Taking logarithm on both sides of the equation and using the rules of logarithm we have,

log y = $\frac{3}{4}log\ x + \frac{1}{2}log(x^2+1) - 5log(3x+2)$

Differentiating implicitly

$\frac{y'}{y} = \frac{3}{4x} + \frac{1}{2}\frac{2x}{(x^2+1)} - \frac{5\times3}{3x+2}$

$= \frac{3}{4x} + \frac{x}{(x^2+1)} - \frac{15}{3x+2}$

Therefore, $\frac{dy}{dx} = y' = \frac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^5}[\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}]$

15) Find y', y''and y''' if y = x^3 - $6x^2$ - 5x + 3.

Answer : We have, y = x^3 - $6x^2$ - 5x + 3 and

y' = $3x^2$ - 12x - 5

y'' = 6x - 12

y''' = 6.

16) Find y''' if y = $\frac{1}{x}$

Answer : We have, y = $\frac{1}{x} = x^{-1}$

y' = $-1x^{-2}$ = $-\frac{1}{x^2}$

y'' = $(-1)(-2)x^{-3} = \frac{(-1)^2 2!}{x^3}$

and y''' = $(-1)(-2)(-3)x^{-4} = \frac{(-1)^3 3!}{x^4}$

17) Find the derivatives of the following : y = $x^{\cos x}$

Answer : $y = x^{\cos x}$

Take log on both sides.

$\log y = \log(x^{\cos x})$

$\log y = \cos x \cdot \log x$

$\frac{1}{y}\frac{dy}{dx} = \cos x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(\cos x)$

$\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \frac{1}{x} + \log x(-\sin x)$

$\frac{dy}{dx} = y\left[\frac{\cos x}{x} - \log x(\sin x)\right]$

$= x^{\cos x}\left[\frac{\cos x}{x} - \log x(\sin x)\right]$

18) Find the derivatives of the following : y = $x^{\log x}$ + $(\log x)^x$

Answer : $y = x^{\log x} + (\log x)^x$

Take log on both sides

$\log y = \log x^{\log x} + \log(\log x)^x$

$\log y = \log x(\log x) + x\log(\log x)$

$\frac{1}{y}\cdot\frac{dy}{dx} = \log x\left(\frac{1}{x}\right) + \log(x)\cdot\frac{1}{x} + \log(\log x) + x\frac{1}{\log x}\cdot\frac{1}{x}$

$\frac{1}{ydx} = 2\log x\left(\frac{1}{x}\right) \div \log(\log x) \div \frac{1}{\log x}$

$\frac{dy}{dx} = y\left[\frac{2}{x}\log x + \log(\log x) \div \frac{1}{\log x}\right]$

$= \left[x^{\log x} \div (\log x)^z\right]\left[\frac{2}{x}\log x \div \log(\log x) \div \frac{1}{\log x}\right]$

19) Find the derivatives of the following : $\sqrt{xy} = e^{(x-y)}$

Answer : $\sqrt{xy} = e^{(z-y)}$

$(xy)^{1/2} = e^{x-y}$

Take log on both sides

$\frac{1}{2}\log xy = (x-y)\log e$

$\frac{1}{2}\log xy = x - y$

$\frac{1}{2}\cdot\frac{1}{xy}\left(x\frac{dy}{dx} + y\right) = 1 - \frac{dy}{dx}$

$\frac{1}{2y}\frac{dy}{dx} + \frac{1}{2x} = 1 - \frac{dy}{dx}$

$\frac{1}{2y}\frac{dy}{dx} + \frac{dy}{dx} = 1 - \frac{1}{2x}$

$\frac{dy}{dx}\left(\frac{1}{2y} + 1\right) = 1 - \frac{1}{2x}$

$\frac{dy}{dx} = \frac{\frac{2x-1}{2x}}{\frac{1+2y}{2y}} = \frac{(2x-1)}{2x}\cdot\frac{2y}{(1+2y)}$

$= \frac{y(2x-1)}{x(1+2y)}$

20) Find the derivatives of the following : $x^y = y^x$

Answer : $x^y = y^x$

Take log on both sides

y log x = x log y

$\frac{y}{x} \div \log x \frac{dy}{dx} = \frac{xdy}{ydx} + \log y$

$\log x \frac{dy}{dx} - \frac{xdy}{ydx} = \log y - \frac{y}{x}$

$\frac{dy}{dx}\left(\log x - \frac{x}{y}\right) = \frac{x \log y - y}{x}$

$\frac{dy}{dx} = \frac{x \log y - y}{x} = \frac{y(x \log y - y)}{x(y \log x - x)}$

21) Differentiate $\sin\left(\sqrt{3}\sin x + \cos x\right)$ with respect to x.

Answer : Let y = $\sin\left(\sqrt{3}\sin x + \cos x\right)$

$\frac{dy}{dx} = \cos\left(\sqrt{3}\sin x + \cos x\right)\frac{d}{dx}\left(\sqrt{3}\sin x + \cos x\right) = \cos\left(\sqrt{3}\sin x + \cos x\right)\left[\sqrt{3}\cos x - \sin x\right]$

22) Differentiate $x^2 (x + 1)^3 (x + 2)^4$ with respect to 'x'.

Answer : Let y = $x^2 (x + 1)^3 (x + 2)^4$

Taking logarithm on both sides we have,

$log \quad y = logx2 + log(x+1)3 + log(x+2)4 = 2logx + 3log(x+1) + 4log(x+2)$

Differentiating both sides with respect to 'x' we have,

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} + \frac{3}{x+1} + \frac{4}{x+2}$

$\Rightarrow \frac{dy}{dx} = y\left[\frac{2}{x} + \frac{3}{x+1} + \frac{4}{x+2}\right] \Rightarrow \frac{dy}{dx} = x^2(x+1)^3(x+2)^4\left[\frac{2}{x} + \frac{3}{x+1} + \frac{4}{x+2}\right]$

23) If $y = 500e^{7x} + 600e^{-7x}$ show that $\frac{d^2y}{dx^2} = 49y.$

Answer : Given y = $y = 500e^{7x} + 600e^{-7x}$ ....(1)

Differentiating both sides with respect to 'x' we have

$\frac{dy}{dx} = 500e^{7x}(7) + 600e^{-7x}(-7)$

$\frac{dy}{dx} = 7(500e^{7x} - 600e^{-7x})$

Differentiating again with respect to 'x' we have

$\frac{d^2y}{dx^2} = 7(500(e^{7x})(7) - 600(e^{-7x})(-7))$

$= 49(500 \quad e^{7x} + 600 \quad e^{-7x}) = \quad 49 \quad y \quad (from \quad (1))$

$\therefore \frac{d^2y}{dx^2} = 49y$ Hence proved.

24) Find the derivation : sin 5 + $\log_{10}$ x + 2 sec x

Answer : y = sin 5 + $\log_{10}$ x + 2 sec x

$\therefore \frac{dy}{dx} = 0 + \left(\frac{1}{x}\right)log_{10}e + 2\left[secxtanx\right] = \frac{log_{10}e}{x} + 2secxtanx$

25) Find the derivation $(3x^2 + 1)^2$

Answer : y = $(3x^2 + 1)^2 = (3x^2 + 1)(3x^{2\,+}\,1)$

u = $3x^2$ + 1 and v = $3x^2$ + 1

∴ u' = 3(2 . x) = 6x and v' = 6x

y' = uv' + vu'

(i.e.,) $\frac{dy}{dx}$ = $(3x^2 + 1)(6x) + (3x^2 + 1)$ 6x = 12 . $x(3x^2 + 1)$

26) Find the derivation : $x^2 e^x \sin x$

Answer : y = $x^2 e^x \sin x$

u = x^2 , v = e^x and w = sin x

u' = 2 . x, v' = e^x and w' = cos x

y' = uvw' + vwu' + uwv'

= $(x^2 e^x) \cos x + (e^x \sin x)(2x) + (x^2 \sin x)e^x$

= $x^2 e^x \cos x + 2 . x e^x \sin x + x^2 e^x \sin x$

= $xe^x\{x \cos x + 2 \sin x + x \sin x\}$